Team notebook

November 27, 2018

Contents

1 Basic

1.1 Auxiliar Comparer

```
// returns true if the first argument goes before the second argument
// in the strict weak ordering it defines, and false otherwise.
struct classcomp {
    bool operator() (const int& lhs, const int& rhs) const
    {return lhs > rhs;}
};

int main() {
    set<int> set1;
    set<int, classcomp> set2;
    set1.insert(26); set1.insert(93); set1.insert(42); // 26, 42, 93
    set2.insert(26); set2.insert(93); set2.insert(42); // 93, 42, 26

    for (auto it=set1.begin(); it!=set1.end(); ++it) cout << *it << " ";
    cout << "\n";
    for (auto it=set2.begin(); it!=set2.end(); ++it) cout << *it << " ";
}
```

1.2 Libraries

```
#include <bits/stdc++.h>
```

algorithm	heap, sort	map	map<S, T>
cfloat	DBL_MAX	queue	priority_queue
cmath	pow, sqrt	set	set<S>
cstdlib	abs, rand	sstream	istringstream, ostringstream
iostream	cin, cout	string	string
iomanip	setprecision	utility	pair<S, T>
list	list<T>	vector	vector<T>

1.3 Macros

```
#define X first
#define Y second
#define LI long long
#define MP make_pair
#define PB push_back
#define SZ size()
#define SQ(a) ((a)*(a))
#define MAX(a,b) ((a)>(b)?(a):(b))
#define MIN(a,b) ((a)<(b)?(a):(b))
#define FOR(i,x,y) for(int i=(int)x; i<(int)y; i++)
#define RFOR(i,x,y) for(int i=(int)x; i>(int)y; i--)
#define SORT(a) sort(a.begin(), a.end())
#define RSORT(a) sort(a.rbegin(), a.rend())
#define IN(a,pos,c) insert(a.begin()+pos,1,c)
#define DEL(a,pos,cant) erase(a.begin()+pos,cant)
```

1.4 Permutations

```
int N = 3;
int a[] = {1,2,3};
do {
    for (int i = 0; i < N; ++i) cout << a[i] << " ";
    cout << "\n";
}
while (next_permutation(a, a + N));
```

1.5 Precision cout

```
cout.setf(ios::fixed);
cout.precision(8);
```

2 Data Structures

2.1 Big Numbers

```
#include <cassert>
```

```
#define BASE 1000000000

struct big {
    vector<int> V;
    big(): V(1, 0) {}
    big(int n): V(1, n) {} // supone n < 1000000000 !!!
    big(const big &b): V(b.V) {}

    bool operator==(const big &b) const { return V==b.V; }
    int &operator[](int i) { return V[i]; }
    int operator[](int i) const { return V[i]; }
    int size() const { return V.SZ; }
    void resize(int i) { V.resize(i); }

    bool operator<(const big &b) const {
        for (int i = b.SZ-1; SZ == b.SZ && i >= 0; i--)
            if (V[i] == b[i]) continue;
            else return (V[i] < b[i]);
        return (SZ < b.SZ);
    }

    void add_digit(int l) {
        if (l > 0) V.PB(l);
    }
};

inline big suma(const big &a, const big &b, int k) {
    LI l = 0;
    int size = MAX(a.SZ, b.SZ+k);
    big c; c.resize(size);
    for (int i = 0; i < size; ++i) {
        l += i < a.SZ ? a[i] : 0;
        l += (k <= i && i < k + b.SZ) ? b[i-k] : 0;
        c[i] = l%BASE;
        l /= BASE;
    }
    c.add_digit(int(l));
    return c;
}

inline big operator+(const big &a, const big &b) {
    return suma(a, b, 0);
}
inline big operator+(const big &a, int b) {return a+big(b);}
inline big operator+(int b, const big &a) {return a+big(b);}
```

```
inline big operator-(const big &a, const big &b) {
    assert(b < a || a == b);
    LI l = 0, m = 0;
    big c; c.resize(a.SZ);
    for (int i = 0; i < a.SZ; ++i) {
        l += a[i];
        l -= i < b.SZ ? b[i] + m : m;
        if (l < 0) { l += BASE; m = 1; }
        else m = 0;
        c[i] = l%BASE;
        l /= BASE;
    }
    if (c[c.SZ-1] == 0 && c.SZ > 1) c.resize(c.SZ-1);
    return c;
}
inline big operator-(const big &a, int b) {return a-big(b);}

inline big operator*(const big &a, int b) {
    if (b == 0) return big(0);
    big c; c.resize(a.SZ);
    LI l = 0;
    for (int i = 0; i < a.SZ; ++i) {
        l += (LI)b*a[i];
        c[i] = l%BASE;
        l /= BASE;
    }
    c.add_digit(int(l));
    return c;
}
inline big operator*(int b, const big &a) {return a*b;}
inline big operator*(const big &a, const big &b) {
    big res;
    for (int i = 0; i < b.SZ; ++i)
        res = suma(res, a*b[i], i);
    return res;
}

inline void divmod(const big &a, int b, big &div, int &mod) {
    div.resize(a.SZ);
    LI l = 0;
    for (int i = a.SZ-1; i >= 0; --i) {
        l *= BASE;
        l += a[i];
        div[i] = l/b;
```

```
        l %= b;
    }
    if (div[div.SZ-1] == 0 && div.SZ > 1) div.resize(div.SZ-1);
    mod=int(l);
}

inline big operator/(const big &a, int b) {
    big div; int mod;
    divmod(a, b, div, mod);
    return div;
}

inline int operator%(const big &a, int b) {
    big div; int mod;
    divmod(a, b, div, mod);
    return mod;
}

inline istream &operator>>(istream &is, big &b) {
    string s;
    if (is >> s) {
        b.resize((s.SZ - 1)/9 + 1);
        for (int n = s.SZ, k = 0; n > 0; n -= 9, k++) {
            b[k] = 0;
            for (int i = MAX(n-9, 0); i < n; i++)
                b[k] = 10*b[k] + s[i]-'0';
        }
    }
    return is;
}

inline ostream &operator<<(ostream &os, const big &b) {
    os << b[b.SZ - 1];
    for (int k = b.SZ-2; k >= 0; k--)
        os << setw(9) << setfill('0') << b[k];
    return os;
}

void p10519() { //10519: calcula 2+2+4+6+8+10+...+2*n
    for (big n; cin >> n; ) {
        if (n == big(0)) cout << 1 << endl;
        else cout << 2 + n*(n-1) << endl;
    }
}

int main(){
    p10519();
}
```

2.2 Binary Indexed Tree

```
/* Binary indexed tree. Supports cumulative sum queries in O(log n) */
#define N (1<<18)
#define LL long long

LL bit[N]={0}; //Binary Indexed Tree , nElements +1 positions
int arr[N]={0}; //Array that represents the BIT (simple data, no
    cumulative) , nElements +1 positions
//CAUTION !! INDEX STARTS IN 1
void update(LL* bit, int* arr,int x,int val) { //add or update a value
    int dif = val - arr[x]; //diference between previous value and new
        value
    arr[x] = val;          //set new value in the array
    for(; x<N; x+=x&-x)    //jumps through indexes by jumps of the last 1
        bit adding
        bit[x]+=dif;       //uploads the tree values
}
LL query(LL* bit,int x) { //acumula desde x hasta 0
    LL res=0;
    for(;x;x-=x&-x)        //salta quitando el bit de menor peso
        res+=bit[x];
    return res;
}
```

2.3 Square Root Trick

```
/* Partitions an array in sqrt(n) blocks of size sqrt(n) to support
 * O(sqrt(n)) range sum queries, O(sqrt(n)) range sum updates, and O(1)
 * point updates */
void update(LL *S, LL *A, int i, int k, int x) {
    S[i/k] = S[i/k] - A[i] + x;
    A[i] = x;
}

LL query(LL *S, LL *A, int lo, int hi, int k) {
        int sum=0, i=lo;
```

```
        while((i+1)%k != 0 && i <= hi)
                sum += A[i++];
        while(i+k <= hi)
                sum += S[i/k], i += k;
        while(i <= hi)
                sum += A[i++];
        return sum;
}
```

3 Dynamic Programming

3.1 Change Making Problem

```
int N = 8; // numero de monedas
int m[] = {1,2,5,10,20,50,100,200}; // monedas
int A[100001]; // vector de resultados

int main() {
    int C; // monto C <= 100000
    cin >> C;
    A[0] = 0;
    for (int i = 1; i <= C; i++) {
        A[i] = 1000000;
        for (int j = 0; j < N && m[j] <= i; j++)
            A[i] = MIN(A[i], A[i-m[j]] + 1);
    }
    cout << A[C] << endl;
}
```

3.2 Cocke-Younger-Kasami (Context-free parsing)

```
// O(n^3 |G|) worst case, bigger constant factor
int rules[3][MAX_RULES], nrules;

struct {
    char t;
    int nt;
} nonterminals[MAX_RULES];
int n_nt;
int len;
bool parsed[N_CHARS][MAX_LEN][MAX_LEN];

bool mark(int c, int e, int d) {
    if(parsed[c][e][d])
        return false;
    if(c == ROOT_NONTERMINAL && e == 0 && d == len-1)
        return true;
    parsed[c][e][d] = true;
    int i;
    for(i=0; i<nrules; i++) {
        if (c == rules[1][i]) {
            int k, j = rules[2][i], d_1 = d+1;
            for(k = d_1; k < len; k++)
                if(parsed[j][d_1][k] && mark(rules[0][i], e, k))
                    return true;
        }
        if (c == rules[2][i]) {
            int k, j = rules[1][i], e_1 = e-1;
            for(k = e_1; k >= 0; k--)
                if(parsed[j][k][e_1] && mark(rules[0][i], k, d))
                    return true;
        }
    }
    return false;
}

scanf("%s", str);
// scan rules
// rules[0][i] := rules[1][i] rules[2][i]

len = strlen(str);
memset(parsed, 0, sizeof(parsed));
int j;
for(j=0; j<n_nonterminals; j++) {
    int i;
    for(i=0; i<len; i++) {
        if(str[i] == nonterminals[j].t && mark(nonterminals[j].nt, i, i)) {
            putchar('1');
            goto finish;
        }
    }
}
putchar('0');
finish: putchar('\n');
```

```
// O(n^3 |G|) worst case, smaller constant factor. Can parse n=1000 with
    about
// 20 rules in less than 5s
for(i=0; i<len; i++) {
    int a = str[i]-'a';
    int b;
    for(b=0; b<N_CHARS; b++)
        parsed[b][i][i] = nonterminals[b][a];
    int j;
    for(j=i-1; j >= 0; j--) {
        int l;
        for(l=0; l<N_CHARS; l++)
            parsed[l][i][j] = false;
        int k;
        for(k=j; k<i; k++) {
            int r;
            for(r=0; r<nrules; r++) {
                if(parsed[rules[1][r]][k][j] &&
                    parsed[rules[2][r]][i][k+1])
                    parsed[rules[0][r]][i][j] = true;
            }
        }
    }
}

if(parsed['S'-'A'][len-1][0])
    putchar('1');
else
    putchar('0');
putchar('\n');
```

3.3 Edit Distance (Damerau-Levenshtein)

```
unsigned int levenshtein_distance(const std::string& s1, const
    std::string& s2) {
    const std::size_t len1 = s1.size(), len2 = s2.size();
    std::vector<unsigned int> col(len2+1), prevCol(len2+1);
    for (unsigned int i = 0; i < prevCol.size(); i++)
        prevCol[i] = i;
    for (unsigned int i = 0; i < len1; i++) {
        col[0] = i+1;
        for (unsigned int j = 0; j < len2; j++)
            col[j+1] = std::min({ prevCol[1 + j] + 1, col[j] + 1,
                prevCol[j] + (s1[i]==s2[j] ? 0 : 1) });
        col.swap(prevCol);
    }
    return prevCol[len2];
}
```

3.4 Knapsack Problem

```
int N = 8; // numero de objetos N <= 1000
int v[] = {1,6,7,1,8,3,7,5}; // valor de objetos
int p[] = {5,3,7,1,8,2,7,3}; // peso de objetos
int A[1001][1001]; // matriz de resultados

int main() {
    int C = 7; // capacidad C <= 1000

    for (int j = 0; j <= C; j++)
        A[0][j] = 0;
    for (int i = 1; i <= N; i++) {
        A[i][0] = 0;
        for (int j = 1; j <= C; j++) {
            A[i][j] = A[i-1][j];
            if (p[i-1] <= j) {
                int r = A[i-1][j-p[i-1]] + v[i-1];
                A[i][j] = MAX(A[i][j], r);
            }
        }
    }
    cout << A[N][C] << endl; // output: 12
}
```

3.5 Longest Common Subsequence

```
table[_][0] = 0;
for(int i=1; i<n+1; i++) {
    table[i][0] = 0;
    for(int j=1; j<n+1; j++) {
        if(x[i-1] == y[j-1])
            table[i][j] = table[i-1][j-1] + 1;
        else
```

```
        table[i][j] = max(table[i-1][j], table[i][j-1]);
    }
}
```

3.6 Longest Increasing Subsequence

```
// O(n^2)

for(int i=0; i<N; i++) {
    inc[i] = 1;
    for(int j=0; j<N; j++) {
        if(seq[j] < seq[i]) {
            int v = inc[j] + 1;
            if(v > inc[i])
                inc[i] = v;
        }
    }
        if(inc[i] > max)
            max = inc[i];
}

// O(n log n)

ind[0] = 0;
ind_sz = 1;
while(scanf("%d", &seq[seq_sz++]) == 1) {
    /* Add next element if it's bigger than the current last */
    int i = seq_sz-1;
    if (seq[ind[ind_sz-1]] < seq[i]) {
        predecessor[i] = ind[ind_sz-1];
        ind[ind_sz++] = i;
        continue;
    }
    /* bsearch to find element immediately bigger */
    int u = 0, v = ind_sz-1;
    while(u < v) {
        int c = (u + v) / 2;
        if (seq[ind[c]] < seq[i])
            u = c+1;
        else
            v = c;
    }
    /* Update b if new value is smaller then previously referenced value
       */
    if (seq[i] < seq[ind[u]]) {
        if (u > 0)
            predecessor[i] = ind[u-1];
        ind[u] = i;
    }
}
```

3.7 Maximum Subarray Sum (Kadane)

```
/* We show the 2D version here. the 1D version is the code block
separated by a newline. You can keep track of where the sequence
starts and ends by messing with the max_here and max assignments
respectively. Use > max_here to keep longer subsequences, >= max_here
to keep shorter ones. Take into account circular arrays by adding the
sum of all elements and the max of the array with sign changed. */
max = mat[0][0];
for(i=0; i<N; i++) {
    memset(aux, 0, sizeof(aux));
    for(k=i; k<N; k++) {
        for(j=0; j<N; j++)
            aux[j] += mat[k][j];

        max_here = aux[0];
        if(max_here > max)
            max = max_here;
        for(j=1; j<N; j++) {
            max_here += aux[j];
            if(aux[j] > max_here)
                max_here = aux[j];
            if(max_here > max)
                max = max_here;
        }
    }
}
```

3.8 Traveling Salesman Problem

```
// TSP in O(n^2 * 2^n). Subset is bitmask, Cost is cost.
// tsp_memoize[subset][j] stores the shortest path starting at node -1,
```

```
// including the nodes in the subset and finishing at node j.
// This is for the TSP with N+1 nodes. We pick the first one arbitrarily.
Cost distances[N][N], tsp_memoize[1 << (N+1)][N];
const Cost sentinel=-0x3f3f3f3f;
#define TSP(subset, i) (tsp_memoize[subset][i] == sentinel ? \
                                                tsp(subset, i) :
                                                    tsp_memoize[subset][i])

Cost tsp(const Subset subset, const int i) {
        Subset without = subset ^ (1 << i);
        Cost minimum = numeric_limits<Cost>::max();
        for(int j=0; j<n_nodes; j++) {
                if(j==i || (without & (1 << j)) == 0)
                        continue;
                Cost v = TSP(without, j);
                v += distances[i][j];
                if(v < minimum)
                        minimum = v;
        }
        return tsp_memoize[subset][i] = minimum;
}

/* fill tsp_memoize with sentinel */
tsp_memoize[1<<i][i] = distance /* from -1 to i */
for(int i=0; i<n_nodes; i++)
        tsp(0xffff >> (16 - n_nodes), i) /* + distance from i to -1 */;
```

4 Geometry

4.1 Convex Hull

```
typedef int T;   // posiblemente cambiar a double
typedef pair<T,T> P;
T xp(P p, P q, P r) {
    return (q.X-p.X)*(r.Y-p.Y) - (r.X-p.X)*(q.Y-p.Y);
}
struct Vect {
    P p, q; T dist;
    Vect(P &a, P &b) {
        p = a; q = b;
        dist = SQ(a.X - b.X) + SQ(a.Y - b.Y);
    }
    bool operator<(const Vect &v) const {
        T t = xp(p, q, v.p);
        return t < 0 || t == 0 && dist < v.dist;
    }
};

vector<P> convexhull(vector<P> v) { // v.SZ >= 2
    sort(v.begin(), v.end());
    vector<Vect> u;
    for (int i = 1; i < (int)v.SZ; i++)
        u.PB(Vect(v[i], v[0]));
    sort(u.begin(), u.end());
    vector<P> w(v.SZ, v[0]);
    int j = 1; w[1] = u[0].p;
    for (int i = 1; i < (int)u.SZ; i++) {
        T t = xp(w[j-1], w[j], u[i].p);
        for (j--; t < 0 && j > 0; j--)
            t = xp(w[j-1], w[j], u[i].p);
        j += t > 0 ? 2 : 1;
        w[j] = u[i].p;
    }
    w.resize(j+1);
    return w;
}

int main() {
    vector<P> v;
    v.PB(MP(0, 1)); v.PB(MP(1, 2)); v.PB(MP(3, 2)); v.PB(MP(2, 1));
    v.PB(MP(3, 1)); v.PB(MP(6, 3)); v.PB(MP(7, 0));
    vector<P> w = convexhull(v);
} // resultado: (0,1) (7,0) (6,3) (1,2)
```

4.2 Line Intersection

Intersection between two lines: here is the system solved. Swap all xs and ys to avoid dividing by zero if $p_x = 0$.

$$s = \frac{P_y - Q_y + \frac{p_y}{p_x}(Q_x - P_x)}{q_y - \frac{p_y}{p_x} q_x}$$

$$x = Q_x + q_x s; \; y = Q_y + q_y s$$

$$t = \frac{Q_x - P_x + q_x * s}{p_x}$$

4.3 Routines

```
double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y) {}
  PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)  const { return PT(x*c, y*c ); }
  PT operator / (double c)  const { return PT(x/c, y/c ); }
};

double dot(PT p, PT q)  { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p)  { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
```

```
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c,
       c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y -
          p[i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
      return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
```

```
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}
```

5 Graphs

5.1 Bellman-Ford (Shortest Path with Negative Weights)

```
// Complexity: E * V - Input: directed graph
typedef pair<pair<int,int>,int> P; // par de nodos + coste
int N;                             // numero de nodos
vector<P> v;                       // representacion aristas

int bellmanford(int a, int b) {
    vector<int> d(N, 1000000000);
    d[a] = 0;
    for (int i = 1; i < N; i++)
        for (int j = 0; j < (int)v.SZ; j++)
            if (d[v[j].X.X] < 1000000000 && d[v[j].X.X] + v[j].Y <
                d[v[j].X.Y])
                d[v[j].X.Y] = d[v[j].X.X] + v[j].Y;
    for (int j = 0; j < (int)v.SZ; j++)
        if (d[v[j].X.X] < 1000000000 && d[v[j].X.X] + v[j].Y < d[v[j].X.Y])
            return -1000000000; // existe ciclo negativo
    return d[b];
}

int main(){
        N=8;
        v.PB(MP(MP(0, 1), +2)); v.PB(MP(MP(1, 2), -1)); v.PB(MP(MP(1, 3),
            +1));
        v.PB(MP(MP(2, 3), +1)); v.PB(MP(MP(6, 4), -1)); v.PB(MP(MP(4, 5),
            -1));
        v.PB(MP(MP(5, 6), -1));

        // min distance, negative cycle, unreachable
        cout << bellmanford(0, 3) << " " << bellmanford(4, 6) << " "
             << bellmanford(0, 7) << endl;
}
```

5.2 Bron-Kerbosch (Maximum Clique in Undirected Graph)

```
#define U unsigned int
typedef vector<short int> V;

vector<vector<U> > graf; // vertices/aristas del grafo
U numv, kmax; // # conjuntos/tamano grupo independiente

int evalua(V &vec) {
    for (int n = 0; n < vec.size(); n++)
        if (vec[n] == 1) return n;
    return -1;
}

void Bron_i_Kerbosch() {
    vector<U> v;
    U i, j, aux, k = 0, bandera = 2;
    vector<V> I, Ve, Va;
    I.PB(V()); Ve.PB(V()); Va.PB(V());
    for (i = 0; i < numv; i++) {
        I[0].PB(0); // conjunto vacio
        Ve[0].PB(0); // conjunto vacio
        Va[0].PB(1); // contiene todos
    }
    while(true) {
        switch(bandera) {
        case 2: // paso 2
            v.PB(evalua(Va[k]));
            I.PB(V(I[k].begin(), I[k].end()));
            Va.PB(V(Va[k].begin(), Va[k].end()));
            Ve.PB(V(Ve[k].begin(), Ve[k].end()));
            aux = graf[v[k]].size();
            I[k+1][v[k]] = 1; Va[k+1][v[k]] = 0;
            for (i = 0; i < aux; i++) {
                j = graf[v[k]][i]; Ve[k+1][j] = Va[k+1][j] = 0;
            }
            k = k + 1; bandera = 3;
            break;
        /**********************************************/
```

```
      case 3: // paso 3
        for (i = 0, bandera = 4; i < numv; i++) {
          if (Ve[k][i] == 1) {
            aux = graf[i].size();
            for (j = 0; j < aux; j++)
              if (Va[k][graf[i][j]] == 1)
                break;
            if (j == aux) { i = numv; bandera = 5; }
          }
        }
        break;
      /***********************************************/
      case 4: // paso 4
        if (evalua(Ve[k]) == -1 && evalua(Va[k]) == -1) {
          for (int n = 0; n < numv; n++)
            if (I[k][n] == 1) cout<< n << " ";
          cout << endl;
          if (k > kmax) kmax = k;
          bandera = 5;
        }
        else bandera = 2; // ir a paso 2
      break;
      /***********************************************/
      case 5: // paso 5
        k = k - 1; v.pop_back(); I[k].clear();
        I[k].assign(I[k+1].begin(), I[k+1].end());
        I[k][v[k]] = 0; I.pop_back(); Ve.pop_back();
        Va.pop_back(); Ve[k][v[k]] = 1; Va[k][v[k]] = 0;
        if (k == 0) {
          if (evalua(Va[0]) == -1) return;
          bandera = 2; // ir a paso 2
        }
        else bandera = 3; // ir a paso 3
      break;
      }
    }
}

int main() {
  U idx, i; stringstream ss; string linea;
  while (cin >> numv) {
    getline(cin, linea);
    for (i = 0; i < numv; i++) { // Lectura del grafo
      // vertices adjacentes al i-esimo vertice
      vector<U> bb; graf.PB(bb);
      getline(cin, linea);
      ss << linea;
      while (ss >> idx) graf[i].PB(idx);
      ss.clear();
    }
    // Llamada al algoritmo
    kmax = 0;
    cout << "Conjuntos independientes: "<< endl;
    if (numv > 0)
      Bron_i_Kerbosch();
    cout << "kmax: " << kmax << endl;
    // Limpieza variables
    for (i = 0; i < numv; i++) graf[i].clear();
    graf.clear();
  }
}
```

5.3 Dijkstra (Shortest Path)

```
// Complexity: ElogV - Input: undirected graph
typedef int V;         // tipo de costes
typedef pair<V,int> P; // par de (coste,nodo)
typedef set<P> S;      // conjunto de pares

int N;                 // numero de nodos
vector<P> A[10001];    // listas adyacencia (coste,nodo)

// int prec[201]; // predecesores (nodes from s to t)
// another way to obtain a path (above all, if there is
// more than one, consists in using BFS from the target
// and add to the queue those nodes that lead to the
// minimum cost in the preceeding node)

V dijkstra(int s, int t) {
  S m;                         // cola de prioridad
  vector<V> z(N, 1000000000); // distancias iniciales
  z[s] = 0;                    // distancia a s es 0
  m.insert(MP(0, s));          // insertar (0,s) en m
  while (m.SZ > 0) {
    P p = *m.begin(); // p=(coste,nodo) con menor coste
    m.erase(m.begin()); // elimina este par de m
    if (p.Y == t) return p.X; // cuando nodo es t, acaba
    // para cada nodo adjacente al nodo p.Y
```

```
        for (int i = 0; i < (int)A[p.Y].SZ; i++) {
            // q = (coste hasta nodo adjacente, nodo adjacente)
            P q(p.X + A[p.Y][i].X, A[p.Y][i].Y);
            // si q.X es la menor distancia hasta q.Y
            if (q.X < z[q.Y]) {
                m.erase(MP(z[q.Y], q.Y)); // borrar anterior
                m.insert(q);          // insertar q
                z[q.Y] = q.X;         // actualizar distancia
                          // prec[q.Y] = p.Y;    // actualizar
                              predecesores
            }
        }
    }
    return -1;
}

int main() {
    N = 6;            // solucion 0-1-2-4-3-5, coste 11
    A[0].PB(MP(2, 1)); // arista (0, 1) con coste 2
    A[0].PB(MP(5, 2)); // arista (0, 2) con coste 5
    A[1].PB(MP(2, 2)); // arista (1, 2) con coste 2
    A[1].PB(MP(7, 3)); // arista (1, 3) con coste 7
    A[2].PB(MP(2, 4)); // arista (2, 4) con coste 2
    A[3].PB(MP(3, 5)); // arista (3, 5) con coste 3
    A[4].PB(MP(2, 3)); // arista (4, 3) con coste 2
    A[4].PB(MP(8, 5)); // arista (4, 5) con coste 8
    cout << dijkstra(0, 5) << endl;
}
```

5.4 Floyd-Warshall (All Pairs Shortest Path)

```
// Complexity: n^3
// A: matriz n*n de adyacencia con costes
// ausencia de arista representada por un numero grande
for (int k = 0; k < n; k++)
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            A[i][j] = MIN(A[i][j], A[i][k] + A[k][j]);
```

5.5 Kruskal (Minimum Spanning Tree)

```
// Complexity: ElogV - Input: undirected graph
typedef vector<pair<int,pair<int,int> > > V;

int N, mf[2000]; // numero de nodos N <= 2000
V v;             // vector de aristas
                 // (coste, (nodo1, nodo2))

// vector< pair<long, int> > K[3001]; // kruskal tree

int set(int n) { // conjunto conexo de n
    if (mf[n] == n) return n;
    else mf[n] = set(mf[n]); return mf[n];
}

int kruskal() {
    int a, b, sum = 0;
    sort(v.begin(), v.end());
    for (int i = 0; i < N; i++)
        mf[i] = i; // inicializar conjuntos conexos
    for (int i = 0; i < (int)v.SZ; i++) {
        a = set(v[i].Y.X), b = set(v[i].Y.Y);
        if (a != b) { // si conjuntos son diferentes
            mf[b] = a; // unificar los conjuntos
            sum += v[i].X; // agregar coste de arista
                  // K[v[i].Y.X].PB(MP(v[i].X, v[i].Y.Y));
                  // K[v[i].Y.Y].PB(MP(v[i].X, v[i].Y.X));
        }
    }
    return sum;
}

int main() {
    N = 5; // solucion 13 (0,3),(1,2),(2,3),(3,4)
    v.PB(MP(4,MP(0,1))); // arista (0,1) coste 4
    v.PB(MP(4,MP(0,2))); // arista (0,2) coste 4
    v.PB(MP(3,MP(0,3))); // arista (0,3) coste 3
    v.PB(MP(6,MP(0,4))); // arista (0,4) coste 6
    v.PB(MP(3,MP(1,2))); // arista (1,2) coste 3
    v.PB(MP(7,MP(1,4))); // arista (1,4) coste 7
    v.PB(MP(2,MP(2,3))); // arista (2,3) coste 2
    v.PB(MP(5,MP(3,4))); // arista (3,4) coste 5
    cout << kruskal() << endl;
}
```

5.6 Max Flow (Dinic's blocking flow)

```
// Running time: O(|V|^4)
// INPUT: graph, constructed using AddEdge(), source, sink
// OUTPUT: maximum flow value,
//        To obtain the actual flow, look at positive values only.
// From Stanford University's notebook.

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

struct MaxFlow {
   int N;
   VVI cap, flow;
   VI dad, Q;

   MaxFlow(int N) :
      N(N), cap(N, VI(N)), flow(N, VI(N)), dad(N), Q(N) {}

   void AddEdge(int from, int to, int cap) {
      this->cap[from][to] += cap;
   }

   int BlockingFlow(int s, int t) {
      fill(dad.begin(), dad.end(), -1);
      dad[s] = -2;

      int head = 0, tail = 0;
      Q[tail++] = s;
      while (head < tail) {
         int x = Q[head++];
         for (int i = 0; i < N; i++) {
            if (dad[i] == -1 && cap[x][i] - flow[x][i] > 0) {
               dad[i] = x;
               Q[tail++] = i;
            }
         }
      }

      if (dad[t] == -1) return 0;

      int totflow = 0;
      for (int i = 0; i < N; i++) {
         if (dad[i] == -1) continue;
         int amt = cap[i][t] - flow[i][t];
         for (int j = i; amt && j != s; j = dad[j])
            amt = min(amt, cap[dad[j]][j] - flow[dad[j]][j]);
         if (amt == 0) continue;
         flow[i][t] += amt;
         flow[t][i] -= amt;
         for (int j = i; j != s; j = dad[j]) {
            flow[dad[j]][j] += amt;
            flow[j][dad[j]] -= amt;
         }
         totflow += amt;
      }

      return totflow;
   }

   int GetMaxFlow(int source, int sink) {
      /* to clean for subsequent executions
      fill(Q.begin(), Q.end(), 0);
      for (int i = 0; i < N; ++i)
      {
         fill(flow[i].begin(), flow[i].end(), 0);
      }
      */

      int totflow = 0;
      while (int flow = BlockingFlow(source, sink))
         totflow += flow;
      return totflow;
   }
};

int main() {
   MaxFlow mf(5);
   mf.AddEdge(0, 1, 3);
   mf.AddEdge(0, 2, 4);
   mf.AddEdge(0, 3, 5);
   mf.AddEdge(0, 4, 5);
   mf.AddEdge(1, 2, 2);
   mf.AddEdge(2, 3, 4);
   mf.AddEdge(2, 4, 1);
   mf.AddEdge(3, 4, 10);

   // should print out "15"
```

```
    cout << mf.GetMaxFlow(0, 4) << endl;
}
```

5.7 Maximum Bipartite Matching

```
// This code performs maximum bipartite matching. with Hopcroft-Karp
// https://sites.google.com/site/indy256/algo_cpp/hopcroft_karp
//
// Running time: O(|E| sqrt(|V|) ) -- often much faster in practice
//
// INPUT: addEdge(izquierda,derecha)
// OUTPUT: marching[i] nodo i de la izquierda unido al matching[i] de la
    derecha
//        function returns number of matches made
const int MAXN1 = 50000;
const int MAXN2 = 50000;
const int MAXM = 150000;

//n1,n2 dimensiones izquierda y derecha
int n1, n2, edges, last[MAXN1], prev[MAXM], head[MAXM];
//matching tiene los matches izquierda derecha
int matching[MAXN2], dist[MAXN1], Q[MAXN1];
bool used[MAXN1], vis[MAXN1];

void init(int _n1, int _n2) {
    n1 = _n1;
    n2 = _n2;
    edges = 0;
    fill(last, last + n1, -1);
}

void addEdge(int u, int v) {
    head[edges] = v;
    prev[edges] = last[u];
    last[u] = edges++;
}

void bfs() {
    fill(dist, dist + n1, -1);
    int sizeQ = 0;
    for (int u = 0; u < n1; ++u) {
        if (!used[u]) {
            Q[sizeQ++] = u;
            dist[u] = 0;
        }
    }
    for (int i = 0; i < sizeQ; i++) {
        int u1 = Q[i];
        for (int e = last[u1]; e >= 0; e = prev[e]) {
            int u2 = matching[head[e]];
            if (u2 >= 0 && dist[u2] < 0) {
                dist[u2] = dist[u1] + 1;
                Q[sizeQ++] = u2;
            }
        }
    }
}

bool dfs(int u1) {
    vis[u1] = true;
    for (int e = last[u1]; e >= 0; e = prev[e]) {
        int v = head[e];
        int u2 = matching[v];
        if (u2 < 0 || !vis[u2] && dist[u2] == dist[u1] + 1 && dfs(u2)) {
            matching[v] = u1;
            used[u1] = true;
            return true;
        }
    }
    return false;
}

int maxMatching() {
    fill(used, used + n1, false);
    fill(matching, matching + n2, -1);
    for (int res = 0;;) {
        bfs();
        fill(vis, vis + n1, false);
        int f = 0;
        for (int u = 0; u < n1; ++u)
            if (!used[u] && dfs(u))
                ++f;
        if (!f)
            return res;
        res += f;
    }
}
```

```
int main() {
    init(2, 2);
    addEdge(0, 0); addEdge(0, 1); addEdge(1, 1);
    cout << (2 == maxMatching()) << endl;
}
```

5.8 Min Cost Max Flow

```
/* From Stanford University's notebook.
 * To perform minimum weighted bipartite matching:
 *  - Capacity between nodes = 1 (cost whatever given by the problem)
 *  - Capacity from source = 1 and cost = 0
 *  - Capacity to sink = 1 and cost = 0
 * Output: <maximum flow value - minimum cost value>
 * Complexity: O(|V|^2) per augmentation
 *            max flow: O(|V|^3) augmentations
 *            min cost max flow: O(|V|^4 * MAX_EDGE_COST) augmentations
 */
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
    int N;
    VVL cap, flow, cost;
    VI found;
    VL dist, pi, width;
    VPII dad;

    MinCostMaxFlow(int N) :
        N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}

    void AddEdge(int from, int to, L cap, L cost) {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    }

    void Relax(int s, int k, L cap, L cost, int dir) {
        L val = dist[s] + pi[s] - pi[k] + cost;
        if (cap && val < dist[k]) {
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
        }
    }

    L Dijkstra(int s, int t) {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);
        fill(width.begin(), width.end(), 0);
        dist[s] = 0;
        width[s] = INF;

        while (s != -1) {
            int best = -1;
            found[s] = true;
            for (int k = 0; k < N; k++) {
                if (found[k]) continue;
                Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
                Relax(s, k, flow[k][s], -cost[k][s], -1);
                if (best == -1 || dist[k] < dist[best]) best = k;
            }
            s = best;
        }

        for (int k = 0; k < N; k++)
            pi[k] = min(pi[k] + dist[k], INF);
        return width[t];
    }

    pair<L, L> GetMaxFlow(int s, int t) {
        L totflow = 0, totcost = 0;
        while (L amt = Dijkstra(s, t)) {
            totflow += amt;
            for (int x = t; x != s; x = dad[x].first) {
                if (dad[x].second == 1) {
                    flow[dad[x].first][x] += amt;
                    totcost += amt * cost[dad[x].first][x];
                }
                else {
                    flow[x][dad[x].first] -= amt;
```

```
                totcost -= amt * cost[x][dad[x].first];
            }
        }
    }
    return make_pair(totflow, totcost);
  }
};
```

5.9 Minimum Cut

Mientras queden > 2 vértices

- Selecciona arista al azar
- Fusiona u y v en un único vértice
- Retorna el coste representado por los dos vértices finales

Si repetimos $N = c \cdot n^2 \cdot \log n$ veces —¿ $P(fracas) \leq \frac{1}{n^c}$

5.10 Stable Marriage Problem

```
short W[1000][1000], M[1000][1000];
short wP[1001], mP[1001], coW[1001];
short listing[50000];
int N;

void stableMarriage(){
    int ac = 0, total = 0, ws, woman;
    for( int i = 0; i < N; i++ ){
        listing[total++] = i;
        coW[i] = 0;
    }

    while( ac < total ){
        ws = listing[ac++];
        if( wP[ws] != -1 )
            continue;

        for( ; coW[ws] < N; coW[ws]++ ){
            if (mP[ W[ws][ coW[ws] ] ] == -1 ) {
                wP[ws] = W[ws][ coW[ws] ];
                mP[ W[ws][ coW[ws] ] ] = ws;
                break;
            }
            else {
                woman = W[ws][ coW[ws] ];
                if( M[woman][ mP[woman] ] > M[woman][ws] ){
                    listing[total++] = mP[woman];
                    wP[ mP[woman] ] = -1;
                    mP[woman] = ws;
                    wP[ws] = woman;
                    break;
                }
            }
        }
    }
}
```

5.11 Tarjan (Strongly Connected Components)

```
// Complexity |V| + |E|
int index, ct;
vector<bool> I;
// L indica el indice del conjunto fuertemente conexo al que pertenece
    cada nodo
vector<int> D, L, S;
vector<vector<int> > V; // listas de adyacencia (grafo dirigido)

void tarjan (unsigned n) {
    D[n] = L[n] = index++;
    S.push_back(n);
    I[n] = true;
    for (unsigned i = 0; i < V[n].size(); ++i) {
        if (D[V[n][i]] < 0) {
            tarjan(V[n][i]);
            L[n] = MIN(L[n], L[V[n][i]]);
        }
        else if (I[V[n][i]])
            L[n] = MIN(L[n], D[V[n][i]]);
    }
    if (D[n] == L[n]) {
        ++ct;
        // todos los nodos eliminados de S pertenecen al mismo scc
        while (S[S.size() - 1] != n) {
            I[S.back()] = false;
            S.pop_back();
```

```
        }
        I[n] = false;
        S.pop_back();
    }
}

void scc() {
    index = ct = 0;
    I = vector<bool>(V.size(), false);
    D = vector<int>(V.size(), -1);
    L = vector<int>(V.size());
    S.clear();
    for (unsigned n = 0; n < V.size(); ++n)
        if (D[n] < 0)
          tarjan(n);
    // ct = numero total de scc
}
```

5.12 Topological Sort

```
vector<int> A[101]; // adjacency list (directed graph without cycles)
int inbound[101]; // number of nodes that point to each node
vector<int> fo; // final order

// M = number of nodes (there might be 'lonely' nodes)
void toposort(int M) {
    stack<int> order;
    int current;

    // Search for roots (identifiers might change between
    // problems (e.g. 1 to M))
    for(int m = 0; m < M; m++){
        if(inbound[m] == 0)
            order.push(m);
    }

    // Start topsort from roots
    while(!order.empty()){
        // Pop from stack
        current = order.top();
        order.pop();
        // Save order in fo
        fo.push_back(current);
        // Add childs only if inbound is 0
        for (int i = 0; i < A[current].size(); ++i)
        {
            inbound[A[current][i]]--;
            if (inbound[A[current][i]] == 0)
                order.push(A[current][i]);
        }
    }
}

int main() {
    A[0].push_back(1); A[0].push_back(2); A[2].push_back(1);
    inbound[0] = 0; inbound[1] = 2; inbound[2] = 1;
    toposort(3);
    for (int i = 0; i < fo.size(); ++i) cout << fo[i] << " ";
    // 0 2 1
}
```

6 Math

6.1 Catalan Numbers

```
unsigned long long v[34]; // 1, 1, 2, 5, 14, 42, 132, 429, 1430, ...
// Cn = number of strings of n*2 consistent parentheses.
// ((())) ()(()) ()()() (())() (()())
// Cn = number of non-isomorphic ordered trees with n vertices.
// Cn = number of full binary trees with n + 1 leaves, and n internal
    nodes
// Cn = number of ways to tile a stairstep shape of height n with n
    rectangles
/* Cn = number of monotonic lattice paths along the edges of a grid with
    n  n
   square cells, which do not pass above the diagonal */
void catalan(){
    v[0] = 1;
    for (int i = 1; i < 34; ++i){
        unsigned long long sum = 0;
        for (int j = 0; j < i; ++j){
            sum += v[j] * v[i-j-1];
        }
        v[i] = sum;
    }
```

```
}
```

6.2 Complex Numbers

```
// Complex number class, from Stanford's Notebook. Required for FFT
struct cpx {
    cpx(){}
    cpx(double aa):a(aa){}
    cpx(double aa, double bb):a(aa),b(bb){}
    double a, b;
    double modsq(void) const { return a * a + b * b; }
    cpx bar(void) const { return cpx(a, -b); }
};
cpx operator +(cpx a, cpx b) { return cpx(a.a + b.a, a.b + b.b); }
cpx operator *(cpx a, cpx b) {
    return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}
cpx operator /(cpx a, cpx b) {
    cpx r = a * b.bar();
    return cpx(r.a / b.modsq(), r.b / b.modsq());
}
cpx EXP(double theta) { return cpx(cos(theta),sin(theta)); }
```

6.3 Exponent

```
template <typename T,typename U> T expo(T &t, U n) {
    if (n == U(0)) return T(1);
    else {
        T u = expo(t, n/2);
        if (n%2 > 0) return u*u*t;
        else return u*u;
    }
}
```

6.4 Fast Fourier Transform

```
// from Stanford's notebook:
    https://web.stanford.edu/~liszt90/acm/notebook.html
// in:    input array
// out:   output array
// step:  {SET TO 1} (used internally)
// size:  length of the input/output {MUST BE A POWER OF 2}
// dir:   either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i * j *
    k / size)
const double two_pi = 4 * acos(0);
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
    if(size < 1) return;
    if(size == 1)
    {
        out[0] = in[0];
        return;
    }
    FFT(in, out, step * 2, size / 2, dir);
    FFT(in + step, out + size / 2, step * 2, size / 2, dir);
    for(int i = 0 ; i < size / 2 ; i++)
    {
        cpx even = out[i];
        cpx odd = out[i + size / 2];
        out[i] = even + EXP(dir * two_pi * i / size) * odd;
        out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) /
            size) * odd;
    }
}
```

6.5 Fibonacci with matrices

```
// O(log n) ops. to compute nth fibonacci number
// use methods 'matriz' and 'expo' of the notebook
matriz m;
m.v[0][0] = 1;
m.v[0][1] = 1;
m.v[1][0] = 1;
m.v[1][1] = 0;

int n = 2; // find 2nd fibo number
matriz res = expo(m, n);
res.v[0][1]
```

6.6 Greatest Common Divisor and Least Common Multiple

```
// in algorithm library: __gcd(a, b)
int gcd(int a, int b) {
    if (a < b) return gcd(b, a);
    else if (a%b == 0) return b;
    else return gcd(b, a%b);
}

gcd(a,b)*lcm(a,b) = a*b
```

6.7 Matrix Multiplication

```
#define SIZE 15  // tamano de matriz cuadrado
#define MOD 10007 // modulo de la multiplicacion
struct matriz {
    int v[SIZE][SIZE];
    matriz() { init(); } // matriz de 0's
    matriz(int x) {    // matriz con x's en la diagonal
        init();
        for (int i = 0; i < SIZE; i++) v[i][i] = x;
    }
    void init() {
        for (int i = 0; i < SIZE; i++)
            for (int j = 0; j < SIZE; j++) v[i][j] = 0;
    }
    // multiplicacion de matrices modulo MOD
    matriz operator*(matriz &m) {
        matriz n;
        for (int i = 0; i < SIZE; i++)
            for (int j = 0; j < SIZE; j++)
                for (int k = 0; k < SIZE; k++)
                    n.v[i][j] = (n.v[i][j] + v[i][k]*m.v[k][j])%MOD;
        return n;
    }
};
```

6.8 Modular Linear Equations

```
// returns d = gcd(a,b); finds x,y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
  int xx = y = 0;
  int yy = x = 1;
  while (b) {
    int q = a/b;
    int t = b; b = a%b; a = t;
    t = xx; xx = x-q*xx; x = t;
    t = yy; yy = y-q*yy; y = t;
  }
  return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
  int x, y;
  VI solutions;
  int d = extended_euclid(a, n, x, y);
  if (!(b%d)) {
    x = mod (x*(b/d), n);
    for (int i = 0; i < d; i++)
      solutions.push_back(mod(x + i*(n/d), n));
  }
  return solutions;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
  int x, y;
  int d = extended_euclid(a, n, x, y);
  if (d > 1) return -1;
  return mod(x,n);
}

// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b. Here, z is unique modulo M = lcm(x,y).
// Return (z,M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
  int s, t;
  int d = extended_euclid(x, y, s, t);
  if (a%d != b%d) return make_pair(0, -1);
  return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}

// Chinese remainder theorem: find z such that
```

```
// z % x[i] = a[i] for all i. Note that the solution is
// unique modulo M = lcm_i (x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
  PII ret = make_pair(a[0], x[0]);
  for (int i = 1; i < x.size(); i++) {
    ret = chinese_remainder_theorem(ret.second, ret.first, x[i], a[i]);
    if (ret.second == -1) break;
  }
  return ret;
}

// computes x and y such that ax + by = c; on failure, x = y =-1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
  int d = gcd(a,b);
  if (c%d) {
    x = y = -1;
  } else {
    x = c/d * mod_inverse(a/d, b/d);
    y = (c-a*x)/b;
  }
}
```

6.9 Newton Method

```
long double tolerance = 1E-6;
long double c0 = 1.0;
long double c1 = 1.0;
bool solutionFound = false;

// find the value of 'c' that makes the function equal to = 0
// might also be used in optimization problems setting y as
// the first derivative and yprime as the second
while (true)
{
        long double y = /* formula of the original function */;
        long double yprime = /* formula of the first derivative respect to
             c */;
        c1 = c0 - y / yprime;
        if ((fabs(c1 - c0) / fabs(c1)) < tolerance)
        {
                solutionFound = true;
                break;
        }
        c0 = c1;
}
```

6.10 Polynomial Multiplication

```
const int MAX_LEN = 262144 * 2;
cpx A[MAX_LEN], B[MAX_LEN], C[MAX_LEN];
int A_len, B_len, C_len;

/* set the appropriate coefficients in the inputs A and B's real-valued
    part,
 * and their length in A_len and B_len. */

for(C_len = 1; !(C_len > A_len + B_len - 1); C_len *= 2);
assert(C_len < MAX_LEN);
memset(A + A_len, 0, (C_len - A_len) * sizeof(cpx));
memset(B + B_len, 0, (C_len - B_len) * sizeof(cpx));
FFT(A, C, 1, C_len, 1);
FFT(B, A, 1, C_len, 1);
for(int i=0; i<C_len; i++)
    A[i] = A[i] * C[i];
FFT(C, A, 1, C_len, -1);
for(int i=0; i<C_len; i++)
    C[i].a /= C_len;
// now C[i].a (the real-valued parts) contain the result
```

6.11 PrimeFactors

```
#include <bits/stdc++.h>
using namespace std;


vector<int> getPrimosSuma (int c) {
    int z = 2;
    vector<int> primeFactors;
    // Se obtiene la factorizacion de c.
    while (z * z <= c) {
        if (c % z == 0) {
            primeFactors.push_back(z);
```

```
            c /= z;
        }
        else z++;
    }

    if (c > 1) primeFactors.push_back(c);
    return primeFactors;
}

int main() {
    vector<int> res = getPrimosSuma(4937775);
    for(int i = 0; i < res.size(); ++i) cout << res[i] << " ";
    cout << endl;
}
```

6.12 Primes

```
int v[10000]; // primes

void savePrimes()
{
    int k = 0;
    v[k++] = 2;
    for (int i = 3; i <= 10010; i += 2) {
        bool b = true;
        for (int j = 0; b && v[j] * v[j] <= i; j++)
            b = i%v[j] > 0;
        if (b)
            v[k++] = i;
    }
}

bool isPrime(int x){
        bool prime = true;
        for (int j = 0; prime && v[j] * v[j] <= x; j++)
                prime = x%v[j] > 0;
        return prime;
}

// probar si un numero x <= 100000000 es primo
int main()
{
    savePrimes();
    cout << isPrime(4);
}
```

7 Sequences

7.1 Binary Search

```
// binary_search function can be found at algorithm library
// devuelve el i mas pequeno tal que t <= v[i]
// si no existe tal i, devuelve v.SZ
template<typename T> int bb(T t, vector<T> &v) {
    int a = 0, b = v.SZ;
    while (a < b) {
        int m = (a + b)/2;
        if (v[m] < t) a = m+1; else b = m;
    }
    return a;
}
```

7.2 Ternary Search

```
double E = 0.0000001; // tolerance
double L = 200000; // R and L are extreme possible values...
double R = -200000; // ... for the optimized parameter
while (1) {
    double dist = R - L;
    if (fabs(dist) < E) break;
    double leftThird = L + dist / 3;
    double rightThird = R - dist / 3;
    // f is the function which we are optimizing
    if (f(leftThird) < f(rightThird))
        R = rightThird;
    else
        L = leftThird;
}
```

7.3 Vector Partition

```
bidirectional_iterator partition(bidirectional_iterator start,
                          bidirectional_iterator end,
                          Predicate p);

bool IsOdd(int i) {return (i%2==1);}

int main () {
   vector<int> myvector;
   vector<int>::iterator it, bound;

   // set some values:
   for (int i=1; i<10; ++i)
      myvector.push_back(i); // 1 2 3 4 5 6 7 8 9

   bound = partition(myvector.begin(), myvector.end(), IsOdd);

   // print out content:
   cout << "odd members:";
   for (it=myvector.begin(); it!=bound; ++it)
      cout << " " << *it;
   cout << "\neven members:";
   for (it=bound; it!=myvector.end(); ++it)
      cout << " " << *it;
   cout << endl;
}
```

8 Strings

8.1 Knuth-Morris-Pratt

```
/*Search of substring in O(n+k)*/
void TablaKMP(string T,vector<int> &F)
{
   int pos = 2; // posicion actual en F
   int cnd = 0; // ndice en T del siguiente carcter del actual candidato
       en la subcadena

   F[0] = -1;
   while(pos <= T.size())
   {
      if(T[pos - 1] == T[cnd] )
      {//siguiente candidato coincidente en la cadena
         cnd++;
         F[pos] = cnd;
         pos++;
      }else if(cnd > 0)
      {//si fallan coincidencias consecutivas entonces asignamos valor
          conocido la primera vez
         cnd = F[cnd];
      }else{
         F[pos] = 0 ;
         pos++;
      }
   }
}
vector<int> KMPSearch(string T, string P)//T: texto donde se busca ,P:
    palabra a buscar ,salida: vector de posiciones match
{
   int k = 0 ; //puntero de T
   int i = 0 ; //avance en P

   vector<int> F(T.size(),0),sol;

   if(T.size() >= P.size())
   {
      TablaKMP(T,F);//optimizacin para no repetir busquedas de
          subcadenas que no hacen match
      while(k+i < T.size())
      {
         if(P[i] == T[k+i])
         {
            if(i == P.size()-1)
            {
               sol.push_back(k); //modificando el return podemos
                   devolver todos los matches
            }
            i++;
         }else{
            k += i-F[i];
            if(i > 0)
            {
               i = F[i];
            }
         }
      }
   }
   return sol;
```

```
}

int main(){
    string T = "PARTICIPARIA CON MI PARACAIDAS PARTICULAR";
    string P = "A";
    vector<int> founds = KMPSearch(T,P);
    for(int i = 0 ; i < founds.size();++i)
    {
        cout<<founds[i]<<endl;
    }
}
```

8.2 Regular Expressions

```
#include <regex>
#include <iostream>
using namespace std;
int main ()
{
  if (regex_match("subject", regex("(sub)(.*)")))
    cout << "yes\n";
}
```

8.3 Suffix Arrays

```
// Suffix array construction in O(L log^2 L) time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:  string s
//
// OUTPUT: array suffix[] such that suffix[i] = index (from 0 to L-1)
//         of substring s[i...L-1] in the list of sorted suffixes.
//         That is, if we take the inverse of the permutation suffix[],
//         we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
    const int L;
    string s;
    vector<vector<int> > P;
    vector<pair<pair<int,int>,int> > M;

    SuffixArray(const string &s) : L(s.length()), s(s), P(1,
        vector<int>(L, 0)), M(L) {
        for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
        for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
            P.push_back(vector<int>(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ?
                    P[level-1][i + skip] : -1000), i);
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[level][M[i].second] = (i > 0 && M[i].first ==
                    M[i-1].first) ? P[level][M[i-1].second] : i;
        }
    }

    vector<int> GetSuffixArray() { return P.back(); }

    // returns the length of the longest common prefix of s[i...L-1] and
        s[j...L-1]
    int LongestCommonPrefix(int i, int j) {
        int len = 0;
        if (i == j) return L - i;
        for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
};

int main() {
    // bobocel is the 0'th suffix
    //  obocel is the 5'th suffix
    //   bocel is the 1'st suffix
    //    ocel is the 6'th suffix
```

```
    //    cel is the 2'nd suffix
    //     el is the 3'rd suffix
    //      l is the 4'th suffix
    SuffixArray suffix("bobocel");
    vector<int> v = suffix.GetSuffixArray();

    // indices of the first character in the ith suffix
        // 0th suffix (bobocel) -> 0
        // 1st suffix (bocel) -> 2
        // 2nd suffix (cel) -> 4
        vector<int> s(v.size());
        for (int i = 0; i < v.size(); ++i)
        {
                s[v[i]] = i;
        }

    // with the 's' vector we would compare whether suffix i
    // has a common prefix with all suffixes from i + 1 to
    // i + M by doing the LCP between just i and i + M.
    // for (int i = 0; i <= N - M; ++i)
    // {
    //  int s1 = S[i];
    //  int s2 = S[i + M - 1];
    //  int length = suffix.LongestCommonPrefix(s1, s2);
    // }

    // Expected output: 0 5 1 6 2 3 4
    //                2
    for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
    cout << endl;
    cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
```

9 Summary

1.4 Generar las n! ordenaciones posibles.

1.5 Precisión de decimales.

2.2 Fenwick (mantiene las frecuencias o suma acumulativa).

2.3 Dividir en trozos raíz(n) para acumular características en intervalo.

3.3 Distancia mínima entre strings.

3.5 Subsecuencia común más larga.

3.6 Subsecuencia creciente el segundo.

3.7 Subsecuencia 1D o 2D con mayor suma.

4.3 Funcions geometriques útils.

5.4 Camino más corto entre todos los pares de nodos.

5.5 Árbol que une todos los nodos con menor coste.

5.6 Máxima cantidad de líquido sin desbordar el sistema.

5.7 Encontrar el máximo numero de parejas monogamicas de nodos en grafo bipartito (a cada nodo solo le corresponde una pareja y no más)

5.8 5.6 pero volem minimitzar el cost.

5.9 Minim numero de talls (borrar branca) perquè quedin 2 grafs inconexos.

5.10 Encontrar parejas de forma que se maximize lo contenta que queda cada persona dependiendo de un ranking de gustos.

5.11 Trobar tots els cicles tancats en un graf dirigit.

5.12 Ordenar els nodes d'un graf dirigit tenint en compte el nombre de branques que incideigen en cada node.

6.1 Número de grafos distintos con n vértices.

6.2 Funcions per nombres complexos

6.3 Funció eficient per poténcies

6.8 Funcions d'aritmetica modular

6.9 Métode iteratiu per trobar zeros (y=0)

6.11 Comprobación de número primo.

6.12 Trobar els factors primers d'un numero

7.2 Trobar máxim o mínim d'una funció continua amb 1 només 1 máxim o mínim

7.3 Sacar en 2 vectores información de un vector dado un comparador de cada elemento.

8.1 Compta ocurrències de substring en string.

8.2 Com utilitzar regex.

8.3 Funcions per identificar sufixos i prefixos.