Mass distribution model of the dwarf galaxy DDO154

We want to perform a mass distribution model of the spiral galaxy DDO154 (Fig. 1)

Figure 1. Composite image of the dwarf galaxy DDO154

For that, we will use the kinematic data fron de Blok et al. 2008 (AJ, 136, 2648). They derived the HI rotation curve of the dwarf galaxy from a titled-ring model of the HI velocity field. They also derived the mass density profiles of stars and gas, yielding the velocity contribution of luminous matter to the total gravitational potential.

1- Derivation of the velocity contributions of luminous matter

Stellar disc

The surface brightness profile of DDO154 measured at 3.6 µm with the Spitzer satellite is shown in Figure 2. Assume that the stellar mass surface density of DDO154 is exponential, and that the total stellar mass of the disc is $\log(M_{*,tot}/M_{sun})$ = 7.27 and the mass-to-light ratio of the disc is $\Upsilon_{3.6}$=0.23 (in M_{sun}/L_{sun}). The distance to DDO154 is 4.3 Mpc.

Q1- Infer the total luminosity (in L_{sun})
Q2- From Figure 2, derive the scale-length h of the stellar disc (in kpc)
Q3- Deduce the central mass surface density Σ_0 of the stellar disc (in $M_{sun}\,pc^{-2}$)
Q4- Deduce the radial profiel of circular velocity of the stellar disc (km/s)

Reminder:

The total mass of an exponential disc is given by
$$M_{\rm tot} = 2\pi\Sigma_0 h^2$$
The squared circular velocity of an exponential disc is given by
$$v^2 = \pi G\Sigma_0 r^2/h \times (I_0(r/(2h))K_0(r/(2h)) - I_1(r/(2h))K_1(r/(2h)))$$
where Σ_0 is the central mass surface density, h the scale-length, and I_k and K_k are the I and K Bessel functions of the 0th and 1st orders.

G = 6.673 10^{-11} $m^3\,kg^{-1}\,s^{-2}$
1 M_{sun} = 1.989 10^{30} kg
1 pc = 3.085678 10^{13} km
Hint: recast G so that h and Σ_0 can be passed as input parameters of a function (developed by yourself) directly into kpc and $M_{sun}\,pc^{-2}$

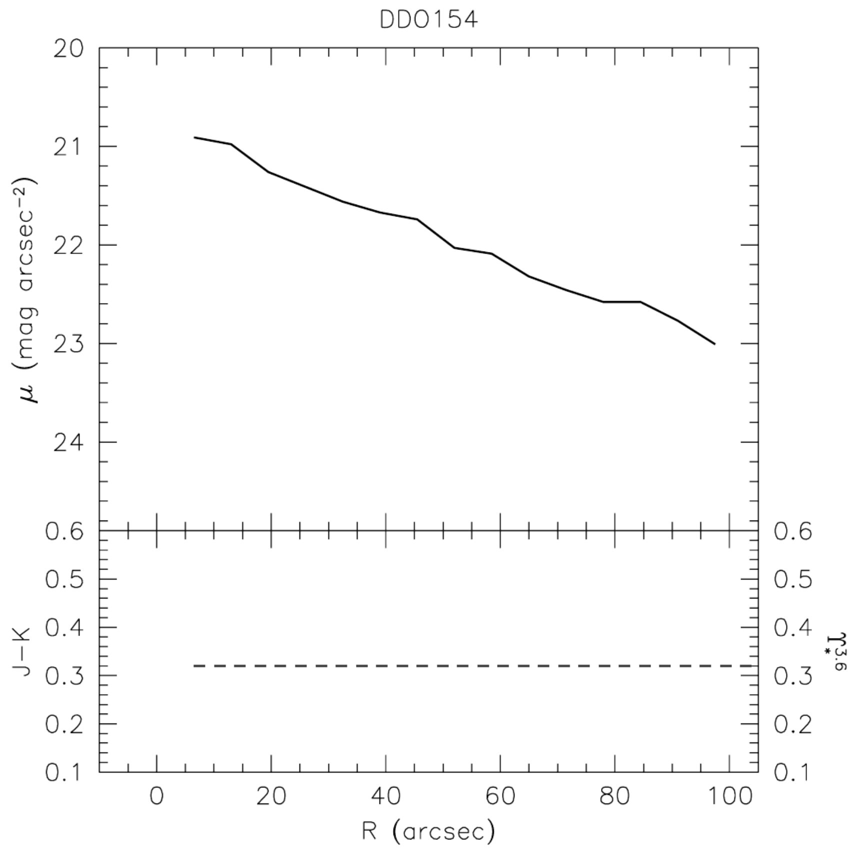

Figure 2. Surface brightness profile at 3.6 μm of the dwarf galaxy DDO154

Gaseous disc

The velocity contribution of the gaseous disc is obtained by integration of the surface density of the atomic gas (the HI density has been scale by a factor of 1.4 to take into account the cosmic contributions of atoms heavier than H). It is given in the second column of the file rcddo154.dat.

2- Rotation curve and baryonic velocity contributions

The rotation curve of HI gas in DDO154 is given in the 3rd column of the file rcddo154.dat. The associated velocity uncertainties δ_v are given in the 4th column.

Q5- Draw the rotation curve with uncertainties as a function of radius
Q6- Overlay the velocity contributions from the stellar disc and the gaseous disc obtained in previous questions

3- Mass distribution modeling

The goal here is to fit the mass distribution model to the rotation curve of DDO154 to constrain the structural parameters of the dark matter density profile, using the velocity contributions from the stellar disc and the gaseous disc.

Q7- Fit the model of rotation curve assuming a (i) pseudo-isothermal sphere of dark matter (PIS model), and (ii) a Navarro-Frenk-White density profile for dark matter (NFW model). To do that, you can assume that the weightings of the velocity points are the inverse of the squared uncertainties $w = 1/\delta_v^2$
Q8- What is the model that best fit the rotation curve of DDO154?

Q9- Plot the rotation curve and the model of rotation curve for the PIS and NFW models, as well the the contributions from stars, gas and dark matter
Q10- Describe the differences between results from the 2 models

Reminder:
The circular velocity for a NFW model is given by $v^2_{\mathrm{NFW}}(r) = v^2_{200}\,(\log(1+\eta) - \eta/(1+\eta))/(x(\log(1+c) - c/(1+c)))$
with 2 characteristic parameters:

- scale velocity v_{200}= velocity (in km/s) of a sphere at a radius r_{200}, with r_{200} being the radius at which the density of the sphere is 200 times the critical density of the closure of the Universe, $r_{200}=100\ v_{200}/H_0$, with r_{200} in kpc, H_0 the Hubble constant (68 km/s/Mpc)
- c = concentration of the halo, unitless
- η= cx, with $x=r/r_{200}$

The circular velocity for a PIS model is given by $v^2_{\mathrm{PIS}}(r) = 4\pi G\rho_0 r_c^2(1 - r_c/r\,\arctan(r/r_c))$
with 2 characteristic parameters:

- scale density ρ_0 of the sphere is the central mass density, in $M_{sun}\,pc^{-3}$
- r_c = core radius of the sphere, in kpc

Hint: recast G so that r_c and ρ_0 can be passed as input parameters of a function (developed by yourself) directly into kpc and $10^{-3}\ M_{sun}\,pc^{-2}$