Disciplina: Processamento Digital de Sinais

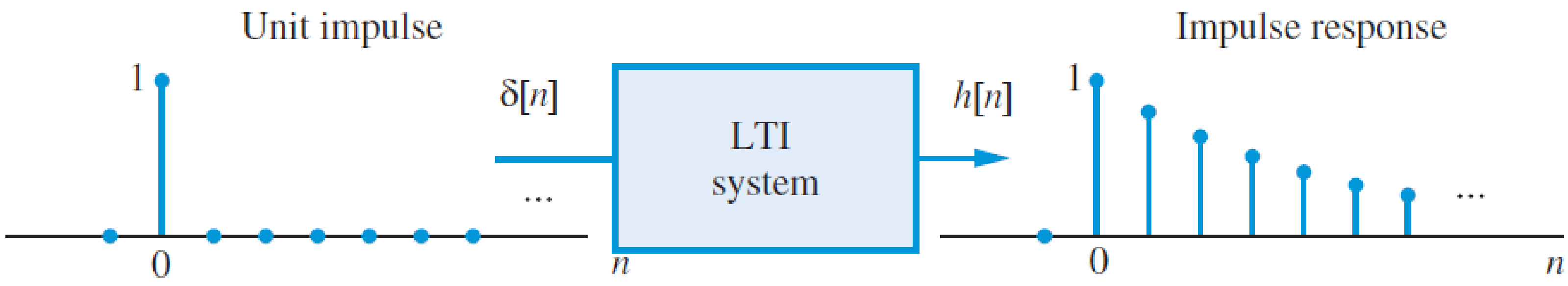
Material _aula 4

Ambiente Blackboard

Apresentação

- 1) Convolução
- 2) Tarefas
- 3) Transformada de Fourier Discreta
- 4) Transformada Z
- 5) Lista de exercícios

Convolução



Convolução

The operation described by the convolution sum takes two sequences $x[n]$ and $h[n]$ and generates a new sequence $y[n]$. We usually say that sequence $y[n]$ is the *convolution* of sequences $x[n]$ and $h[n]$ or that $y[n]$ is obtained by convolving $x[n]$ with $h[n]$. Convolution

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]. \qquad -\infty < n < \infty$$

Convolução – Resumo do cálculo

In summary, the computation of convolution of two sequences involves the following steps:

1. Change the index of the sequences $h[n]$, $x[n]$ from n to k and plot them as a function of k.
2. Flip (or fold) the sequence $h[k]$ about $k = 0$ to obtain the sequence $h[-k]$.

3. Shift the flipped sequence $h[-k]$ by n samples to the right, if $n > 0$, or to the left, if $n < 0$.
4. Multiply the sequences $x[k]$ and $h[n-k]$ to obtain the sequence $z_n[k] = x[k]h[n-k]$.
5. Sum all nonzero samples of $z_n[k]$ to determine the output sample at the given value of the shift n.
6. Repeat steps $3-5$ for all desired values of n.

Convolução – Resumo do cálculo

- Considere as sequências

$$x[n] = \{\underset{\uparrow}{1}\ 1\ 1\ 1\ 1\ 1\}, \quad h[n] = \{\underset{\uparrow}{1}\ 0.5\ 0.25\ 0.125\},$$

Convolucão – Forma gráfica

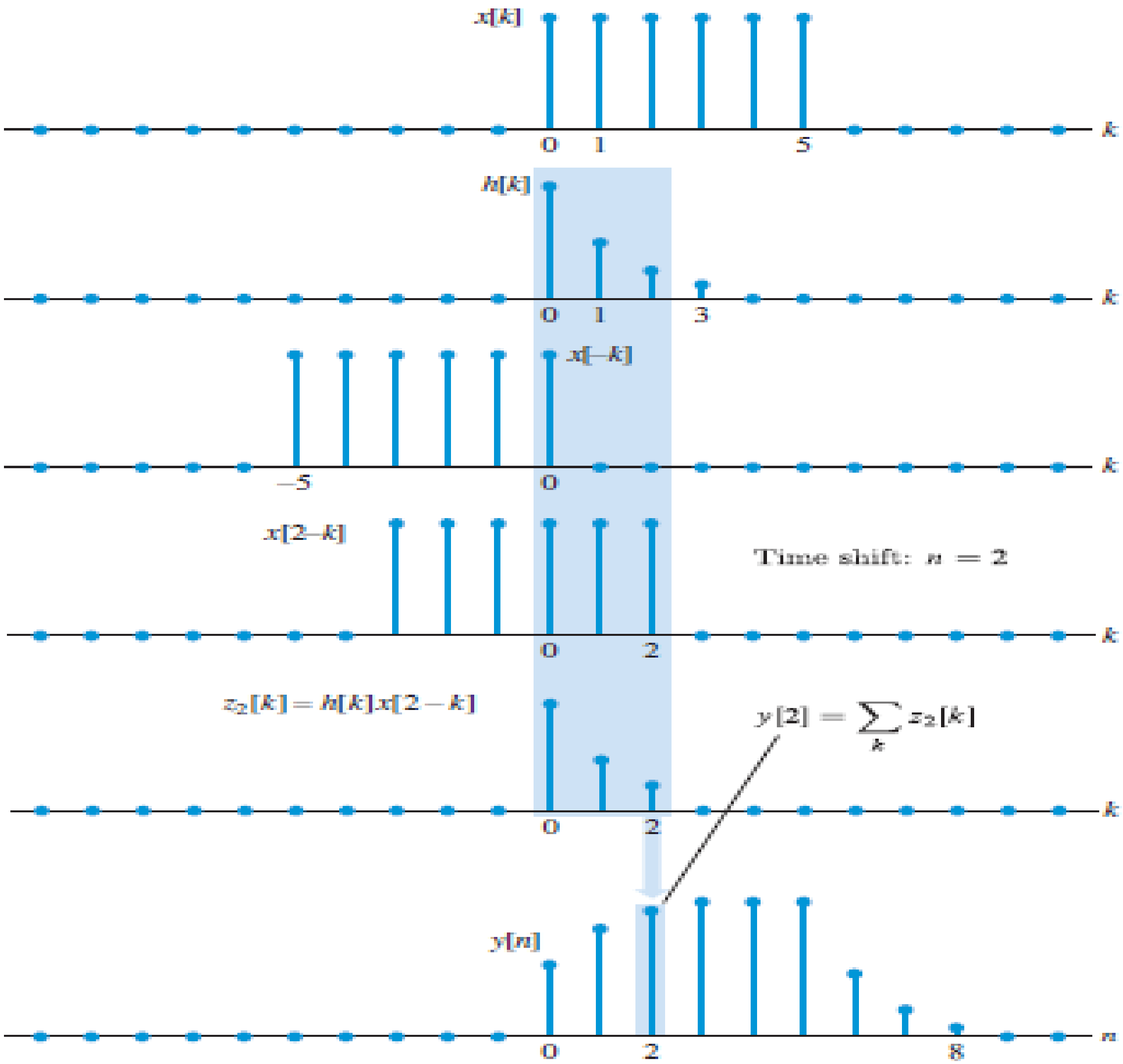


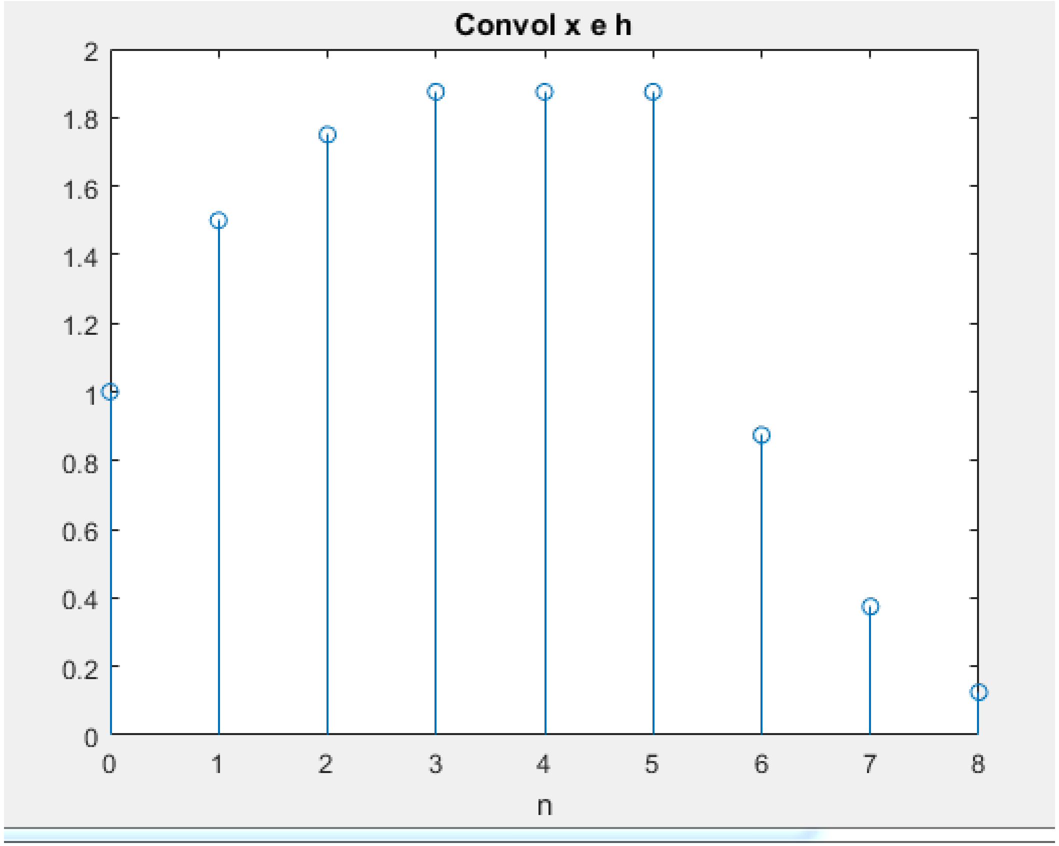
Figure 2.12 Graphical illustration of convolution as a scanning operation.

Convolução – Forma gráfica

Programa exemplo no Matlab

- % Exemplo do calculo da convolução
- clear all;close all; clc;
-
- x = [1 1 1 1 1 1];
- h = [1 .5 .25 .125];
-
- y = conv(x,h)
-
- tama = length(x) + length(h) -1;
-
- n = (0:1:tama-1);
- stem(n,y)
-
- title('Convol x e h');
- xlabel('n');

Convolução – Forma gráfica



Convolução – Forma gráfica

- **Tarefas**:
- Considere o filtro média móvel da aula passada com k = 8, obtenha:
- A saída do filtro para as seguintes entradas: impulso unitário, degrau unitário e o vetor x[n] = [1, 0.5, 0.25, 0.125]

Transformada de Fourier Discreta

- Definição: A TFD de uma sequência x[n] é dada por:

$$X(e^{jw}) = \sum_{n=-\infty}^{n=+\infty} x[n]\, e^{-j\omega n}$$

Transformada de Fourier Discreta

- Fórmulas úteis para séries

$$S = \sum_{n=0}^{\infty} x^n$$

- A série converge para |x| < 1, por exemplo para x = 0,5; temos:

- $$S = 1 + 0{,}5 + 0{,}25 + 0{,}125 + \ldots.$$

$$S = \sum_{n=0}^{\infty} x^n = \frac{1}{1 - x}$$

Transformada de Fourier Discreta

- Fórmulas úteis – Série Geométrica Finita

$$S = \sum_{n=0}^{n=L-1} x^n = \frac{1 - x^L}{1 - x}$$

- A série converge para x≠1, por exemplo, x = 0,5 e L = 4, temos:

- $$S = \sum_{n=0}^{n=3} 0{,}5^n = \frac{1 - 0{,}5^4}{1 - 0{,}5} = 1{,}875$$

Transformada de Fourier Discreta

- Calculando a TFD de alguns sinais.

$$x[n] = a^n u[n]\ ;\ para\ |a| < 1$$

$$X(e^{jw}) = \sum_{n=0}^{n=+\infty} a^n e^{-j\omega n}$$

$$X(e^{jw}) = \sum_{n=0}^{n=+\infty} (ae^{-jw})^n$$

Transformada de Fourier Discreta

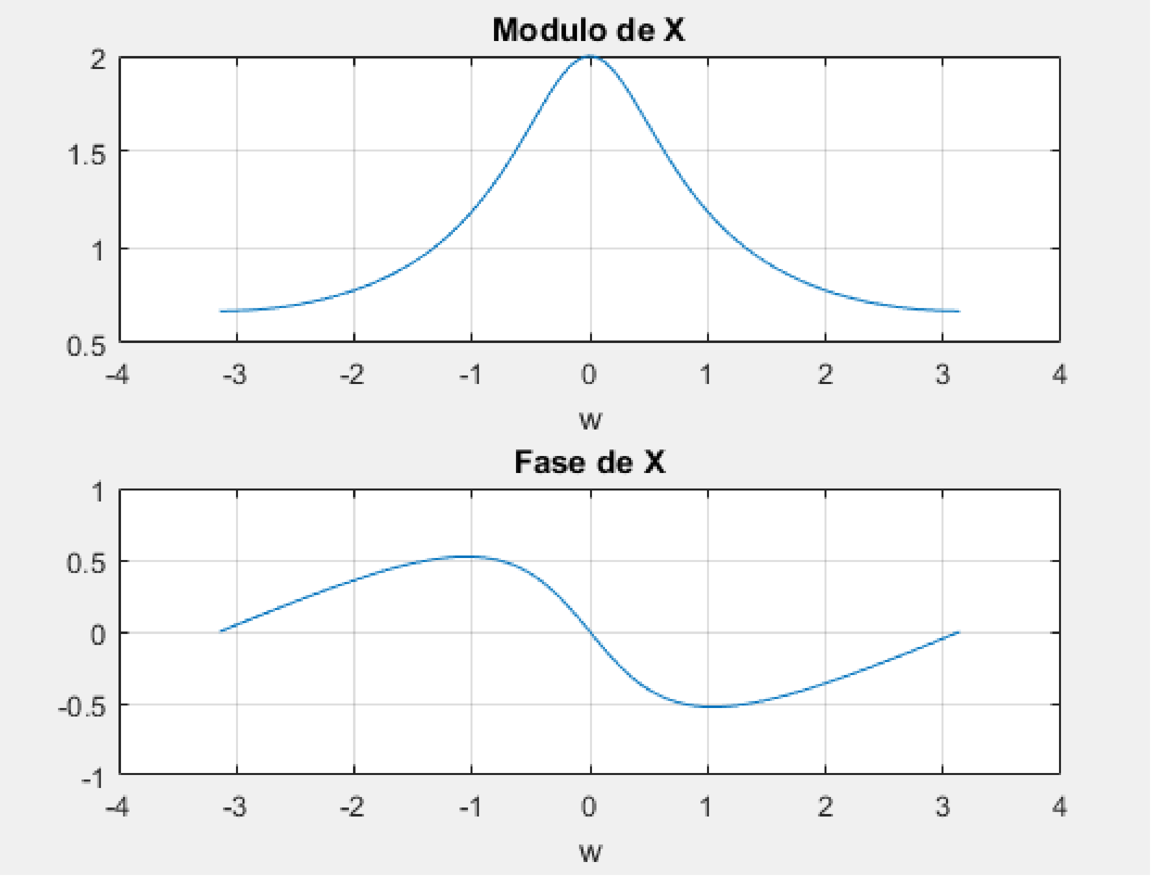
- Calculando a TFD de alguns sinais.

$$x[n] = a^n u[n]\,;\, para\ |a| < 1$$

$$X(e^{jw}) = \frac{1}{1 - ae^{-j\omega}}$$

Transformada de Fourier Discreta

- Plotando o módulo e a fase da função.



Transformada de Fourier Discreta

```
% Exemplo para plotar a Magnitude e Fase da TFD
clear all;close all; clc;

a = .5;

w = [-1*pi:pi/100:1*pi];

Num = 1;
Den = 1 - a*exp(-j*w);
X = Num./Den;

Mod_X = abs(X);
Fase_X = angle(X);
% plotando o módulo e a fase
subplot(2,1,1)
plot(w,Mod_X)
title('Modulo de X');xlabel('w');
grid on;

subplot(2,1,2)
plot(w,Fase_X)
title('Fase de X');xlabel('w');
grid on;
```

Transformada de Fourier Discreta

- Calculando a TFD de alguns sinais.

$$x[n] = \delta[n]$$

- $$X(e^{jw}) = \sum_{n=-\infty}^{n=+\infty} \delta[n] e^{-j\omega n}$$

- $$X(e^{jw}) = 1$$

Transformada de Fourier Discreta

- TAREFA: Calcule e plote a magnitude e fase da TFD do sinal.

$$x[n] = \delta[n+1] + \delta[n] + \delta[n-1].$$

- $$X(e^{j\omega}) = \sum_{n=-1}^{1} x[n]e^{-j\omega n} = e^{j\omega} + 1 + e^{-j\omega} :$$

-

Transformada de Fourier Discreta

- TAREFA: Calcule e plote a magnitude e fase da TFD do sinal.
- Relembrando a equação de Euler

$$e^{jw} = cos(w) + jsin(w)$$

$$e^{-jw} = cos(w) - jsin(w)$$

$$cos(w) = \frac{e^{jw} + e^{-jw}}{2}$$

Transformada Z

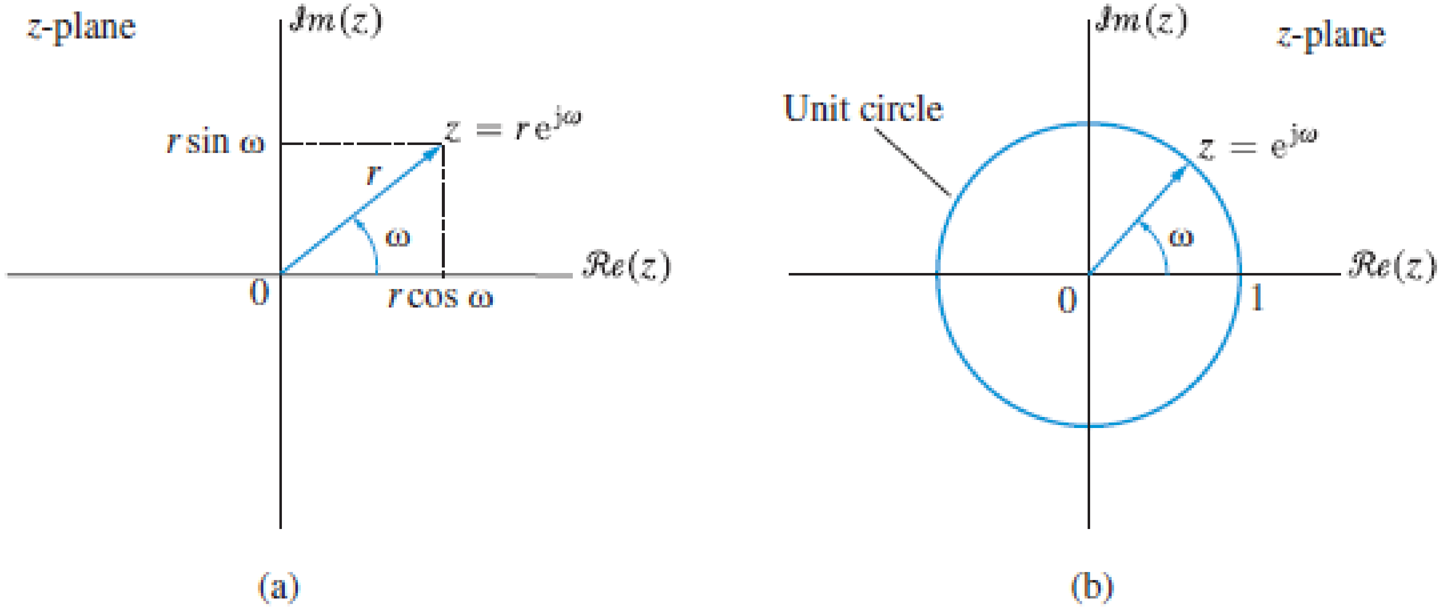
- Definição: A TZ de uma sequência x[n] é dada por:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n},$$

- $$z = re^{j\omega}$$
-

Transformada Z

- Plano Z
-



- **Figure 3.1** (a) A point $z = re^{j\omega}$ in the complex plane can be specified by the distance r from the origin and the angle ω with the positive real axis (polar coordinates) or the rectangular coordinates $r\cos(\omega)$ and $r\sin(\omega)$. (b) The unit circle, $|z| = 1$, in the complex plane.

Transformada Z

- Calculando a TZ de alguns sinais.
- Impulso unitário δ[n]

$$X(z) = \sum_{n=-\infty}^{\infty} \delta[n] z^{-n} = z^0 = 1.$$

Transformada Z

- Calculando a TZ de alguns sinais.

$$x[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

$$X(z) = \sum_{n=0}^{M} 1 z^{-n} = \frac{1 - z^{-(M+1)}}{1 - z^{-1}}.$$

Transformada Z

- Calculando a TZ de alguns sinais.

$$x[n] = \begin{cases} a^n, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$$

$$X(z) = \sum_{n=0}^{M} a^n z^{-n} = \sum_{n=0}^{M} (az^{-1})^n = \frac{1 - a^{M+1} z^{-(M+1)}}{1 - az^{-1}}.$$

Transformada Z

- Calculando a TZ de alguns sinais.

$$x[n] = a^n u[n]$$

$$X(z) = \sum_{n=0}^{\infty} (az^{-1})^n = \frac{1}{1 - az^{-1}} = \frac{z}{z - a}. \quad \text{ROC: } |z| > |a|$$

Transformada Z

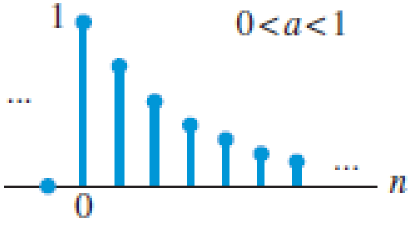
- Calculando a TZ de alguns sinais.

The infinite geometric series converges if $|az^{-1}| < 1$ or $|z| > |a|$. Since $X(z) = 1/(1 - az^{-1}) = z/(z-a)$, there is a zero at $z = 0$ and a pole at $p = a$. For $a = 1$ we obtain the z-transform of the unit step sequence

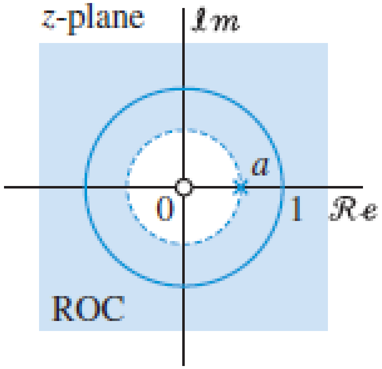
$$X(z) = \frac{1}{1 - z^{-1}}, \quad \text{ROC: } |z| > 1$$

Transformada Z

- Diagrama pólo zero - exponencial

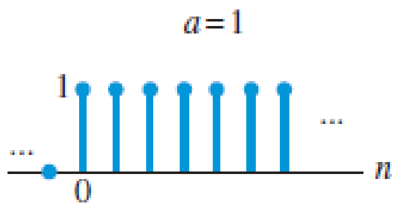


Decaying exponential

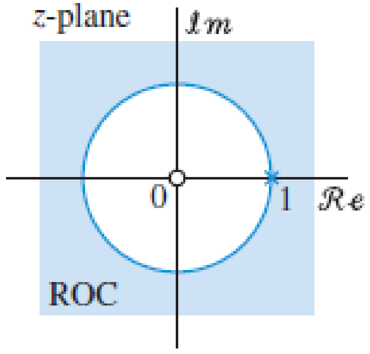


Transformada Z

- Diagrama pólo zero – Degrau unitário



Unit step



Transformada Z - Tabela

Table 3.1 Some common z-transform pairs

	Sequence $x[n]$	z-Transform $X(z)$	ROC
1.	$\delta[n]$	1	All z
2.	$u[n]$	$\dfrac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3.	$a^n u[n]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4.	$-a^n u[-n-1]$	$\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
5.	$na^n u[n]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
6.	$-na^n u[-n-1]$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
7.	$(\cos\omega_0 n)u[n]$	$\dfrac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
8.	$(\sin\omega_0 n)u[n]$	$\dfrac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
9.	$(r^n\cos\omega_0 n)u[n]$	$\dfrac{1-(r\cos\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
10.	$(r^n\sin\omega_0 n)u[n]$	$\dfrac{(\sin\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

Transformada Z

- TAREFA: Faça o exercício 1 da lista enviada.

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