CSC 355. Discrete Structures & Basic Algorithms

Homework 3

Instructions: Solve the following questions.

Question 1. Written Response.

(a) Write a recursive algorithm that takes a linked list of size N elements and splits it into two linked lists where the first list contains all the elements that are less than or equal to the first element in the list and the second list contains all the elements that are greater than the first element in the list.

(b) Show that your algorithm works by running it on example list given below.

(c) Write and explain a recurrence relation for the runtime of the algorithm and give the runtime in big-Oh notation.

For this assignment, assume that you have a pointer to the head of a list but not to the tail of the list. That means adding and removing an element from the front of the list is O(1), but adding or removing from the end of the list is not.

Example:

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Input: L = 3 → 4 → 1 → 0 → 2 → 9
Output: L1 = 3 → 1 → 0 → 2 and L2 = 4 → 9
```

Question 2. Written Response.

Let *A* be an array of size *N* that is made up of two separate subarrays that are already sorted in increasing order. Let *i* be the index where the first sorted subarray ends. Write an algorithm that runs in *O(N)* time and uses *O(N)* extra space that merges the two sorted subarrays into a single sorted array.

Example:

```
Input: [0, 2, 5, 6, 2, 6, 6, 7, 8, 9]
Output: [0, 2, 2, 5, 6, 6, 6, 7, 8, 9]
```

Question 3. Fill in the blank.

The proof below is sparse but valid. For each item, determine the correct line number and type it into the answer space in D2L. Note that when we say something is used in line X, X should be the line in which the item was applied and, in a more detailed proof, would include the item as justification.

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Conjecture: For full binary tree (FBT) T, h(T) <= i(T).
Proof by structural induction.
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1  An FBT with just one node has a height of 0 and has 0 internal
nodes.
2  T1 and T2 are FBTs.
3  T3 = T1·T2.
4  h(T3) = max(h(T1), h(T2)) + 1
5  <= h(T1) + h(T2) + 1
6  <= i(T1) + i(T2) + 1
7  = i(T3)
```

(a) The inductive hypothesis is applied in line __________.

(b) The recursive step of the definition of h(T) for an FBT is used in line _________.

(c) The basis step is proven in line __________.

(d) The recursive step of the definition of i(T) for an FBT is used in line ________.

Question 4. Written Response.

Using structural induction, prove that for any full binary tree T, $e(T) = i(T) + 1$.

Submission Instructions

You must upload your homework in a **pdf** file in the designated area in D2L.

Grading Points

Total Score: 25 points

**Each question has a value of 6.25 points*