Math 243 – Discrete Mathematics **Homework 7: Functions**

Please submit one set of well-written solutions per group. Upload your completed solution set to Gradescope in PDF format. At the beginning of your document, include a brief statement of who contributed to the work. Each solution should be written in paragraph form using complete sentences. For each question, be sure to state any assumptions you have made and discuss your conclusions.

1. Consider the function $h : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined by $h(n) = |\{k \in \mathbb{Z}^+ : k|n\}|$. The bars around the set mean that we are taking the size of the set. Thus $h(n)$ is the number of positive divisors of n.
   (a) Make a table of values for $h(n)$ for $1 \leq n \leq 10$. Write one or two sentences describing how you found the values in the table.
   (b) Find the value of $h(90)$. Explain how you found your answer.
   (c) Find the value of $h(2^m)$. The answer will be a simple formula involving m.
   (d) Is h a one-to-one function? Why or why not?
   (e) Is h an onto function? Why or why not?
   (f) Describe the set $h^{\leftarrow}(2)$. Explain your reasoning. (The superscript left arrow means the preimage.)
2. Consider the function $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ defined by $f(x, y) = (x + y, x - y)$.
   (a) Show that f is one-to-one and onto.
   (b) Find the inverse function $f^{-1}(a, b)$.
3. Let $S = \{a, b, c, d\}$ and let S^* be the set of all possible finite strings using letters from S. (Remember that strings can use repeated letters, and the order of the letters matters.) Define the function $L(w) =$ the length of string w, where $w \in S^*$.
   (a) Identify the domain, target (codomain), and range of L. (There is more than one reasonable target; pick your favorite.)
   (b) Is L one-to-one? Explain.
   (c) Find $L^{\leftarrow}(\{1, 2\})$.
   (d) Let p be a function that turns any string into a palindrome by concatenating the string with its reverse. For example, $p(ab) = abba$ and $p(\lambda) = \lambda$, where λ denotes the null string. Is $p : S^* \to S^*$ well-defined? Why or why not? If not, change the domain and/or target to make p well-defined.
   (e) Using your well-defined version of p, which of the compositions $p \circ L$ or $L \circ p$ is well-defined? Explain, and describe the domain and range of the well-defined function(s).