Homework 1 (due on Sep. 8)

1. Hydrostatic Balance

Using the following expression for hydrostatic balance

$$\frac{\partial P}{\partial z} = -\rho g$$

a. Derive an expression for pressure at some depth -H, assuming that $\rho = \rho_0$ (a constant)

b. Derive an expression for pressure at some depth -H for $\rho = \frac{\partial \rho}{\partial z} z + \rho_0$, where $\frac{\partial \rho}{\partial z}$ is a constant

c. Derive an expression for the percentage difference between your two expressions from a) and b) at depth -H

d. If ρ_0=1027 kg m^{-3}, and $\frac{\partial \rho}{\partial z}$=-0.005 kg m^{-4} (a reasonable value for the range of density with depth in the ocean), what is the percentage difference between your two pressures if H=4000m? Given your answer, do you think it is reasonable to ignore the dependence of density with depth in the hydrostatic balance equation?

2. The Equation of State

Suppose you have two equal water parcels, one with temperature Θ_1 and salinity S_{A1} and another with temperature $\Theta_1+\Delta\Theta$ and salinity $S_{A1}+\Delta S_A$

a. Using the following equation for density, show mathematically that the resulting density of the mixture of the two parcels is always greater than the average density of the two parcels. Assume that each parcel contributes equally to the mixture.

$$\rho(\Theta, S_A, p) \approx \rho_0 \left[1 - \frac{gz}{c_0^2} - \alpha(1-\gamma_b z)(\Theta - \Theta_0) - \frac{\gamma_c}{2}(\Theta - \Theta_0)^2 + \beta(S_A - S_{A0})\right]$$

b. Which term(s) in the density equation give rise to the difference?

c. What is different about this/these term(s) relative to the other terms in the equation?

3. North Atlantic Deep Water and Antarctic Bottom Water

Given the following average properties for NADW and AABW

NADW: S_A=34.9 g/kg, Θ=1℃

AABW: S_A=34.6 g/kg, Θ=−1.5℃

a. Explain in a few sentences why AABW should sit below NADW in the ocean

b. Show mathematically that this is the case by calculating the potential density of each water mass with respect to the surface and with respect to 4000 m. Use the same equation you used for density in 2) and assume the following ρ_0=1027 kg/m^3, Θ_0=10℃, S_{A0}=35 g/kg, g=9.8 m/s^2, c_0=1500 m/s, and γ_c, and γ_b defined as in the class notes. Something to think about before you do your calculation: what sign should values of z have?

c. Which water mass is denser at each of the reference levels you used in b)? How do your results relate to your answer in a)?

d. How do your answers change if γ_b=0?