Temporal Conditional Reasoning with Weighted Knowledge Bases

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Abstract

In this paper we introduce a temporal, conditional logic with typicality, which combines a multi-preferential conditional logic, allowing for defeasible reasoning, with the temporal modalities of Linear Time Temporal Logic. The combination provides a formalism which is capable of capturing the dynamics of a system, trough its strict and defeasible temporal properties. The paper also studies weighted temporal conditional knowledge bases for strengthening preferential entailment.

Keywords

Preferential and Conditional reasoning, Temporal logic, Typicality

1. Introduction

Preferential logics [1, 2, 3, 4, 5, 6, 7, 8, 9] have been proposed to provide axiomatic foundations of non-monotonic or defeasible reasoning, and to deal with commonsense reasoning. They allow to capture the defeasible properties of a domain through conditional formulas (simply, *conditionals*), of the form $\alpha \mathrel{|\!\sim} \beta$, meaning that "normally if α holds, β holds".

In this short paper we aim at extending a conditional logic with typicality, based on a multi-preferential semantics, with the temporal operators X (next), $\mathcal{U}$ (until), $\Diamond$ (eventually) and $\Box$ (always) of Linear Time Temporal Logic (LTL) [10]. Preferential extensions of LTL with defeasible temporal operators have been studied [11, 12, 13] to enrich temporal formalisms with non-monotonic reasoning features, by considering defeasible versions of the LTL operators. Our approach, instead, adds the standard LTL operators to a conditional logic with typicality, an approach which has been previously pursued in the definition of a preferential extension of the temporal description logic $LTL_{\mathcal{ALC}}$ [14] with a *typicality operator* to allow for conditional reasoning in temporal Description Logics (DLs) [15]. In [16] a many-valued conditional temporal logic has also been proposed, for the verification of conditional properties of an argumentation graph in gradual argumentation semantics. In this paper we consider the two-valued case and develop an approach for combining preferences based on weights.

As in the Propositional Typicality Logic (PTL) by Booth et al. [17] (and in the DLs with typicality [18]) the conditionals are formalized based on material implication (resp., concept inclusions) plus the *typicality operator* $\mathbf{T}$. A *conditional implication* $\mathbf{T}(\alpha) \rightarrow \beta$ means that "normally if α holds, β holds", or that "in the typical situations in which α holds, β also holds". For instance, the temporal conditional implication: $\mathbf{T}(Student) \rightarrow \Diamond get\_degree$ means that, normally, students will eventually get a degree.

Conditional implications $\mathbf{T}(\alpha) \rightarrow \beta$ correspond to conditionals $\alpha \mathrel{|\!\sim} \beta$ in KLM logics [4, 6], when α and β are propositional formulas. In this paper, we exploit a *multi-preferential semantics*, based on *multiple preference relations* $<_{A_i}$ with respect to different *distinguished formulas* A_i, along the lines of previous semantics exploiting preferences with respect to different aspects [19], and including ranked and weighted knowledge bases (KBs) for Description Logics [20, 21]. The idea is that a world w may

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represent a more typical situation describing a student, compared to w' ($w <_{stud} w'$) but, vice-versa, world w' may represent a more typical situation describing an employee, compared to w ($w' <_{emp} w$). Under this respect, the semantic we consider is a generalization of the KLM preferential semantics, which exploits a single preference relation on worlds. Decidability of the satisfiability problem is proven.

The paper provides a construction for strengthening preferential entailment in the temporal conditional logic, based on weighted temporal KBs, in which conditional implications are associated a weight representing its plausibility or implausibility. For instance, for the proposition $student$, we may have a set of weighted temporal conditionals $K_{student} = \{(\mathbf{T}(student) \rightarrow has\_Classes, +50)$, $(\mathbf{T}(student) \rightarrow \Diamond holds\_Degree, +30)$, $(\mathbf{T}(student) \rightarrow has\_Boss, -40)\ \}$, that describes *prototypical properties* of students, i.e., that a student normally has classes and will eventually get a degree, but she usually does not have a boss (negative weight). Accordingly, a student having classes, but not a boss, is regarded as being more typical than a student having classes and a boss.

An approach for combining preferences is also presented, defining the preference relations for boolean combinations of distinguished propositions (e.g., $student \wedge employee$, if $student$ and $employee$ are distinguished propositions). These preference relations are defined by combining the weights of the distinguished propositions at each time point, by generalizing an approach recently proposed for propositional conditional logic (and Conditional ASP) [22].

The schedule of the paper is the following. Section 2 develops a many-valued preferential logic with typicality. Section 2.2 extends the logic with LTL modalities to develop a temporal conditional logic. In Section 3 weighted temporal conditional KBs are introduced, and a construction for combining preferences is developed. Section 4 concludes the paper.

2. A temporal multi-preferential logic with typicality

In this section we define a two-valued preferential logic with typicality, which generalizes Kraus Lehmann and Magidor's preferential semantics [4, 6], by allowing for multiple preference relations (i.e., preferences with respect to multiple aspects), rather than a single preference relation.

We consider a propositional language L, whose formulae are built from a set $Prop$ of propositional variables using the boolean connectives $\wedge$, $\vee$, $\neg$ and $\rightarrow$ of propositional logic. We assume that $\bot$ (representing falsity) and $\top$ (representing truth) are formulae of L.

A propositional language with a typicality operator is introduced following the approach used in the description logic $\mathcal{ALC} + \mathbf{T}$ [23] as well as in the Propositional Typicality Logic (PTL) [17]. Let $L^{\mathbf{T}}$ be the language with typicality. Intuitively, "a sentence of the form $\mathbf{T}(\alpha)$ is understood to refer to the *typical situations in which α holds*" [17]. As in PTL [17], the typicality operator cannot be nested. An implication of the form $\mathbf{T}(\alpha) \rightarrow \beta$ is called a *defeasible implication*, meaning that "normally, if α then β". An implication $\alpha \rightarrow \beta$ is called *strict*, if it does not contain occurrences of the typicality operator.

The KLM preferential semantics [4, 6, 3] exploits *a set of worlds $\mathcal{W}$*, with their valuation and a preference relation $<$ among worlds (where $w < w'$ means that world w is more normal than world w'). A conditional $A \mathrel{|\!\sim} B$ is satisfied in a KLM preferential interpretation, if B holds in all the most normal worlds satisfying A, i.e., in all $<$-minimal worlds satisfying A. Here, instead, we consider a *multi-preferential semantics*, where preference relations are associated with distinguished propositional formulas $A_1, \ldots, A_m$ (*distinguished propositions*). In the semantics, a preference relation will be associated with each distinguished proposition A_i, where $w <_{A_i} w'$ means that world w is less atypical than world w' concerning aspect A_i.

2.1. Multi-preferential semantics

In the following, we shortly recall the multi-preferential semantics from [22]. We consider finite KBs, and a finite set of *distinguished propositions* $A_1, \ldots, A_m$. Preferential interpretations are equipped with a set of worlds $\mathcal{W}$ and a finite set of preference relations $<_{A_1}, \ldots, <_{A_n}$, where, for each distinguished proposition A_i, $<_{A_i}$ is a *strict partial order* on the set of worlds $\mathcal{W}$. For the moment, we assume that, in any typicality formula $\mathbf{T}(A)$, A is a distinguished proposition.

Definition 1. *A* (multi-)preferential interpretation *is a triple* $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v\rangle$ *where:*

- $\mathcal{W}$ *is a non-empty set of worlds;*
- *each* $<_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ *is an irreflexive and transitive relation on* $\mathcal{W}$*;*
- $v : \mathcal{W} \longrightarrow 2^{Prop}$ *is a valuation function, assigning to each world* w *a set of propositional variables in* $Prop$.

A *ranked interpretation* is a (multi-)preferential interpretation $\mathcal{M} = \langle \mathcal{W}, \{<_{A_i}\}, v\rangle$ for which all preference relations $<_{A_i}$ are *modular*, that is: for all x, y, z, if $x <_{A_i} y$ then $x <_{A_i} z$ or $z <_{A_i} y$. A relation $<_{A_i}$ is *well-founded* if it does not allow for infinitely descending chains of worlds $w_0, w_1, w_2, \ldots$ with $w_1 <_{A_i} w_0,\ w_2 <_{A_i} w_1$, etc. The valuation v is inductively extended to all formulae:

$$\mathcal{M}, w \models \top \qquad\qquad \mathcal{M}, w \not\models \bot$$
$$\mathcal{M}, w \models p \text{ iff } p \in v(w), \quad \text{for all } p \in Prop$$
$$\mathcal{M}, w \models A \wedge B \text{ iff } \mathcal{M}, w \models A \text{ and } \mathcal{M}, w \models B$$
$$\mathcal{M}, w \models A \vee B \text{ iff } \mathcal{M}, w \models A \text{ or } \mathcal{M}, w \models B$$
$$\mathcal{M}, w \models \neg A \text{ iff } \mathcal{M}, w \not\models A$$
$$\mathcal{M}, w \models A \rightarrow B \text{ iff } \mathcal{M}, w \models A \text{ implies } \mathcal{M}, w \models B$$
$$\mathcal{M}, w \models \mathbf{T}(A_i) \text{ iff } \mathcal{M}, w \models A_i \text{ and } \nexists w' \in \mathcal{W} \text{ s.t. } w' <_{A_i} w \text{ and } \mathcal{M}, w' \models A_i.$$

Whether $\mathbf{T}(A_i)$ is satisfied at a world w also depends on the other worlds of the interpretation $\mathcal{M}$. Restricting our consideration to modular interpretations, leads to the notions of satisfiability and validity of a formula in the *ranked (or rational)* multi-preferential semantics. Unlike [22] here we do not assume well-foundedness of the preference relations.

When an implication has the form $\mathbf{T}(A) \rightarrow B$, with B in $\mathcal{L}$, it stands for a conditional $A \mathrel{|\!\sim} B$ in KLM logics [4]. It can be easily proven that, when all the preference relations $<_{A_i}$ coincide with a single well-founded preference relation $<$, a multi-preferential interpretation $\mathcal{M}$ corresponds to a KLM preferential interpretation, and a defeasible implication $\mathbf{T}(A) \rightarrow B$ (with A and B in $\mathcal{L}$) has the semantics of a KLM conditional $A \mathrel{|\!\sim} B$. The multi-preferential semantics is, therefore, a generalization of the KLM preferential semantics.

It is well known that preferential entailment and rational entailment are weak. As with rational closure [6] and lexicographic closure [24] for KML conditionals, also in the multi-preferential case entailment can be strengthened, restricting to specific preferential models, based on some *closure construction*, which allow for defining preference relations $<_{A_i}$ from a knowledge base K, e.g., exploiting the ranks and weights of conditional implications, when available [19, 20, 25, 22], in the two-valued and many-valued case. In Section 3 we will consider a construction for reasoning from weighted KBs in the language $LTL^{\mathbf{T}}$.

2.2. A temporal multi-preferential semantics

Compared with the preferential semantics above, the semantics of $LTL^{\mathbf{T}}$ also considers the temporal dimension, through a set of time points in $\mathbb{N}$. The valuation function assigns, at each time point $n \in \mathbb{N}$, a truth value to each propositional variable in a world $w \in \mathcal{W}$; the preference relations $<^n_{A_i}$ (with respect to each A_i) are relative to time points. Evolution in time may change the valuation of propositions at the worlds, and it may also change the preference relations between worlds (w might represent a typical situation for a student at time point 0, but not at time point 50).

Definition 2. *A* temporal (multi-)preferential interpretation *(or* $LTL^{\mathbf{T}}_D$ *interpretation) is a triple* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n\in\mathbb{N}}, v\rangle$ *where:*

- $\mathcal{W}$ *is a non-empty set of worlds;*
- *each* $<^n_{A_i} \subseteq \mathcal{W} \times \mathcal{W}$ *is an irreflexive and transitive relation on* $\mathcal{W}$*;*

- $v : \mathbb{N} \times \mathcal{W} \longrightarrow 2^{Prop}$ *is a valuation function assigning, at each time point* n*, a set of propositional variables in* $Prop$ *to each world* w*.*

For $w \in \mathcal{W}$ and $n \in \mathbb{N}$, $v(n, w)$ is the set of the propositional variables which are true in world w at time point n. If there is no $w' \in \mathcal{W}$ s.t. $w' <^n_A w$, we say that w is a normal situation for A at time point n.

Given an $LTL^{\mathbf{T}}$ interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$, we define inductively the *truth value of a formula* A *in a world* w *at time point* n (written $\mathcal{I}, n, w \models A$), as follows:

$$\mathcal{I}, n, w \models \top \qquad\qquad \mathcal{I}, n, w \not\models \bot$$
$$\mathcal{I}, n, w \models p \text{ iff } p \in v(n, w), \quad \text{for all } p \in Prop$$
$$\mathcal{I}, n, w \models A \wedge B \text{ iff } \mathcal{I}, n, w \models A \text{ and } \mathcal{I}, n, w \models B$$
$$\mathcal{I}, n, w \models A \vee B \text{ iff } \mathcal{I}, n, w \models A \text{ or } \mathcal{I}, n, w \models B$$
$$\mathcal{I}, n, w \models \neg A \text{ iff } \mathcal{I}, n, w \not\models A$$
$$\mathcal{I}, n, w \models A \rightarrow B \text{ iff } \mathcal{I}, n, w \models A \text{ implies } \mathcal{I}, n, w \models B$$
$$\mathcal{I}, n, w \models XA \text{ iff } \mathcal{I}, n+1, w \models A$$
$$\mathcal{I}, n, w \models \Diamond A \text{ iff there is an } m \geq n \text{ such that } \mathcal{I}, m, w \models A$$
$$\mathcal{I}, n, w \models \Box A \text{ iff for all } m \geq n,\ \mathcal{I}, m, w \models A$$
$$\mathcal{I}, n, w \models A\mathcal{U}B \text{ iff there is an } m \geq n \text{ such that } \mathcal{I}, m, w \models B \text{ and, for all } k \text{ such that } n \leq k < m,\ \mathcal{I}, k, w \models A$$
$$\mathcal{I}, n, w \models \mathbf{T}(A_i) \text{ iff } \mathcal{I}, n, w \models A_i \text{ and } \nexists w' \in \mathcal{W} \text{ s.t. } w' <^n_{A_i} w \text{ and } \mathcal{I}, n, w' \models A_i.$$

Note that whether a world w represents a typical situation for A_i at a time point n depends on the preference between worlds at time point n.

A *temporal conditional KB* is a set of $LTL^{\mathbf{T}}$ formulas. We evaluate the satisfiability of a temporal formula at the initial time point 0 of a temporal preferential interpretation $\mathcal{I}$, as in LTL.

Definition 3 (Satisfiability and entailment)**.** *A* $LTL^{\mathbf{T}}$ *formula* α *is* satisfied *in a temporal preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ *if* $\mathcal{I}, 0, w \models \alpha$ *for some world* $w \in \mathcal{W}$*. A* $LTL^{\mathbf{T}}$ *formula* α *is* valid *in the temporal preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ *(written* $\mathcal{I} \models \alpha$*) if* $\mathcal{I}, 0, w \models \alpha$*, for all worlds* $w \in \mathcal{W}$*. A preferential interpretation* $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ *is a* model *of a temporal conditional knowledge base* K*, if* $\mathcal{I} \models \alpha$ *holds, for all the formulas* α *in* K*.*

A temporal conditional knowledge base K preferentially (rationally) entails *a formula* α *if* α *is satisfied in all the (ranked) models* $\mathcal{I}$ *of* K*.*

It can be shown that the problem of deciding the satisfiability of a $LTL^{\mathbf{T}}$ formula α can be polynomially reduced to the problem of deciding the satisfiability of a concept C_α in the description logic $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ introduced in [15], which extends the temporal description logic $LTL_{\mathcal{ALC}}$ [14] with the typicality operator. $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ has been proven to be decidable when a finite set of well-founded preference relations $<_{A_1}, \ldots, <_{A_m}$ is considered, and concept inclusions are regarded as global temporal constraints. The decidability of concept satisfiability in $LTL^{\mathbf{T}}_{\mathcal{ALC}}$ relies on the result that concept satisfiability for $LTL_{\mathcal{ALC}}$ w.r.t. TBoxes is in ExpTime (and, actually, it is ExpTime-complete), both with expanding domains [26] and with constant domains [14].

As for KLM logics, the notion of preferential and rational entailment of temporal conditionals is weak. Preferential entailment can be strengthened by restricting to a subset of the temporal preferential models of a conditional knowledge base K.

In the next section we will consider weighted conditionals and extend to the temporal case the construction for reasoning with weighted conditional knowledge bases developed in [22]. In a companion paper [27], dealing with a deontic temporal conditional logic, a construction exploiting temporal deontic conditionals with ranks is developed.

3. Weighted temporal conditional knowledge bases

A *weighted temporal conditional knowledge base* includes *weighted temporal conditionals* of the form $(\mathbf{T}(A_i) \rightarrow B_j, w_{ij})$, where A_i and B_j are LTL formulas, and the weight w_{ij} is a real number, representing the plausibility or implausibility of the conditional implication. For instance, for the proposition $student$, we may have the set of weighted temporal conditionals $K_{student}$ in Section 1, while for $employee$ the set of conditionals $K_{employee} = \{\mathbf{T}(student) \rightarrow has\_Boss$, +40), $(\mathbf{T}(student) \rightarrow \Diamond get\_wage$,+50), $(\mathbf{T}(student) \rightarrow has\_Classes, -30)\}$.

A set of weighted conditionals K_{A_i} is introduced for each distinguished formula A_i. They coexist with a strict part of the knowledge base, K_S, containing formulas with no occurrences of the typicality operator. A *weighted temporal conditional knowledge* base (with respect to the distinguished formulas $A_1, \ldots, A_m$) is a tuple $K = \langle K_S, K_{A_1}, \ldots, K_{A_m} \rangle$, where K_S is the strict part of the knowledge base, and $K_{A_1}, \ldots, K_{A_m}$ are the sets of weighted conditionals (the defeasible part of the KB).

Given a temporal conditional interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$, the preference relation $<^n_{A_i}$ associated with the distinguished proposition A_i at time point n is determined based on the weights of worlds at n, depending on the weights of the conditionals in K_{A_i}.

More precisely, we define for each world $w \in \mathcal{W}$ and time point n, the weight $W_{A_i}(w, n)$ of world w at time point n with respect to proposition A_i, as follows:

$$W_{A_i}(w, n) = \sum_{j: \mathcal{I}, n, w \models B_{i,j}} h_{i,j} \tag{1}$$

Informally, the weight $W_{A_i}(w, n)$ of w at n wrt A_i is the sum of the weights of all conditional implications $\mathbf{T}(A_i) \rightarrow B_{i,j}$ for A_i, such that $B_{i,j}$ is satisfied by w at time point n in the interpretation $\mathcal{I}$. The more plausible are the conditional properties for A_i satisfied in w at n, the higher is the weight of w at time point n with respect to A_i. We let $W_{A_i}(w, n) = 0$, when $K_{A_i} = \emptyset$ (there are no defeasible implications $(\mathbf{T}(A_i) \rightarrow B_{i,j}\ , h_{i,j})$ for the distinguished proposition A_i). We can define a preorder relation $\leq^n_{A_i}$ associated with proposition A_i at time point n as follows: for all $w_1, w_2 \in \mathcal{W}$,

$$w_1 \leq^n_{A_i} w_2 \quad \text{iff} \quad W_{A_i}(w_1, n) \geq W_{A_i}(w_2, n). \tag{2}$$

The strict partial order $<^n_{A_i}$ induced by $\leq^n_{A_i}$ is defined as usual: $w_1 <^n_{A_i} w_2$ iff $w_1 \leq^n_{A_i} w_2$ and $w_2 \not\leq^n_{A_i} w_1$.

Definition 4 (Model of a weighted temporal KB). *A temporal multi-preferential interpretation $\mathcal{I} = \langle \mathcal{W}, \{<^n_{A_i}\}_{n \in \mathbb{N}}, v \rangle$ is an $LTL^{\mathbf{T}}$* model *of a weighted KB $K = \langle K_S, K_{A_1}, \ldots, K_{A_m} \rangle$ if it is a model of K_S and, for all distinguished formulas A_i, $<^n_{A_i}$ is the strict partial order induced by the preorder $\leq^n_{A_i}$.*

A formula α is entailed from a weighted $LTL^{\mathbf{T}}$ knowledge base K *if α is satisfied in all the $LTL^{\mathbf{T}}$ models of K.*

For instance, from the knowledge base containing the set of conditionals $K_{student}$ and $K_{employee}$, the preference relations $<^n_{student}$ are determined as above, and we are able to conclude that $\mathbf{T}(student) \rightarrow has\_Classes \wedge \Diamond holds\_Degree$, while we cannot conclude that $\mathbf{T}(student) \rightarrow has\_Classes \wedge has\_Boss$.

3.1. Combining preferences from weighted knowledge bases

So far, we have only considered typicality formulas of the form $\mathbf{T}(A_i)$, with A_i a distinguished formula. To evaluate whether formulas like:

$$\mathbf{T}(student \wedge employee) \rightarrow has\_Classes \wedge \Diamond holds\_Degree \tag{3}$$

are entailed from a weighted KB, we extend the formalism to deal with general typicality formulas $\mathbf{T}(A)$, with A a boolean combination of distinguished proposition. To deal with such formulas, in this section we extend to the temporal case the approach proposed in [22] for combining preferences in a Conditional ASP based on weighted knowledge bases.

First, for the language of $LTL^{\mathbf{T}}$, which includes typicality formulas $\mathbf{T}(A)$ with A a boolean combination of distinguished formulas, the truth of $\mathbf{T}(A)$ in a world w at time point n in an interpretation $\mathcal{I}$ is defined, as for distinguished formulas A_i (see Definition 2), as follows:

$$\mathcal{I}, n, w \models \mathbf{T}(A) \text{ iff } \mathcal{I}, n, w \models A \text{ and } \nexists w' \in \mathcal{W} \text{ s.t. } w' <^n_A w \text{ and } \mathcal{I}, n, w' \models A.$$

For evaluating the truth of a typicality formula $\mathbf{T}(A)$ at time point n, we need to define the preference relation $<^n_A$ associated to the formula A at time point n. For instance, evaluating formula (3) above in an interpretation $\mathcal{I}$ at timepoint 0, requires determining the most normal situations for an employed student at a time point 0. This requires the preference relation $<^0_{student \wedge employee}$ to be determined. Assume that the weighted knowledge base K, includes the sets of conditionals $K_{student}$ and $K_{employee}$. Then the weight $W_{student \wedge employee}(w, 0)$ of world w at time point 0 with respect to proposition $student \wedge employee$ can be computed from the weights $W_{student}(w, 0)$ and $W_{employee}(w, 0)$. We define the preference relation $<^n_A$ associated with a complex formula A (a boolean combination of the A_i's) at time point n, by inductively extending to complex formulae the notion of the weight of a world w, at time point n.

Given a weighted temporal knowledge base K, let Max and Min be, resp., the maximum and the minimum value of the weight $W_{A_i}(w, n)$, for each distinguished proposition A_i in K, world w and time point n. These values can be determined from the weights of the conditionals in K. We let:

$$W_{A_1 \wedge A_2}(w, n) = min(W_{A_1}(w, n), W_{A_2}(w, n))$$
$$W_{A_1 \vee A_2}(w, n) = max(W_{A_1}(w, n), W_{A_2}(w, n))$$
$$W_{\neg A_i}(w, n) = Max - W_{A_i}(w, n) + Min$$

The preference relation $<^n_A$ associated with a complex formula A can be defined by exploiting the weight function W_A, as for distinguished propositions: For all $w_1, w_2 \in \mathcal{W}$, we let:

$$w_1 <^n_A w_2 \quad \text{iff} \quad W_A(w_1, n) > W_A(w_2, n).$$

The preference relation $<_A$ is modular. By generalizing the result in [22] to the temporal case, the following proposition can be proved.

Proposition 1. *Let A and B be boolean combinations of the distinguished propositions $A_1, \ldots, A_m$. If A and B are equivalent in propositional logic, then, for all worlds $w_1 w_2 \in \mathcal{W}$, $w_1 <^n_A w_2$ iff $w_1 <^n_B w_2$.*

The notion of *model of a weighted temporal KB*, and the notion of entailment from a weighted knowledge base K, can be suitably extended, by letting the preference relation $<^n_A$ be defined as above, for any boolean combinations A of the distinguished propositions $A_1, \ldots, A_m$.

This logic can be used in conjunction with a formalism which generates a set of trajectories from a set of possible initial states (e.g., the runs of a business process). The verification of conditional properties over a set of runs is based on the preference relations computed from a weighted temporal KB.

4. Conclusions

The short paper presents a formalism which combines a preferential logic with typicality and the temporal logic LTL. The interpretation of the typicality operator is based on a multi-preferential semantics, and an extension of weighted conditional knowledge bases to the temporal case is proposed. The extension exploits the weights of conditionals for combining preferences between worlds with respect to different formulas.

On a different route, a preferential logic with defeasible LTL operators has been studied in [12, 28]. The decidability of different fragments of the logic has been proven, and tableaux based proof methods for such fragments have been developed [11, 28]. Our approach does not consider defeasible temporal operators nor preferences over time points, but it combines standard LTL operators with the typicality operator in a temporal logic.

In [16] we considered a many-valued temporal logic with typicality for the verification of temporal properties of gradual argumentation graphs in gradual argumentation semantics.

Future work includes: developing proof methods, e.g., based on ASP encodings, as done for the non-temporal case [29, 22]; exploiting the formalism for explainability and for defeasible reasoning about dynamic systems and, specifically, for reasoning about the dynamics of argumentation graphs in gradual semantics (see [30], but also to reason about obligations and permissions in a deontic temporal conditional logic [27].

While conditional weighted KBs have been shown to capture the stationary states of some neural networks (or their finite approximation) [25], and allow for combining empirical knowledge with elicited knowledge for post-hoc verification, adding a temporal dimension also opens to the possibility of verifying properties concerning the dynamic behavior of a network.

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Declaration on Generative AI

The authors have not employed any Generative AI tool.

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