

The One-Body and Two-Body Density Matrices of Finite Nuclei and Center-of-Mass Correlations

A. Shebeko*, P. Papakonstantinou[†] and E. Mavrommatis**

*NCS "Kharkov Institute of Physics and Technology", Akademicheskaya Str. 1,
61108 Kharkov, Ukraine (shebeko@kipt.kharkov.ua)

[†]Institute of Nuclear Physics, T.U.Darmstadt, Schlossgartenstr. 9, D-64289 Darmstadt, Germany
(panagiota.papakonstantinou@physik.tu-darmstadt.de)

**University of Athens, Physics Department, Nuclear and Particle Physics Division,
Panepistimiopoli, Ilissia, GR-15771 Athens, Greece (emavrom@cc.uoa.gr)

Abstract. A method is presented for the calculation of the one-body (1DM) and two-body (2DM) density matrices and their Fourier transforms in momentum space, that is consistent with the requirement for translational invariance (TI), in the case of a nucleus (a finite self-bound system). We restore TI by using the so-called fixed center-of-mass (CM) approximation for constructing an intrinsic nuclear ground state wavefunction (WF) by starting from a non-translationally invariant (nTI) WF and applying a projection prescription. We discuss results for the one-body (OBMD) and two-body (TBMD) momentum distributions of the ^4He nucleus calculated with the Slater determinant of the harmonic oscillator (HO) orbitals, as the initial nTI WF. Effects of such an inclusion of CM correlations are found to be quite important in the momentum distributions.

INTRODUCTORY REMARKS

The last years the interest in the study of nuclei from both experimental and theoretical point of view involves, besides the 1DM

$$\rho^{[1]}(\vec{r}_1, \vec{r}_{1'}) \equiv A \int \Psi^*(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) \Psi(\vec{r}_{1'}, \vec{r}_2, \dots, \vec{r}_A) d^3 r_2 \dots d^3 r_A \quad (1)$$

and the OBMD

$$\eta(\vec{p}) \equiv \int e^{i\vec{p} \cdot (\vec{r}_1 - \vec{r}_{1'})} \rho^{[1]}(\vec{r}_1, \vec{r}_{1'}) d^3 r_1 d^3 r_{1'}, \quad (2)$$

also the 2DM

$$\rho^{[2]}(\vec{r}_1, \vec{r}_2; \vec{r}_{1'}, \vec{r}_{2'}) \equiv A(A-1) \int \Psi^*(\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_A) \Psi(\vec{r}_{1'}, \vec{r}_{2'}, \vec{r}_3, \dots, \vec{r}_A) d^3 r_3 \dots d^3 r_A \quad (3)$$

and its Fourier transforms, for instance the TBMD

$$\eta^{[2]}(\vec{p}, \vec{k}) \equiv \int e^{i\vec{p} \cdot (\vec{r}_1 - \vec{r}_{1'})} e^{i\vec{k} \cdot (\vec{r}_2 - \vec{r}_{2'})} \rho^{[2]}(\vec{r}_1, \vec{r}_2; \vec{r}_{1'}, \vec{r}_{2'}) d^3 r_1 d^3 r_{1'} d^3 r_2 d^3 r_{2'}. \quad (4)$$

The above quantities provide among others information on the short-range correlations (SRC) in nuclei.

A prominent role towards the experimental investigation of the 2DM and related quantities is played by the study of electromagnetically induced two-nucleon emission (γ, NN), ($e, e'NN$), carried out with high precision in photon facilities (ELSA, MAMI) and electron accelerators with high energy 100% duty-cycle beams (Jlab, MAMI) [1, 2].

One of the theoretical issues still under discussion is a proper consideration of TI and the respective separation of spurious CM effects. The latter contaminate the calculated observables, when the independent-particle shell model and theories which take also dynamical correlations into account (e.g., Brueckner-Hartree-Fock, Variational Monte Carlo) are used, and inhibit the extraction of reliable information on the intrinsic properties of nuclei directly from the experimental data (see, e.g., [3, 4, 5, 6] and Refs. therein).

In this context, we prefer to deal with the intrinsic OBMD and TBMD and the associated 1DM and 2DM [7]. They appear naturally when calculating the cross sections of nuclear emission in the plane-wave impulse approximation, i.e., neglecting the final-state interaction effects and the meson-exchange current contributions to the electromagnetic interactions with nuclei. Eqs. (1)-(4) are modified by properly replacing the A space/momentum vectors by the respective Jacobi coordinates [7]. In particular, we have calculated the expectation value for the TBMD

$$\begin{aligned}\eta_{\text{int}}^{[2]}(\vec{p}, \vec{k}) &= A(A-1) \langle \Psi_{\text{int}} | \delta(\hat{\vec{p}}_{A-1} - \frac{1}{A}\hat{\vec{P}} - \vec{p}) \delta(\hat{\vec{p}}_A - \frac{1}{A}\hat{\vec{P}} - \vec{k}) | \Psi_{\text{int}} \rangle \\ &\equiv A(A-1) \langle \Psi_{\text{int}} | \hat{\eta}_{\text{int}}^{[2]}(\vec{p}, \vec{k}) | \Psi_{\text{int}} \rangle,\end{aligned}\quad (5)$$

where $\hat{\vec{P}} = \sum_{\alpha=1}^A \hat{\vec{p}}_{\alpha}$ is the total momentum operator for a nucleus of A nucleons and Ψ_{int} is the intrinsic WF of the ground state.

EVALUATION OF THE INTRINSIC OBMD AND TBMD

The intrinsic quantities of interest are determined as the expectation values of appropriate A -particle operators, that depend on the Jacobi variables, in intrinsic nuclear states (see Eq. (5)). An algebraic technique [4, 8] is applied for their evaluation, based on the Cartesian representation, in which the coordinate and momentum operators are linear combinations of the creation \hat{a}^{\dagger} and destruction \hat{a} operators for oscillating quanta in the three different space directions (see, e.g., [9]). By following a normal ordering procedure, these intrinsic operators can be reduced to the form: an exponential of the set $\{\hat{a}^{\dagger}\}$ times other exponential of the set $\{\hat{a}\}$. For example, in the case of the intrinsic TBMD we get the representation

$$\hat{\eta}_{\text{int}}^{[2]}(\vec{p}, \vec{k}) = (2\pi)^{-6} \int d^3\lambda_1 d^3\lambda_2 e^{-i\vec{p}\cdot\vec{\lambda}_1} e^{-i\vec{k}\cdot\vec{\lambda}_2} \hat{E}_{\text{int}}(\vec{\lambda}_1, \vec{\lambda}_2), \quad (6)$$

$$\hat{E}_{\text{int}}(\vec{\lambda}_1, \vec{\lambda}_2) = e^{-\frac{p_0^2 \lambda^2}{8}} e^{-\frac{A-2}{A} \frac{p_0^2 \Lambda^2}{2}} \hat{O}_1(\vec{z}) \dots \hat{O}_{A-2}(\vec{z}) \hat{O}_{A-1}(\vec{x}_2) \hat{O}_A(\vec{x}_1), \quad (7)$$

where $\hat{O}_{\alpha}(\vec{y}) = e^{-\vec{y}^* \cdot \hat{\vec{a}}_{\alpha}^{\dagger}} e^{-\vec{y} \cdot \hat{\vec{a}}_{\alpha}}$ ($\alpha = 1, \dots, A$), $\vec{\Lambda} = (\vec{\lambda}_1 + \vec{\lambda}_2)/2$, $\vec{\lambda} = \vec{\lambda}_1 - \vec{\lambda}_2$ and

$$\vec{x}_1 = \frac{p_0}{\sqrt{2}} \left(\frac{A-2}{A} \vec{\Lambda} - \frac{1}{2} \vec{\lambda} \right), \quad \vec{x}_2 = \frac{p_0}{\sqrt{2}} \left(\frac{A-2}{A} \vec{\Lambda} + \frac{1}{2} \vec{\lambda} \right), \quad \vec{z} = -\sqrt{2} \frac{p_0}{A} \vec{\Lambda},$$

with p_0 the oscillator parameter in the momentum space. The Tassie-Barker-type factors in Eq. (7) appear in a model-independent way, i.e. they result from the intrinsic operator structure and do not depend on the intrinsic WF Ψ_{int} , which is yet to be determined.

In the following, we use the intrinsic unit-normalized WF constructed from a given nTI WF Ψ , following Ernst-Shakin-Thaler (EST) (fixed-CM approximation) [3],

$$|\Psi_{\text{int}}^{\text{EST}}\rangle = (\vec{R} = 0|\Psi\rangle)/[\langle\Psi|\delta(\hat{\vec{R}})|\Psi\rangle]^{1/2}, \quad (8)$$

where $|\vec{R}\rangle$ is an eigenvector of the CM operator $\hat{\vec{R}} = A^{-1}\sum_{\alpha=1}^A\hat{\vec{r}}_{\alpha}$ and $\hat{\vec{R}}|\vec{R} = 0\rangle = 0$. Here the bracket $|\)$ is used to represent a vector in the space of the CM coordinates only.

CALCULATION WITHIN THE INDEPENDENT PARTICLE MODEL: APPLICATION TO ^4He AND DISCUSSION

One can show that if $|\Psi\rangle$ is the Slater determinant $|\text{Det}\rangle$ composed of single-particle (s.p.) orbitals $\phi_i(\alpha)$ ($\alpha = 1, \dots, A$), then the evaluation of the distribution $\eta_{\text{EST}}^{[2]}(\vec{p}, \vec{k})$ is reduced to the evaluation of the matrix element

$$\langle\text{Det}|\hat{O}_1(\vec{z}')\cdots\hat{O}_{A-2}(\vec{z}')\hat{O}_{A-1}(\vec{x}'_2)\hat{O}_A(\vec{x}'_1)|\text{Det}\rangle = \langle\text{Det}(-\vec{x}'_1, -\vec{x}'_2, -\vec{z}')|\text{Det}(\vec{x}'_1, \vec{x}'_2, \vec{z}')\rangle,$$

that depends on some new complex vectors $\vec{x}'_1, \vec{x}'_2, \vec{z}'$ [7]. The OBMD is evaluated in a similar way. The Slater determinant $|\text{Det}(\vec{x}'_1, \vec{x}'_2, \vec{z}')\rangle$ is deduced from the original $|\text{Det}\rangle$ via a substitution $|\phi_i(\alpha)\rangle \rightarrow \hat{O}_{\alpha}|\phi_i(\alpha)\rangle$. Further analytical evaluations are simplified with the HO orbitals. For example, in the simplest case of the $0s^4$ configuration, which we encounter in the ^4He nucleus, one can easily see that the matrix element is equal to unity since the s.p. state $|0s\rangle$ coincides with the vacuum state of the Cartesian representation, viz., $\hat{a}|000\rangle = 0$, so that $\exp\{\hat{a}\}|000\rangle = |000\rangle$. Moreover, the results obtained in this case are independent of the projection treatment used (see Eq. (8)).

In this way we have obtained the CM-corrected OBMD of ^4He

$$\eta_{\text{EST}}(\vec{p}) = \eta_{\text{EST}}(p) = 4\frac{4^{3/2}b_{\text{cm}}^3}{3^{3/2}\pi^{3/2}}e^{-\frac{4}{3}p^2b_{\text{cm}}^2} \quad (9)$$

vs. the OBMD in the HO model without CM corrections

$$\eta_{\text{sp}}(\vec{p}) = \eta_{\text{sp}}(p) = 4\frac{b_0^3}{\pi^{3/2}}e^{-p^2b_0^2} \quad (10)$$

and the CM-corrected TBMD of ^4He

$$\eta_{\text{EST}}^{[2]}(\vec{p}, \vec{k}) = 12\frac{2^{3/2}b_{\text{cm}}^6}{\pi^3}e^{-\frac{3}{2}p^2b_{\text{cm}}^2}e^{-\frac{3}{2}k^2b_{\text{cm}}^2}e^{-\vec{p}\cdot\vec{k}b_{\text{cm}}^2} \quad (11)$$

vs. the TBMD in the simple HO model [10]

$$\eta_{\text{sp}}^{[2]}(\vec{p}, \vec{k}) = 12\frac{b_0^6}{\pi^3}e^{-p^2b_0^2}e^{-k^2b_0^2}. \quad (12)$$

The value for b_0 ($b_{\text{cm}} = \sqrt{4/3}b_0$) is obtained by equating the charge rms radius of ^4He , $r_{\text{rms}}^2 = \frac{3}{2}b_0^2 + b_p^2$ in the simple HO model ($r_{\text{rms}}^2 = \frac{3}{2}\frac{A-1}{A}b_{\text{cm}}^2 + b_p^2$ in the HO model with CM corrections), with its experimental value ($r_{\text{rms}} = 1.67$ fm) and by taking the proton rms radius b_p equal to 0.8 fm. We find $b_0 = 1.197$ fm and $b_{\text{cm}} = 1.382$ fm.

From Eqs. (9)-(12) a shrinking of the distribution $\eta_{\text{EST}}(p)$ ($\eta_{\text{EST}}^{[2]}(\vec{p}, \vec{k})$) with respect to $\eta_{\text{sp}}(p)$ ($\eta_{\text{sp}}^{[2]}(\vec{p}, \vec{k})$) follows, i.e., each of these distributions, after being CM-corrected, increases in its central but decreases in its peripheral region. More exactly, in the case of the TBMD this effect is related to the two-dimensional surface given by the function $\eta(p, k, \cos \gamma) \equiv \eta_{\text{EST}}^{[2]}(\vec{p}, \vec{k})$ of the variables p and k at each value of the angle γ between the vectors \vec{p} and \vec{k} . As shown in [4], the shrinking of the OBMD plays an essential role in getting a fair treatment of the data on the inclusive electron scattering in the GeV region. Another prominent feature of $\eta_{\text{EST}}^{[2]}(\vec{p}, \vec{k})$ is its asymmetry due to the γ -dependence. Fig. 1 demonstrates these changes for $\vec{k} = k_p \hat{p}$, where k_p is positive (negative) for \vec{k} in the same (opposite) direction with respect to \vec{p} . The HO2 curves correspond to the calculations in Ref. [10] (b_0 replaced by b_{cm} in Eq. (12)). Among the evident quantitative changes we observe the shift of the peak from $k_p = 0$ towards negative k_p 's, for $p \neq 0$, due to a specific correlation induced by the CM fixation.

In Ref. [10] the dimensionless quantity

$$\xi(\vec{p}, \vec{k}) \equiv \eta^{[2]}(\vec{p}, \vec{k}) / \eta(\vec{p})\eta(\vec{k}) \quad (13)$$

was introduced as a measure of different correlations. In the complete absence of correlations ξ should be equal to $1 - 1/A$. For a finite, self-bound interacting fermion system, deviations of ξ from the above value is a measure of CM and(or) statistical and(or) dynamical correlations. For the nucleus ^4He in the simple HO (where A equals the level degeneracy of the only occupied state, thus statistical correlations are not active),

$$\xi = 1 - 1/A = 0.75 \quad (14)$$

for all \vec{p} and \vec{k} . After fixing the CM, ξ depends on p , k and γ

$$\xi_{\text{EST}}(\vec{p}, \vec{k}) = 0.89493e^{-\frac{1}{6}p^2b_{\text{cm}}^2}e^{-\frac{1}{6}k^2b_{\text{cm}}^2}e^{-\vec{p} \cdot \vec{k}b_{\text{cm}}^2}. \quad (15)$$

In Fig. 2 $\log_{10} \xi$ is plotted as a function of $\cos \gamma$ for selected values of p and k . ξ is significantly reduced in forward angles. The EST TBMD favors momenta of opposite directions as compared to the product of the two OBMD. The same holds for the TBMD of ^4He if Jastrow-type SRC are included, as in Ref. [10] (see the corresponding ξ for the case $p = k = 4$ fm, plotted in Fig. 2 (dotted line)). It is anticipated that within the EST approach additional corrections due to SRC will appear at high values of p and/or k and that they will be larger when \vec{p} and \vec{k} are antiparallel.

PROSPECTS

The method presented here is sufficiently flexible to be applied to a combined consideration of the CM and SRC. The latter, being introduced by means of Jastrow correlations,

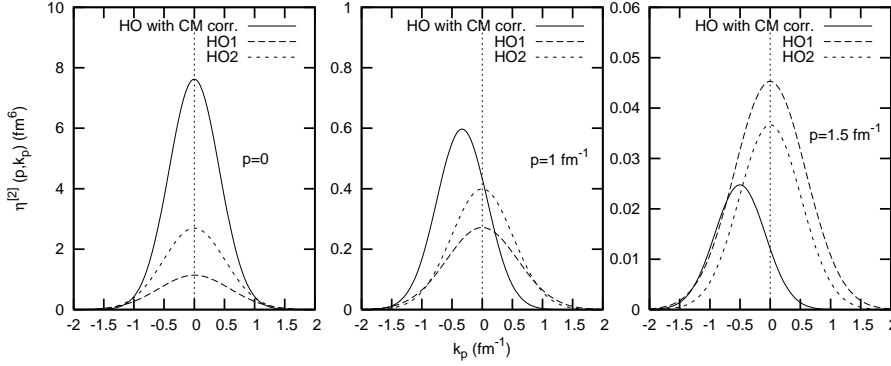


FIGURE 1. The TBMD of ${}^4\text{He}$ for $\vec{p} \parallel \vec{k}$ in the HO model with and without CM correlations (HO1: $b = b_0$; HO2: $b = b_{\text{cm}}$).

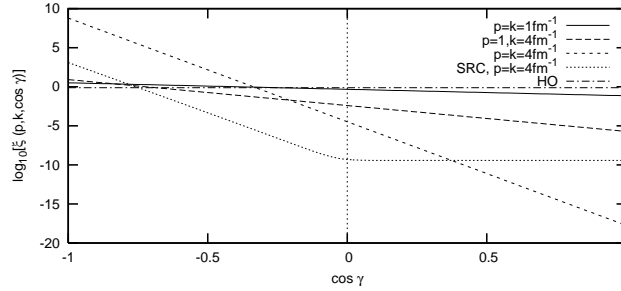


FIGURE 2. The quantity $\log_{10} \xi$ of ${}^4\text{He}$ for the indicated values of p and k as a function of $\cos \gamma$, where γ is the angle between \vec{p} and \vec{k} . Full, long and short-dashed lines: with CM correlations; dot-dashed: HO model without CM correlations; dotted: including SRC but not CM correlations ($b = b_{\text{cm}}$).

do not violate the TI. We are also planning to extend our elaborations to other $Z = N$ light-medium nuclei. This general formalism can be helpful in studying other two-body and many-body quantities.

This research has been supported in part by the University of Athens under Grant No 70/4/3309 and by the Deutsche Forschungsgemeinschaft within SFB 634.

REFERENCES

1. A. Braghieri, C. Giusti and P. Grabmayr (Eds), *Proceedings of the 6th Workshop on Electromagnetically Induced Two-Hadron Emission*, Pavia, Italy September 24-27, 2003.
2. A.V. Stavinsky *et al.* (CLAS Collaboration), *Phys. Rev. Lett.* **93**, 192301 (2004).
3. D.J. Ernst, C.M. Shakin, and R.M. Thaler, *Phys. Rev. C* **7**, 925 (1973).
4. S. Dementij, V. Ogurtzov, A. Shebeko and N.G. Afanasiev, *Sov. J. Nucl. Phys.* **22**, 6 (1976).
5. D. Van Neck and M. Waroquier, *Phys. Rev. C* **58**, 3359 (1998).
6. K.W. Schmid, *Eur. Phys. J. A* **12**, 29 (2001).
7. A.V. Shebeko, P. Papakonstantinou and E. Mavrommatis, submitted to *Eur. Phys. J. A*.
8. A.V. Shebeko and N.N. Goncharov, *Sov. J. Nucl. Phys.* **18**, 532 (1974).
9. A. Bohr and B.R. Mottelson, *Nuclear Structure I*, A. Benjamin, New York, 1969.
10. P. Papakonstantinou, E. Mavrommatis and T.S. Kosmas, *Nucl. Phys. A* **713**, 81 (2003).