TTK4190 Guidance and Control of Vehicles

Assignment 1 part 2

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Problem 1 - Open-loop analysis

1a) Ground Speed

From chapter 2, slide 15 of [1], the relation between the airspeed V_a, ground speed V_g and wind speed V_w is given by the equation:

$$V_a = V_g - V_w \tag{1}$$

With the windspeed being zero, $V_g = V_a = \underline{\underline{637 \text{ km/h}}}$

1b)Expressions for Sideslip

In the absence of wind, the sideslip angle β equals the crab angle χ_c. From [1], chapter 2, slide 16 β is given as a function of relative velocity. Since $\boldsymbol{V}_a^b = [u_r v_r w_r]^T$, β can be expressed as a function of velocity as in eq. (2a). From slide 18 the equation for crab angle is expressed as a function of heading Ψ and course angle χ. Since $\beta = \chi_c$ the equation can be given in eq. (2b).

$$\beta = \underline{\underline{sin^{-1}\left(\frac{v_r}{V_a}\right)}} \tag{2a}$$

$$\beta = \underline{\underline{\chi - \Psi}} \tag{2b}$$

1c) Dutch-Roll Natural Frequency and Relative Damping Ratio

The following equation for dutch-roll is given in page 91 of [2]:

$$\lambda_{dutch\ roll} = \frac{Y_v + N_r}{2} \pm \sqrt{\left(\frac{Y_v + N_r}{2}\right)^2 - (Y_v N_r - N_v Y_r)} \tag{3}$$

Comparing matrix (5.44) from [2] with matrix (5) from [3], the numerical value of $Y_v = -0.322$ and $N_r = -0.32$. We also get the equations:

$$\frac{Y_r}{V_a cos\beta} = -1.12 \tag{4a}$$

$$N_v V_a cos\beta = 6.87 \tag{4b}$$

By multiplying the equations of eq. (4) $N_v Y_r = -7.69$. Inserting the numerical values for Y_v, N_r and $N_v Y_r$ in eq. (3), the eigenvalues are calculated as $\underline{\lambda_1 = -0.3210 + 2.77j}$ and $\underline{\lambda_2 = -0.3210 - 2.77j}$

The characteristic equation is:

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0 \tag{5}$$

Solving for ζ:

$$\zeta = -\left(\frac{\lambda^2 + \omega_n^2}{2\omega_n \lambda}\right) \tag{6}$$

Given that $\zeta(\lambda_1) = \zeta(\lambda_2)$, we can solve for ω_n with eq. (6). This yields:

$$\omega_n = \frac{\lambda_1 \lambda 2^2 - \lambda_1^2 \lambda_2}{\lambda_2 - \lambda_1} \tag{7}$$

Inserting the λ-values in eq. (7) and disregarding negative frequencies, $\underline{\underline{\omega_n = 2.79}}$.

From eq. (6) we can derive $\zeta \pm 0.115$

Can you very briefly describe how the dutch roll mode affect the yaw and roll motion?

How would the motion change with increased relative damping ratio

1d) Spiral Divergence Mode

From [2], page 91, the eigenvalues of the spiral divergence mode is given by:

$$\lambda_{spiral} = \frac{N_r L_v - N_v L_r}{L_v} \tag{8}$$

Comparing the A-matrix (5.44) from [2] with matrix (5) from [3] the parameter values in eq. (8) are:

$$N_r = -0.32 \tag{9a}$$
$$L_r = 0.46 \tag{9b}$$
$$N_v = \frac{6.87}{V_a cos\beta} \tag{9c}$$
$$L_v = \frac{-10.6}{V_a cos\beta} \tag{9d}$$

Inserting these values in eq. (8) yields:

$$\begin{aligned}\lambda_{spiral} &= \frac{-0.32\left(\frac{-10.6}{V_a cos\beta}\right) - \left(\frac{6.87}{V_a cos\beta}\right)0.46}{\left(\frac{-10.6}{V_a cos\beta}\right)} \\ &= \underline{\underline{-0.0219}}\end{aligned} \tag{10}$$

This is in the left hand plane, and therefore the Spiral Divergence Mode is in this case $\underline{\underline{\textit{stable}}}$.

1e) Roll Mode

From [2], page 90, the eigenvalues of the spiral divergence mode is given by:

$$\lambda_{rolling} = L_p \tag{11}$$

Comparing the A-matrix (5.44) from [2] with matrix (5) from [3] $L_p = -2.87$. From eq. (11) we have that $\underline{\underline{\lambda_{rolling} = -2.87}}$.

This is much faster than the role mode.

Hvorfor er det snn? (forklar gjennom det fysiske systemet)

Problem 2 -Autopilot for Course Hold using Aileron and Successive Loop Closure

2a) Finding $a_{\Phi 1}$ and $a_{\Phi 2}$

From [3], figure 1, the following transferfunction can be derived:

$$p = \left(\frac{a_{\phi 2}}{s + a_{\phi 1}} \right) \delta_a \tag{12}$$

Further, from the state space model in [3], the function of $\dot{p}$ in the time domain is given as:

$$\dot{p} = -10.6\beta - 2.87p + 0.46r - 0.65\delta_a \tag{13}$$

From [2], page 90, the rolling mode is obtained by assuming that $\beta = r = 0$. Rewriting eq. (13) in the frequency domain gives us:

$$\mathcal{L}\left(\dot{p}\right) = \mathcal{L}\left(-2.87p - 0.65\delta_a\right) \tag{14a}$$

$$sp(s) = -2.87p(s) - 0.65\delta_a(s) \tag{14b}$$

Solving for p we get:

$$p = \left(\frac{-0.65}{s + 2.87} \right) \tag{15}$$

Comparing eq. (12) with eq. (15) it is trivial to calculate the numerical values:

$$a_{\phi 1} = \underline{\underline{2.87}}$$
$$a_{\phi 2} = \underline{\underline{-0.65}}$$

2b) Find gains

Based on the given values: $\delta_a^{max} = 25°$,$e_\phi^{max} = 15°$, $\zeta_\phi = 0.707$, and the values for $a_{\phi 1}$ and $a_{\phi 2}$we get:

$$K_{p\phi} = sign(\alpha_{\phi 2}) \frac{\delta_a^{max}}{e_a^{max}} = -\frac{25°}{15°} = -\frac{5}{3} \tag{17}$$

From equation 6.2 in [2], we have:

$$\omega_{n\phi} = \sqrt{|\alpha_{\phi 2}| \frac{\delta_a^{max}}{e_a^{max}}} = 2.19\,\frac{\text{rad}}{\text{s}} \tag{18}$$

This gives us a $K_{d\phi}$ equal to:

$$K_{d\phi} = \frac{2\zeta_\phi \omega_{n\phi} - \alpha_{\phi 1}}{\alpha_{\phi 2}} = \frac{2\zeta_\phi \sqrt{|\alpha_{\phi 2}| \frac{\delta_a^{max}}{e_a^{max}}} - \alpha_{\phi 1}}{\alpha_{\phi 2}} = \frac{2 \cdot 0.707 \sqrt{|-0.65| \frac{5}{3}} - 2.87}{-0.65} = 2.15 \tag{19}$$

To find a suitable gain $K_{i\phi}$, we need to analyze the poles of our system. As shown in subchaper 6.3.1 in [2] the poles of the inner system can be found from:

$$1 + k_{i\phi} \left(\frac{\alpha_{\phi 2}}{s^3 + (\alpha_{\phi 1} + \alpha_{\phi 2} k_{d\phi})s^2 + \alpha_{\phi 2} k_{p\phi} s} \right) \tag{20}$$

To use this in the Matlab function *rlocus*() [4], we need to transform this to a feedback loop, with $k_{i\phi}$ as the feedback gain, as shown in figure 1. Using this system in *rlocus*()[4], we get that the system is stable, as long as $k_{i\phi} \in [-2.4, 0]$. To choose an actual value for $k_{i\phi}$, we would need to do simulations of the aircraft. Figure 2a and 2b show how the poles of the system changes with $k_{i\phi}$.

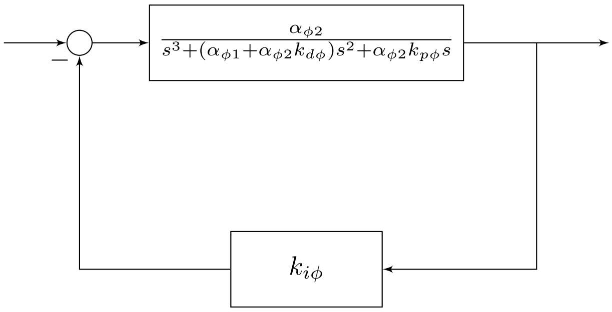
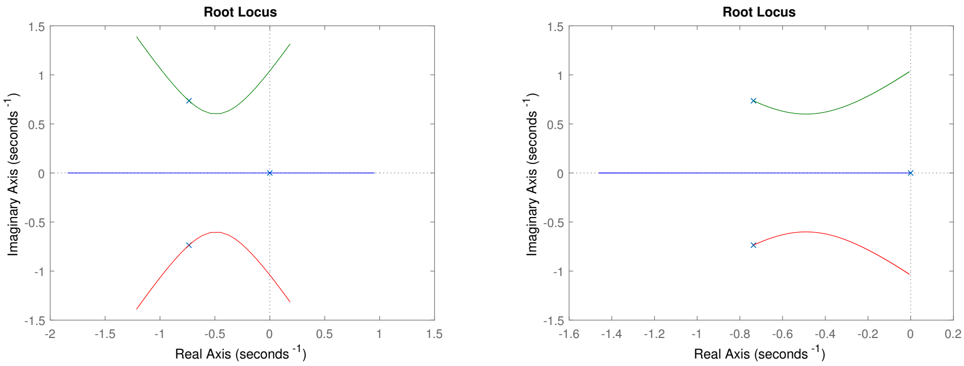


Figure 1: restructured feedback loop for rlocus()



(a) Root locus with $k_{i\phi} \in [-5, 5]$

(b) Root locus with $k_{i\phi} \in [-2.4, 0]$

In chapter 6.3.2 in the book [2], it is described how to choose the gains $k_{p\chi}$ and $k_{i\chi}$. To do this we need to decide where we want to place the natural frequency of the outer loop. We want the outer control loop to be slower than the inner loop. This is done because we need the inner loop to stabilize before the error in the outer loop, to get a more reliable control of the vehicle. We choose the bandwith separation W_χ to be greater than 5. For now we choose $W_\chi = 10$, to see how that will work. The relationship between the natural frequencies ω_{n_χ} and ωn_ϕ is shown in equation 21.

$$\omega_{n_\chi} = \frac{1}{W_\chi}\omega_{n_\phi} = \frac{1}{10}2.19 = 0.22\,\frac{\text{rad}}{\text{s}} \tag{21}$$

From this we can choose $k_{i\chi}$, with

$$k_{i\chi} = \omega_{n_\chi}^2 \frac{V_g}{g} = 0.22^2 \frac{\left(\frac{637}{3.6}\right)}{9.81} = 3.97. \tag{22}$$

To choose $k_{p\chi}$, we need to choose the relative dampening of the outer loop. Figure 6.12 in the book [2] shows the frequency responce of the system, based on the chosen values for ζ_χ. A value $\zeta_\chi = 5$, looks good initially, since it has the fastest response. This requirement may however be to ambitious. A high relative dampening, causes a much higher strain on the actuators, and may also be uncomfortable to fly. As a basis for simulation, we then choose $\zeta_\chi = 2$. This gives us

$$k_{p\chi} = 2 \cdot \zeta_\chi \omega_{n_\chi} \frac{V_g}{g} = 2 \cdot 2 \cdot 0.22 \frac{\left(\frac{637}{3.6}\right)}{9.81} = 15.87 \tag{23}$$

2c) Integral Action in Roll Loop

Integrators add delay and instability and is therefore not desired for the inner most loop. The steady state values will be corrected for in the course loop. Switching between regulators will not transfer the integral. One more parameter to tune.

A stationary offset in roll will cause a stationary offset in course. The integration feedback in course will correct for offsets in roll.

Finskriv.

2d) Open-Loop Transfer Functions

The integration of the roll is not used in this assignment. This is explained in section 2c). From the block diagram in figure (1) of [3] the open loop block diagram from ϕ^c to ϕ can be rewritten as in fig. 3. Using block reduction, the open loop transfer function becomes:

$$\begin{aligned} \frac{\phi}{\phi^c}_{openloop}(s) &= k_{p_\phi} \frac{\left(\frac{a_{\phi_2}}{s+a_{\phi_1}}\right)}{\left(1 + k_{d_\phi} \frac{a_{\phi_2}}{s+a_{\phi_1}}\right)} \frac{1}{s} \\ &= \underline{\underline{\frac{k_{p_\phi} a_{\phi_2}}{s^2 + (a_{\phi_1} + k_{d_\phi} a_{\phi_2})s}}} \end{aligned} \tag{24}$$

With eq. (24), the open loop system from control course (χ^c) to actual course (χ), can be written as in fig. 4. The transfer function of the course open loop is calculated as shown:

$$\begin{aligned} \frac{\chi}{\chi^c}_{openloop}(s) &= (k_{p_\chi} + \frac{k_{i_\chi}}{s}) \frac{k_{p_\phi} a_{\phi_2}}{s^2 + (a_{\phi_1} + k_{d_\phi} a_{\phi_2})s} \frac{g}{V_g s} \\ &= \underline{\underline{\frac{\left(\frac{k_{p_\phi} a_{\phi_2} g}{V_g}\right) k_{p_\chi} s + \left(\frac{k_{p_\phi} a_{\phi_2} g}{V_g}\right) k_{i_\chi}}{s^4 + (a_{\phi_1} + k_{d_\phi} a_{\phi_2})s^3}}} \end{aligned} \tag{25}$$

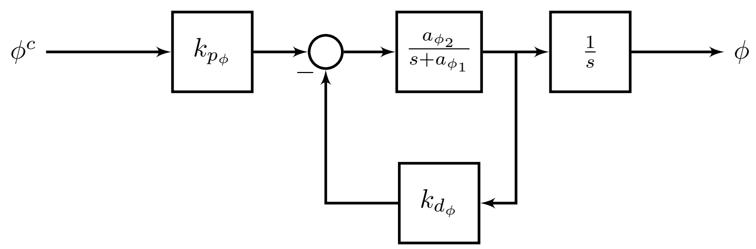


Figure 3: Open loop roll dynamics

Comment on stability

Roll stable for all Heading unstable for all phase

What happens with the stability of the system if the natural frequencies of the course loop is designed with the frequency as the roll loop?

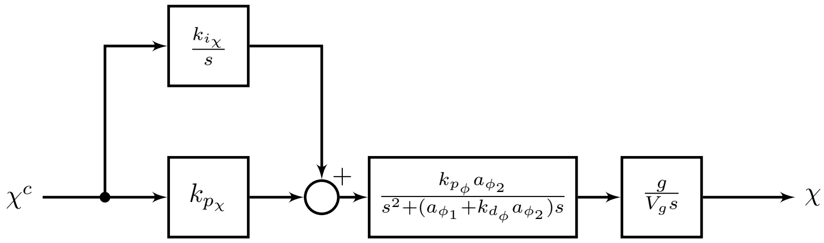


Figure 4: Open loop course dynamics

The inner loop can't stabilize.

Spr studass

2e) Simulation of results

Husk at det skal vre en serie med kontrollinput.

Valg av zeta. Legg til figur (denne er laget Sveinung)

References

[1] R. Beard and T. McLain, "Small unmanned vehicle," 2012.

[2] ——, *Small Unmanned Aircraft*. Princeton University Press, 2012.

[3] T. I. Fossen, "Ttk4190 - assignment 2 - aircraft autopilot design with state estimation," 2017.

[4] "Matlab documentation for rlocus()," https://se.mathworks.com/help/control/ref/rlocus.html, accessed: 2017-10-03.