TTK4190 Guidance and Control of Vehicles

Assignment 1 Part 2

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Problem 2 - Underwater Vehicles

In this part of the assignment, we will study the kinematics of an underwater vehicle, with and without current.

Problem 2.1

Assuming there are no currents and that the sideslip angle is zero degrees, the heading angle ψ is the same as the course angle χ, which is 30°. With a pitch angle θ of 2^o, the BODY-fixed linear velocities are

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} U\cos(\theta)\cos(\chi-\psi) \\ U\sin(\theta)\sin(\chi-\psi) \\ U\sin(\theta) \end{bmatrix} = \begin{bmatrix} 1.499 \\ 0 \\ 0.052 \end{bmatrix} m/s \tag{1}$$

Because the depth is constant, the flight path angle γ is 0.

Problem 2.2

The turning rate r, is defined as the time derivative of the course, χ. Assuming constant heading:

$$\begin{aligned} \dot{\chi} &= \dot{\beta} \\ &= \frac{d}{dt} sin^{-1}(\frac{v}{U}) \\ &= \frac{d}{dt} sin^{-1}(\frac{U sin(\omega t)}{U}) \\ &= \frac{d(\omega t)}{dt} \\ &= \omega \end{aligned} \tag{2}$$

ω is the angular velocity around the circle. The numerical value of ω is given by

$$\omega = \frac{U}{R} = 0.015 rad/s \tag{3}$$

The crab angle, β, is given by

$$\beta = sin^{-1}\left(\frac{U sin(\omega t)}{U}\right) = \omega t \tag{4}$$

Problem 2.3

The current velocities given in the NED frame, is given by the rotation of the current velocity from FLOW to NED, which is the product of two basic rotations[1]:

$$\begin{aligned}(\boldsymbol{v_c})^{\boldsymbol{n}}_{\boldsymbol{f\backslash n}} &= \boldsymbol{R}^T_{y,\alpha}\boldsymbol{R}^T_{z,-\beta}v^f_c \\ &= \begin{bmatrix} cos(\alpha) & 0 & -sin(\alpha) \\ 0 & 1 & 0 \\ sin(\alpha) & 0 & cos(\alpha) \end{bmatrix} \begin{bmatrix} cos(\beta) & sin(\beta) & 0 \\ -sin(\beta) & cos(\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_c \\ 0 \\ 0 \end{bmatrix}\end{aligned} \tag{5}$$

The rotation from the NED to the BODY frame is given by

$$\boldsymbol{R}^b_n = \begin{bmatrix} cos(\phi) & -sin(\phi) & 0 \\ sin(\phi) & cos(\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & 0 & sin(\theta) & \\ 0 & & 1 & 0 \\ -sin(\theta) & 0 & cos(\theta) & \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\psi) & -sin(\psi) \\ 0 & sin(\psi) & cos(\psi \end{bmatrix} \tag{6}$$

Thus, the current velocities in the BODY frame is given by

$$\boldsymbol{v_c} = \boldsymbol{R}^T_{x,\phi}\boldsymbol{R}^T_{y,\theta}\boldsymbol{R}^T_{z,\psi}\boldsymbol{R}^T_{y,\alpha}\boldsymbol{R}^T_{z,-\beta}(v_c)^f \tag{7}$$

This is calculated in MATLAB using the mss toolbox' Rzyx function:

$$\boldsymbol{v_c} = \begin{bmatrix} 0.571 \\ 0.159 \\ 0.094 \end{bmatrix} m/s \tag{8}$$

The relative velocities, i.e. the vehicle's velocities relative to the water, given in the BODY frame, are

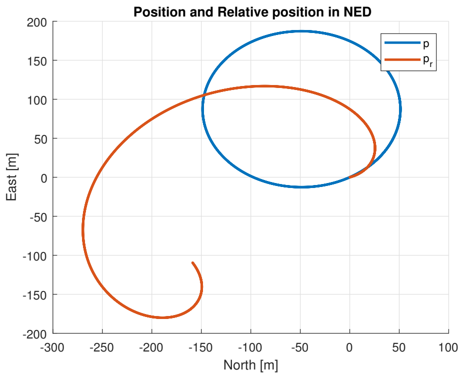
$$\boldsymbol{v_r} = \boldsymbol{v} - \boldsymbol{v_c} \quad = \begin{bmatrix} Ucos(\omega t) - 0.571 \\ Usin(\omega t) - 0.159 \\ -0.094 \end{bmatrix} m/s \tag{9}$$

Problem 2.4

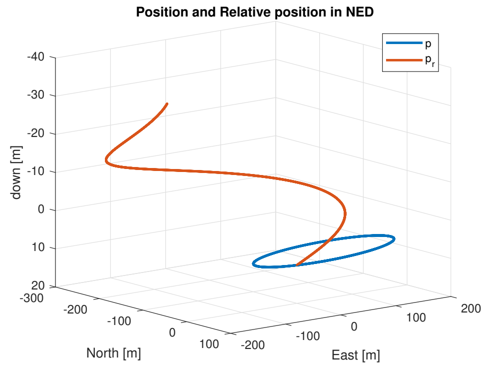
The crab angle β is defined as in equation (4), and the sideslip angle is defined as

$$\beta_r = \beta_r(t) = arcsin\left(\frac{v_r(t)}{\sqrt{u_r(t)^2 + v_r(t)^2}}\right) = arcsin\left(\frac{v(t) - v_c(t)}{U_r(t)}\right). \tag{10}$$

We note that $u_r = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\boldsymbol{v}_r$, $v_r = \begin{bmatrix}0 & 1 & 0\end{bmatrix}\boldsymbol{v}_r$ and $v_c = \begin{bmatrix}0 & 1 & 0\end{bmatrix}\boldsymbol{v}_c$. With the heading, pitch and roll being constant, $\phi = 0°$, $\theta = 2°$ and $\psi = 30°$, the movement of the submarine from origin was simulated over 500 seconds. This resulted in the following plots on page 3:



(a) North-East movement



(b) 3D movement

Figure 1: Movement of the submarine

we see from figure 1b how the rotation of the body axis causes the position in NED to gain a time-varying z-component, but maintains it's circular shape. Since there is a current, we also see that relative velocity, or the velocity relative to the movement of the water, forms a spiral. This is expected as the current moves at a constant rate $U_c = 0.6\,\mathrm{m/s}$, at a constant direction in the NED frame. This can also be seen in figure 2, where the relative velocities are offset at a constant distance relative to the velocities.

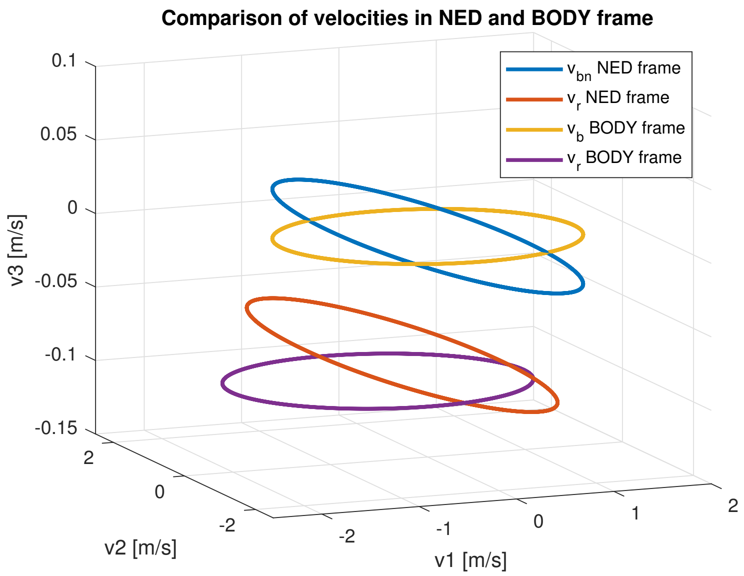


Figure 2: velocity $v(t)$ compared to relative velocity v_r, in body and NED frame

In figure 3 the relationship between crab β, sideslip β_r and course angle χ is shown. Since β and β_r is defined as an arcsine, the resulting crab and sideslip angle will be between $-90°$ and $90°$. As expected, the course angle follows $\chi = \beta + \psi = wt + 30$. The sideslip oscillates around the crab angle, crossing four times. Two of these crossings, at $t \approx 18$ and $t \approx 227$, are caused by the current being alligned with the course, making $\beta = \beta_r$. The other two crossings come from the arcsine calulations.

Figure 3: Crab, sideslip and course angle

Problem 2.5

At steady-state turning, r_s can be found using the final value theorem [2]:

$$\frac{r_s}{\delta} = \lim_{t\to\infty} \frac{r}{\delta}(t) = \lim_{s\to 0} \frac{r}{\delta}(s) = \lim_{s\to 0} \frac{K}{Ts+1} = K = 0.1, \tag{11}$$

for $K = 0.1\,\mathrm{s}^{-1}$. Since $\delta(t) = \delta = constant$, r_s becomes:

$$r_s = K\delta = 0.1\delta. \tag{12}$$

Further, $\dot{p} = \dot{q} = 0$ at steady state, which means that

$$\begin{aligned} q_s &= -\frac{\omega_q \theta_s}{2\zeta_q} = -0.125\theta \\ p_s &= -\frac{\omega_p \phi_s}{2\zeta_p} = -0.5\phi, \end{aligned} \tag{13}$$

for $\zeta_p = 0.1$, $\zeta_q = 0.2$, $\omega_p = 0.1$ and $\omega_q = 0.125$. Since $q = \dot{\theta}$ and $p = p\dot{h}i$, this implies that $q_s = \theta_s = p_s = \phi_s = 0$.

Problem 2.6

from the Nomoto model, we find the dynamics regarding the turning rate to be:

$$\mathcal{L}^{-1}\left(\frac{r}{\delta}(s) = \frac{K}{Ts+1}\right) \to \mathcal{L}^{-1}\left(Tsr(s) + r(s) = K\delta(s)\right). \tag{14}$$

since $\mathcal{L}(\frac{\partial r}{\partial t}(t)) = sr(s)$, we get:

$$\dot{r} + \frac{1}{T}r = \frac{K\delta}{T}. \tag{15}$$

Using equation 15, in addition to equation (12) in the assignment text, we get the model:

$$\begin{bmatrix} p \\ \dot{p} \\ q \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega_p^2 & -2\zeta_p\omega_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\omega_q^2 & -2\zeta_q\omega_q & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{T} \end{bmatrix} \begin{bmatrix} \phi \\ p \\ \theta \\ q \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{K}{T} \end{bmatrix} \delta \tag{16}$$

with ω_q, ω_p, ζ_q, ζ_p and K as in problem 2.5.

Using the initial conditions:

$$\mathbf{\Theta}(0) = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} -1,0° \\ 2,0° \\ 0,0° \end{bmatrix} \quad and \quad p(0) = q(0) = r(0) = 0,0 \tag{17}$$

the system was simulated, starting out with a rudder angle $\delta = 5°$ and increasing it to $10,0°$ after 700 seconds. The results are presented in the following plots, and discussed below the relevant figures.

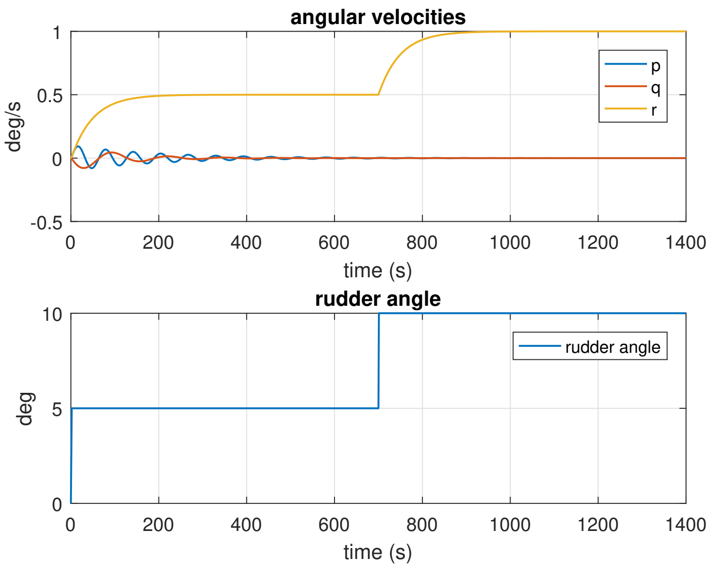
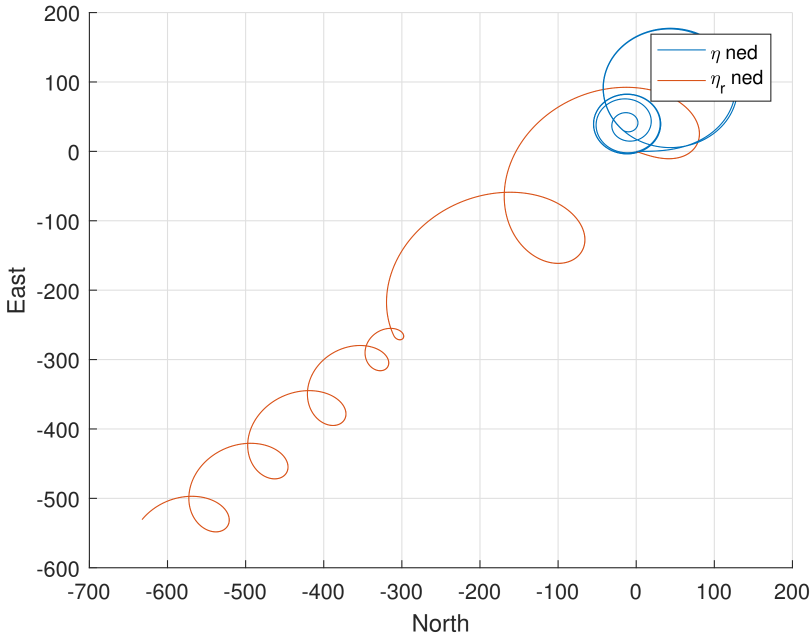
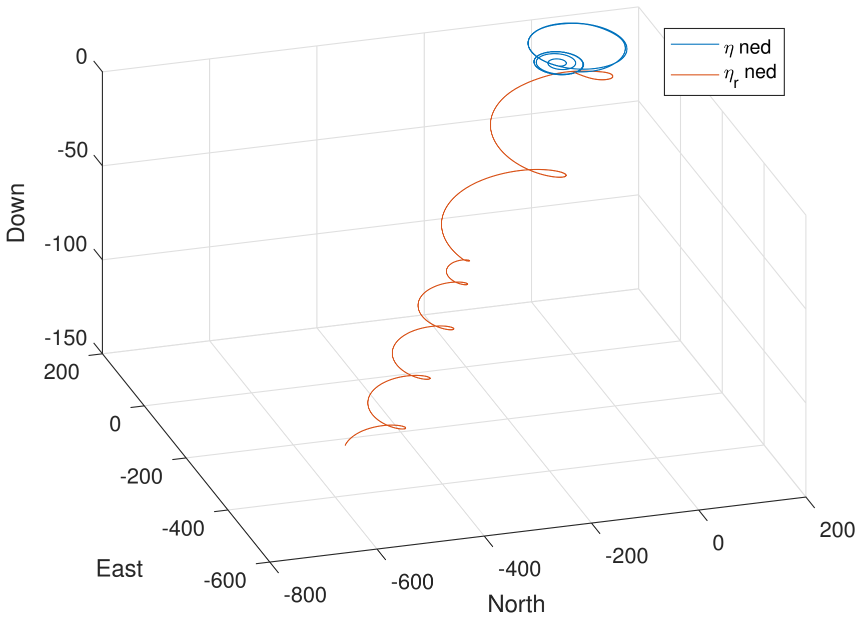


Figure 4: Angular velocities of the vehicle

We can clearly see from figure 4 how the turning rate follows a first order response, stabilizing to the expected values of $0,1\delta$. Since the damping constants ζ_p and ζ_q are both ≤ 1, oscillations are expected. Here er can also see that p and q stabilizes to the expected values: $p = q = 0$.



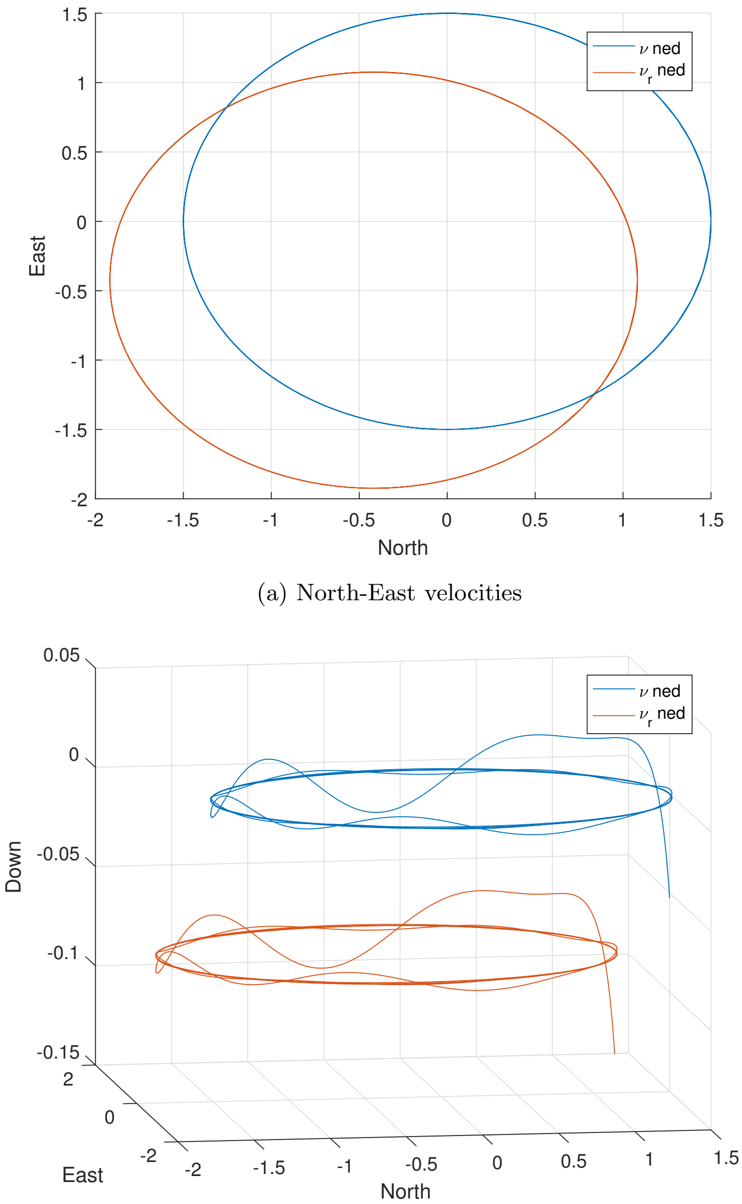
(a) North-East movement



(b) 3D movement

Figure 5: Translational movement of the submarine

As in problem 2.4, we assume that the ship's velocity is given in a fixed frame. When the velocities in north and east directions are given by sine and cosine functions of the same angle, the resulting movement is a circular path fixed in space, which is expected when the rudder is set at a fixed angle. This can be seen in figure 5. The relative velocity, v_r, gives the vehicle's movement relative to the water. From this we can assume that the vehicle is perfectly controlled to negate the effects of the current.



(a) North-East velocities

(b) 3D velocities

Figure 6: Translational velocities of the submarine

Figures figure 4,figure 6 and figure 7 clearly show the spring-damper behaviour of the pitch and roll motion. The same figures show that the yaw motion reaches a steady-state yaw rate, r_s, which depends on the rudder angle. For both values of rudder angle, the yaw rate satisfies equation (11).

When the simulation was run with no current, the crab angle and the sideslip angle were equal. Obviously, all movement relative to the flow becomes equal to the absolute (NED) movement.

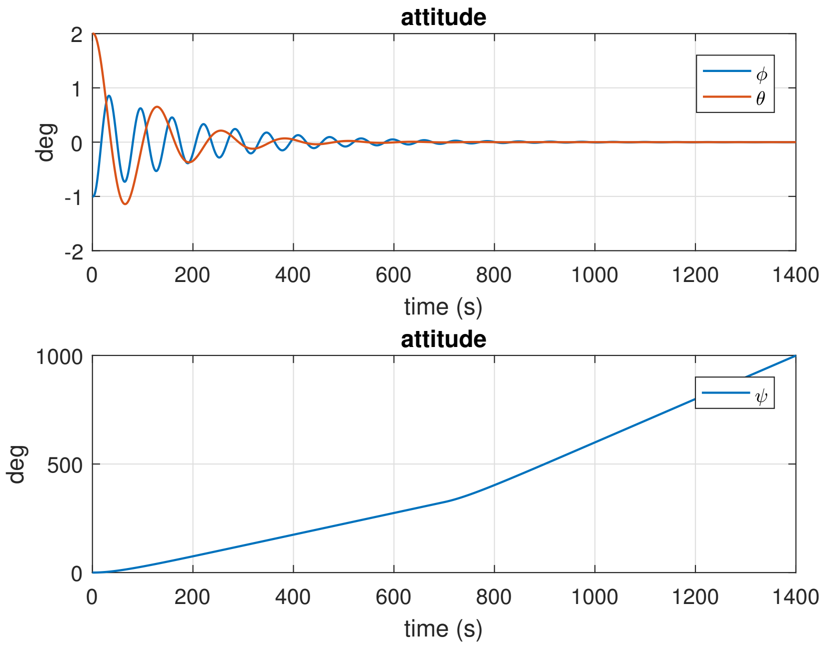


Figure 7: Attitude of the vehicle

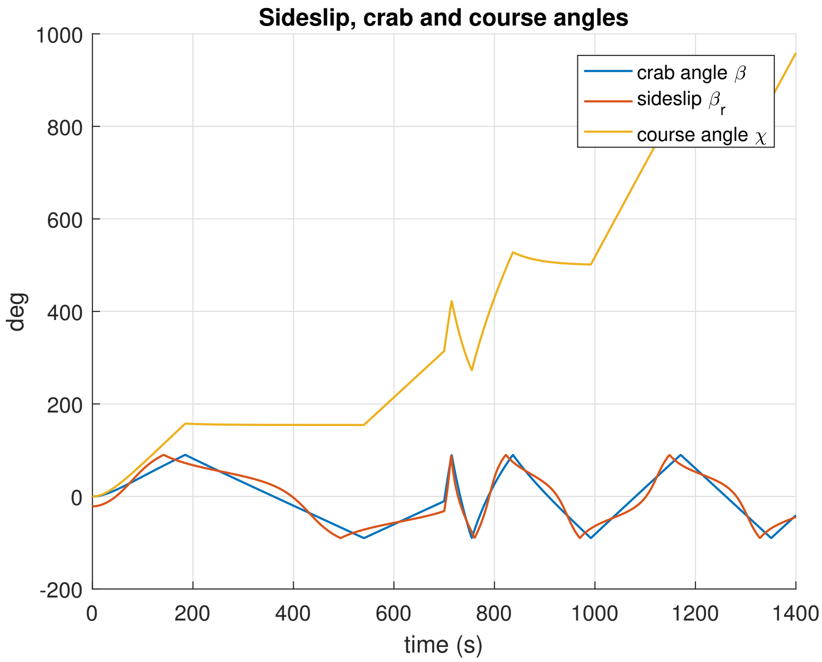


Figure 8: Crab, sideslip and course angle

References

[1] T. Fossen, *Handbook of Marine Craft Hydrodynamics and Motion Control.* John Wiley & Sons, 2011.

[2] J. Balchen, T. Andresen, and B. Foss, "Reguleringsteknikk department of engineering cybernetics," *Norwegian University of Science and Technology, Trondheim, Norway*, 2003.