1) 1) $P(A \cup B) = 1 - P(\overline{A \cup B}) = 1 - .45 = \boxed{.55}$

2) [redacted]

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= (.5) + (.2) - (.55) = \boxed{.15}$

3) $P(A \cap \bar{B}) = P(A) - P(A \cap B) = (.5) - (.15) = \boxed{.35}$

4) A & B are NOT mutually exclusive because $P(A \cap B) = .15$ and not 0.

2) 1) A = top face will be 4 atleast once on 4 rolls of a fair 6-sided die

- can NOT use $1/6 \cdot 4$ because you would be overcounting the cases when more than a single 4 is present

$P(A) = 1 - P(A^c) = 1 - (5/6)^4 = \boxed{.518}$

2) A = top face will be 4 atleast once on 20 rolls of a fair 6-sided die

$P(A) = 1 - P(A^c) = 1 - (5/6)^{20} = \boxed{.974}$

3) A = top face will be 4 atleast once on $X = \boxed{13 \text{ rolls}}$ of a fair 6-sided die

$P(A) = .9 = 1 - (5/6)^x \rightarrow .9 - 1 = -(5/6)^x$

$\ln(.1) = x \ln(5/6)$

$x = \dfrac{\ln(.1)}{\ln(5/6)} = 12.629$

3) • each even face 2x likely

• six sided: 1 , 2 , 3 , 4 , 5 , 6

1 2 1 2 1 2

$(1+2)\cdot 3 = 9$ $\quad \underbrace{1/9 \quad 2/9 \quad 1/9}_{\boxed{4/9}} \quad \underbrace{2/9 \quad 1/9 \quad 2/9}_{5/9}$

4) • 3 books on Prob & Stats

• 2 books on Linalg

• 2 books on ML

• 3 books on culinary

1) binomial coefficient: select k out of n objects

* order does not matter *

$$\binom{n}{k} = C_n^k = \frac{n!}{(n-k)!k!}$$

$$= \frac{10!}{(10-3)!3!} = \frac{10\cdot 9\cdot 8}{3!}$$

$$= \boxed{120}$$

2) A = for 3 randomly selected books, 1 is Prob & Stats, 1 is Linalg, & 1 is culinary

Prob & Stats: $\binom{n}{k} = C_n^k = \frac{3!}{(3-1)!1!} = 3$ ways

Linalg: $\binom{n}{k} = C_n^k = \frac{2!}{(2-1)!1!} = 2$ ways $\qquad \frac{3\cdot 2\cdot 3}{210} =$

Culinary: $\binom{n}{k} = C_n^k = \frac{3!}{(3-1)!1!} = 3$ ways $\qquad \boxed{P(A) = .0857}$

NOT $P(D \cap X)$, $X = A, B, C$

5) 1) D = deffective chip, $P(D|A) = .002$, $P(D|B) = .02$, $P(D|C) = .001$

$P(D) = (.002)(1/3) + (.02)(1/3) + (.001)(1/3)$ ← Law of Total Probability

$= \boxed{.007\bar{6}}$

2) $P(A \cap D) = P(A|D)P(D)$

$P(D \cap A) = P(D|A)P(A)$

$P(A|D)P(D) = P(D|A)P(A)$

$P(A|D) = \dfrac{P(A|D)P(A)}{P(D)}$ ← Bayes' Theorem

$P(A|D) = \dfrac{(.002)(1/3)}{(.007\bar{6})} = \boxed{.0870}$

$P(B|D) = \dfrac{(.02)(1/3)}{(.007\bar{6})} = \boxed{.870}$

$P(C|D) = \dfrac{(.001)(1/3)}{(.007\bar{6})} = \boxed{.0435}$

3) $P(D) = (.002)(.5) + (.02)(.1) + (.001)(.4)$ ← Law of Total Probability

$= \boxed{.0034}$

4) Bayes' Theorem

$P(A|D) = \dfrac{(.002)(.5)}{(.0034)} = \boxed{.294}$

$P(B|D) = \dfrac{(.02)(.1)}{(.0034)} = \boxed{.588}$

$P(C|D) = \dfrac{(.001)(.4)}{(.0034)} = \boxed{.118}$

6) a) four aces in a 52 card deck:

$4/52 = \boxed{.0769}$

b) 4 jacks (1 spade, 1 clover, 1 heart, 1 diamond)

jack of spade: $1/52 = \boxed{.0192}$

c) jack of spade or six of diamond

$1/52 + 1/52 = 2/52 = 1/26 = \boxed{.0385}$

d) 13 cards in each suit & 4 suits

$$\frac{(4-2)\cdot 13}{52} = \frac{26}{52} = \boxed{.5}$$

7) 1) $P(H_1) = \frac{13}{52} = \boxed{.25}$ bc 13 cards per suit (heart)

2) $P(H_2) = P(H_1)\cdot(12/51) + (1-P(H_1))\cdot(4/51)$

$= (13/52)\cdot(12/51) + (39/52)(13/51)$

$= .0588 + .1911$

$= \boxed{.25}$