Lecture 18: Classification

Big Data and Machine Learning for Applied Economics
Econ 4676

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Agenda

Elastic Net

- Naive Elastic Net

$$min_{\beta} NEL(\beta) = \sum_{i=1}^{n}(y_i - x_i'\beta)^2 + \overbrace{\lambda_1 \sum_{s=2}^{p} |\beta_s|}^{\text{Lasso}} + \overbrace{\lambda_2 \sum_{s=2}^{p} \beta_s^2}^{\text{Ridge}} \tag{1}$$

- Elastic Net: reescaled version
- Double Shrinkage introduces "too" much bias, *final* version "corrects" for this

$$\hat{\beta}_{EN} = \frac{1}{\sqrt{1+\lambda_2}} \hat{\beta}_{naive\,EN} \tag{2}$$

- Careful sometimes software asks. Zou & Hastie

Lasso for Causality

Inference with Selection among Many Controls

tratem (0,1)
cont (ℝ)

$g(w_i) = x_i \theta_y$

$$y_i = \alpha d_i + x_i' \theta_y + r_{yi} + \zeta_i \tag{3}$$

- We apply variable selection methods to each of the two reduced form equations and then use all of the selected controls in estimation of α.
- We select
  1. A set of variables that are useful for predicting y_i, say x_{yi}, and
  2. A set of variables that are useful for predicting d_i, say x_{di}.
- We then estimate α by ordinary least squares regression of y_i on d_i and the union of the variables selected for predicting y_i and d_i, contained in x_{yi} and x_{di}.
- We thus make sure we use variables that are important for either of the two predictive relationships to guard against OVB

Classification

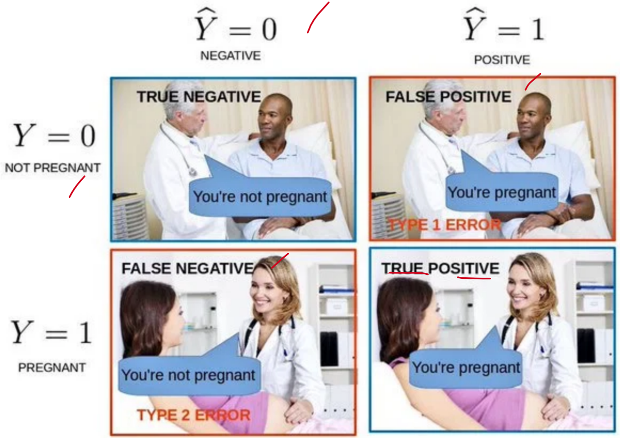
Classification: Motivation

- Admit a student to *PEG* based on their grades and LoR
- Give a credit, based on credit history, demographics?
- Classifying emails: spam, personal, social, based on email contents
- Aim is to classify y based on $X's$
- y can be
  - qualitative (e.g., spam, personal, social)
  - Not necessarily ordered
  - Not necessarily two categories, but will start with the binary case

Lecture 2 Teoria de la Decision

Motivation

- Two states of nature $y \rightarrow i \in \{0, 1\}$
- Two actions $(\hat{y}) \rightarrow j \in \{0, 1\}$



Source: https://dzone.com/articles/understanding-the-confusion-matrix

Probability, Cost, and Classification

- Two states of nature $y \to i \in \{0,1\}$
- Two actions $(\hat{y}) \to j \in \{0,1\}$
- Probabilities
  - $p = Pr(Y = 1|X)$
  - $1 - p = Pr(Y = 0|X)$
- Loss: $L(i,j)$, penalizes being in bin i, j
- Risk: expected loss of taking action i

$L(a,\theta) = (a-\theta)^2$

$= |a-\theta|$

$L(\;) \to [0,\infty)$

Probability, Cost, and Classification

$$E(L) = \int x f(x)\, dx$$

- Risk: expected loss of taking action i

$$E[L(i,j)] = \sum_j p_j L(i,j) \tag{4}$$

- The objective is the same as before: minimize the risk
- We have to define $L(i,j)$

Probability, Cost, and Classification

$$L(i,j) = \begin{cases} 1 & i \neq j \\ 0 & i = j \end{cases}$$

- Risk: expected loss of taking action i

$$E[L(i,j)] = \sum_j p_j L(i,j) \tag{5}$$

$$R(i) = (1-p)L(i,0) + pL(i,1)$$

- Loss 0-1: $L(i,j) = 1[i \neq j]$
- the expected loss will be negative (i.e. you will expect to make a profit (min costs))

$$R(1) = (1-p)1 + p0$$
$$R(0) = (1-p)0 + p1$$

$$\begin{aligned} R(1) &< R(0) \\ 1-p &< p \\ \boxed{p > \frac{1}{2}} \end{aligned} \tag{6}$$

- This is known as the Bayes Classifier

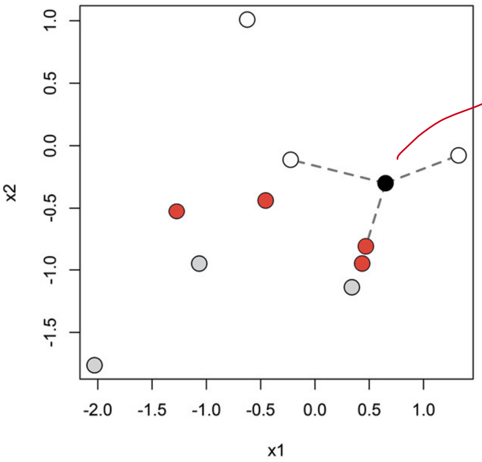
Probability, Cost, and Classification

- Under a 0-1 penalty the problem boils down to finding $p = PR(Y = 1|X)$ r
- We then predict 1 if $p > 0.5$ and 0 otherwise (Bayes classifier)
- We can think 3 ways of finding this probability in binary cases
  - K-Nearest Neighbors
  - Logistic
  - LDA
- Why not $p = X\beta$? $\to$ 4.2 ISLR
  pg 129

K-Nearest Neighbors

K-Nearest Neighbors

- K nearest neighbor (K-NN) algorithm predicts class $\hat{y}$ for x by asking *What is the most common class for observations around x?*



$K = 3$

Blanco

$p(\text{white}) = \frac{2}{3}$

Source: Taddy (2019)

K-Nearest Neighbors

- K nearest neighbor (K-NN) algorithm predicts class $\hat{y}$ for x by asking *What is the most common class for observations around x?*
- Algorithm: given an input vector x_f where you would like to predict the class label
  - Find the K nearest neighbors in the dataset of labeled observations, $x_i, y_i{}_{i=1}^{n}$, the most common distance is the Euclidean distance:

$$d(x_i, x_f) = \sqrt{\sum_{j=1}^{p}(x_{ij} - x_{fj})^2} \tag{7}$$

  - This yields a set of the K nearest observations with labels:

$$[x_{i1}, y_{i1}], \ldots, [x_{iK}, y_{iK}] \tag{8}$$

  - The predicted class of x_f is the most common class in this set

$$\hat{y}_f = mode\{y_{i1}, \ldots, y_{iK}\} \tag{9}$$

K-Nearest Neighbors

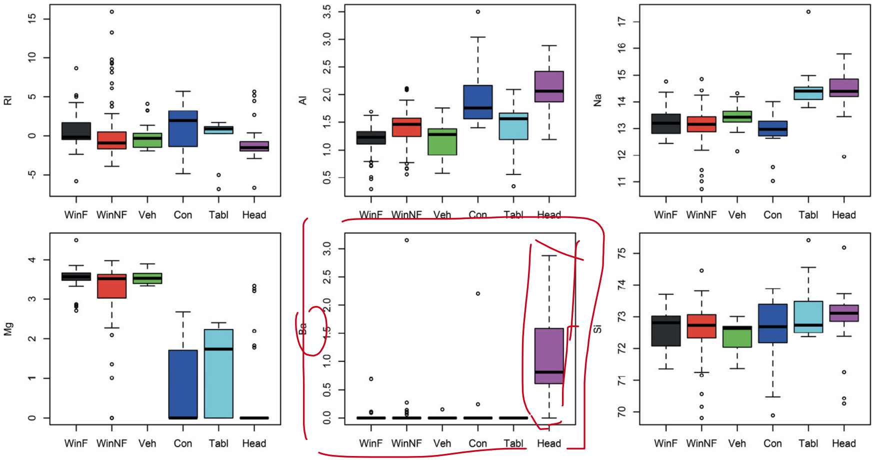
caret ?
HW

```
#Load the required packages
library("class") #for KNN
library("MASS") # a library of example datasets
#Read the data
data(fgl) ## loads the data into R; see help(fgl)
str(fgl)
```

```
## 'data.frame':    214 obs. of  10 variables:
##  $ RI  : num  3.01 -0.39 -1.82 -0.34 -0.58 ...
##  $ Na  : num  13.6 13.9 13.5 13.2 13.3 ...
##  $ Mg  : num  4.49 3.6 3.55 3.69 3.62 3.61 3.6 3.61 3.58 3.6 ...
##  $ Al  : num  1.1 1.36 1.54 1.29 1.24 1.62 1.14 1.05 1.37 1.36 ...
##  $ Si  : num  71.8 72.7 73 72.6 73.1 ...
##  $ K   : num  0.06 0.48 0.39 0.57 0.55 0.64 0.58 0.57 0.56 0.57 ...
##  $ Ca  : num  8.75 7.83 7.78 8.22 8.07 8.07 8.17 8.24 8.3 8.4 ...
##  $ Ba  : num  0 0 0 0 0 0 0 0 0 0 ...
##  $ Fe  : num  0 0 0 0 0 0.26 0 0 0 0.11 ...
##  $ type: Factor w/ 6 levels "WinF","WinNF",..: 1 1 1 1 1 1 1 1 1 1 ...
```

Refractive index and chemical composition for six possible glass types: float glass window (WinF), nonfloat glass window (WinNF), vehicle window (Veh), container (Con), tableware (Tabl), vehicle headlamp (Head)

K-Nearest Neighbors



K-Nearest Neighbors

- Units matter
  - Since distance is measured on the raw x values, units matter.
  - As we did for regularization, we will standarized observations.
  - R `scale` function does this, i.e., convert columns to mean-zero sd-one

```
x <- scale(fgl[,1:9]) # column 10 is the class label
apply(x,2,sd) # see ?apply
```

```
## RI Na Mg Al Si  K Ca Ba Fe
##  1  1  1  1  1  1  1  1  1
```

K-Nearest Neighbors

- Before running Knn
  - Make sure you have numeric matrices of training data x values, with labels y
  - Also need to provide new *test* values where you would like to predict
  - Note that there's no model do fit, Knn, just counts neighbors for each observation in test

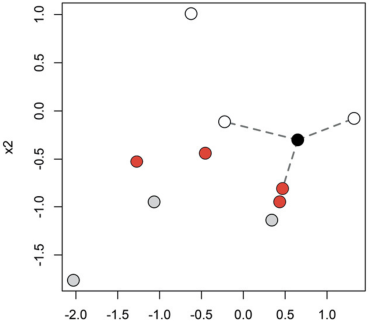
```
set.seed(1010101)
test <- sample(1:214,10)
nearest1 <- knn(train=x[-test,], test=x[test,], cl=fgl$type[-test], k=1)
nearest5 <- knn(train=x[-test,], test=x[test,], cl=fgl$type[-test], k=5)
data.frame(fgl$type[test],nearest1,nearest5)
```

```
##    fgl.type.test. nearest1 nearest5
## 1            WinF     WinF    WinNF
## 2            Head     Head     Head
## 3           WinNF    WinNF    WinNF
## 4            WinF     WinF     WinF
## 5           WinNF    WinNF    WinNF
## 6           WinNF    WinNF    WinNF
## 7            Head      Con      Con
## 8            Head    WinNF    WinNF
## 9           WinNF    WinNF    WinNF
## 10          WinNF     WinF     WinF
```

K-Nearest Neighbors

- In this case
  - 1-Knn manages 70% accuracy
  - 5-Knn manages 60% accuracy
  - H.W. try for different seed, (Taddy's is 80% and 70%)
- There are some major problems with practical implications
  - Knn predictions are unstable as a function of K

$$K = 1 \implies \hat{p}(white) = 0$$
$$K = 2 \implies \hat{p}(white) = 1/2$$
$$K = 3 \implies \hat{p}(white) = 2/3$$
$$K = 4 \implies \hat{p}(white) = 1/2$$



K-Nearest Neighbors

- In this case
  - 1-Knn manages 70% accuracy
  - 5-Knn manages 60% accuracy
  - H.W. try for different seed, (Taddy's is 80% and 70%)
- There are some major problems with practical implications
  - Knn predictions are unstable as a function of K
  - This instability of prediction makes it hard to choose the optimal K and cross validation doesn't work well for KNN
  - Since prediction for each new x requires a computationally intensive counting, KNN is too expensive to be useful in most big data settings.
  - KNN is a good idea, but too crude to be useful in practice

Logit

Logit

Lineal $= f(x'\beta) = x\beta$

We have a conditional probability

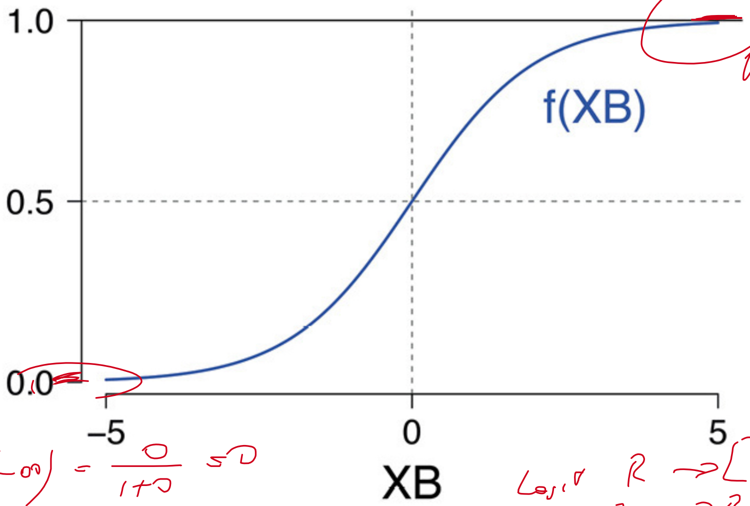
$$Pr(y = 1|X) = f(X'\beta) \tag{10}$$

Logistic regression uses a *logit* (sigmoid, softmax) link function

$$p(y = 1|X) = \frac{e^{X'\beta}}{1 + e^{X'\beta}} = \frac{exp(\beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k)}{1 + exp(\beta_0 + \beta_1 x_1 + \cdots + \beta_k x_k)} \tag{11}$$

$$= \frac{1}{1 + e^{-x'\beta}}$$

Logit



$f(\infty) = \frac{\infty}{1+\infty} = 1$

$f(-\infty) = \frac{0}{1+0} = 0$

Logit $R \to [0,1]$

OLS $R \to R$

Source: Taddy (2019)

Logit Demo

```
#Load the required packages
library("dplyr") #for data wrangling
library("gamlr") #ML

#Read the data
credit<-readRDS("credit_class.rds")
dim(credit)
```

```
## [1] 1000    9
```

```
head(credit)
```

```
##   Default duration amount installment age  history      purpose foreign  rent
## 1       0        6   1169           4  67 terrible goods/repair foreign FALSE
## 2       1       48   5951           2  22     poor goods/repair foreign FALSE
## 3       0       12   2096           2  49 terrible          edu foreign FALSE
## 4       0       42   7882           2  45     poor goods/repair foreign FALSE
## 5       1       24   4870           3  53     poor       newcar foreign FALSE
## 6       0       36   9055           2  35     poor          edu foreign FALSE
```

Logit Demo

```r
mylogit <- glm(Default~duration + amount + installment + age +
                      factor(history) + factor(purpose) + factor(foreign) + factor(rent),
                      data = credit, family = "binomial")
summary(mylogit,type="text")
```

```
## ...
##
## Coefficients:
##                            Estimate Std. Error z value Pr(>|z|)
## (Intercept)              -1.936e-01  4.579e-01  -0.423  0.67239
## duration                  2.666e-02  8.152e-03   3.270  0.00108 **
## amount                    9.793e-05  3.670e-05   2.668  0.00763 **
## installment               2.361e-01  7.687e-02   3.072  0.00213 **
## age                      -1.598e-02  7.348e-03  -2.175  0.02964 *
## factor(history)poor      -1.101e+00  2.490e-01  -4.424 9.67e-06 ***
## factor(history)terrible  -1.849e+00  2.837e-01  -6.518 7.13e-11 ***
## factor(purpose)usedcar   -1.702e+00  3.273e-01  -5.201 1.98e-07 ***
## factor(purpose)goods/repair -7.551e-01  1.867e-01  -4.044 5.25e-05 ***
## factor(purpose)edu       -1.473e-01  3.263e-01  -0.451  0.65166
## factor(purpose)biz       -8.501e-01  2.801e-01  -3.036  0.00240 **
## factor(foreign)german    -1.322e+00  5.814e-01  -2.274  0.02298 *
## factor(rent)TRUE          5.702e-01  1.944e-01   2.934  0.00335 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## ...
```

Logit Demo: Build a design matrix

```r
str(credit$foreign)
```

```
## Factor w/ 2 levels "foreign","german": 1 1 1 1 1 1 1 1 1 1 ...
```

```r
source("naref.R")
credit<-naref(credit)
str(credit$foreign)
```

```
## Factor w/ 3 levels NA,"foreign","german": 2 2 2 2 2 2 2 2 2 2 ...
```

$X = (x_1, x_2, \quad)$

```r
credx <- model.matrix( Default ~ .^2, data=credit)[,-1]
dim(credx)
```

```
## [1] 1000  121
```

```r
colnames(credx)[c(1,2,16,17,18)]
```

```
## [1] "duration"            "amount"                   "rentTRUE"
## [4] "duration:amount"     "duration:installment"
```

poly(duration, 2)

Logit Demo (Matrix package)

```r
credx <- sparse.model.matrix( Default ~ .^2, data=credit)[,-1]
head(credx)
```

```
## 6 x 121 sparse Matrix of class "dgCMatrix"
##
## 1  6 1169 4 67 . . 1 . . 1 . . 1 . 1 .   7014 24  402 .  .  6 . .  6 . .  6 .
## 2 48 5951 2 22 . 1 . . . 1 . . 1 . 1 . 285648 96 1056 . 48  . . . 48 . . 48 .
## 3 12 2096 2 49 . . 1 . . . 1 . 1 . 1 .  25152 24  588 .  . 12 . . . 12 . 12 .
## 4 42 7882 2 45 . 1 . . . 1 . . 1 . 1 . 331044 84 1890 . 42  . . . 42 . . 42 .
## 5 24 4870 3 53 . 1 . 1 . . . . 1 . 1 . 116880 72 1272 . 24  . 24 .  . . . 24 .
## 6 36 9055 2 35 . 1 . . . . 1 . 1 . 1 . 325980 72 1260 . 36  . . .  . 36 . 36 .
##
```

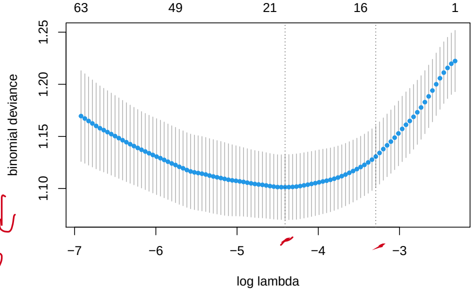
$$\begin{bmatrix} -4 & 0 \\ 0 & 10 \\ 5 & 0 \end{bmatrix} \rightarrow \left(\begin{array}{l} i = 1, 3, 2 \\ j = 1, 1, 2 \\ x = -4, 5, 10 \end{array} \right)$$

Logit Demo

```
default <- credit$Default
credscore <- cv.gamlr(credx, default, family="binomial", verb=TRUE)
```

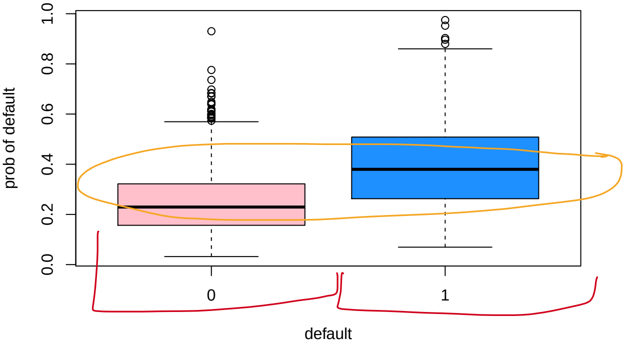
```
## fold 1,2,3,4,5,done.
```

```
plot(credscore)
```



Logit Demo

```
## What are the underlying default probabilities
## In sample probability estimates
pred <- predict(credscore$gamlr, credx, type="response")
pred <- drop(pred) # remove the sparse Matrix formatting
boxplot(pred ~ default, xlab="default", ylab="prob of default", col=c("pink","dodgerblue"))
```



Logit Demo Misclassification Rates

- A classification rule, or cutoff, is the probability p at which you predict
  - $\hat{y}_i = 0$ if $p_i < p$
  - $\hat{y}_i = 1$ if $p_i < p$
- We have two types of error associated with this that we can use as a measure of performance

$$
\begin{aligned}
\textit{False Positive Rate} &= \frac{\textit{expected \# false positives}}{\textit{\# classified positive}} \\
\textit{False Negative Rate} &= \frac{\textit{expected \# false negatives}}{\textit{\# classified negative}}
\end{aligned}
\tag{12}
$$

- Another measure of performance is using the number of *correct* classifications
  - *Sensitivity* proportion of true $y = 1$ classified as such
  - *Specificity* proportion of true $y = 0$ classified as such

Logit Demo

```
rule <- 1/2
sum( (pred>rule)[default==0] )/sum(pred>rule) ## false positive rate
```

```
## [1] 0.3189655
```

```
sum( (pred<rule)[default==1] )/sum(pred<rule) ## false negative rate
```

```
## [1] 0.25
```

```
sum( (pred>rule)[default==1] )/sum(default==1) ## sensitivity
```

```
## [1] 0.2633333
```

```
sum( (pred<rule)[default==0] )/sum(default==0) ## specificity
```

```
## [1] 0.9471429
```

Logit Demo

```
rule <- 1/5

sum( (pred>rule)[default==0] )/sum(pred>rule) ## false positive rate
```

```
## [1] 0.6059744
```

```
sum( (pred<rule)[default==1] )/sum(pred<rule) ## false negative rate
```

```
## [1] 0.07744108
```

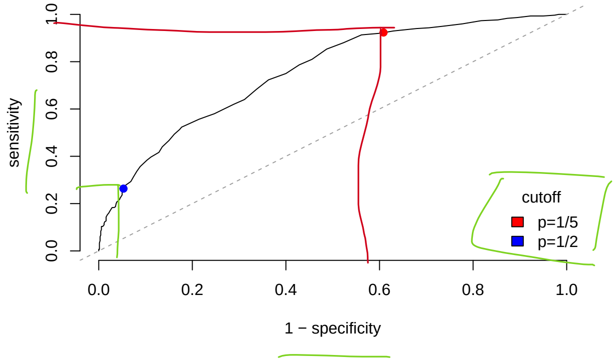
```
sum( (pred>rule)[default==1] )/sum(default==1) ## sensitivity
```

```
## [1] 0.9233333
```

```
sum( (pred<rule)[default==0] )/sum(default==0) ## specificity
```

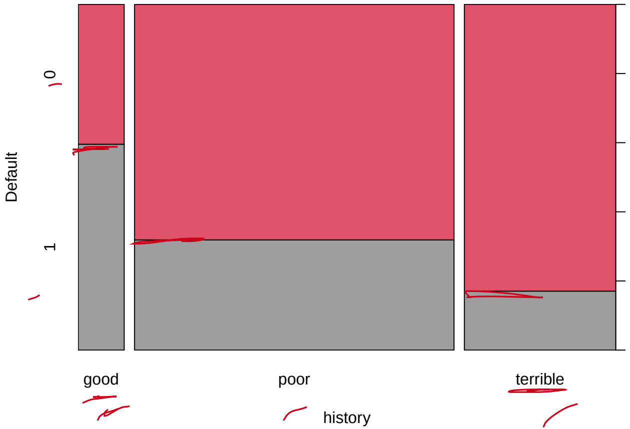
```
## [1] 0.3914286
```

ROC



- Lorenz
- Gini

Logit Demo



OVB → Selection Bias

Review & Next Steps

- Today:
  - KNN
    - Intuitive
    - Not very useful in practice, curse of dimensionality
  - Logit
    - `gamlr`
    - Exploit sparcity
    - Model evaluation
    - Careful with naive models
- Next class: Classification (cont.)
- Questions? Questions about software?

Further Readings

- Friedman, J., Hastie, T., & Tibshirani, R. (2001). The elements of statistical learning (Vol. 1, No. 10). New York: Springer series in statistics.
- James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). An introduction to statistical learning (Vol. 112, p. 18). New York: springer.
- Kuhn, M. (2012). The caret package. R Foundation for Statistical Computing, Vienna, Austria. https://topepo.github.io/caret/index.html
- Taddy, M. (2019). Business data science: Combining machine learning and economics to optimize, automate, and accelerate business decisions. McGraw Hill Professional.
- Zou, H. y Hastie, T., 2005, Regularization and variable selection via the elastic net, Journal of the Royal Statistical Society, 67, 2, 301-320.