Lecture 16:
Regularization/Shrinkage Methods

Big Data and Machine Learning for Applied Economics
Econ 4676

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Agenda

, PS 2 on github ~~15 oct~~ 22 oct

1. Recap & Implementation
   - K-Fold Cross Validation
   - Model selec

2. Regularization
   - Lasso
   - Ridge
   - Extension
   - Demo Regularization

3. Further Readings

Overfit and out of Sample Prediction

- ML we care about prediction out of sample
- Overfit: complex models predict very well inside a sample but ”bad” outside
- Choose the right complexity level
- Trade off Bias/Variance
- Challenge: accept some bias to decrease variance

Motivation

- Estimating test error: two approaches
  1. We can directly estimate the test error, using either a validation set approach or a cross-validation approach
  2. We can indirectly estimate test error by making an adjustment to the training error to account for overfitting.
     - AIC, BIC, C_p and Adjusted R^2

       1) $\ell_c(\hat{\theta})$

       2) AIC $\quad \ell_i(\hat{\theta}) - P_i$
     - These techniques adjust the training error for the model size, and can be used to select among a set of models with different numbers of variables.
     - AIC and BIC are very closely related to classical notions of hypothesis testing.

Demo: Prelogomenon

ETA → 20%
↓
100%
PS
3/4
→ Boots

```r
#Load the required packages
library("McSpatial") #loads the package
library("dplyr") #for data wrangling
library("caret") #ML
data(matchdata) #loads the data set
matchdata <- matchdata %>% mutate(price=exp(lnprice)) %>% select(-lnprice)
#transforms log prices to standard prices
set.seed(123) #set the seed for replication purposes
str(matchdata) #compact display
```

1995 - 2005

```
## 'data.frame':    3204 obs. of  18 variables:
##  $ year     : num  1995 1995 1995 2005 1995 ...
##  $ lnland   : num  8.23 8.63 8.7 8.63 8.63 ...
##  $ lnbldg   : num  6.98 7.02 7.22 6.87 7.2 ...
##  $ rooms    : int  5 5 5 4 6 7 7 6 4 6 ...
##  $ bedrooms : int  3 3 3 2 3 3 4 3 2 3 ...
##  $ bathrooms: num  1 1.5 1.5 1 1 2 1 2 1.5 1.5 ...
##  $ centair  : int  0 1 1 0 1 0 1 1 1 1 ...
##  $ fireplace: int  0 0 0 0 0 1 0 0 0 0 ...
##  $ brick    : num  1 1 1 1 1 1 1 1 1 1 ...
##  $ garage1  : num  1 0 0 1 0 1 0 0 1 0 ...
##  $ garage2  : num  0 1 1 0 1 0 1 0 0 0 ...
##  $ dcbd     : num  13.6 13.5 13.6 13.5 13.6 ...
##  $ rr       : num  0 0 0 0 0 0 0 0 0 0 ...
##  $ yrbuilt  : num  1953 1952 1952 1949 1953 ...
##  $ carea    : Factor w/ 11 levels "Albany Park",..: 3 3 3 3 3 3 3 3 3 3 ...
##  $ latitude : num  42 42 42 42 42 ...
##  $ longitude: num  -87.8 -87.8 -87.8 -87.8 -87.8 ...
##  $ price    : num  170750 168000 192500 398000 215000 ...
```

Demo: K-fold CV

```
model2 <- train(price ~ bedrooms,    # model to fit
                data = matchdata,
                trControl = trainControl(method = "cv", number = 10),
                # Method: crossvalidation, 10 folds
                method = "lm")# specifying regression model

model2
```

formula (modelo)

- 10 / 5
- LOOCV
- Validation

```
## Linear Regression
##
## 3204 samples
##    1 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 2883, 2884, 2884, 2883, 2884, 2885, ...
## Resampling results:
##
##   RMSE      Rsquared    MAE
##   147308.8  0.01385314  122117.8
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

$$\sum (y - x\beta)^2$$

$$\sum |y_i - x_i\beta|$$

Demo: K-fold CV

```
model2 <- train(price ~ bedrooms+bathrooms+ rooms+centair+fireplace+brick+
                        lnland+lnbldg+garage1+garage2+
                        dcbd+ rr +
                        yrbuilt+ factor(year) +
                        carea+ latitude+longitude,
                        data = matchdata,
                        trControl = trainControl(method = "cv", number = 10),
                        method = "lm")

model2
```

→ dummies

```
## Linear Regression
##
## 3204 samples
##   17 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 2883, 2884, 2884, 2883, 2883, 2884, ...
## Resampling results:
##
##   RMSE      Rsquared   MAE
##   74297.54  0.7490674  48170.37
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
```

Mas complex
Smaller RMSE
Av pred error

Model Subset Selection

- We have M_k models
- We want to find the model that best predicts out of sample
- We have a number of ways to go about it
  - Best Subset Selection
  - Stepwise Selection
    - Forward selection
    - Backward selection

Best Subset Selection

$p = 20$ $+1,000,000$

$$y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + u \quad (1)$$

1. Estimate **all** possible models with $k = 0, 1, ..., p$ predictors.
2. Compute the prediction error using cross validation
3. Pick the one with the smallest prediction error

Demo: Best Subset Selection

- `Caret` is usually the solution but it doesn't have everything =(

Linear Regression with Backwards Selection	leapBackward	Regression	leaps	nvmax
Linear Regression with Forward Selection	leapForward	Regression	leaps	nvmax
Linear Regression with Stepwise Selection	leapSeq	Regression	leaps	nvmax

Note: `https://topepo.github.io/caret/available-models.html` 238 mod

Demo: Best Subset Selection

```r
require("leaps")
```

```r
class(matchdata$carea)
```

```
## [1] "factor"
```

```r
class(matchdata$year)
```

```
## [1] "numeric"
```

```r
matchdata$year<-factor(matchdata$year)
```

```r
best<-regsubsets(price ~ ., method="exhaustive",data = matchdata)
summary(best)
```

default
up to 8
número todas
la variables

price ~ .
→ todas las demás variables

Demo: Best Subset Selection

nvmax = 26 vars

```
## Subset selection object
## Call: regsubsets.formula(price ~ ., method = "exhaustive", data = matchdata)
## 26 Variables  (and intercept)
## ...
## Selection Algorithm: exhaustive
##          year2005 lnland lnbldg rooms bedrooms bathrooms centair fireplace
## 1  ( 1 ) "*"      " "    " "    " "   " "      " "       " "     " "
## 2  ( 1 ) "*"      " "    "*"    " "   " "      " "       " "     " "
## 3  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
## 4  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
## 5  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
## 6  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
## 7  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
## 8  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     "*"
##          brick garage1 garage2 dcbd rr  yrbuilt careaEdgewater careaEdison Park
## 1  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 2  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 3  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 4  ( 1 ) " "   " "     " "     " "  " " " "     "*"            " "
## 5  ( 1 ) " "   " "     " "     " "  " " " "     "*"            " "
## 6  ( 1 ) " "   " "     " "     " "  " " " "     "*"            " "
## 7  ( 1 ) " "   " "     " "     " "  " " " "     "*"            " "
## 8  ( 1 ) " "   " "     " "     " "  " " " "     "*"            " "
```

Demo: Best Subset Selection

```
##          careaForest Glen careaJefferson Park careaLincoln Square
## 1  ( 1 ) " "              " "                 " "
## 2  ( 1 ) " "              " "                 " "
## 3  ( 1 ) " "              " "                 " "
## 4  ( 1 ) " "              " "                 " "
## 5  ( 1 ) "*"              " "                 " "
## 6  ( 1 ) "*"              " "                 " "
## 7  ( 1 ) "*"              " "                 "*"
## 8  ( 1 ) "*"              " "                 "*"
##          careaNorth Park careaNorwood Park careaRogers Park careaUptown
## 1  ( 1 ) " "             " "               " "              " "
## 2  ( 1 ) " "             " "               " "              " "
## 3  ( 1 ) " "             " "               " "              " "
## 4  ( 1 ) " "             " "               " "              " "
## 5  ( 1 ) " "             " "               " "              " "
## 6  ( 1 ) " "             " "               " "              "*"
## 7  ( 1 ) " "             " "               " "              "*"
## 8  ( 1 ) " "             " "               " "              "*"
##          careaWest Ridge latitude longitude
## 1  ( 1 ) " "             " "      " "
## 2  ( 1 ) " "             " "      " "
## 3  ( 1 ) " "             " "      " "
## 4  ( 1 ) " "             " "      " "
## 5  ( 1 ) " "             " "      " "
## 6  ( 1 ) " "             " "      " "
## 7  ( 1 ) " "             " "      " "
## 8  ( 1 ) " "             " "      " "
```

Stepwise Selection

1. Forward Stepwise Selection
    - Start with no predictors
    - Test all models with 1 predictor. Choose the one with smallest prediction error using cross validation (ISLR AIC, BIC, C_p)
    - Add 1 predictor at a time, without taking away.
    - Of the p+1 models, choose the one with smallest prediction error using cross validation
2. Backward Stepwise Selection
    - Same idea but start with a complete model and go backwards, taking one at a time.

Demo Stepwise Selection

Caret

```
forward <- train(price ~ ., data = matchdata,
                 method = "leapForward",     (f ~.)
                 trControl = trainControl(method = "cv", number = 10))
forward
```

(AIC, BIC, Cp)

```
## Linear Regression with Forward Selection
##
## 3204 samples
##   17 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 2884, 2884, 2884, 2883, 2883, 2883, ...
## Resampling results across tuning parameters:
##
##   nvmax  RMSE      Rsquared   MAE
##   2      84219.28  0.6768962  55184.58
##   3      79212.49  0.7147519  51009.82
##   4      78595.00  0.7193113  50631.17
##
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was nvmax = 4.
```

Demo Stepwise Selection

```
summary(forward$finalModel)
```

```
## Subset selection object
## 26 Variables  (and intercept)
##                    Forced in Forced out
## ...

## 1 subsets of each size up to 4
## Selection Algorithm: forward
##          year2005 lnland lnbldg rooms bedrooms bathrooms centair fireplace
## 1  ( 1 ) "*"      " "    " "    " "   " "      " "       " "     " "
## 2  ( 1 ) "*"      " "    "*"    " "   " "      " "       " "     " "
## 3  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
## 4  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
##          brick garage1 garage2 dcbd rr  yrbuilt careaEdgewater careaEdison Park
## 1  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 2  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 3  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 4  ( 1 ) " "   " "     " "     " "  " " " "     "*"            " "
##          careaForest Glen careaJefferson Park careaLincoln Square
## 1  ( 1 ) " "              " "                 " "
## 2  ( 1 ) " "              " "                 " "
## 3  ( 1 ) " "              " "                 " "
## 4  ( 1 ) " "              " "                 " "
##          careaNorth Park careaNorwood Park careaRogers Park careaUptown
## 1  ( 1 ) " "             " "               " "              " "
## 2  ( 1 ) " "             " "               " "              " "
## 3  ( 1 ) " "             " "               " "              " "
## 4  ( 1 ) " "             " "               " "              " "
##          careaWest Ridge latitude longitude
## 1  ( 1 ) " "             " "      " "
## 2  ( 1 ) " "             " "      " "
## 3  ( 1 ) " "             " "      " "
```

Demo Stepwise Selection

```
backwards <- train(price ~ ., data = matchdata,
                   method = "leapBackward",
                   trControl = trainControl(method = "cv", number = 10))
backwards
```

```
## Linear Regression with Backwards Selection
##
## 3204 samples
##   17 predictor
##
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 2884, 2882, 2884, 2885, 2883, 2884, ...
## Resampling results across tuning parameters:
##
##   nvmax  RMSE      Rsquared   MAE
##   2      84353.39  0.6769755  55217.19
##   3      79280.53  0.7153318  51000.22
##   4      78137.63  0.7235894  49820.72
##
## RMSE was used to select the optimal model using the smallest value.
## The final value used for the model was nvmax = 4.
```

Demo Stepwise Selection

```r
summary(backwards$finalModel)
```

```
## Subset selection object
## 26 Variables  (and intercept)
## ...
## Selection Algorithm: backward
##          year2005 lnland lnbldg rooms bedrooms bathrooms centair fireplace
## 1  ( 1 ) "*"      " "    " "    " "   " "      " "       " "     " "
## 2  ( 1 ) "*"      " "    "*"    " "   " "      " "       " "     " "
## 3  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
## 4  ( 1 ) "*"      "*"    "*"    " "   " "      " "       " "     " "
##          brick garage1 garage2 dcbd rr  yrbuilt careaEdgewater careaEdison Park
## 1  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 2  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 3  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
## 4  ( 1 ) " "   " "     " "     " "  " " " "     " "            " "
##          careaForest Glen careaJefferson Park careaLincoln Square
## 1  ( 1 ) " "              " "                 " "
## 2  ( 1 ) " "              " "                 " "
## 3  ( 1 ) " "              " "                 " "
## 4  ( 1 ) "*"              " "                 " "
##          careaNorth Park careaNorwood Park careaRogers Park careaUptown
## 1  ( 1 ) " "             " "               " "              " "
## 2  ( 1 ) " "             " "               " "              " "
## 3  ( 1 ) " "             " "               " "              " "
## 4  ( 1 ) " "             " "               " "              " "
##          careaWest Ridge latitude longitude
## 1  ( 1 ) " "             " "      " "
## 2  ( 1 ) " "             " "      " "
## 3  ( 1 ) " "             " "      " "
## 4  ( 1 ) " "             " "      " "
```

mlr
CRAN
Machine learning
Task Force
→ todos los paquetes

Python
reticulate
(correr python desde R)

Regularization

- Isn't it worth starting with the general model (p-variables) and crossing out the non-significant coefficients?
- Or we could go the following path:
  - Run model, get coefficients and p-values
  - Take out those with p-values bellow a certain α (we can adjust for FDR)
  - Why is this a bad idea? → Hint (Multicollinearity) ✓

    $V(\beta_k)$? → p-value
- Backward selection approximates that idea (not exactly) and does a better job that the previous point

Regularization

- The key of modern statistics is regularization
- The strategy involves penalizing complexity so as to depart from optimality and stabilize the system
- 'Crossing out' variables / coefficients is an extreme way to 'shrink' them.
- Lasso: a formal and algorithmic way of accomplishing this task.

Bayesiano y la posible conexión con
Regularización

Lasso

- For $\lambda \geq 0$ given, consider minimizing the following objective function

$|\beta_s - 0|$

$$min_\beta L(\beta) = \underbrace{\sum_{i=1}^{n}(y_i - x_i'\beta)^2}_{\text{costo de ajustar}} + \underbrace{\lambda \sum_{s=2}^{p} |\beta_s|}\tag{2}$$

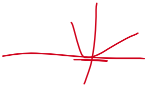
- Note:
  - First coef. constant $\quad s=1$ constante
  - $\lambda = 0$? $\quad \rightarrow$ OLS
  - $\lambda = \infty$?

$\lambda \rightarrow \infty \qquad \lambda \underbrace{\sum |\beta_s|}_{\geq 0} \qquad \beta_s \rightarrow 0 \quad \forall s$

Lasso

$$min_{\beta}L(\beta) = \underbrace{\sum_{i=1}^{n}(y_i - x_i'\beta)^2}_{\text{cost to optimize}} + \lambda \underbrace{\sum_{s=2}^{p}}_{\text{penal}} |\beta_s| \tag{3}$$

- LASSO magic: it automatically choses which variables go in ($\beta_s \neq 0$) and which are out ($\beta_s = 0$)
- Why? Coefficients that don't go in are corner solutions
- $L(\beta)$ in non differentiable



Lasso Intuition

1 predictor $s = 1$

no constante

(FWL)

- Lasso Intuition

$$min_\beta L(\beta) = \sum_{i=1}^{n} (y_i - x_i\beta)^2 + \cancel{\lambda|\beta|} \tag{4}$$

$\lambda = 0$

- Only one predictor, i.e., one coefficient.
- Standardize predictor $\sum x_i^2 = 1$ so

$$\hat{\beta}_{OLS} = \frac{\sum_i x_i y_i}{\sum_i x_i^2} = \sum x_i y_i \tag{5}$$

Lasso Intuition

$0 < \lambda < \infty$

$$min_{\beta} L(\beta) = \sum_{i=1}^{n} (y_i - x_i\beta)^2 + \lambda|\beta| \tag{6}$$

$$L(\beta) = \sum_{i=1}^{n} (y_i - x_i\beta)^2 + \lambda|\beta| = \begin{cases} \sum_{i=1}^{n} (y_i - x_i\beta)^2 + \lambda\beta & \text{if } \beta \geq 0 \\ \sum_{i=1}^{n} (y_i - x_i\beta)^2 - \lambda\beta & \text{if } \beta \leq 0 \end{cases} \tag{7}$$

- Non differentiable at $\beta = 0$
- Differentiable otherwise $\beta \neq 0$

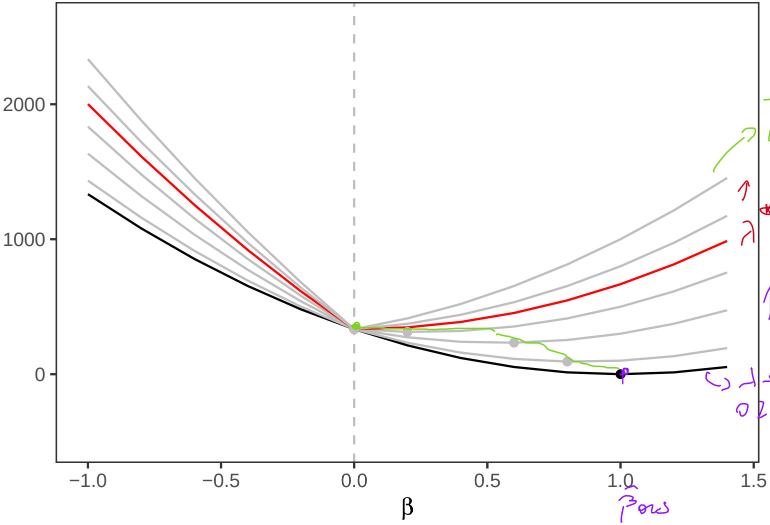
es cuadratic →
y diff

→ Analisis convexo
– Lasso
– Quantile reg

Boyd
Stanford

Lasso Intuition

$\beta > 0$ $\quad L(\beta)$



Incre
lambda
$\uparrow \lambda$
$\hookrightarrow \lambda = 0$
OLS
β_{OLS}

Lasso Intuition

$\mathcal{L}(\beta) = \Sigma (y_i - x_i \beta) + \lambda |\beta|$

$$\frac{dL(\beta)^+}{d\beta} = -2\sum y_i x_i + 2\beta \sum x_i^2 + \lambda$$
$$= \underbrace{-2\sum y_i x_i + 2\beta + \lambda} \tag{8}$$

$$\frac{dL(0)^+}{d\beta} = -2\sum y_i x_i + \lambda$$

then, if

$$\frac{dL(0)^+}{d\beta} > 0$$

$$\boxed{\lambda \geq 2\sum y_i x_i} \tag{9}$$

we have $\hat{\beta}_{lasso} = 0$

Lasso Intuition

λ^*

if $\lambda < 2\sum y_i x_i$ we have an interior solution

$$-2\sum y_i x_i + \hat{\beta}_{lasso} + \lambda = 0 \tag{10}$$

$$\hat{\beta}_{lasso} = \sum y_i x_i - \frac{\lambda}{2} \tag{11}$$

$\sum x_i^2 = 1$

$$\hat{\beta}_{lasso} = \hat{\beta}_{OLS} - \frac{\lambda}{2} \tag{12}$$

$\beta > 0$ → How Beach $\beta < 0$

Ridge

- For $\lambda \geq 0$ given, consider minimizing the following objective function

$$min_{\beta} R(\beta) = \sum_{i=1}^{n} (y_i - x_i'\beta)^2 + \lambda \sum_{s=2}^{p} \underline{(\beta_s)^2} \tag{13}$$

$|\beta_s|$ Lasso

- Note:
  - Intuition is similar to Lasso, however the problem is completely different

Ridge

1 variable
Regresor estandarizado $\sum x_i^2 = 1$

$$min_{\beta} R(\beta) = \sum_{i=1}^{n} (y_i - x_i'\beta)^2 + \lambda(\beta_s)^2 \tag{14}$$

$s = 1$

FOC

$$-2\sum_{i=1}^{n} y_i x_i + 2\beta + 2\lambda\beta_s = 0 \tag{15}$$

$$\begin{aligned} \hat{\beta}_{ridge} &= \frac{\sum_{i=1}^{n} y_i x_i}{(1+\lambda)} \\ &= \frac{\hat{\beta}_{OLS}}{(1+\lambda)} \end{aligned} \tag{16}$$

$\frac{\hat{\beta}}{1+\lambda}$

- Solution is *always* interior (unlike Lasso)
- Solutions is "shrunken" $(\lambda \to \infty)$

Lasso and Ridge Intuition

$$\underbrace{\sum (y - x_i \beta)^2}_{SSR} + \lambda \underbrace{\text{Penalización}}$$

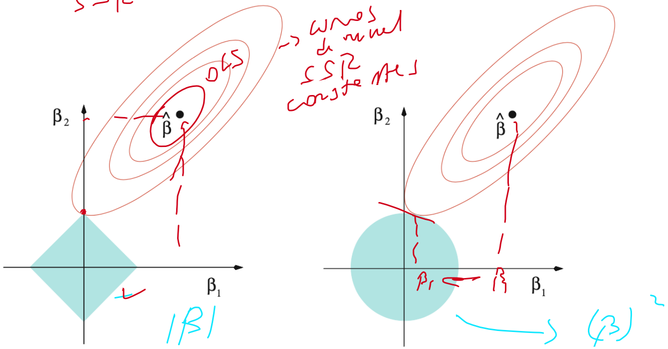


FIGURE 3.11. *Estimation picture for the lasso (left) and ridge regression (right). Shown are contours of the error and constraint functions. The solid blue areas are the constraint regions* $|\beta_1| + |\beta_2| \leq t$ *and* $\beta_1^2 + \beta_2^2 \leq t^2$*, respectively, while the red ellipses are the contours of the least squares error function.*

Technical comments

- We showed that Ridge is biased, but for certain λ $MSE(\beta_{ridge}) < MSE(\beta_{OLS})$
- Not possible to derive an exact result for Lasso, but ridge approximates to lasso
- Lasso shrinks everything towards zero, Ridge not quite so
- Application wise:
  - Standardize the data
  - Selection of λ?

HW pg 217 ISLR

$$\tilde{x} = \frac{x_{ij} - \bar{x}_j}{\sqrt{\frac{1}{n}\sum(x_{ij} - \bar{x}_j)^2}}$$

$$\sum x_i^2 = 1$$

Technical comments: λ Selection

- Selection of λ?
- Use CV
- Choose a grid of λ values, and compute the CV error for each value
- Select the λ^* that minimizes the prediction error
- Estimate using all the observations and the selected λ^*

Extension

- Family of penalized regressions

$$min_\beta R(\beta) = \sum_{i=1}^{n}(y_i - x_i'\beta)^2 + \overbrace{\lambda \sum_{s=2}^{p} |\beta_s|^p}^{Pen} \tag{17}$$

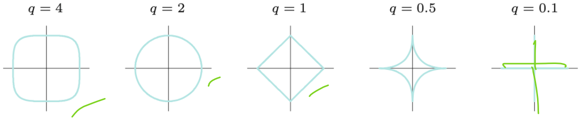


FIGURE 3.12. *Contours of constant value of* $\sum_j |\beta_j|^q$ *for given values of* q.

- Combination? Elastic Net

$$\begin{aligned} min_\beta EL(\beta) &= \sum_{i=1}^{n}(y_i - x_i'\beta)^2 + \lambda_1 \sum_{s=2}^{p} |\beta_s| + \lambda_2 \sum_{s=2}^{p} \beta_s^2 \\ &= \sum_{i=1}^{n}(y_i - x_i'\beta)^2 + \lambda \sum_{s=2}^{p} \left((1-\alpha)\beta_s^2 + \alpha|\beta_s|\right) \end{aligned} \tag{18}$$

$\alpha = 0$ $\alpha = 1$ HW

Demo Regularization

100

ISLR glmnet

```
lambda <- 10^seq(-2, 3, length = 10)
lasso <- train(
  price ~., data = matchdata, method = "glmnet",
  trControl = trainControl("cv", number = 10),
  tuneGrid = expand.grid(alpha = 1,
  lambda=lambda), preProcess = c("center", "scale")
  )
lasso
```

```
## 3204 samples
##   17 predictor
##
## Pre-processing: centered (26), scaled (26)
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 2884, 2885, 2882, 2883, 2885, 2883, ...
## Resampling results across tuning parameters:
##
##   lambda        RMSE      Rsquared   MAE
##   1.000000e-02  74410.32  0.7487304  48120.44
##   3.593814e-02  74410.32  0.7487304  48120.44
##   1.291550e-01  74410.32  0.7487304  48120.44
##   4.641589e-01  74410.32  0.7487304  48120.44
##   1.668101e+00  74410.32  0.7487304  48120.44
##   5.994843e+00  74410.32  0.7487304  48120.44
##   2.154435e+01  74410.32  0.7487304  48120.44
##   7.742637e+01  74411.91  0.7487131  48102.41
##   2.782559e+02  74485.72  0.7481557  48011.74
##   1.000000e+03  74702.02  0.7467291  47865.45
##
## Tuning parameter 'alpha' was held constant at a value of 1
## RMSE was used to select the optimal model using the smallest value.
## The final values used for the model were alpha = 1 and lambda = 21.54435.
```

Demo Regularization

```
ridge <- train(
  price ~., data = matchdata, method = "glmnet",
  trControl = trainControl("cv", number = 10),
  tuneGrid = expand.grid(alpha = 0,lambda=lambda), preProcess = c("center", "scale")
  )
ridge
```

ridge

```
## 3204 samples
##   17 predictor
##
## Pre-processing: centered (26), scaled (26)
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 2884, 2883, 2883, 2884, 2884, 2883, ...
## Resampling results across tuning parameters:
##
##   lambda        RMSE      Rsquared   MAE
##   1.000000e-02  75133.92  0.7479252  47361.73
##   3.593814e-02  75133.92  0.7479252  47361.73
##   1.291550e-01  75133.92  0.7479252  47361.73
##   4.641589e-01  75133.92  0.7479252  47361.73
##   1.668101e+00  75133.92  0.7479252  47361.73
##   5.994843e+00  75133.92  0.7479252  47361.73
##   2.154435e+01  75133.92  0.7479252  47361.73
##   7.742637e+01  75133.92  0.7479252  47361.73
##   2.782559e+02  75133.92  0.7479252  47361.73
##   1.000000e+03  75133.92  0.7479252  47361.73
##
## Tuning parameter 'alpha' was held constant at a value of 0
## RMSE was used to select the optimal model using the smallest value.
## The final values used for the model were alpha = 0 and lambda = 1000.
```

Demo Regularization

```
el <- train(
  price ~., data = matchdata, method = "glmnet",
  trControl = trainControl("cv", number = 10), preProcess = c("center", "scale")
  )
el
```

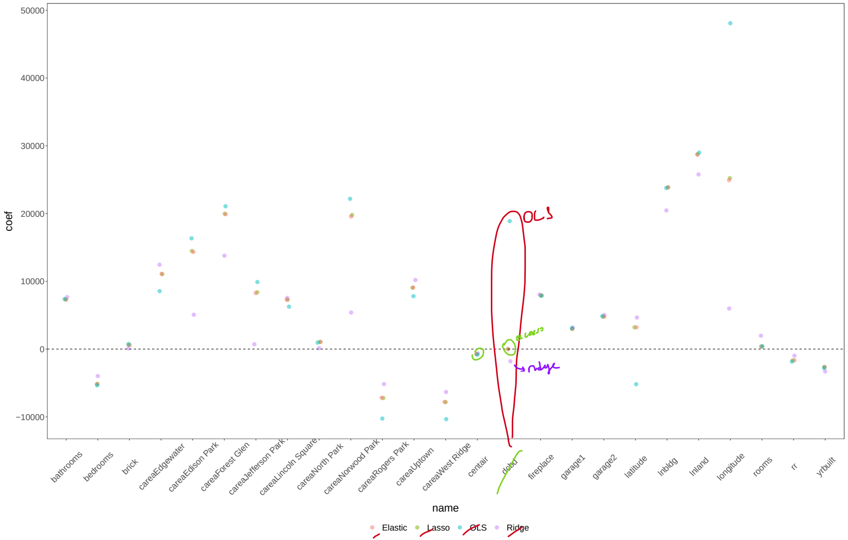
```
## glmnet
##
## 3204 samples
##   17 predictor
##
## Pre-processing: centered (26), scaled (26)
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 2885, 2885, 2883, 2884, 2883, 2884, ...
## Resampling results across tuning parameters:
##
##   alpha  lambda       RMSE      Rsquared   MAE
##   0.10     230.7569  74163.82  0.7525328  48112.63
##   0.10    2307.5689  74241.34  0.7518849  47772.50
##   0.10   23075.6893  76941.03  0.7489116  48065.77
##   0.55     230.7569  74161.27  0.7524710  48056.67
##   0.55    2307.5689  74420.59  0.7505525  47656.06
##   0.55   23075.6893  82931.37  0.7161268  52413.46
##   1.00     230.7569  74189.84  0.7522040  48029.77
##   1.00    2307.5689  74622.97  0.7493269  47527.50
##   1.00   23075.6893  88016.04  0.6846917  57561.95
##
## RMSE was used to select the optimal model using the smallest value.
## The final values used for the model were alpha = 0.55 and lambda = 230.7569.
```

Demo Regularization

```r
models <- list(ridge = ridge, lasso = lasso, elastic = el)
resamples(models) %>% summary( metric = "RMSE")
```

```
##
## Call:
## summary.resamples(object = ., metric = "RMSE")
##
## Models: ridge, lasso, elastic
## Number of resamples: 10
##
## RMSE
##             Min.  1st Qu.   Median     Mean  3rd Qu.     Max. NA's
## ridge   64485.44 70582.29 74874.23 75133.92 77717.74 89880.65    0
## lasso   66245.34 71294.79 74792.57 74410.32 78080.16 80946.67    0
## elastic 59004.20 69395.53 71960.20 74161.27 78485.49 88578.06    0
```

Demo Regularization



Further Readings

- Friedman, J., Hastie, T., & Tibshirani, R. (2001). The elements of statistical learning (Vol. 1, No. 10). New York: Springer series in statistics.
- James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). An introduction to statistical learning (Vol. 112, p. 18). New York: springer.
- Kuhn, M. (2012). The caret package. R Foundation for Statistical Computing, Vienna, Austria. https://topepo.github.io/caret/index.html
- Taddy, M. (2019). Business data science: Combining machine learning and economics to optimize, automate, and accelerate business decisions. McGraw Hill Professional.