Chapter 5: Probability

Section 5.1: Probability Rules

Def **Probability** – a numerical measure of the chance / likelihood that a specific event will occur.

Statistics versus **Probability**

SET NOTATION

set: collection of things { things } Ex {H, T}

TERMINOLOGY

1. An **event** is any collection of results (or outcomes) of a procedure.
2. A **simple event** is an outcome that cannot be further broken down into simpler components.
3. The **sample space** for a procedure consists of all possible simple events.

NOTATION

Symbol	*Represents*
P	probability
A, B, C	specific events
$P(A)$	probability of event A occurring

S = sample space

Ex flip a coin: $S = \{H, T\}$

Ex: A probability experiment consists of rolling a single six-sided *fair* die.

(a) Determine the sample space.

$S = \{1, 2, 3, 4, 5, 6\}$

(b) What is $P(5)$?

$P(5) = 1/6 = 0.167$

(c) What is $P(even)$?

$P(even) = \frac{3}{6} = \frac{1}{2} = 0.5$

(d) What is P(at least 3)

$P(\text{at least } 3) = 4/6 = 2/3 = 0.667$

Important Notes

1. For any event A, $0 \le P(A) \le 1$.
2. The probability of an impossible event is zero.
3. The probability of an event that is certain to occur is one.
4. **Rounding Rule:** If necessary, round probabilities to three significant digits.

Def **Unusual Events** An event that has a low probability. *Typically, an event with less than* 5 *% is considered unusual, but it depends on the problem.*

Coin Flip Activity

We will investigate whether or not flipping a coin is fair. This means there's a 50% chance it will land on "Heads" (H) and a 50% chance it will land on "Tails" (T). Everyone take out a quarter and flip the coin 10 times and add your results to the class spread-sheet: link here

Relative Frequency

n = 7 (70 flips)

Coin Flips	Total	Probability
Heads	36	36/70 = 0.514
Tails	34	34/70 = 0.486
Total	70	

51.4%
48.6%

p lowercase P uppercase

THE LAW OF LARGE NUMBERS

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the ~~actual~~ theoretical probability.

Three Approaches to Probability

Ideal / Theoretical

Empirical Probability: *Based on data.* Formula: relative freq $P(E) = \frac{\#\text{ observ. } E}{\text{total observ.}}$	**Classical Probability:** *Based on (hypothetical) equally likely outcomes.* Formula: $P(E) = \frac{\#\text{ in } E}{\#\text{ in } S}$	**Subjective Probability:** *The probability of event A is found by simply guessing or estimating its value based on knowledge of the relevant circumstances.*
Empirical: As of March 24, 2020 over 416 thousand people have contracted Covid19 (coronavirus) worldwide. Of those people, 18,574 have died. What is the probability that someone affected by Covid19 dies? dying from Covid19 $P(E) = \frac{18574}{416000} \approx 44649$ $= 0.0446$ (3 sig fig)	***Classical:*** When two children are born, what's the probability that both are the same gender? $S = \{GG, GB, BB\}$ E = same gender $= \{GG, BB\}$ $P(E) = \frac{\#E}{\#S} = \frac{2}{3} = 0.67$	***Subjective:*** What is the probability that the next dollar bill you spend was previously spent by Jeff Goldblum? IDK $P(1\text{JG}) \stackrel{?}{=} \frac{1}{1{,}000{,}000}$

Ex: Suppose we flip a fair coin three times and record the outcomes of the three tosses. one outcome: HHT, THT #S = 8

(a) Write the sample space S for this experiment. How many outcomes are in S?

$S = \{$ HHH (0 T), THH, HTH, HHT (1 T), TTH, THT, HTT (2 T), TTT (3 T) $\}$

(b) Let E be the event that we get exactly one heads in three tosses. What is $P(E)$? $P(E) = 3/8 = 0.375$

(c) Find:

P(No H) = $P(TTT) = 1/8 = 0.125$

P(exactly one H) = $3/8 = 0.375$

P(exactly two H) = $= P(\text{exactly 1 T}) = 3/8 = 0.375$

P(exactly three H) = $P(HHH) = 1/8 = 0.125$

(d) Find the sum of all the probabilities in part (c).

$0.125 + 0.375 + 0.375 + 0.125 = 1$

#S = 5

Ex: A bag has 1 red marble, 1 blue marble, 1 yellow marble, 1 orange marble, and 1 purple marble. The table below shows the results of choosing a marble out of the bag and replacing it each trial. Give answers as decimals and percentages.

→ Empirical

Outcome of the Draw	100 trials	600 trials
Red	33	120
Blue	24	121
Yellow	18	119
Orange	17	122
Purple	8	118

↑ not Red

(a) Find the relative frequency probability of drawing a yellow or an orange for the 100 trials.

$P(Y) = \frac{18}{100} = 0.18 = 18\%$

(b) Find the relative frequency probability of drawing a marble that is not red for 100 trials.

$P(\text{not } R) = \frac{24+18+17+8}{100} = \frac{77}{100} = 0.77 = 77\%$

(c) Find the relative frequency probability of drawing a blue for the 100 trials.

$P(B) = \frac{24}{100} = 0.24 = 24\%$

(d) Find the relative frequency probability of drawing a blue for the 600 trials.

$P(B) = \frac{121}{600} = 0.202 = 20.2\%$

(e) What is the classical probability of choosing a blue?

$P(B) = \frac{\#B}{\#S} = \frac{1}{5} = 0.2 = 20\%$

$S = \{R, B, Y, O, P\}$ (one of each)

$\#S = 5$

$\#B = 1$