

Chapter 12: Inference on Categorical Data

Section 12.3: Inference about Two Population Proportions: Dependent Samples

MCNEMAR'S TEST

11.1 → Two Proportions: Independent
12.3 → Two Proportions: Matched Pairs

Def **McNemar's Test** tests the null hypothesis that two proportions from matched-pairs data (dependent samples) are equal.

		Treatment A	
		Success column 1	Failure column 2
Treatment B	Success row 1	f_{11}	f_{12}
	Failure row 2	f_{21}	f_{22}

Hypothesis Test Regarding Two Proportions p_1 and p_2 (Matched-Pairs)

Step 0: Check Requirements

- The samples are simple random and dependent (Matched Pairs)

- The total number of observations where the outcomes differ are greater than or equal to 10.

That is $f_{12} + f_{21} \geq 10$.

Step 1: State Hypotheses

$$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 \neq p_2 \end{cases} \quad (\text{Right Tailed Test})$$

Step 2: Level of Significance

$$\alpha$$

$\alpha = 0.05$ if not given.

Step 3: Test Statistic

$$\chi_0^2 = \frac{(f_{12} - f_{21})^2}{f_{12} + f_{21}} \quad \text{and} \quad df = 1$$

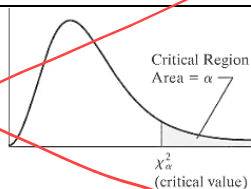
Step 4: Find P-Value to check using the P-value method.

Step 5: Make a decision and draw a conclusion.

CRITICAL REGION METHOD

* Table VIII

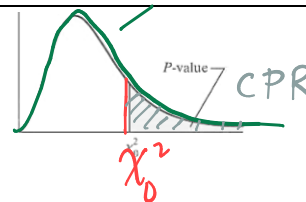
$$df = 1$$



- Reject H_0 if χ^{**} lies in the critical region
- Fail to Reject H_0 if χ^{**} doesn't lie in the critical region

P-VALUE METHOD

$$df = 1$$



- Reject H_0 if $P\text{-value} \leq \alpha$
- Fail to Reject H_0 if $P\text{-value} > \alpha$

GRAPHING CALCULATOR (TI-83 OR 84)

- Instructions:
- Compute the test statistic
 - Find the area under the χ^2 -distribution using $\chi^2 - cdf$ (2nd Vars + $\chi^2 - cdf$)
 - Use: a = test stat; $b = 1e99$, $df = 1$

P-Value: $P(\chi^2 > \chi_0^2) = \chi^2 cdf(\text{low}, \text{high}, 1)$

Ex 1: The General Social Survey asked a random sample of adult Americans with children two questions:

p_1 (1) Do you believe there should be paid leave for childcare?

p_2 (2) Do you believe children are a financial burden on parents?

Results of the survey are in the following table.

Does the sample evidence suggest there is a difference in the proportion of adult Americans with children who feel there should be paid leave for childcare and the proportion who feel children are a financial burden on parents? Use an $\alpha = 0.05$ level of significance.

		(2) Financial Burden	
		Agree $cd 1$	Disagree $cd 2$
(1) Paid Leave	Yes $row 1$	259 f_{11}	616 f_{12}
	No $row 2$	64 f_{21}	101 f_{22}

3) $f_{12} + f_{21} \geq 10$
 $64 + 616 \geq 10 \checkmark$

Check requirements

1) SRS \checkmark 2) Dependent? Matched Pairs? yes! 2 questions to same person!

Null and Alternative Hypothesis

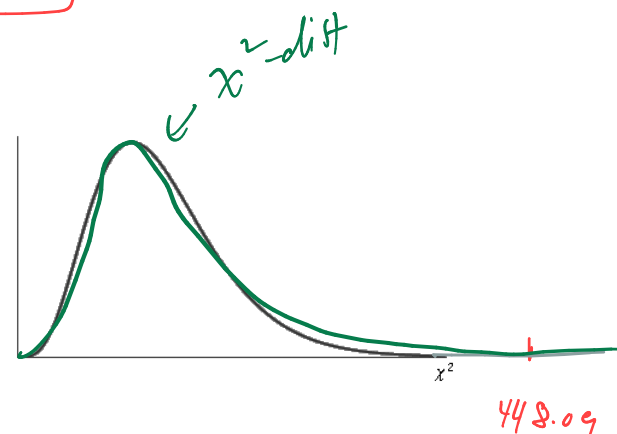
$$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 \neq p_2 \end{cases} \text{ (Right Tailed Test)}$$

Test Statistic

$$\chi^2_0 = \frac{(f_{12} - f_{21})^2}{f_{12} + f_{21}} = \frac{(616 - 64)^2}{616 + 64} = 448.09$$

P-Value

$$\begin{aligned} P &= P(\chi^2 > 448.09) \\ &= \chi^2 cdf(448.09, 1E99, 1) \\ &= 1.88 \times 10^{-99} = 0+ \end{aligned}$$



Decision about Null Hypothesis

$$\alpha = 0.05$$

$P < \alpha \rightarrow P \text{ low, Null go}$

$$P = 0+$$

Reject H_0

Conclusion

"There is enough statistical evidence to support the claim that the difference of Adult Americans with children who feel there should be paid leave for childcare & the proportion who feel children are a financial burden are not equal."