binompdf(n, p, x)

Last chapter Ch6 Probability Dist. ~~discrete~~ discrete

X RV ① $\sum P(x)=1$ ② $0 \le P(x) \le 1$

x	P(x)

This chapter continuous

Chapter 7: The Normal Probability Distribution

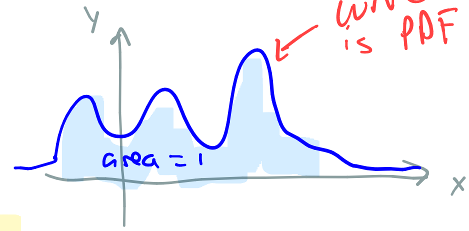
Section 7.1: Properties of the Normal Distribution

CONTINUOUS PROBABILITY DISTRIBUTIONS

Def A ***probability density function (pdf) (density curve)*** is the graph of a continuous probability distribution:

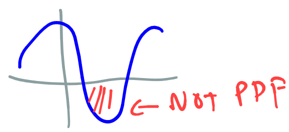
REQUIREMENTS

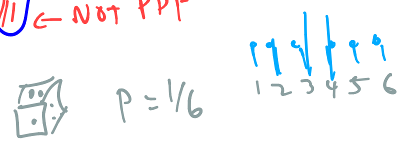
1. The total area under the curve must equal 1. (c.f. $\sum P(x) = 1$)



2. Every point on the curve must have a vertical height that is 0 or greater.

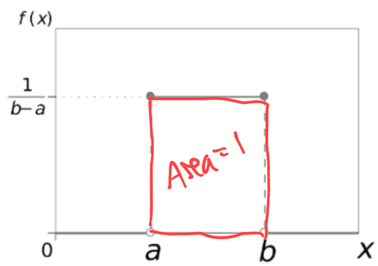
curve above x-axis always





UNIFORM PROBABILITY DISTRIBUTION

A random variable X is said to have a **uniform probability distribution** if all intervals of equal length are equally likely.



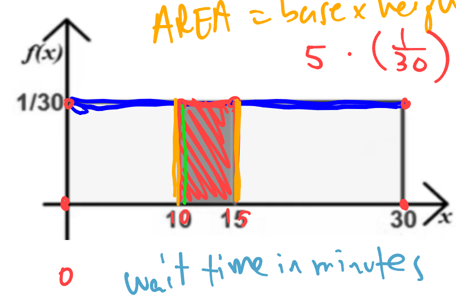
Area = (b − a) · height = 1

⇒ height = $\frac{1}{b-a}$ ⇒ explanation

Ex: The bus to Union Station leaves every 30 minutes and is uniformly distributed. Find the probability that a randomly chosen person arriving at a random time will wait between 10 and 15 minutes.

AREA = base × height
$5 \cdot \left(\frac{1}{30}\right)$

$P(10 < x < 15)$ = Area of rectangle
$= (5) \cdot \left(\frac{1}{30}\right)$
$= \frac{1}{6} = \boxed{0.167}$



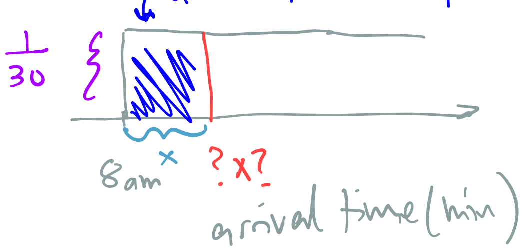
Discrete vs Continuous:

- How can we adapt the classical probability formula for continuous random variables?
- Probability a randomly chosen person arrives with a wait time of exactly 10.13245234234 minutes?
  ↳ line segment = no Area
  ⇒ Probability is zero!
- $P(10 < x < 15)$ vs $P(10 \le x \le 15)$ ← are the same!
  b/c line segments added at the ends have 0 area
- Probability = Area

Ex: It is 8 a.m.. There is a 30% probability that your friend will arrive within the next ___9___ minutes.

still bus example

area = probability = 0.3



Area = 0.3

(x) · height = 0.3

$30\left(x\left(\frac{1}{30}\right)\right) = (0.3)30$

$x = 30(0.3) = 9$

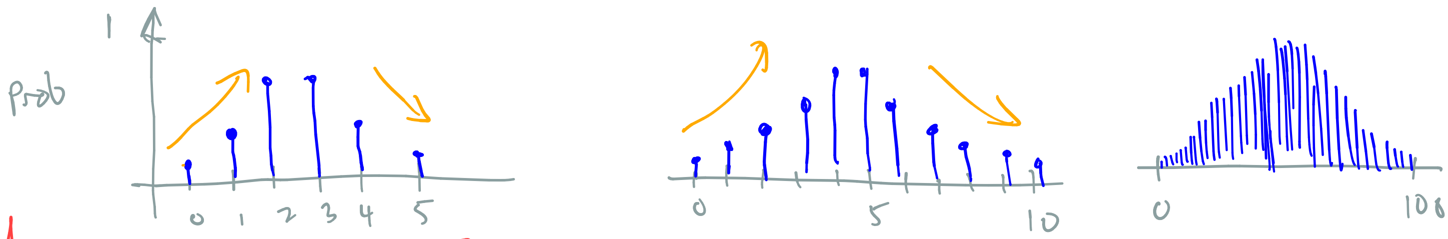
PRO TIP Generate sequence: 0,1,2,...,n
LIST (2nd stat) >> OPS >> 5.Seq

EXP: I ALPHA
Variable: I
start: 0
end: 0
step: 1

Simulating Normal Distributions

A binomial distribution is discrete and will have the number of successes $x = 0, 1, \dots, n$ where n is the max number of trials. Remember the values that are allowed are whole numbers.

If we flip a fair coin with $p = 0.5$ a total of $n = 10, 50, 100$. We can look at the probability distributions for $n = 10, 50, 100$.



Normal Distributions

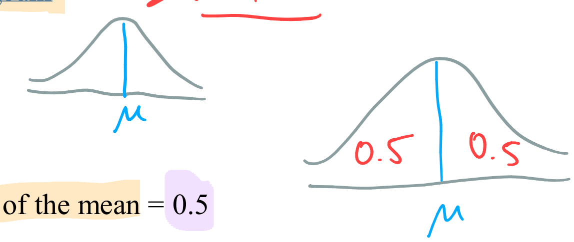
Def A continuous random variable X has a **normal distribution** if its density curve is symmetric and bell-shaped.

PDF

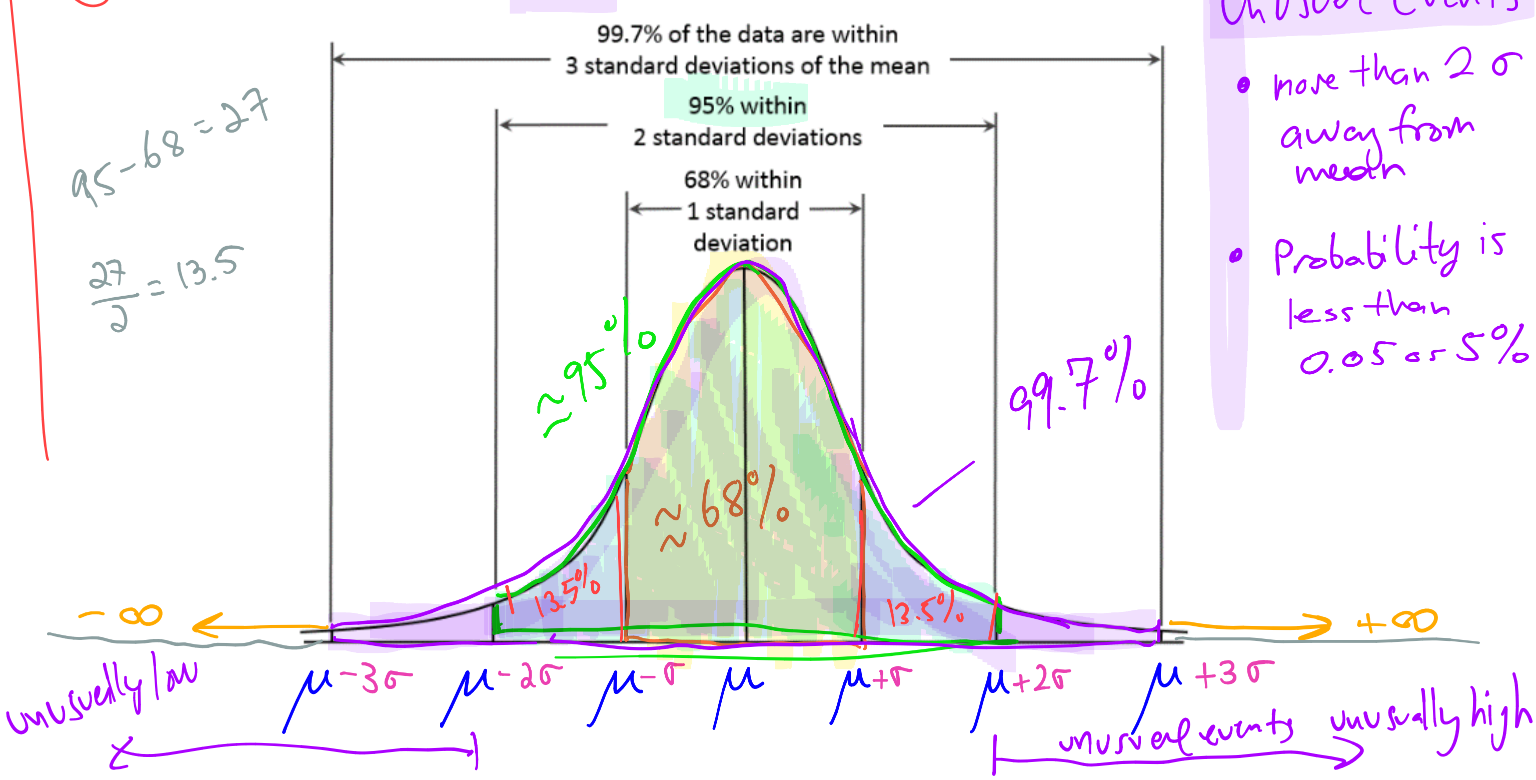
Specifically, the curve is given by: $y = \dfrac{1}{\sigma\sqrt{2\pi}} e^{\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right)}$ *(Don't worry, we'll never use it.)*

Desmos: https://www.desmos.com/calculator/pooefgeuhi → MANY normal curves!

Properties of Normal Distributions



1. The normal curve is symmetric about its mean
2. Mean = Median = Mode
3. Area under curve is 1.
4. Area to the left of the mean = Area to the right of the mean = 0.5
5. The graph never touches the horizontal axis
6. Area under normal curve for any interval of values of the random variable X represents
   A. The proportion of the population with the characteristics described by the interval of values
   B. The probability that a randomly selected individual from the population will have the characteristic described by interval of values
7. The **Empirical Rule** gives us an estimate for the area between standard deviations.

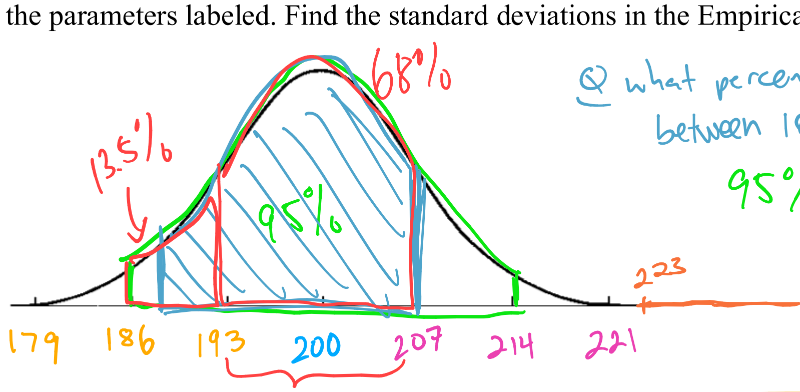


Unusual Events
- more than 2σ away from mean
- Probability is less than 0.05 or 5%

$\bar{x}$

Ex: The weights of all firefighters are normally distributed with a mean of 200 lbs and a standard deviation of 7 lbs.

(a) Draw a normal curve with the parameters labeled. Find the standard deviations in the Empirical Rule.



Q what percentage of FF weigh between 186 lbs & 214 lbs?

95% Empirical Rule

(b) Shade the region under the normal curve representing firefighters with weights greater than 223 lbs.

(c) The **notation** for the probability that a randomly selected firefighter weighs more than 223 lbs is: $P(x > 223)$

(Note: you don't need to find the probability only write the notation)

(d) Use the Empirical Rule to estimate the probability that a randomly chosen firefighter weighs between188 and 207 lbs.

$P(188 < x < 207)$ = prob a single randomly FF

$\approx 13.5\% + 68\%$

$= 0.135 + 0.68 = \boxed{0.815}$

$P(186 < x < 193) + P(193 < x < 207)$

(e) If there are 1000 firefighters in Glendale, how many would you estimate weigh 188 and 207 lbs?

FF w/ weight = 1000 * 0.815

= $\boxed{\text{815 firefighters}}$