Chapter 6: Discrete Probability Distributions

Ex Dice Roll "experiment"
↳ determined by chance
RV = create a game based on die

Section 6.1: Probability Distributions

R.V.

Random Variable (X): is a numerical measure of the outcome of a probability experiment, so its value is determined by chance.

Ch 6

Discrete Random Variable: either finite or countable number of values.

0 1 2 3 4 ...

Ch 7

Continuous Random Variable: has measurably infinite many values.

0 1 2 3 4

Ex: Identify the random variable and sample space.

(a) Coin toss for Heads
X = # of Heads in a coin toss,
the sample space x = 0, 1.

(b) An experiment that measures the time between arrivals of cars at a drive-through.
X = time b/w arrivals @ drive-thru,
the sample space x ≥ 0.

0 → time

Notation X = x values X takes

PROBABILITY DISTRIBUTIONS

Def A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a graph, table, or formula.

REQUIREMENTS

sum

1. $\sum P(x) = 1$ (where x assumes all possible values.)
2. $0 \le P(x) \le 1$ (for every individual value of x.)

Ex: Are the following a probability distribution? If not, state why.

A.

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	0.01

2) √ $0 \le P \le 1$
1) $\Sigma P = 1$
$\boxed{NO}$
$\Sigma = 0.97$

B.

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	– 0.01

C.

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	0.04

2) √
1) ? √
$\Sigma = 1$

Ex: A couple plans to have four children. Let x be the number of boys the couple will have. Find the probability distribution for the number of boys.

GTQ

Experiment: having 4 kids
RV: # of boys out of 4 kids → order matters
x = # of B

Prob. Dist

x	$P(x)$
0	1/16 = 0.0625
1	4/16 = 0.25
2	6/16 = 0.375
3	4/16 = 0.25
4	1/16 = 0.0625

$S = \{ \underbrace{GGGG}_{x=0}, \underbrace{BGGG, GBGG, GGBG, GGGB}_{x=1},$

$\underbrace{BBGG, GBBG, GGBB, BGGB, BGBG, GBGB}_{x=2},$

$\underbrace{BBBG, BBGB, BGBB, GBBB}_{x=3}, \underbrace{BBBB}_{x=4} \}$

$\# S = 16$

Formulas for Probability Distributions

Mean Value

Mean

Formula: $\boxed{\mu = \sum[x \cdot P(x)]}$

Note: The Greek letter μ is read "mu"

Variance

Formula: $\sigma^2 = \sum\left[(x-\mu)^2 \cdot P(x)\right]$ or $\boxed{\sigma^2 = \underbrace{\left(\sum\left[x^2 \cdot P(x)\right]\right)}_{9.36} - \mu^2}$

Standard Deviation

Formula: $\boxed{\sigma = \sqrt{\sum\left[x^2 \cdot P(x)\right] - \mu^2}}$

Note: The Greek letter σ is read "sigma"

Rounding Rule: *Carry one more decimal place than is used for the random variable, i.e. use "stats law of rounding"* x

Ex: In a study of brand recognition, random groups of four people are interviewed. Let x be the number of people who recognize Jeff Bezos when shown a picture. The following probability distribution gives the likelihood of the random variable.

✓Probability Dist.

L3 = L1 * L2 L4 = L1² * L2

L1 x	L2 $P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	0.01	0	0
1	0.10	0.1	0.1
2	0.24	0.48	0.96
3	0.30	0.9	2.7
4	0.35	1.4	5.6
	$\sum P(x) =$ 1.00	$\sum[x \cdot P(x)] =$ 2.88 mean	$\sum[x^2 \cdot P(x)] =$ 9.36

(a) What is the probability that more than two people will recognize a picture of Jeff Bezos?

more than 2 $x = 3, 4$

$P(x > 2) = P(x = 3 \text{ or } 4) = P(3) + P(4) = 0.30 + 0.35 = \boxed{0.65}$

(b) What is the probability that at most three people will recognize a picture of Jeff Bezos?

at most 3 use complement rule $x = 0, 1, 2, 3$

$P(x \le 3) = 1 - P(x=4) = 1 - 0.35 = \boxed{0.65}$

(c) Find the mean number of people who recognize a picture ~~of Jeff Bezos~~.

unit $\frac{\sum x}{n}$ $\mu = \sum[x \cdot P(x)] = 2.88 = \boxed{2.9 \text{ people per four}}$

(d) Find the standard deviation of the given probability distribution.

$\sigma = \sqrt{\sigma^2} = \sqrt{\sum[x^2 \cdot P(x)] - \mu^2} = \sqrt{9.36 - (2.88)^2} = 1.03227$

"$\sqrt{\text{var}}$"

$\boxed{\sigma = 1.0 \text{ people}}$

EXPECTED VALUE

Def The **expected value** (denoted $E(X)$) of a discrete random variable represents the mean value of the outcomes.

FORMULA: $\boxed{E(X) = \sum [\, x \cdot P(x) \,]}$ $= \sum [x \cdot P(x)] = \mu$

GTQ

EX: There is a game in Vegas where you can win \$4 or \$9 but it costs \$1 to play the game. The probability of winning \$4 is 0.3 and the probability of winning \$9 is 0.1. Find the expected value for this game.

	x	$P(x)$
win	9	0.1
win	4	0.3
lose	−1	0.6

$\sum P(x) = 1.00$

$1.00 - 0.1 - 0.3$

RV X = the amount (\$) won or lost in Vegas game.

x = \$9, \$4, −\$1

EV $E(X) = \sum x \cdot P(x) = (9)\cdot(0.1) + (4)\cdot(0.3) + (-1)\cdot(0.6)$

$\boxed{E(X) = 1.5 \text{ \$/game}}$ ← in the long run! (theoretical limit)

Interpretation of μ: Over the long run (if we play this game MANY times) we **expect** the mean profit to be \$1.50.

EX: When someone buys a life insurance policy, that policy will pay out a sum of money to a benefactor upon the death of the policyholder. Suppose a 25-year-old male buys a \$150,000 1-year term life insurance policy for \$250. The probability that the male will not survive the year is 0.0013.

The "experiment" has two possible outcomes: ___live___ or ___die___. Let the random variable X represent the money lost or gained by the life insurance company for the 25-year-old male after many years. What is the expected value for the company?

Prob. Dist

	x	$P(x)$
\$ (lives)	250	$1 - 0.0013 = 0.9987$
\$ (die)	−149,750	0.0013

x = \$250 (lives), ~~−\$150,000~~ (die) −149,750

$E(X) = (250)(0.9987) + (-149{,}750)(0.0013)$

$= \boxed{55 \text{ \$/policy}}$

Over 1,000 policies, how much should they expect to make? ($M \to E$)

↳ lots of policies "long run"

"The insurance company expects to gain \$55 over 1000 such policies"

EX: Find the expected value of the random variable. Round to the three decimal places.

A contractor is considering a sale that promises a profit of \$29,000 with a probability of 0.7 or a loss (due to bad weather, strikes, and such) of \$3,000 with a probability of 0.3. What is their expected profit?

A. \$19,400
B. \$21,200
C. \$20,300
D. \$22,400
E. \$26,000

A

x	$P(x)$
29000	0.7
−3000	0.3

X = profit for construction worker

x = \$29,000, −\$3000

$E(X) = (29000)(0.7) + (-3000)(0.3)$

$= \$19{,}400$