# MATH 460: Probability & Statistics — January_2019

## Ch 4 Special Probability Distribution Functions

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Mon Jan_28 ∪ Tues Jan_29

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### Guiding Question(s)

(1) In a normal distribution, given a probability $p$ can we find the value of $x$ so that $p = P(X \leq x)$?

(2) How can we find the probability for discrete random variables with respect to intervals of time?

(3) What are Poisson distributions and how can we find their probabilities?

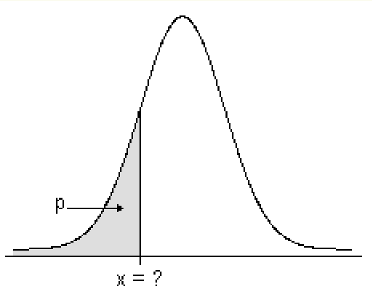
## Chapter 4: Probability Distribution Functions

## Inverse Normal Distribution

### Definition 1: Inverse-Normal-Distribution

This is an informal name to the process of working backwards from a known (or given) probability to find an $x$-value.

Given probability $p$ of the area under the curve of the PDF to the left of the value $x_p$, we want to find what the value $x_p$ is. It really helps to draw a picture to help explain what we are doing:




Essentially, we solving for $x$ in the equation: $P(X \leq x) = p$.

USING CALCULATOR TI83: invNorm(p,$\mu$,$\sigma$)

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- Given probability $p$ of the area under the curve of the PDF to the left of the value $x_p$, we can find what the value $x_p$ is by using the “inverse normal distribution:”

$$\text{invNorm}(p, \mu, \sigma) = x_p \tag{1}$$

### Activity 1: Inverse-Normal-Distribution

Find the 90th percentile for a normal distribution with a mean of 70 and a standard deviation of 4.5.

### Activity 2: Inverse-Normal-Distribution

The time it takes employees to get to work from home (in minutes) is normally distributed with a mean of 30 minutes, and a standard deviation of 5 minutes. Find:

(a) the percentage of employees that take between 28 and 37 minutes to get to work (Hint: this is not an inverse problem)

(b) The number of minutes the longest it would take the bottom employee in the bottom 5% of the data to get to work. (Hint: this is an inverse problem)

### Activity 3: Inverse-Normal-Distribution

An average light bulb manufactured in a factory lasts 280 days with a standard deviation of 45 days. Assume that bulb life is normally distributed.

(a) What is the probability that an Acme light bulb will last at most 360 days? (Hint: this is not an inverse problem)

(b) What bulb life separates the bottom 12%? (Hint: this is an inverse problem)

## Poisson Distribution

### Definition 2: Poisson-Distribution

For a Poisson distribution, we make the following assumptions:

- $X$ is a discrete random variable with (possibly infinite) non-negative values, i.e. values 0,1,2, $\ldots$.
  Similar to the other distributions, we let $X$ = the number of successes of a Poisson random variable.
- the number of successes in two disjoint **time intervals** is independent
- the probability of success during a small time interval is proportional to the entire length of the time interval

Then, if $\lambda$ is the mean of the number of successes in the given time interval, the **probability that $X$ is successful exactly $x$ times** is given by

$$\boxed{P(X = x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!}} \tag{2}$$

### Theorem 1: Properties of Poisson Distributions

(a) Note that $\lambda = \mu$ (that is, the mean).

(b) If $n$ is an interval of time, and $p$ is the probability of success, then $\lambda = np$.

(c) Similarly, if $r$ is a rate of time and $t$ an interval of time, then $\lambda = rt$.

(d) A Poisson Distribution is often used to approximate the Binomial Distribution, when $n$ is "large" and $p$ is "small" (general rule is $n \geq 30$ and $p < 0.05$)

(e) A Poisson distribution has $\sigma = \sqrt{\lambda} = \sqrt{\mu}$.

NOTE: It will be noticed that in the formula, the only variable quantity is the rate $\lambda$. That number is the only way in which one Poisson situation differs from another, and it is the only determining variable (parameter) of the Poisson equation. Nothing else enters in.

## Definition 3: Poisson-Distribution

USING CALCULATOR TI83: poissonpdf($\mu$,$x$)

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- Given mean $\mu$ and the number of successes $x$, the probability can be calculated by the "poisson distribution:"

$$\text{poissonpdf}(\mu, x) = P(X = x) \tag{3}$$

## Example 1: Poisson Distribution

For the 55-year period since 1960, there were 336 Atlantic hurricanes. Assume that the Poisson distribution is a suitable model. Let $X =$ the number of hurricanes in a year.

(a) Find $\lambda$, the mean number of hurricanes per year.

(b) Find the probability that in a randomly selected year, there are exactly 8 hurricanes. That is, find $P(X = 8)$, where $P(X = x)$ is the probability of $x$ Atlantic hurricanes in a year.

(c) In this 55-year period, there were actually 5 years with 8 Atlantic hurricanes. How does this actual result compare to the probability found in part (b)? Does the Poisson distribution appear to be a good model in this case?

*SOLUTION:*
(a) The Poisson distribution applies because we are dealing with the occurrences of an event (hurricanes) over some interval (a year). The mean number of hurricanes per year is $\lambda = 336/55 = 6.1$. So, there are 6.1 hurricanes per year.

(b) With $\lambda = 6.1$, we set $x = 8$ and use the formula

$$P(X = 8) = \frac{e^{-6.1} \cdot 6.1^8}{8!} = \text{poissonpdf}(6.1, 8) = 0.107.$$

(c) From part (b), the likelihood of getting 8 Atlantic hurricanes in 1 year is 0.107. In 55 years, the expected number of years with 8 hurricanes is $55 \times 0.107 = 5.9$ years. The expected number of years with 8 hurricanes is 5.9, which is reasonably close to the 5 years that actually had 8 hurricanes. So, in this case, the Poisson model appears to work reasonably well. □

## Activity 4: Poisson-Distribution

ACME Realty reports it sells 75 homes in 25 days. What is the probability that exactly 2 homes will be sold tomorrow? *(Note: this is problem 8 from our Midterm review)* Round answers to four decimal places.

## Activity 5: Poisson-Distribution

A company makes electrical motors. The probability an electrical motor is defective is 0.01. What is the probability that a sample of 415 electrical motors will contain exactly five defective motors? Round answers to four decimal places.
*(Hint: use $\lambda = n \cdot p$)*

## Activity 6: Poisson-Distribution

Round answers to four decimal places. A Life Insurance (LI) salesman sells on average 3 LI policies per week. Assuming a Poisson Distribution, calculate the probability that in a given week she will sell:

(a) some policies

(b) 2 or more but less than 5 policies

(c) Assuming a five day workweek, what is the probability that in a given day, she will sell a policy?

## Activity 7: Visualizing-Poisson-Distribution

A 911 operator receives about six telephone calls between 8 a.m. and 10 a.m.

(a) What is the probability that she receives more than one call in the next 15 minutes?

(b) Plot the histogram for the probability $P(x) = P(X = x)$ for $x = 0, 1, 2, 3, 4 \ldots$

| x | $P(x)$ |
|---|---|
| | |
| | |
| | |
| | |
| | |
| $\vdots$ | $\vdots$ |