Section 10.2: Hypothesis Test for a Population Proportion

p - population
$\hat{p}$ - sample

GOAL: Make a decision about p based on $\hat{p}$ using probability (sampling dist).

STRUCTURE OF A HYPOTHESIS TEST

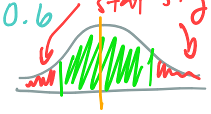
1) Make an assumption about reality	• state Hypotheses H_0 & H_A
2) Look at a sample evidence	• sample evidence $\hat{p}$
3) Determine whether it contradicts our assumption.	• Use P-Value Method or Critical Value Method

We won't be 100% certain, we will just be able to tell if sample data supports a statement or not.

BIG QUESTION: Is this STATISTICALLY SIGNIFICANT?

Def ***Statistically Significant*** When observed results are unlikely under the assumption that the null hypothesis is true and we reject the null hypothesis.

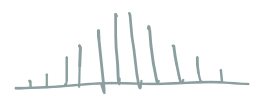
Ex $\begin{cases} H_0: p = 0.5 \\ H_A: p > 0.5 \end{cases}$ sample $\hat{p} = 0.57$, $\hat{p} = 0.6$, $\hat{p} = 0.55$ stat sig



HYPOTHESIS TESTING: CLAIM ABOUT A PROPORTION

Requirements

1. The sample observations are a simple random sample. SRS
2. The conditions for a BINOMIAL distribution are satisfied.
3. If $n \cdot \hat{p} \cdot \hat{q} \geq 10$, then the normal distribution can be used to approximate the distribution of sample proportions with mean μ = np and standard deviation σ= $\sqrt{npq}$ exactly as found when using the binomial distribution.



New Notation: "past proportion" p_0 "new" sample $\hat{p}$

Steps for *a* Hypothesis Test When Applied to Testing Population Proportion *p*	
Step 0: *Check Requirements* • It is a valid SRS sample • The requirements are met to use the needed distribution.	BINOMIAL DIST 1) fixed trials (n) 2) independent events 3) "bi" S F 4) probability of events are constant. $P(\text{success}) = p$ $P(\text{failure}) = q = 1-p$
Step 1: *Hypotheses* parameter "past" H_0: $p = p_0$ H_1: $p < p_0$ or $p \neq p_0$ or $p > p_0$	***Step 2:*** *Level of Significance* $\alpha = P(\text{RH}_0 \mid \text{H}_0\text{T})$ Convention: choose $\alpha = 0.05$ if LOS is not specified.
Step 3: *Test Statistic* (Find a *z*-score, ~~*t*-value~~ or ~~χ^2-value~~) $z^* = \dfrac{\hat{p} - p_0}{\sigma_{p_0}}$ $\boxed{z^* = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}}$ • p_0 = past prop ($q_0 = 1 - p_0$) • $\hat{p}$ = new sample prop • n = sample size Sampling: z^* is saying the # of standard deviations $\hat{p}$ is away from $\mu_{p_0} = p_0$ (& $\sigma_{p_0} = \sqrt{\frac{p_0 q_0}{n}}$).	
Step 4: *Find a critical value or P-value to check using either the Critical Value Method OR P-value Method.*	
Step 5: *Make a* ***decision*** *AND draw a* ***conclusion***	

Null and Alternative Hypothesis

Left-Tailed	Two-Tailed	Right-Tailed
$H_0: \ p = p_0$	$H_0: \ p = p_0$	$H_0: \ p = p_0$
$H_1: \ p < p_0$	$H_1: \ p \neq p_0$	$H_1: \ p > p_0$

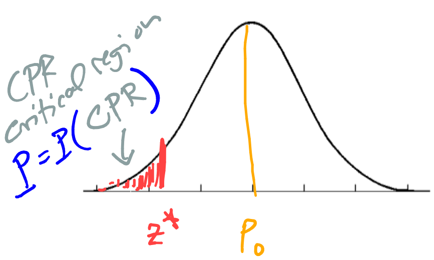
Null Hyp. now

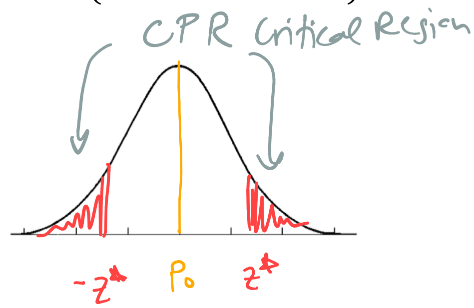
P-Value Method

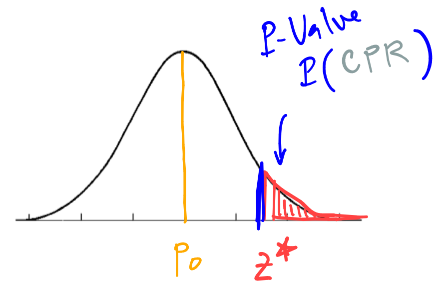
Def ***P-Value***: *probability that a sample is as extreme as our test statistic or more extreme assuming H_0 is true.*

Key: *use test statistic z^* to draw the **critical region (CPR)** and compute probability of it.*

$P = P(z < z^*)$ = normalcdf(-1E99, z^*, 0, 1) $\quad P = P(z < -z^* \text{ or } z > z^*) \quad P = P(z > z^*)$







P-Val = P(CPR)

Make a Decision: $\begin{cases} \textbf{If } P \le \alpha, \textbf{ then we "Reject } H_0\text{"} \\ \textbf{If } P > \alpha, \textbf{ then we "Fail to Reject } H_0\text{"} \end{cases}$

SLOGAN

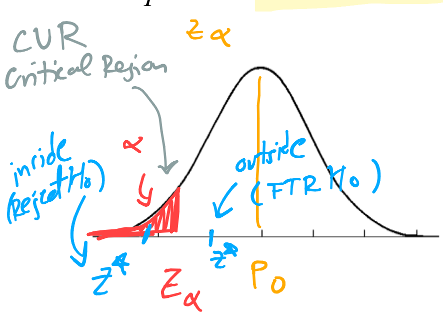
If P low, Null must go! — Reject H_0

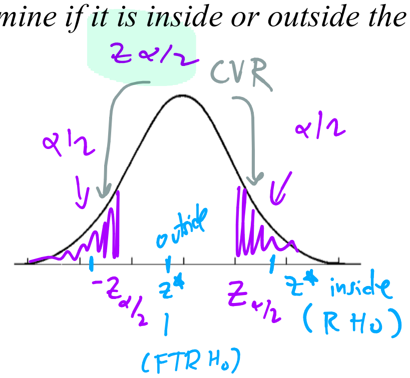
If P high, Null will fly! — Fail to Reject H_0

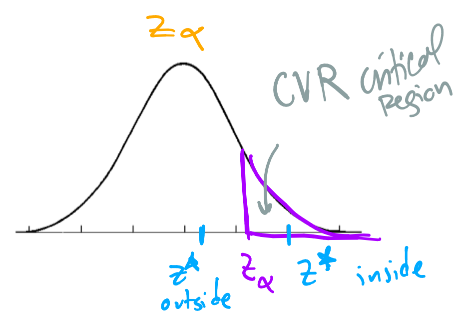
Critical Value Method

Key: *Use the level of significance, α, to compute the critical value z_α or $z_{\alpha/2}$ (depending on one or two-tailed test) which determines the **critical region (CVR)**.*

Compute the test statistic z^* and determine if it is inside or outside the CVR.







z_α = invNorm(α, 0, 1, LEFT)

Make a Decision: $\begin{cases} \textit{If } z^* \textit{ is INSIDE the CVR, then we "}\textbf{Reject } H_0\text{"} \\ \textit{If } z^* \textit{ is OUTSIDE the CVR, then we "}\textbf{Fail to Reject } H_0\text{"} \end{cases}$

Parameter: proportion p $\quad p_0 \quad \hat{p}$ new $\quad$ past

Ex1: According to the Census Bureau, 8.8% of the U.S. population had no health insurance coverage in 2017. Suppose that in a recent random sample of 1200 Americans, 130 had no health insurance. Use a 0.02 significance level (α) to test the claim that the current percentage of Americans who have no health insurance coverage is greater than 8.8%. Use the ___P-Value___ method.

Step 0 Check Requirements: 1) SRS 2) Indep 3) Binomial dist

Null and Alternative Hypothesis

$$\begin{cases} H_0: p = 0.088 & (p_0 = 0.088) \\ H_A: p > 0.088 & (\text{Right Tailed Test}) \end{cases}$$

Test Statistic

$$z^* = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.108 - 0.088}{\sqrt{\frac{0.088 * 0.912}{1200}}} = \boxed{2.45}$$

z^*

$p_0 = 0.088 \rightarrow q_0 = 1 - p_0 = 0.912$

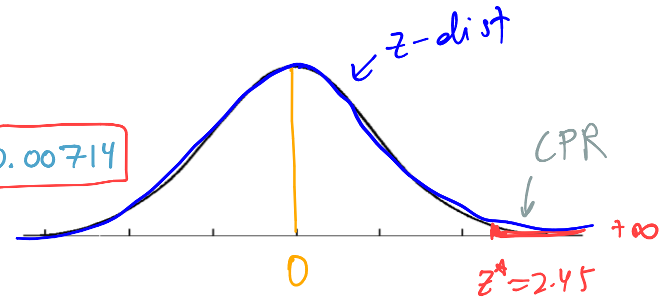
$\hat{p} = \frac{130}{1200} = 0.108$

$n = 1200$

P-value/~~Critical Region~~

P-Value: $P = P(\text{CPR}) = P(z > 2.45)$

$= \text{normalcdf}(\overset{a}{2.45}, \overset{b}{1E99}, \overset{\mu}{0}, \overset{\sigma}{1}) = \boxed{0.00714}$



Decision about Null Hypothesis

- $P = 0.00714$
- $\alpha = 0.02$

$\Rightarrow P < \alpha$ "P is low Null must go" $\Rightarrow$ $\boxed{\text{Reject } H_0}$ (support H_A)

Conclusion (M→E)

use as template

" There is enough statistical evidence to support the claim that the percentage of Americans who have no health insurance coverage is greater than 8.8%. "

Identify the Type I error

Identify the Type II error

Two Tailed Test

past p_0 $\qquad$ $\hat{p}$ new

Ex2: According to NPR, in 2016 32.1% of adults aged 18-34 lived at home with their parents. A sociologist recently randomly surveyed 500 people aged 18-34 and found that 143 of them did. At $\alpha = 0.05$, do we think the proportion has changed? Use the ___Critical Value___ method.

↳ Hyp Test

Null and Alternative Hypothesis $\qquad$ step 0 check Req

$$\begin{cases} H_0: p = 0.321 \quad (p_0 = 0.321) \\ H_A: p \neq 0.321 \quad (\text{Two Tailed Test}) \end{cases}$$

Test Statistic

- $p_0 = 0.321 \longrightarrow q_0 = 0.679$
- $n = 500$
- $\hat{p} = \frac{143}{500} = 0.286$

$$z^* = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.286 - 0.321}{\sqrt{\frac{0.321 * 0.679}{500}}} = \boxed{-1.68}$$

z^*

~~P-value~~/Critical Region

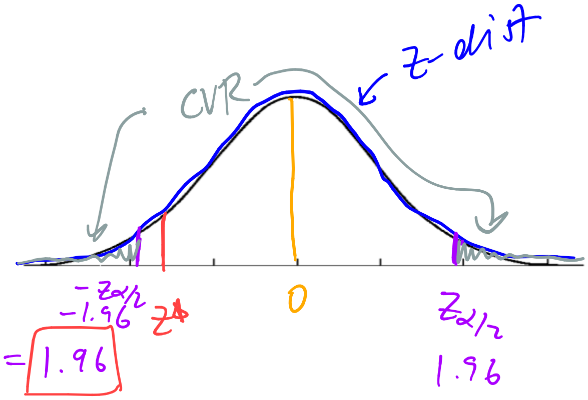
$\alpha = 0.05$

Two-Tailed Test

CVR: cut up by $-z_{\alpha/2}$ & $z_{\alpha/2}$

$\alpha/2 = 0.025$

$$z_{\alpha/2} = \text{invNorm}(\underset{\alpha/2}{0.025}, \underset{\mu}{0}, \underset{\sigma}{1}, \underset{\text{TAIL}}{\text{RIGHT}}) = \boxed{1.96}$$



Decision about Null Hypothesis

z^* is OUTSIDE CVR $\Longrightarrow$ $\boxed{\text{Fail to Reject } H_0}$

Conclusion

"There is not enough statistical evidence to support the claim that the proportion of adults (18-34 yo) who live at home with their parents has changed from 0.321 as in 2016."

Graphing Calculator (TI-83 or 84)

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ 1-PropZTest

(b) Enter
- p_0 = population proportion stated in H_0
- x = number of successes
- n = number of trials
- *prop* alternative hypothesis

P-Value Method

$P = 0.0937$

$\alpha = 0.05$

$P > \alpha \rightarrow$ P high, Null fly

$\Rightarrow$ FTR H_0

ie same conclusion!

Screen

prop $\neq 0.321$ (H_A)

$z = -1.67$ (Test Statistic!) Tricky!

$p = 0.0937$ (P-Value)

$\hat{p} = 0.286$ $\quad n = 500$ (sample prop & sample size) (new) $\hat{p}$ α p_0

Ex3: In a 2016 Gallup poll, 34% of people said that it was morally acceptable to clone animals. In 2017, a survey found that 192 out of 600 randomly selected people believed that it was morally acceptable to clone animals. Use a 0.10 significance level to test the claim that less than 34% of all adults say that it is morally acceptable to clone animals. Use the ___P-Value___ method.

Step 0 Check Requirements
1) SRS ✓ 2) Indep. ✓ 3) BINOMIAL DIST
i) $n = 600$ fixed trials ✓
ii) indep ✓
iii) "bi" 2 outcomes ✓
iv) prob of success const.? ✓

Null and Alternative Hypothesis

$$\begin{cases} H_0: p = 0.34 & (p_0 = 0.34) \\ H_A: p < 0.34 & (\text{Left Tailed Test}) \end{cases}$$

Test Statistic

$$z^* = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.32 - 0.34}{\sqrt{\frac{0.34 * 0.66}{600}}} = \boxed{-1.03}$$

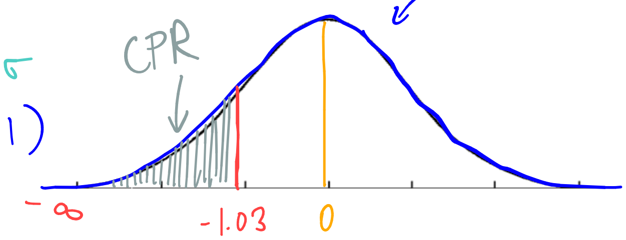
- $p_0 = 0.34$
  $q_0 = 1 - p_0 = 0.66$
- $n = 600$
- $\hat{p} = \frac{x}{n} = \frac{192}{600} = 0.32$

P-value/~~Critical Region~~

P-Value

$P = P(z < -1.03) = \text{normalcdf}(-1E99, -1.03, 0, 1)$ (a, b, μ, σ; CPR)

$\boxed{P = 0.152}$



Decision about Null Hypothesis

$\alpha = 0.1$
$P = 0.152$ $\Rightarrow$ $P > \alpha$ "P high, Null fly" $\Rightarrow$ $\boxed{\text{Fail to Reject } H_0}$

Conclusion (M→E)

" There is not enough statistical evidence to support the claim that the proportion of people (adults) that believe cloning is morally acceptable is less than 0.34. "

Big Question: Is this ___statistically___ ___significant___?

Def ***Statistically Significant*** When observed results are unlikely under the assumption that the null hypothesis is true and we reject the null hypothesis.

Logic of Hypothesis Tests (Cover this at the end of the chapter)

Classical Approach

We now explain how Hypothesis Tests work. It starts with a sampling distribution. Recall that we have a past claim, p_0. We want to know if it has changed so we find a new sample, $\hat{p}$, of size n. Then we know what the sampling distribution should look like assuming the null hypothesis, $p = p_0$, is true. In Chapter 8 we learned that the sampling distribution is a normal distribution with $\mu_{p_0} = p$ and $\sigma_{p_0} = \sqrt{p_0 \cdot q_0/n}$.

The key is we want to know whether our new sample, $\hat{p}$, could reasonably be determined by chance if H_0 is true.

If the null hypothesis is true, then we expect that our sample, $\hat{p}$, come from the "middle" of the sampling distribution. That means it should be, say, within 2 standard deviations from the mean (recall this corresponds to approximately 95% of the samples). To compute how many standard deviations away from the mean it is, we simply compute the z-score! We call this z-score the test statistic, z^*.

In symbols, this looks like this: $z^* = \frac{\hat{p}-p_0}{\sigma_{p_0}}$. We can use this number to make our decision.

If the test statistic is greater than 2 or less than -2, then this is saying that our sample is unlikely to have occurred by chance! So we reject the null hypothesis since have statistical evidence that our sample, $\hat{p}$, is unlikely to occur if the null hypothesis is true.

If the test statistics is between -2 and 2, then this is exactly what we would expect assuming the null hypothesis is true. That is, our sample is likely to have occurred if the null hypothesis is true.

P-value Approach

This is very similar to the classical approach. This time we compute the probability of obtaining a sample that is as extreme as $\hat{p}$, or more extreme. For example, if we are trying to do a Right-Tailed Test, then we compute the probability $P(z > \hat{p})$ using our new sample as the cut-off and the sampling distribution of p_0:

$$P(z > \hat{p}) = normalcdf\left(\hat{p}, 1\text{E}99, \mu_{p_0}, \sigma_{p_0}\right).$$

If P is small (less than 0.05), then it is unlikely that our sample, $\hat{p}$, was determined by chance! So we have statistical evidence that the null hypothesis is not true and we reject H_0. For example, if P=0.02, then 2 samples in 100 will give a sample proportion of $\hat{p}$ or higher.

If P is high (greater than 0.05), then this is not unusual assuming that the null hypothesis is true. We haven't proved the null hypothesis only found evidence that our sample is reasonable assuming it is true.

Confidence Interval Approach

There's actually one more way we can do Hypothesis Tests. We can use a confidence interval to make our decision. We construct a confidence interval using the new sample data, $\hat{p}$.

If the past claim, p_0, is outside the confidence interval then we "reject H_0."

If the past claim, p_0, is inside the confidence interval then we "fail to reject H_0."