§6.5 Exponential Growth & Decay

In-class Activity 6.5

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Activity 1: Bacteria

A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour, the population has increased to 420.

(a) Find an expression for the number of bacteria after t hours.

(b) Find the number of bacteria after 3 hours.

(c) Find the rate of growth after 3 hours.

(d) When will the population reach 10,000?

(e) The doubling-time T_D is defined to be the time it takes a population to double in size, that is: $P(T_D) = 2C = 2P(0)$. Find the doubling-time for the bacteria.

Activity 2: Radioactive Strontium

Strontium-90 decreases at a rate proportional to its mass. Strontium-90 has a half-life of 28 days.

(a) A sample has a mass of 50 mg initially. Find a formula for the mass remaining after t days.

(b) Find the mass remaining after 40 days.

(c) How long does it take the sample to decay to a mass of 2 mg?

Activity 3:

Find the solutions to the DE: $\frac{dT}{dt} = k(T - T_S)$, where k and T_S are constants, by solving:

(a) Make the substitution: $y(t) = T(t) - T_S$. What does the new DE look like?

(b) Solve the new DE from part (a)

(c) Solve for $T(t)$

Activity 4:

A thermometer reading $70° F$ is taken outside where the ambient temperature is $22° F$. Four minutes later the reading is $32° F$.

(a) Write the differential equation (DE) that models the temperature $T = T(t)$ of the thermometer at time t.

(b) Find the general solution of the differential equation (i.e. all of the solutions with C).

(c) Find the particular solution to the differential equation, using the initial condition that when $t = 0\, min$, then $T = T(0) = 70° F$.

(d) Find the thermometer reading $7\, min$ after the thermometer was brought outside.

(e) Find the time it takes for the reading to change from $70° F$ to within $0.5° F$ of the air temperature.

Activity 5:

In this activity, we attempt to answer the question asked by many investors: "How long is it going to take for me to double my money?"

(a) Consider an investment of \$100 invested at 5%, compounded continuously. How long would it take for the investor to have \$200?

(b) What would the doubling-time be if the initial investment were \$1,000? \$10,000? What effect does changing the principal have on the doubling time, and why?

Activity 6: Continued.

One of the first things that is taught in an economics class is the Rule of 72. It can be summarized thusly:

> "The number of years it takes an investment to double is equal to 72 divided by the annual percentage interest rate."

(a) What would the Rule of 72 say the doubling time of a 5% investment is? Is it a good estimate?

(b) Repeat Parts (a) and (c) for investments of 3%, 8%, 12% and 18%. What can you say about the accuracy of the Rule of 72?

(c) Derive a precise formula for the time T to double an initial investment.

Activity 7: Continued.

One of the first things that is taught in an economics class is the Rule of 72. It can be summarized thusly:

> "The number of years it takes an investment to double is equal to 72 divided by the annual percentage interest rate."

(a) There is an integer that gives a more accurate answer for continuous or nearly continuous compounding than the Rule of 72. What is this number? Check your answer by using it to estimate the doubling time of a 5% investment.

(b) It turns out that there is a reason that we use the number 72 in the Rule. It has to do with one of the assumptions we made. Why do economists use the Rule of 72?