

§11.7: Strategies for Testing Series

Ch 11: Infinite Sequences and Series Math 5B: Calculus II

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Guiding Questions for §11.7

Guiding Question(s)

- 1 What are **strategies** for Testing Series?

- Similar to integration (where we studies many techniques), we have many techniques for determining whether a series converges or diverges.
- In this section, we'll develop a [testing series toolbox](#).

Testing Series Toolbox

Memorize all the series tests! Pay close attention to the conditions needed.

- **Tool #1** Try Test for Divergence First
- **Tool #2** Is it a **geometric series** or **p -series**? Is it a **alternating series**?
 - Keep in mind the algebraic rules you can apply to convergent series. Try apply them to get a Geometric series into the correct form.
- **Tool #3** Does the “**Squint Test**” give you a series you know to C or D (p -series, geometric series, or alternating series)? If **YES**, then use the **Limit Comparison Test** (or the **Ratio Test**)
- **Tool #4** Is $\sqrt[n]{|a_n|}$ easy to analyze? Try the **Root Test**.
- **Tool #5** Is $\left| \frac{a_{n+1}}{a_n} \right|$ easy to analyze? Try the **Ratio Test**.
- **Tool #6** Does the series have positive terms?
 - Are there **easy** comparison? Try the **Comparison Test**.
 - Is there a positive, decreasing $f(x)$ with $f(n) = a_n$ and $\int f(x)dx$ doable? Try the **Integral Test**.

Remarks

- **Only use tests you're allowed to use!**
There's no point in trying to use a test if the series doesn't fit the necessary hypotheses.
- **Remember: some tests can be inconclusive!**
For example, Ratio and Root Tests are inconclusive when $L = 1$.
- **Re-write the series terms.** You can use algebra rules to simplify series.

Testing Series Toolbox

§11.7

Dr. Basilio

Outline

Guiding
Questions

Intro

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SERIES CONVERGENCE/DIVERGENCE FLOW CHART

TEST FOR DIVERGENCE

Does $\lim_{n \rightarrow \infty} a_n = 0$?

NO

$\sum a_n$ Diverges

YES

p-SERIES

Does $a_n = 1/n^p$, $n \geq 1$?

YES

Is $p > 1$?

YES

$\sum a_n$ Converges

NO

$\sum a_n$ Diverges

NO

GEOMETRIC SERIES

Does $a_n = ar^{n-1}$, $n \geq 1$?

YES

Is $|r| < 1$?

YES

$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$

NO

$\sum a_n$ Diverges

NO

ALTERNATING SERIES

Does $a_n = (-1)^n b_n$ or $a_n = (-1)^{n+1} b_n$, $b_n \geq 0$?

YES

Is $b_{n+1} \leq b_n$ & $\lim_{n \rightarrow \infty} b_n = 0$?

YES

$\sum a_n$ Converges

NO

NO

TELESCOPING SERIES

Do subsequent terms cancel out previous terms in the sum? May have to use partial fractions, properties of logarithms, etc. to put into appropriate form.

YES

Does $\lim_{n \rightarrow \infty} s_n = s$, s finite?

YES

$\sum a_n$ Converges

NO

$\sum a_n$ Diverges

NO

TAYLOR SERIES

Does $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$?

YES

Is x in interval of convergence?

YES

$\sum_{n=0}^{\infty} a_n = f(x)$

NO

$\sum a_n$ Diverges

NO

Try one or more of the following tests:

COMPARISON TEST

Pick $\{b_n\}$. Does $\sum b_n$ converge?

YES

Is $0 \leq a_n \leq b_n$?

YES

$\sum a_n$ Converges

NO

Is $0 \leq b_n \leq a_n$?

YES

$\sum a_n$ Diverges

NO

NO

LIMIT COMPARISON TEST

Pick $\{b_n\}$. Does $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$, c finite & $a_n, b_n > 0$?

YES

Does $\sum b_n$ converge?

YES

$\sum a_n$ Converges

NO

$\sum a_n$ Diverges

NO

INTEGRAL TEST

Does $a_n = f(n)$, $f(x)$ is continuous, positive & decreasing on $[a, \infty)$?

YES

Does $\int_a^{\infty} f(x) dx$ converge?

YES

$\sum_{n=a}^{\infty} a_n$ Converges

NO

$\sum a_n$ Diverges

NO

RATIO TEST

Is $\lim_{n \rightarrow \infty} |a_{n+1}/a_n| \neq 1$?

YES

Is $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$?

YES

$\sum a_n$ Abs. Conv.

NO

$\sum a_n$ Diverges

NO

ROOT TEST

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$?

YES

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$?

YES

$\sum a_n$ Abs. Conv.

NO

$\sum a_n$ Diverges

NO

Problems 1-38 from Stewart's *Calculus*, page 784

$$1. \sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + n}$$

$$2. \sum_{n=1}^{\infty} \frac{n-1}{n^2 + n}$$

$$3. \sum_{n=1}^{\infty} \frac{1}{n^2 + n}$$

$$4. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n-1}{n^2 + n}$$

$$5. \sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$$

$$6. \sum_{n=1}^{\infty} \left(\frac{3n}{1+8n} \right)^n$$

$$7. \sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$$

$$8. \sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$$

$$9. \sum_{k=1}^{\infty} k^2 e^{-k}$$

$$10. \sum_{n=1}^{\infty} n^2 e^{-n^3}$$

$$11. \sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n \ln(n)}$$

$$12. \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 25}$$

$$13. \sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$$

$$14. \sum_{n=1}^{\infty} \sin(n)$$

$$15. \sum_{n=0}^{\infty} \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$$

$$16. \sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + 1}$$

$$17. \sum_{n=1}^{\infty} (-1)^n 2^{1/n}$$

$$18. \sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$$

$$19. \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$$

$$20. \sum_{k=1}^{\infty} \frac{k+5}{5^k}$$

$$21. \sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$$

$$22. \sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3 + 2n^2 + 5}$$

$$23. \sum_{n=1}^{\infty} \tan(1/n)$$

$$24. \sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$$

$$25. \sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

$$26. \sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$$

$$27. \sum_{k=1}^{\infty} \frac{k \ln(k)}{(k+1)^3}$$

$$28. \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

$$29. \sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n\sqrt{n}}$$

$$30. \sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$$

$$31. \sum_{k=1}^{\infty} \frac{5^k}{3^k + 4^k}$$

$$32. \sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$$

$$33. \sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$$

$$34. \sum_{n=1}^{\infty} \frac{1}{n + n \cos^2(n)}$$

$$35. \sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$$

$$36. \sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$$

$$37. \sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)^n$$

$$38. \sum_{n=1}^{\infty} (\sqrt[3]{2} - 1)$$