

§7.2

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Dutline

Guiding Questions

Int of prod of trig

Summary

ase: ODD

se: EVEN

Case: tan & sec

Case: tan & sec

Case: sin(nx) & cos(mx)

§7.2: Trigonometric Integrals

Ch 7: Techniques of Integration
Math 5B: Calculus II

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Class #10 Notes

March 26, 2019 Spring 2019

Outline



- **Guiding Questions**
- Integrals of products of trigonometric functions
- Summary of Techniques
- Case: At least one ODD
- Case: EVEN
- Case: tan and sec
- Case: sin(nx) & cos(mx)

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Outline

trig

Case: tan & sec

Guiding Questions for §7.2



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Guiding

Questions

trig

Case: tan & sec Case: sin(nx) &

Guiding Question(s)

What are the methods for evaluating integrals with products of powers of trigonometric functions?

Introduction



We study techniques for evaluating integrals of the form:

$$\int \sin^n(x) \cos^m(x) \, dx, \int \tan^n(x) \sec^m(x) \, dx, \int \sin(mx) \cos(nx) \, dx$$

- Why are these types of integrals interesting or useful?
- Many simple geometric problems lead to integrals of products of trigonometric functions.
- Another example involves digital music. It is made possible because we can solve integrals of the above form. In Chapter 11 (which we will study later) we'll lay the foundations for this application (so called, Fourier series).

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Summary of Techniques



Ingredients Needed

Techniques for evaluating integrals of the form:

$$\int \sin^n(x) \cos^m(x) \, dx$$

What ingredients are needed?

- U-Substitution
- Trig Formulas
 - Pythagorean $\cos^2(x) + \sin^2(x) = 1$
 - **Double Angle** $\sin^2(x) = \frac{1 \cos(2x)}{2}$ and $\cos^2(x) = \frac{1 + \cos(2x)}{2}$
 - **Product-to-Sum** $\left[\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A-B) + \sin(A+B)\right) \right]$ & 2 more formulas

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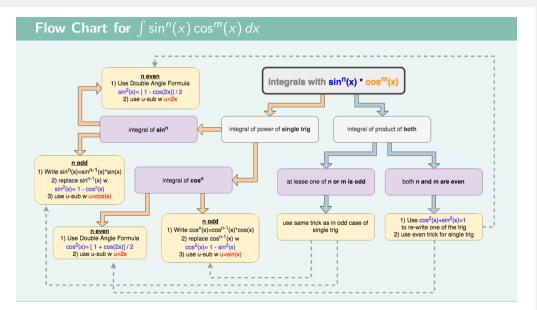
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Case: tan & sec

Case: sin(nx) &

Summary of Techniques





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Outline

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Summary

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Case: tan & sec

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Summary of Techniques

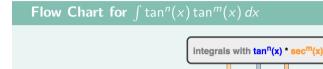




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Summary

Case: tan & sec



Keep in mind two basic integrals:

1) separate out sec²(x)

2) use $\sec^{2}(x)=1+\tan^{2}(x)$

to express the rest in terms of tan(x)

3) use u=tan(x), $du=sec^2(x)$

$$\int \tan(x) \, dx = \ln|\sec(x)| + C \quad \text{and} \quad \int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C$$

n odd

1) separate out sec(x)tan(x)

2) use $tan^{2}(x)=sec^{2}(x)-1$

to express the rest in terms of sec(x)

3) use u=sec(x), du=sec(x)tan(x)

n even, m odd

1) use $tan^{2}(x)=sec^{2}(x)-1$

to express the rest in terms of sec(x)

Powers of trigonometric functions: Odd Case



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Case: ODD

Case: tan & sec

Evaluate:

(a) $\int \sin^3(x) dx$ (b) $\int \sin^4(x) \cos^5(x) dx$

Powers of trigonometric functions: Odd Case



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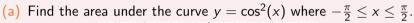
Case: ODD

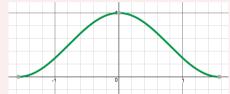
Case: tan & sec

Case: tan & secCase: sin(nx) &

Powers of trigonometric functions: Even case

Activity 2: Case: EVEN





(b) Find: $\int \cos^4(x) dx$

Powers of trigonometric functions: Even Case



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Case: EVEN

Case: tan & sec Case: sin(nx) &



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Case: tan & sec

Ingredients Nedded

We'll study techniques for evaluating integrals of the form:

$$\int \tan^n(x) \sec^m(x) \, dx$$

What are the ingredients needed?

- U-Substitution
- Trig Formulas
 - Pythagorean $1 + \tan^2(x) = \sec^2(x)$
- See flow chart.



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Case: tan & sec

Activity 3:

Evaluate:

(a) Find: $\int \tan^3(x) \sec^5(x) dx$ (b) Find: $\int \tan^2(x) \sec^4(x) dx$



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Case: tan & sec Case: sin(nx) &



$$\int \tan(x) \, dx = \ln|\sec(x)| + C$$

Example 1:

We work this out:

$$\int \tan(x) dx = \int \frac{\sin(x)}{\cos(x)} dx$$

$$= \int \frac{1}{u} \cdot -du \quad (u = \cos(x), du = -\sin(x) dx)$$

$$= -\ln|u| + C$$

$$= -\ln|\cos(x)| + C$$

$$= \ln|\cos^{-1}(x)| + C$$

$$= \ln|\sec(x)| + C$$

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Case: tan & sec



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$\int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C$

This derivation requires a clever trick that comes out of nowhere:

Example 2:

$$\int \sec(x) \, dx = \int \sec(x) \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)} \, dx$$

$$= \int \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)} dx$$

$$= \int \frac{\sec(x) + \tan(x)}{\sec(x) + \tan(x)}$$

$$\int \sec(x) + \tan(x)$$
$$= \int \frac{1}{u} \cdot du$$

$$(u = \sec(x) + \tan(x), du = (\sec(x)\tan(x) + \sec^2(x))dx)$$

$$\ln |u| + C$$

$$= \ln |u| + C$$

$$= \ln |\sec(x) + \tan(x)| + C$$

Case: tan & sec

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Case: sin(nx) &

Activity 4:

Use IBP to evaluate: $\int \sec^3(x) dx$



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Products of trig functions: Case: sin(nx) & cos(mx)



We finish with an important class of integrals. These show up in lots of applications.

Ingredients Needed

We'll study techniques for evaluating integrals of the form:

$$\int \sin(nx)\cos(mx)\,dx$$

What are the ingredients needed?

- Trig Formulas

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Case: tan & sec

Case: sin(nx) & $\cos(mx)$





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cos(mx)

Case: tan & sec Case: sin(nx) &

Activity 5:

Products of trig functions: Case: sin(nx) & cos(mx)



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