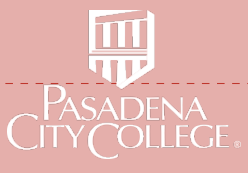
MATH 5B – Calculus II — Spring 2019

FINAL EXAM

Study Guide

Dr. Jorge Basilio

Ch 6, 7, 8, 9, 10, 11



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General Exam Info

Exams are a way for you to show me what you have learned (and please show all your steps so I can see this!) and to give you a sense of accomplishment! They are meant to be challenging and not just homework problems with the numbers changed. I really want to prepare you for university level math classes–so some exams may be longer or more challenging than others. Remember that I do grade fairly and my goal is to push you to succeed and excel in this class.

- Attendance required for all exams–I do NOT drop the lowest exam score.
- **"Make-up Exams"** are given only in extreme cases and at instructor's discretion; a student is allowed at most one make-up exam. (Documentation must be provided for the instructor to even consider a make up exam. This means you would need a doctor's note, etc.) A "Make-Up Exam" means you will be allowed to replace the missing score with the percentage you earn on the final exam. Please contact your instructor as soon as possible should there be a problem.
- **Your student ID is required for all exams.**
- During the exams–you will be required to leave your backpack and all non-test items at the front of the room, including cell phones and smart watches. Only your pencil/eraser and calculator will be allowed during the exam, and there will be a calculator check. Should you need to leave during the exam please ask for permission first before leaving and leave your cell phone with me. Not doing these things could result in a 0 on your exam.
- Once the exam is graded and returned, any problem you would like me to revisit must be brought to my attention by the next class session.
- Always keep your exams!

FINAL EXAM	Date & Time
• **Final Exam**	**Thursday, June 13 from 3:15-5:15 pm**

Final Exam Specific Info

- **Material covered:** Anything from Exam 1, 2, 3, 4, PLUS 8.1, 8.2, 10.1, 10.2
- You will need a calculator (only scientific allowed)–you won't be able to use your phone.
- Almost all questions have multiple parts
- Be prepared for the following types of questions: True-False-Sometimes, Multiple-Choice, and Free-response
- Be prepared to prove certain results (see below for more details)
- I will allow a cheat sheet for this exam but only if they meet certain conditions. Read carefully below.
- Be prepared for questions with multiple parts that build on each other

Rules for an acceptable Cheat Sheet

Please read the document "GoodvsBad" Cheat Sheets which is linked on our webpage. It was written for a different school but the info obviously is school-independent.

The key to making a Good Cheat Sheet (GCS) is that it has to be done by hand and well-organized. Printing one out from the internet will not be allowed and defeats the purpose of a GCS.

I will collect your cheat sheets before the exam begins and you will start the exam promptly. While you are taking the exam I will check them individually to see if they satisfy my requirements for a GCS. If they pass, it will be returned to you and you can use it for the remained of the text (approximately 15-20 minutes after the exam begins). However, if it's a bad cheat sheet then I will not return it and you will not be able to use it on the exam.

Please follow the following rules/requirements:

- Must be hand-written & NEAT & written by YOU!
- SIZE: using a regular 8.5" x 11" piece of white paper (no other paper allowed, no lined paper, graph paper, etc)
- CHEAT SHEET NOTES ARE ONLY ALLOWED ON **BOTH SIDEs of the page**
- You must include: Your name, Course, Instructor, Date. Put this neatly on the top of your page.
- You may include an unlimited number of definitions, theorems, graphs of standard functions, and formulas
- You are allowed to have **AT MOST TWO** example problems with complete solutions. **These must be clearly indicated with a box around them so that I can find them and check that no more than two worked out problems are on your cheat sheet.**
  - If it takes me more than 5 seconds to find these or if you try to sneak more worked out problems outside of the two clearly labeled, then your cheat sheet will not be returned to you.
- Must follow the principles outlined in the GCSvsBCS document discussed in class and available on my website.
  - YES, this is completely subjective and up to my discretion
  - If I don't think you followed the spirit of the document or if you failed to follow any of the above directions then I reserve the right to not allow you to use your cheat sheet
- ***BONUS*** If your cheat sheet is beautiful & complete (meaning it has the main ideas from all the material we covered) you can earn 2% extra credit on your exam.
- If you have any questions, you can email me.

BIG PICTURE REVIEW OF CALC II

LIMITS

- Basic Limit Rules from Calc I: $+, -, \times, \div$
- L'Hôpital's Rule
- Limits of Transcendental functions:
  - polynomials, radical and rational functions (from Calc I)
  - exponential and logarithms
  - trigonometric functions:
    - usual trig functions
    - inverse trigonometric functions
    - hyperbolic trigonometric functions
- Improper Integrals
- Sequences
- Squint Test (aka Limit Comparison Test)
- Ratio and Root Tests

SERIES

- series as numbers:
  - Tests for C or D
  - Geometric, Harmonic, Alternating Harmonic, p-series, and telescoping series
- series as functions:
  - Interval of Convergence, Radius of Convergence
  - power series
  - power series representations
  - Taylor series
  - Differentiating and Integrating power series (Just like Poly Thm)
- Applications:
  - Differentiating and Integrating power series (Just like Poly Thm)
  - Finding power series representations using the Just like Poly Thm
  - Evaluating Integrals via power series
  - Solving Differential Equations using power series

DERIVATIVES

- Basic Derivative Rules from Calc I: Sum/Difference Rule, Product Rule, Quotient Rule, Chain Rule
- Inverse Functions
- Applications:
  - Tangent Lines (and Tangent Line Approximation)
  - Instantaneous Rates of Change
  - ID Test (Increasing/Decreasing Test)
  - Concavity Test
- Derivatives of Transcendental functions:
  - polynomials, radical and rational functions (from Calc I)
  - exponential and logarithms
  - trigonometric functions:
    - usual trig functions
    - inverse trigonometric functions
    - hyperbolic trigonometric functions
- Derivatives of power series

DIFFERENTIAL EQUATIONS

- the language of life
- methods of solving:
  - using integration
  - using separation of variables
  - using Euler's Method
- Slope Fields/Direction Fields
- General Solutions vs Particular Solutions
- Important Models:
  - Law of Natural Growth/Decay
  - Newton's Law of Cooling
  - Logistic Population Growth

INTEGRALS

- Integration Toolbox: (most important)
  - Tool#1: Know lots of DRs/ADRs!
  - Tool#2: u-substitution
  - Tool#3: integration-by-parts
  - Tool#4: trig-sub
- Additional integration techniques:
  - Products of trigonometric integrals ($\cos^2(x)$ and $\sin^2(x)$ are most important)
  - Method of Partial Fractions
- Applications:
  - Lengths
    - of curves given by $y = f(x)$
    - of curves given by parametric eqs $\begin{cases} x &= x'(t) \\ y &= y'(t) \end{cases}$
  - Areas
    - under curves given by $y = f(x)$
    - under curves given by parametric eqs $\begin{cases} x &= x'(t) \\ y &= y'(t) \end{cases}$
    - Surface Areas of Surfaces of Revolution
  - Volumes
    - Surfaces of Revolution:
      - Slicing Method, Disk/Washer Method, Shell Method (Calc I)
  - ⋆ **Differential Equations**⋆
  - Integral Tests for series
- Integration of series

Topics Review: Key ideas

Chapter 6

- **Key ideas:** **inverse functions**, **transcendental functions** and **L'Hôpital's Rule**
- Inverser and Transcendental functions:
  - inverse functions: $f^{-1}(x)$
  - exponential functions: e^x, a^x
  - logarithmic functions: $\ln(x)$, $\log_b(x)$
  - inverse trig: $\sin^{-1}(x) = \arcsin(x)$, $\cos^{-1}(x) = \arccos(x)$, $\tan^{-1}(x) = \arccos(x)$, etc (note: careful with domain! study graphs!)
  - hyperbolic trig functions: $\sinh(x)$, $\cosh(x)$, $\tanh(x)$, etc
- Know all the graphs of the above functions! Helpful for computing limits, answering T/F and short-response questions
- Know how to **differentiate** and **integrate** all of the above functions!!!!!
- Know all the definitions of e (see hand-out)
- Know how to solve exponential and log equations and know the log properties (log rules and inverse properties)
- Know: the **Law of Natural Growth/Decay**. Recognize when DEs are law of natural growth and use the formula to solve them. Know what **general solutions** and **particular solutions** are.
- Applications of Law Natural Growth: **doubling-time**, **half-life**, **Newton's Law of Cooling**, **Continuous Compound Interest**
- Know: **indeterminate forms** when computing limits and how to use various algebraic techniques to evaluate limits using **L'Hôpital's Rule**
- Know: **rates of growth** and using L'Hop

Chapter 9

- **Key ideas:** **differential equations (DEs)**, **general solutions**, **particular solutions**, and techniques for solving DEs
- Know:**techniques for solving DEs:**
  - use formulas derived in class for: Law of Natural Growth/Decay, Newton's Law of Cooling/Heating, Logistics Equation with Carrying Capacity
  - Solving DEs of the form, $\frac{dy}{dx} = f(x)$, via integration: $y(x) = \int f(x)dx$
    - Note: I can ask questions involving any integral we have studied this term. So I can ask you to solve DEs involving integrals from Chapter 7.
  - **separation of variables**. Important. Practice this.
  - Numerically using **Euler's Method**.
    - Important. Know the general idea of how it works. I can ask T/F, fill-in the blank, or Multiple-Choice type questions testing your knowledge of the method but NOT have you compute approximate solutions USING the method.

Chapter 7

- **Key ideas:** **integration toolbox: integration by parts**, **trigonometric integrals**, **trigonometric substitution**, and **method of partial fractions**.
- Know Tool#1: see class notes for integrals you must memorize.
  - Ex: $\int \frac{1}{\sqrt{1-x^2}}dx = \arcsin(x) + C = \sin^{-1}(x) + C$.
  - Ex: $\int \sinh(x)dx = \cosh(x) + C$, etc
- Know Tool#2: **u-sub**. This can be combined with the all the transcendental functions we've learned.
- Know Tool#3 **Integration by Parts:** Important!
  - Be prepared for easy, medium, and hard IBP problems (see ICAs/classnotes).
  - Practice the **I-trick**
- Know **Trigonometric Integrals:** Important!

  - Be sure you have all the necessary trig identities memorized (double-angle formulas for $\sin^2(x)$ and $\cos^2(x)$ and the Pythagorean Identities). I will pick straight-forward problems and not too hard.
  - See flowchart from the classnotes.
- Know Tool#4 **Trig Sub:** Important! Be prepared for easy, medium, and hard TrigSub problems with $x = a\sin(\theta)$, $x = a\tan(\theta)$, or $x = a\sec(\theta)$.
- Know **Method of Partial Fractions:** Important!
  - I will provide the formulas for how to use on a formula sheet (just like in Exam 2).
  - Prepare problems in Case I, II, III, IV, and V
- **Key ideas:** **approximate integration**.
  - Know the formulas for LEA, REA, MPA, TrapA, and SimpaA. Be prepared to approximate an integral with $n = 4$ or $n = 5$.
  - You do not need to have **error estimates** memorized, they will be provided for you in a formula sheet.
- **Key ideas:** **improper integrals**. Important!
  - Be prepared to compute improper integrals of Type I and Type II.
  - Know how to determine whether an improper integral converges or diverges.
  - Know how to use the **comparison test** to determine whether an improper integral converges or diverges.

Chapter 8

- **Key ideas:** **arclength** of a function and the **surface area** of a surface of revolution.
- Know **arclength** formula for computing the length of a curve.
  - Be able to compute arclengths
  - Be able to set-up but not evaluate the integrals for computing arclength.
  - Practice examples with the "clever trick" where inside the square root, the $1 + [f'(x)]^2$, actually can be written a a perfect square.
- Know **surface area** formula for computing the surface area for a surface of revolution.
  - Be able to compute surface areas.
  - Be able to set-up but not evaluate the integrals for computing surface areas.
- The problems I will put on the final will be very similar to the ones completed in class which are similar to the examples from the textbook. So study the examples and you'll be fine. Be sure to have the formulas memorized/on your cheat-sheet.

Chapter 10

- **Key ideas:** **parametric equations**, **tangent lines** to parametric equations, the **arclength** given parametric equations
- Know how to **sketch curves** defined by parametric equations.
  - Study the examples from the classnotes and the textbook.
  - Circles, lines, and parabolas in parametric form
  - Know the equations for a **cycloid** and be to sketch it at various positions.
  - Questions can be in the form of T/F, MC, or fill-in the blank.
- Know the calculus concepts applied to parametric equations: **tangent lines**, **arclengths**, and **areas**
  - Study the examples from the classnotes and the textbook.
  - These problems will be very similar to the textbook examples.

Chapter 11

- **Key ideas:** **power series**, **power series representations** and **Taylor series**.
- The focus will be on **power series representations** and **Taylor series**. Practice these! §11.9, 11.10, and 11.11 are the key sections to study.
  - Know how to find the **interval of convergence** and the **radius of convergence**
  - Be able to find power series representations using the **Just like Poly** theorem (aka Term-by-term differentiation and integration)

  - Memorize/have on your cheat-sheet standard power series/Taylor series:
    * e^x, $\ln(1+x)$, $\arctan(x)$, $\sin(x)$, $\cos(x)$
    * You may be asked to derive these formulas using Taylor's Theorem
  - Applications of Power/Taylor series:
    * Computing **limits**
    * Computing **derivatives** (Just-like-Poly)
    * Computing **integrals** (Just-like-Poly)
- Know the precise statements of the following **series tests:**
  - Tool#1: Test for Divergence
  - Tool#2: Is it **geometric**, **alternating**, or p-**series**?
  - Tool#3: Can you use the **Squint Test** (aka **Limit Comparison Test**)?
  - Tool#4: Is $\sqrt[n]{|a_n|}$ easy to analyze? Use **root test**
  - Tool#5: Is $\left|\frac{a_{n+1}}{a_n}\right|$ easy to analyze? Use **ratio test**
  - Tool#6: Does the series have positive terms? Try the **comparison test** or the **integral test**
- Summing **Series:**
  - Exactly: geometric series or telescoping series
  - Be prepared for examples where you need to do some algebraic manipulations before you can apply the geometric series sum formula to evaluate a series exactly.
  - Know the notation and meaning of **partial sums** (S_N) and **Remainder** (R_n)
- **Estimating errors:**
  - Know the error formula for the Alternating Series Test
  - Know the **remainder estimates** from the integral test (§11.3, Theorem 3)
  - Taylor's Remainder Theorem: you do not need to memorize– I will provide this on a formula sheet. You do need to know how to use it.
- **Sequences:**
  - Know how to find the general term of a sequence
  - Know how to compute limits of sequences using standard limit rules and using **L'Hôpital's Rule**
  - Be sure to know when limits **DNE** (because they equal $+\infty$, $-\infty$, or due to "oscillation")
  - Be able to show that sequence is **increasing** or **decreasing** either using inequalities or by proving a certain function is increasing/decreasing.
  - Know the statement of the **Monotone Convergence Theorem**