MATH 5B – Calculus II — Spring 2019

Exam 1

Study Guide — **Dr. Jorge Basilio**

Chapter 6: §6.1–§6.7 — Pasadena City College — **gbasilio@pasadena.edu**

General Exam Info

Exams are a way for you to show me what you have learned (and please show all your steps so I can see this!) and to give you a sense of accomplishment! They are meant to be challenging and not just homework problems with the numbers changed. I really want to prepare you for university level math classes–so some exams may be longer or more challenging than others. Remember that I do grade fairly and my goal is to push you to succeed and excel in this class.

- Attendance required for all exams–I do NOT drop the lowest exam score.
- **"Make-up Exams"** are given only in extreme cases and at instructor's discretion; a student is allowed at most one make-up exam. (Documentation must be provided for the instructor to even consider a make up exam. This means you would need a doctor's note, etc.) A "Make-Up Exam" means you will be allowed to replace the missing score with the percentage you earn on the final exam. Please contact your instructor as soon as possible should there be a problem.
- **Your student ID is required for all exams.**
- During the exams–you will be required to leave your backpack and all non-test items at the front of the room, including cell phones and smart watches. Only your pencil/eraser and calculator will be allowed during the exam, and there will be a calculator check. Should you need to leave during the exam please ask for permission first before leaving and leave your cell phone with me. Not doing these things could result in a 0 on your exam.
- Once the exam is graded and returned, any problem you would like me to revisit must be brought to my attention by the next class session.
- Always keep your exams!

Exam 1 Date & Time		
• Exam 1	Tuesday, March 12	4:00 pm – 5:20 pm

ALSO, after the exam:

- As the test is only 80 minutes long, we will have a 10 minutes break, then continue with new material.
- I take attendance at the end of class on test days.

Rules for an acceptable Cheat Sheet

Please read the document "GoodvsBad" Cheat Sheets which is linked on our webpage. It was written for a different school but the info obviously is school-independent.

The key to making a Good Cheat Sheet (GCS) is that it has to be done by hand and well-organized. Printing one out from the internet will not be allowed and defeats the purpose of a GCS.

I will collect your cheat sheets before the exam begins and you will start the exam promptly. While you are taking the exam I will check them individually to see if they satisfy my requirements for a GCS. If they pass, it will be returned to you and you can use it for the remained of the text (approximately 15-20 minutes after the exam begins). However, if it's a bad cheat sheet then I will not return it and you will not be able to use it on the exam.

Please follow the following rules/requirements:

- Must be hand-written & NEAT & written by YOU!
- SIZE: using a regular 8.5" x 11" piece of white paper (no other paper allowed, no lined paper, graph paper, etc)
- CHEAT SHEET NOTES ARE ONLY ALLOWED ON **ONE SIDE of the page**
- The other side of the page must include: Your name, Course, Instructor, Date **and nothing else**
- You may include an unlimited number of definitions, theorems, graphs of standard functions, and formulas
- You are allowed to have **AT MOST TWO** example problems with complete solutions. **These must be clearly indicated with a box around them so that I can find them and check that no more than two worked out problems are on your cheat sheet.**
  - If it takes me more than 5 seconds to find these or if you try to sneak more worked out problems outside of the two clearly labeled, then your cheat sheet will not be returned to you.
- Must follow the principles outlined in the GCSvsBCS document discussed in class and available on my website.
  - YES, this is completely subjective and up to my discretion
  - If I don't think you followed the spirit of the document or if you failed to follow any of the above directions then I reserve the right to not allow you to use your cheat sheet
- ***BONUS*** If your cheat sheet is beautiful & complete (meaning it has the main ideas from all the material we covered) you can earn 1% extra credit on your exam.
- If you have any questions, you can email me.

Exam 1 Specific Info

- Material covered: §6.1, 6.2, 6.3, 6.4, 6.6, 6.7, 6.5
- You will need a calculator (only scientific allowed)–you won't be able to use your phone.
- Almost all questions have multiple parts
- Be prepared for the following types of questions: True-False-Sometimes, Multiple-Choice, and Free-response
- Be prepared to prove certain results (see below for more details)
- I will allow a cheat sheet for this exam but only if they meet certain conditions. Read carefully below.

- Section 6.4 is an important section with many types of problems
- Be prepared for questions with multiple parts that build on each other

Section 6.1

- Know the definitions of **inverse functions**, **one-to-one**
- Know the Properties of Inverse Functions (Theorem 1 in §6.1 from my slides)
- You should be able to sketch the graph of an inverse using the symmetry property
- Know that the inverse f^{-1} exists if f is one-to-one
- Know how to prove that the inverse exists using the ID test (Theorem 4 in §6.1 of my slides) of Calc 1
- Know the formula for the **derivative of the inverse** based on the derivative of f (Theorem 3 in §6.1 from my slides)
- The proof of Theorem 3 might be on the test

Section 6.2

- Know the definitions of **exponential functions base** b, $f(x) = b^x$
- Know the Properties of Exponential Functions (Theorem 1 in §6.2 from my slides). Pay attention to: Domain, range, shape of the graphs for the two cases $b > 1$ and $0 < b < 1$
- Know that $b^{-x} = (1/b)^x$ are the same and are decreasing
- You should be able to sketch the graph of an exponential function using the five points at $x = -2, -1, 0, 1, 2$
- Be able to compute limits of exponential functions, especially as $x \to \pm\infty$
- Know the "exponent laws"

- Know the **"official definition of** e**"** and the basic graph of the **natural exponential function**, e^x
- Know the DR and ADR for e^x
- Be able to compute derivatives of functions in the form e^u via the **chain rule** with $u(x)$ built from other known functions
- Be able to compute anti-derivatives of functions in the form e^u via **u-substitution** with $u(x)$ built from other known functions

Section 6.3

- Know the definitions of **logarithmic functions base** b, $g(x) = \log_b(x)$
- Know that they are the inverse functions of the exponential functions, so $f^{-1}(x) = g(x)$ with $f(x) = b^x$
- Know the Properties of Logarithmic Functions (Theorem 1 in §6.3 from my slides). Pay attention to: Domain, range, shape of the graphs for the two cases $b > 1$ and $0 < b < 1$
- You should be able to sketch the graph of a logarithmic function using the five points at $x = -2, -1, 0, 1, 2$ and the graph of it's corresponding exponential function
- Be able to compute limits of log functions, especially as $x \to \pm\infty$
- Know the "Properties of Logs" (Theorem 2 in §6.3 from my slides)
- Know the "Inverse Properties" (Theorem 3 in §6.3 from my slides)
- Know the **"special bases"** $b = 10$ and $b = e$. Especially, $\ln(x)$ the **natural logarithm function** and $\log(x)$ the **common logarithm**
- Know the change of base formula
- Be able to: solve log and exponential equations giving both exact (e.g. $x = \ln(5)$ or $x = e^5$) and approximate answers (using your calculator)

Section 6.4

- Know the derivative rule for $\ln(x)$ (Theorem 1 in §6.4 from my slides)
- The proof of Theorem 1 might be on the test
- Be able to compute derivatives of functions $\ln(u)$ with the chain rule where $u(x)$ are other known functions. NOTE: using the log properties to simplify first is very helpful!
- Know the anti-derivative rule for $\frac{1}{x}$–be careful and notice that we need absolute values! (Theorem 2 in §6.4 from my slides)
- Be able to compute anti-derivatives and definite integrals of functions of the form $1/u$ via **u-substitution**
- Know the DR and ADRs for exponential and logarithms with a general base b (Theorem 3 in §6.4 of my slides)
- The proofs of DR1 and DR2 of Theorem 3 might be on the test
- **(Optional)** Be able to compute derivatives of complicated functions with **logarithmic differentiation**. Since we didn't cover this, I won't hold you responsible for it for Exam 1. However, it might show up as a extra-credit.

Section 6.6

- Know the definitions of the **inverse trig functions:** $\sin^{-1}(x) = \arcsin(x)$, $\cos^{-1}(x) = \arccos(x)$, $\tan^{-1}(x) = \arctan(x)$, $\csc^{-1}(x) = \operatorname{arccsc}(x)$, $\sec^{-1}(x) = \operatorname{arcsec}(x)$, $\cot^{-1}(x) = \operatorname{arccot}(x)$
- Note carefully the domain and range of each of the inverse trig functions
- Know the graphs of $\sin^{-1}(x)$, $\cos^{-1}(x)$, and $\tan^{-1}(x)$
- Be able to evaluate inverse trig functions at specific values. Know how to "draw the triangle" to help you solve these types of problems
- Know the derivative rules for **all six inverse trigonometric functions** (Theorem 1/2 of §6.6 of my slides)
- The proof of DRs 1,2, or 3 might be on the exam (i.e. $\sin^{-1}(x) = \arcsin(x)$, $\cos^{-1}(x) = \arccos(x)$, or $\tan^{-1}(x) = \arctan(x)$)
- Be able to compute derivatives using the DRs for the inverse trig functions and the chain rule
- Be able to evaluate integrals using the ADRs for the inverse trig functions $\sin^{-1}(x)$ and $\tan^{-1}(x)$.

Section 6.7

- Know the definitions of the **hyperbolic (trig) functions:**
  $\sinh(x)$, $\cosh(x)$, $\tanh(x)$, $\operatorname{csch}(x)$, $\operatorname{sech}(x)$, $\coth(x)$
- Know the graphs of $\sinh(x)$, $\cosh(x)$, and $\tanh(x)$ and be able to compute limits based on these graphs
- Know the properties of hyperbolic functions (Theorem 1 in §6.7 of my slides). Especially
  $\boxed{\cosh^2(x) - \sinh^2(x) = 1}$
- Know all six DRs for the hyperbolic trig functions (Theorem 2 in §6.7 of my slides). Pay attention to the signs.
- The proofs of DRs 1,2 or 3 might be on the exam
- Be able to compute derivatives involving the hyperbolic functions
- Know all six ADRs for the hyperbolic trig functions (Theorem 3 in §6.7 of my slides)
- Be able to compute integrals involving the hyperbolic functions
- Know the definition of the **inverse hyperbolic functions**:
  $\sinh^{-1}(x)$, $\cosh^{-1}(x)$, and $\tanh^{-1}(x)$
- Know the graphs of $\sinh^{-1}(x)$, $\cosh^{-1}(x)$, and $\tanh^{-1}(x)$ and be able to compute limits based on their graphs
- Know the formulas for $\sinh^{-1}(x)$, $\cosh^{-1}(x)$, and $\tanh^{-1}(x)$ found in Theorem 4
- Know the DRs and ADRs for the inverse hyperbolic functions (Theorem 5 in §6.7)
- Be able to compute derivative and intergrals involving the inverse hyperbolic functions

Section 6.5

- Know the **Law of Exponential Growth and Decay**
- Know that the phrase "rate of growth/decay of population is proportional to population" translates to the DE "$\frac{dP}{dt} = kP$"
- Be able to recognize when a problem is governed by the law of exponential growth/decay
- Know the general solutions to the law of exponential growth/decay (Theorem 1 in §6.5 in my slides)
- Be able to use Theorem 1 to solve application problems for growth/decay
- Know what a **differential equation** is and the difference between **general solutions** and a **particular solutions**
- Know the definitions of **doubling-time** and **half-life**.
- Be able to find the doubling-time and half-life to problems
- Know what **Newton's Law of Cooling** is and what the general solutions look like.
- Be able to solve cooling/heating problems using Newton's Law of Cooling
- Know what **continuous compound interest** is and the equation $A = Pe^{rt}$.
- Be able to solve problems involving continuous compounding interest