CI
HT

Chapter 11: Inferences on Two Samples

Section 11.1: Inference about Two Population Proportions

Poll Q : variable

Introduction Scenario:

p_1 p_2

Think about whether you plan to vote in the next presidential election in 2020. Let's say I split the responses into two groups: Females and Males

Notice: There are **TWO** proportions here: $p_1 = \frac{8}{11} = 0.727$

$p_2 = \frac{2}{3} = 0.667$

QUESTION: Is there a statistically significant difference?

We don't care what the proportions actually are, we care about whether they're the SAME **or not.**

** In this section (11.1), we will only deal with independent samples with Qualitative (QL) variables.

Independent vs Dependent (Matched-Pairs) Samples

sampling method is **independent** when an individual selected for one sample DOESN'T dictate which individual is to be in a second sample. ↳ M vs F   USA vs Canada

A sampling method is **dependent** when an individual selected to be in one sample is used to determine the individual in the second sample. Dependent samples are often referred to as **matched-pairs** samples. It is possible for an individual to be matched against him- or herself. same person → Height, weight   Married Couples

Ex 1: Independent vs Dependent samples

Scenario #1: Among competing acne medications, does one perform better than the other? To answer this question, researchers applied Medication A to one part of the subject's face and Medication B to a different part of the subject's face to determine the proportion of subjects whose acne cleared up for each medication. The part of the face that received Medication A was randomly determined. Matched Pair Sample → Dependent

Scenario #2: A researcher wishes to determine the effects of alcohol on people's reaction time to a stimulus. She randomly divides 100 people aged 21 or older into two groups. Group 1 is asked to drink 3 ounces of alcohol, while group 2 drinks a placebo. Both drinks taste the same, so the individuals in the study do not know which group they belong to. Thirty minutes after consuming the drink, the subjects in each group perform a series of tests meant to measure reaction time. Independent Samples

The Logic

If $p_1 = p_2$ then $p_1 - p_2 =$ 0.

So we will first collect simple random samples from each group and find $\hat{p}_1$ and $\hat{p}_2$

Then we will see if $\hat{p}_1 - \hat{p}_2$ is anywhere close to 0.

Recall the Logic: Say $H_0: p_1 = p_2$ and $H_A: p_1 < p_2$. If we get… "either by a little or a lot"

- $\hat{p}_1 = \hat{p}_2$ Fail to Reject H_0
- $\hat{p}_1 > \hat{p}_2$ Fail to Reject H_0
- $\hat{p}_1 < \hat{p}_2$ by "a little" Fail to Reject H_0
- $\hat{p}_1 < \hat{p}_2$ by "a lot" then we Reject H_0

$\hat{q} = 1 - \hat{p}$

Hypothesis Test Regarding the Difference between Two Proportions p_1 and p_2 (Indep)

Step 0: *Check Requirements*

- The samples are simple random and independent
- At least five success $\boxed{n\hat{p}_i \geq 5}$ and five failures $\boxed{n\hat{q}_i \geq 5}$ for each sample. $n\hat{p}_1 \geq 5$ & $n\hat{q}_1 \geq 5$; $n\hat{p}_2 \geq 5$ & $n\hat{q}_2 \geq 5$

Step 1: *State Hypotheses*

	Group 1 → Group 2	
$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 < p_2 \end{cases}$	$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 \neq p_2 \end{cases}$	$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 > p_2 \end{cases}$

Step 2: *Level of Significance* α

If it's not given, then use <u>0.05</u>. Choice depends on seriousness of making Type <u>I</u> error.

Step 3: *Test Statistic*

compare w/ one prop: $z^* = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}$

$$\boxed{z^* = \frac{(\hat{p}_1 - \hat{p}_2) - \overbrace{(p_1 - p_2)}^{0 \text{ by } H_0}}{\sqrt{\dfrac{\bar{p}\,\bar{q}}{n_1} + \dfrac{\bar{p}\,\bar{q}}{n_2}}}}$$

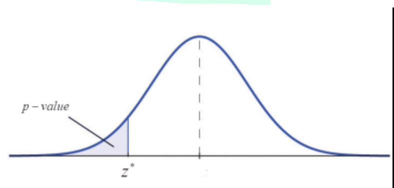
where the **point estimate** is $\boxed{\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}}$ "total proportion"

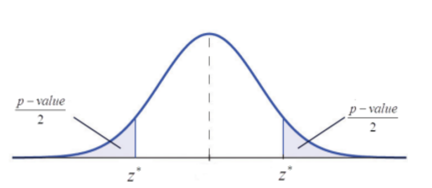
$\bar{q} = 1 - \bar{p}$

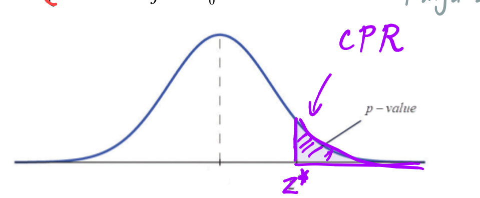
Step 4: *Find a Critical Value or P-Value*

P-VALUE METHOD | **DECISION**

Decision $\begin{cases} \textit{Reject } H_0 \text{ if } P\text{-value} \leq \alpha & P \text{ is low} \to \text{Null go} \\ \textit{Fail to Reject } H_0 \text{ if } P\text{-value} > \alpha & P \text{ High} \to \text{Null fly} \end{cases}$

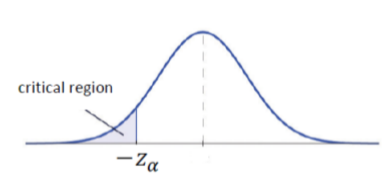


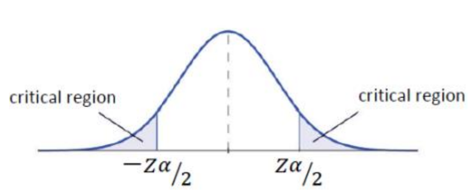


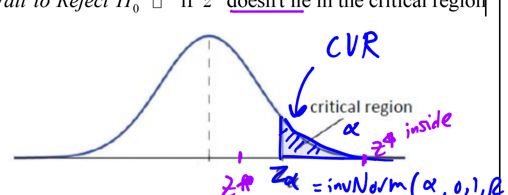


CRITICAL VALUE METHOD | **DECISION**

Decision $\begin{cases} \textit{Reject } H_0 \text{ if } z^* \text{ lies in the critical region} \\ \textit{Fail to Reject } H_0 \text{ if } z^* \text{ doesn't lie in the critical region} \end{cases}$







Step 5: Make a DECISION and write a *CONCLUSION* either rejecting or failing to reject H_0

GRAPHING CALCULATOR (TI-83 OR 84)

Instructions: (a) STAT ⇒ TESTS ⇒ $\boxed{\text{2-PropZTest}}$

(b) Enter $\begin{cases} x_1 \,/\, x_2 = \text{number of success in sample \#1 / \#2} \\ n_1 \,/\, n_2 = \text{size of sample \#1 / \#2} \\ p_1 \text{ alternative hypothesis} \end{cases}$

x_2 $\quad$ x_1 $\quad$ α $\quad$ n_2 $\quad$ n_1

Ex 2: An insurance company is concerned that men are more likely to speed than women. In a sample of 500 randomly selected women, 27 have been ticketed for speeding in the last year. In a sample of 250 randomly selected men, 26 have been ticketed for speeding in the last year. Use a 0.05 significance level to test the insurance company's claim that the percentage of women ticketed for speeding is less than the percentage of men. Use the P-Value Method.

Group 2 $\quad$ Group 1

Check requirements

① SRS? ✓

② Independent? ✓

③ Success & Failures

$n_1\hat{p}_1 = 500\left(\frac{27}{500}\right) = x_1 = 27 \geq 5$ ✓ $\quad$ $n_2\hat{p}_2 = x_2 = 26 \geq 5$ ✓

$n_1\hat{q}_1 = 500\left(\frac{473}{500}\right) = 473 \geq 5$ ✓ $\quad$ $n_2\hat{p}_2 = 250 - x_2$ ✓

Null and Alternative Hypothesis

$$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 < p_2 \end{cases}$$

Group 1: $x_1 = 27$, $n_1 = 500$, $\hat{p}_1 = \frac{27}{500} = 0.054$

Group 2: $x_2 = 26$, $n_2 = 250$, $\hat{p}_2 = \frac{26}{250} = 0.104$

$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$

$\bar{p} = \frac{53}{750} = 0.071$

$\bar{q} = 1 - \bar{p}$

$\bar{q} = 0.929$

Test Statistic

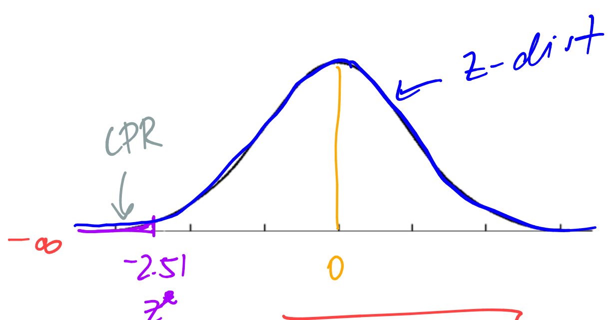
$$z^* = \frac{(\hat{p}_1 - \hat{p}_2) - (\underbrace{p_1 - p_2}_{=0 \text{ by } H_0})}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.054 - 0.104) - 0}{\sqrt{(0.071)(0.929)\left(\frac{1}{500} + \frac{1}{250}\right)}} = \boxed{z^* = -2.51}$$

P-value/Critical Region

$P\text{-Val} = P(\text{CPR})$

$= \text{normalcdf}(-1E99, -2.51, 0, 1)$

$\boxed{P = 0.00604}$



Decision about Null Hypothesis

$\alpha = 0.05$

$P = 0.00604$

$P < \alpha \Rightarrow$ P low, Null go! $\Rightarrow \boxed{\text{Reject } H_0}$

Conclusion

"There is enough statistical evidence to support the claim that the percentage of women ticketed is less than the percentage of men ticketed."

Ex 3: In clinical trials of the anti-inflammatory drug Inflaminex, adult and adolescent allergy patients were randomly divided into two groups. Some patients received 500mcg of Inflaminex, while some patients received a placebo. Of the 2103 patients who received Inflaminex, 520 reported bloody noses as a side effect. Of the 1671 patients who received the placebo, 368 reported bloody noses as a side effect. Is there significant evidence to conclude that the proportion of Inflaminex users who experienced bloody noses as a side effect is greater than the proportion of the placebo group at the $\boxed{\alpha = 0.01}$ level of significance? Use the Critical Value Method.

n_1 x_1 n_2 x_2

claim 2prop

Check requirements

① SRS ✓

② Independent? ✓ Drug vs control

③ $n_1\hat{p}_1 \geq 5$ 520 ✓ $n_2\hat{p}_2 = x_2 = 368 \geq 5$ ✓

$n_1\hat{q}_1 \geq 5$ $n_2\hat{q}_2 \geq 5$ ✓

Group 2: $n_2 = 1671$, $x_2 = 368$, $\hat{p}_2 = \frac{368}{1671} = 0.220$

p_2 = prop of Group 2 bloody noses

Group 1: $n_1 = 2103$, $x_1 = 520$, $\hat{p}_1 = \frac{520}{2103} = 0.247$

p_1 = proportion of Group 1 w/ bloody noses

Null and Alternative Hypothesis

$$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 > p_2 \end{cases}$$ (Right Tailed Test)

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{520 + 368}{2103 + 1671} = 0.235$$

$$\bar{q} = 0.765$$

Test Statistic

$$z^* = \frac{(\hat{p}_1 - \hat{p}_2) - \overbrace{(p_1 - p_2)}^{0 \leftarrow H_0}}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.247 - 0.220) - 0}{\sqrt{0.235 * 0.765\left(\frac{1}{2103} + \frac{1}{1671}\right)}} = 1.94$$

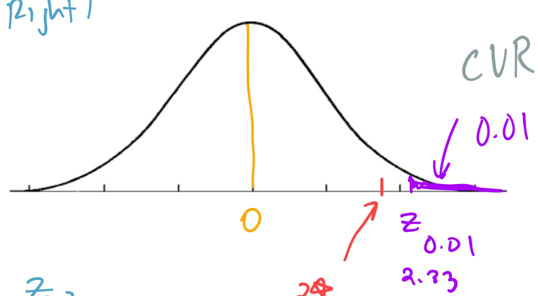
$\boxed{z^* = 1.94}$ Test Statistic

P-value/Critical Region

Critical Value $z_\alpha = z_{0.01} = \text{invNorm}(0.01, 0, 1, \text{Right})$

$\alpha = 0.01$

$z_{0.01} = 2.33$



Decision about Null Hypothesis

$z^* = 1.94$ is OUTSIDE CVR b/c $z^* < z_\alpha$

$1.94 < 2.33$

so we $\boxed{\text{Fail to Reject } H_0}$

Conclusion

"There is not enough statistical evidence to support the claim that the proportion of Inflaminex users who experience bloody noses is greater than the proportion of the placebo group at a level of significance of $\alpha = 0.01$."

p_1 smaller p_2 $\quad$ p_1 is bigger p_2

p_1-p_2 $\quad$ p_1-p_2

$-\#$ $\quad$ 0 $\quad$ $\#$

• $p_1-p_2>0$

• $p_1>p_2$

parameter: difference of (p-p) proportions (p_1-p_2)

CONFIDENCE INTERVAL FOR THE DIFFERENCE OF TWO POPULATION PROPORTIONS

Alternative Forms: $\quad (\hat{p}_1-\hat{p}_2)-E < p_1-p_2 < (\hat{p}_1-\hat{p}_2)+E \quad$ or $\quad (\hat{p}_1-\hat{p}_2) \pm E$

where the margin of error is given by $\boxed{E = z_{\alpha/2}\cdot\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1}+\dfrac{\hat{p}_2\hat{q}_2}{n_2}}}$

compare w/ one prop CI:

$E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

point estimate: $\hat{p}_1-\hat{p}_2$

GRAPHING CALCULATOR (TI-83 OR 84)

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ $\boxed{\text{2-PropZInt}}$

(b) Enter
$$\begin{cases} x_1 \;/\; x_2 = \text{number of successes in sample \#1 / \#2} \\ n_1 \;/\; n_2 = \text{size of sample \#1 / \#2} \\ C-level = \text{confidence level} \end{cases}$$

Ex 4: A study was conducted to test the effectiveness of a sweetener called xylitol in preventing ear infections in preschool children. In a randomized experiment, 159 preschool children took five daily doses of xylitol, and 46 of these children got an ear infection during the three months of the study. Meanwhile, 165 children took five daily doses of placebo syrup, and 68 of these children got an ear infection during the study. Construct a 90% confidence interval for the difference in the proportion of children that got ear infections for the control group and the xylitol group.

x_2 $\quad$ n_2 $\quad$ n_1 $\quad$ x_1

CI

parameter p_1-p_2

p_1: control group

p_2: Xylitol group

p_1

Find the point estimate (difference between sample proportions)

$\hat{p}_1-\hat{p}_2 = \dfrac{68}{165}-\dfrac{46}{159} = 0.123$ $\quad$ $\boxed{\text{point est: } 0.123}$

$\hat{p}_1 = \dfrac{68}{165}$ $\quad$ $\hat{p}_2 = \dfrac{46}{159}$

$\hat{p}_1 = 0.412$ $\quad$ $\hat{p}_2 = 0.289$

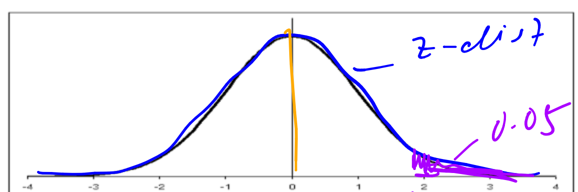
Determine critical value $z_{\alpha/2}$

$CL = 0.9$

$\alpha = 1-CL = 0.1$

$\alpha/2 = 0.05$

$\boxed{z_{0.05} = 1.64}$



$z_{\alpha/2} = z_{0.05} = \text{invNorm}(0.05, 0, 1, R)$

Find margin of error

$E = z_{\alpha/2}\sqrt{\dfrac{\hat{p}_1\hat{q}_1}{n_1}+\dfrac{\hat{p}_2\hat{q}_2}{n_2}} = 1.64\sqrt{\dfrac{0.412(1-0.412)}{165}+\dfrac{0.289(1-0.289)}{159}} = 0.0862$

$\boxed{E = 0.0862}$

Construct confidence interval

point est $\pm E$

0.123 ± 0.0862

~4% $\quad$ ~21%

$\boxed{CI: (0.037, 0.209)}$ $\leftarrow$ estimating p_1-p_2

Does it appear that the sweetener is effective at reducing ear infections?

want p_2 smaller than p_1

pop parameter: $p_1-p_2>0$ if Xylitol reduces ear infections.

$p_1-p_2>0$

"We are 90% confident, that yes it is effective."