MATH 5B - Single Variable Calculus II — Spring 2019

§6.1 Inverse Functions

In-class Activity 6.1

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Activity 1:

For the following functions: show that $f(x)$ is one-to-one (hint: use the equivalent version). After this, find a formula for f^{-1} and determine the domain and range.

(a) $f(x) = x^3 + 5$

(b) $f(x) = \dfrac{1}{x+2}$

(c) $f(x) = \dfrac{x+3}{x-4}$

(d) $f(x) = \dfrac{2x-5}{3x+7}$

(e) $f(x) = \sqrt{3x-8}$

Activity 2:

Consider the function $f(x) = 3 - \sqrt{7 - 2x}$

(a) Sketch the graph and explain why its one-to-one.

(b) Use your graph to find the domain and the range of $f(x)$.

(c) Find a formula for $f^{-1}(x)$ and state its domain and range.

(d) Sketch the graph of $f^{-1}(x)$ along with the graph of $f(x)$.

Activity 3:

Consider the function $f(x) = 2x^2 - 12x + 23$.

(a) Sketch the graph and explain why its not one-to-one.

(b) Find the smallest possible value for a such that $f(x)$ is one-to-one on $[a, \infty)$.

(c) Sketch the graph of f on this restricted domain.

(d) Find a formula for $f^{-1}(x)$ and state its domain and range.

(e) Sketch the graph of $f^{-1}(x)$ along with the graph of $f(x)$.

Activity 4:

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(a) If $f(0) = 4$ and $f'(0) = -2$, find $(f^{-1})'(4)$

(b) Given that $f(x) = \sqrt[3]{x} + 8$, compute: $(f^{-1})'(5)$

Activity 5:

Let's use the derivative formula for the inverse to find the derivatives of the inverse functions from Activity 1. Find $(f^{-1})'(x)$:

(a) $f(x) = x^3 + 5$

(b) $f(x) = \dfrac{1}{x+2}$

(c) $f(x) = \dfrac{x+3}{x-4}$

(d) $f(x) = \dfrac{2x-5}{3x+7}$

(e) $f(x) = \sqrt{3x-8}$

Activity 6:

Consider the function $f(x) = x^3 + 5x - 3$.

(a) Use the ID Test to prove that $f(x)$ is one-to-one on its entire domain.

(b) By virtue of (a), we can construct the inverse function $f^{-1}(x)$. Without explicitly finding a formula for $f^{-1}(x)$, find the values of $f^{-1}(-9)$ and $f^{-1}(15)$. *(Hint: use rational roots theorem)*

(c) Use your answers in (b) and the derivative formula for $f^{-1}(x)$ to find the values of $(f^{-1})'(-9)$ and $(f^{-1})'(15)$.

Activity 7:

Consider the function $f(x) = 2\cos(x) - 5x$.

(a) Use the ID Test to prove that $f(x)$ is one-to-one on its entire domain.

(b) By virtue of (a), we can construct the inverse function $f^{-1}(x)$. Without explicitly finding a formula for $f^{-1}(x)$, find the values of $f^{-1}(5\pi/2)$ and $f^{-1}(-15\pi/2)$. *(Hint: try $x = \frac{\pi}{2}k$ and look for k)*

(c) Use your answers in (b) and the derivative formula for $f^{-1}(x)$ to find the values of $(f^{-1})'(5\pi/2)$ and $(f^{-1})'(-15\pi/2)$.