Section 10.5 Systems of Inequalities

Objectives
- Graphing an Inequality
- Systems of Inequalities
- Systems of Linear Inequalities
- Feasible Regions

$2<5$ or $2=5$

• Graphing an Inequality

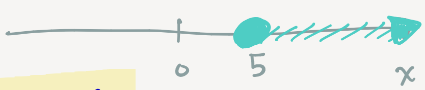
Warm-up True or False: If $x < 5$ then $x \le 5$. if $x=2$. Then $2<5$ so $2 \le 5$. :)

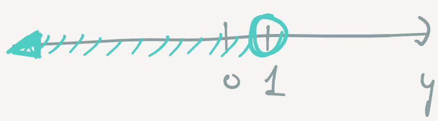
Ex 1 Inequalities in one variable Describe the solution sets both in "set notation" and "by graphing":
(a) $x \ge 5$
(b) $y < 1$
(c) $x^2 - 1 > 0$

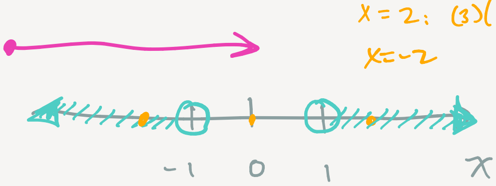
c) $(x+1)(x-1) > 0$

test value
$x=0$: $-1 \cancel{>0}$?
$x=2$: $(3)(1)>0$ ✓
$x=-2$

$y = x^2-1 \rightarrow y>0$







interval notation

$[5, \infty)$ $(-\infty, 1)$ $(-\infty,-1) \cup (1,\infty)$

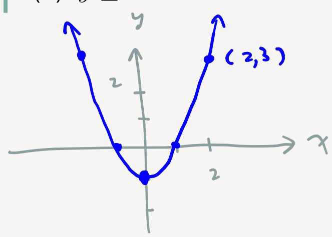
Defn 1
The **solution set** to an inequality in two variables is the set of all points (x, y) which make the inequality true.
By **boundary curve** we mean the graph of the corresponding **equation**.

Ex 2 Inequalities in two variables Sketch the solution set and boundary curves.
(a) $y = x^2 - 1$
(b) $y > x^2 - 1$
(c) $y \ge x^2 - 1$ "or ="
(d) $y < x^2 - 1$
(e) $y \le x^2 - 1$

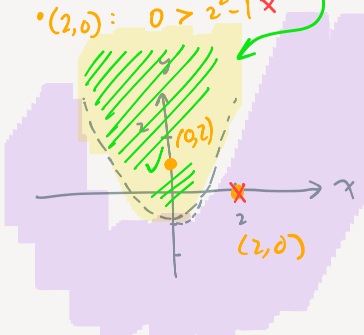
Test Points
• $(0,2)$: $2 > 0^2-1$ ✓
• $(2,0)$: $0 > 2^2-1$ ×

solution set

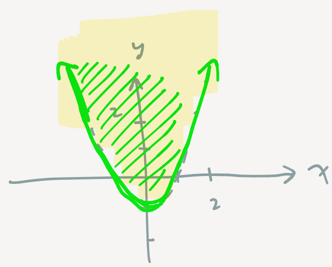
all are Unbounded!



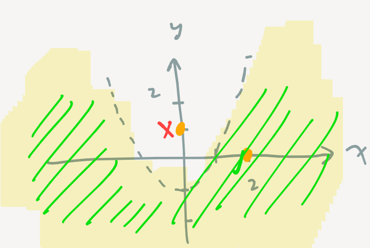
$y = x^2-1$



$y > x^2-1$

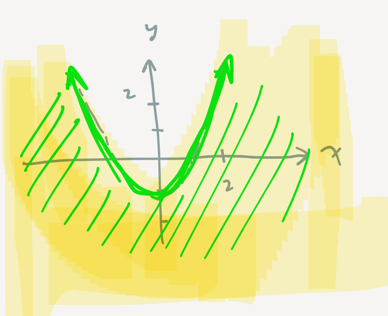


$y \ge x^2-1$



$y < x^2-1$

• $(0,2)$: $2 < 0^2-1$ ×
• $(2,0)$: $0 < 2^2-1$ ✓

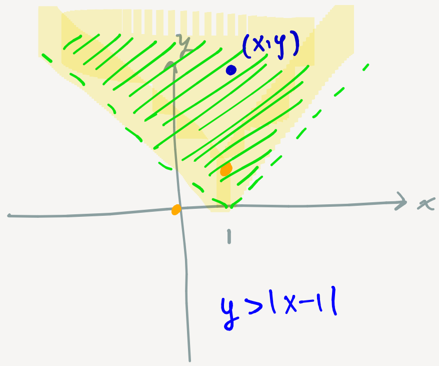


How to graph an inequality To graph an inequality of two variables:

1. **Graph the boundary curve** corresponding to the equation.
   Use a solid line if $\leq$ or $\geq$; otherwise, use a dashed line.
2. **Shade the correct side** of the boundary curve.
   Use the "$y >$" or "$y <$" trick; or
   Use test points to help you determine which side to shade.

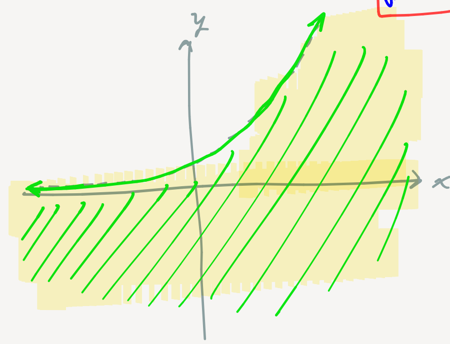
Ex 3 Inequalities in two variables Graph the solution set and boundary curves.

(a) $y > |x - 1|$



$y = |x|$ $\quad y = |x-1|$ HS → 1

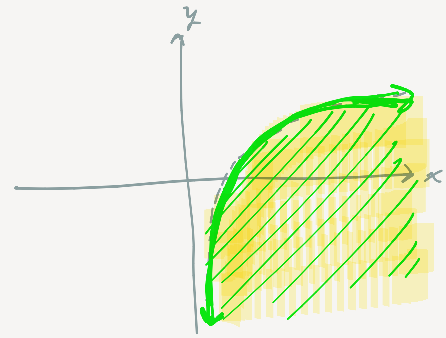
(b) $e^x - y \geq 0$ → $e^x \geq y$ → $\boxed{y \leq e^x}$



solution set

$\boxed{\{(x,y) \mid e^x - y \geq 0\}}$

(c) $y - \ln(x) \leq 0$ → $y \leq \ln(x)$



all 3 are unbounded!

• Systems of Inequalities

Systems of Inequalities combine the ideas of systems of equations and also inequalities.

Defn 2

The **solution set** to a system of inequalities in two variables is the set of all points (x, y) which make every inequality true.
Vertices are the points in the plane where two (or more) inequalities meet.

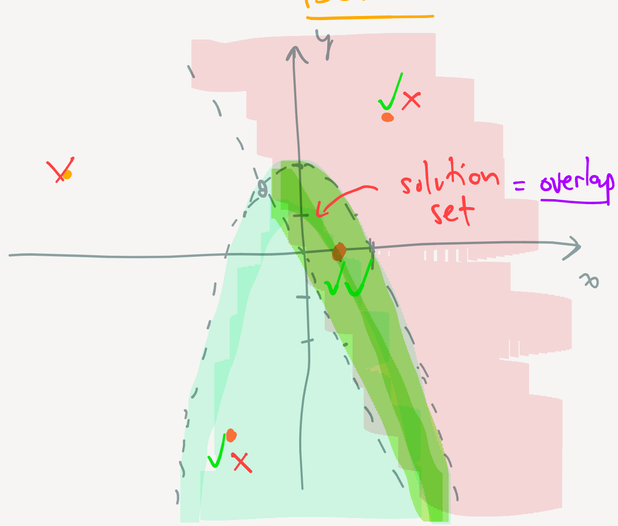
Key Points

- the solution set is the overlap or intersection of the shaded regions for each individual inequality.
- Vertices correspond to the solutions of pairs of the corresponding equations.
- Vertices may or may not be part of the solution set.
  - If vertices are part of the solution set, draw them as solid points.
  - If vertices are NOT part of the solution set, draw them as open circles.

Ex 4 System of Two Inequalities Graph the solution set and label any vertices.

(a) $\begin{cases} y \leq -x^2 + 2 \\ y \geq -2x \end{cases}$

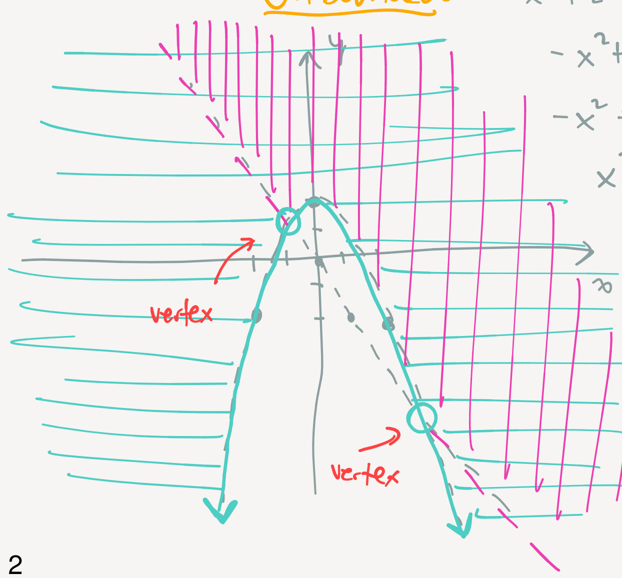
Bounded



(b) $\begin{cases} y \geq -x^2 + 2 \\ y > -2x \end{cases}$

vertices $\begin{cases} y = -x^2 + 2 \\ y = -2x \end{cases}$

Unbounded



$-x^2 + 2 = y = -2x$

$-x^2 + 2 = -2x$

$-x^2 + 2x + 2 = 0$

$x^2 - 2x - 2 = 0$

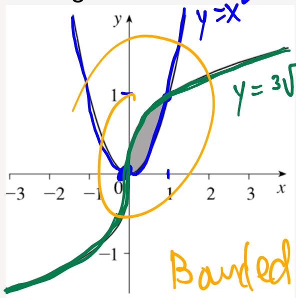
$x = \frac{2 \pm \sqrt{4 - 4(-2)}}{2}$

$= \frac{2 \pm \sqrt{12}}{2}$

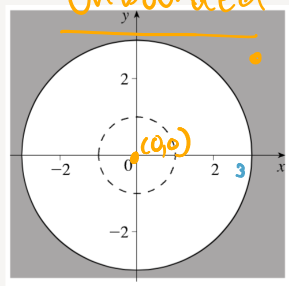
$= 1 \pm \sqrt{3}$

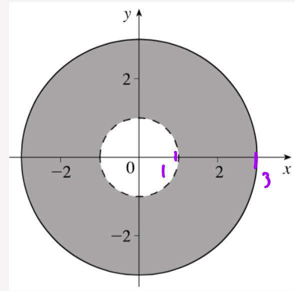
$y = -2x$

Ex 5 "Reverse" System of Two Inequalities Write an inequality or a system of inequalities that describe the following shaded regions.

(a) 

Unbounded

(b) 

(c) 

$$\begin{cases} y \geq x^2 \\ y \leq \sqrt[3]{x} \end{cases}$$

$x^2+y^2=r^2$

$x^2+y^2=9$

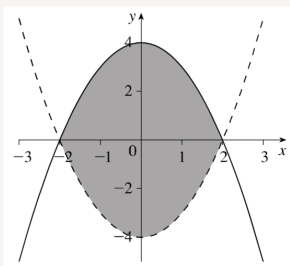
$\cancel{x^2+y^2\leq 9}$ or $\boxed{x^2+y^2\geq 9}$

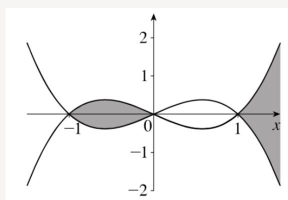
$\checkmark\ \cancel{0^2+0^2\leq 9}$

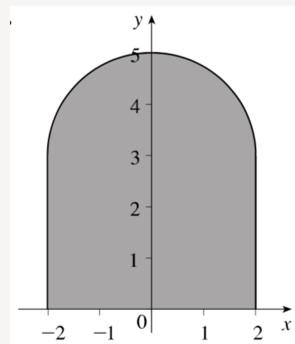
~~inside~~ outside

$$\begin{cases} x^2+y^2>1 \\ x^2+y^2\leq 9 \end{cases}$$

Bounded

(d) 

(e) 

(f) 

GTQ!

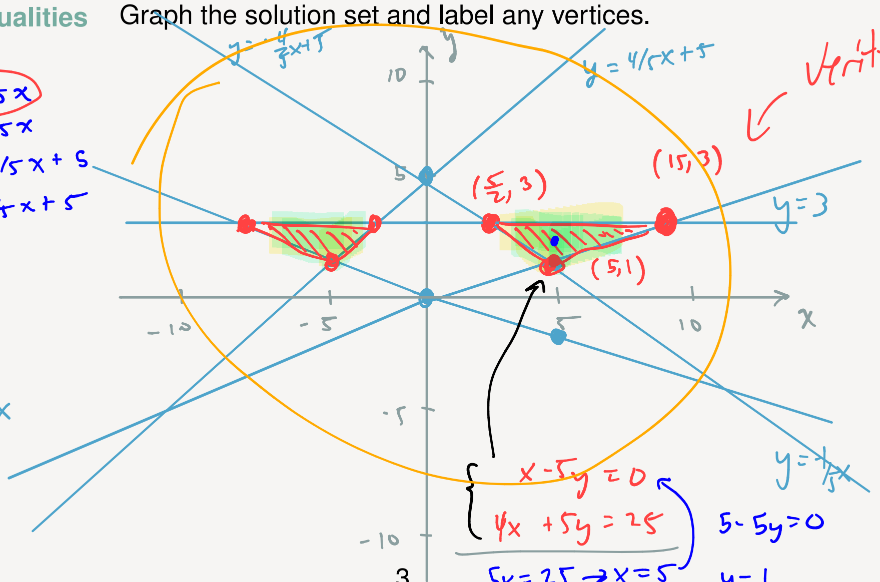
• Systems of Linear Inequalities

$y=mx+b$ or $ax+by=c$
pick $x=0$ or $y=0$

Because lines are simple, we can solve systems of linear inequalities with any number of inequalities.

Ex 6 System of Linear Inequalities Graph the solution set and label any vertices.

$$\begin{cases} y-3 & \leq 0 \rightarrow y\leq 3 \\ x-5y & \leq 0 \rightarrow y\geq 1/5\,x \\ x+5y & >0 \rightarrow y> -1/5\,x \\ 4x+5y & \geq 25 \rightarrow y\geq -4/5\,x+5 \\ 4x-5y & \leq -25 \rightarrow y\geq 4/5\,x+5 \end{cases}$$



Bounded!

$y=3 \rightarrow y=1/5\,x$

$3=1/5\,x$

$15=x$

$y=3 \rightarrow 4x+5(3)=25$

$4x=10$

$x=10/4=5/2$

$$\begin{cases} x-5y=0 \\ 4x+5y=25 \end{cases}$$

$5x=25 \rightarrow x=5$

$5-5y=0$

$y=1$

• Feasible Regions

$0 \le x \le 10$

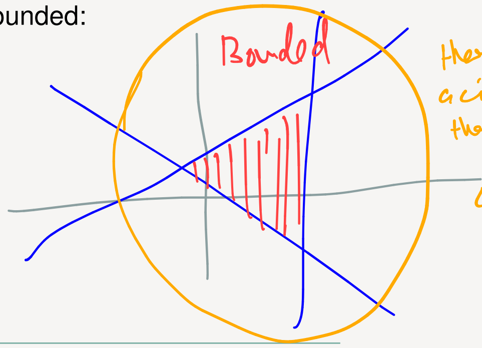
Defn 3

Constraints: Restrictions to the values a variable can take. These are usually expressed as inequalities.

Feasibility region: The solution set to a system of inequalities corresponding to constraints imposed by a real-world problem.

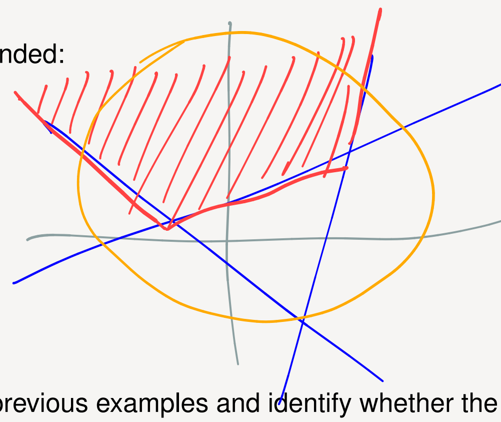
Bounded vs **Unbounded** regions.

Bounded:



there exists a circle that completely contains region

Unbounded:



Unbounded

it is not bounded.

Ex 7 Bounded vs Unbounded Go back through each of the previous examples and identify whether the solution set is bounded or unbounded.

Ex 8 Feasible Region: Financial Planning During the summers, Jorge roasts his own coffee. He roasts 75 pounds of an Ethiopian single-origin coffee and 120 pounds of a Colombian single-origin coffee.

These will be blended into 1 pound bags as follows:
an **economy blend** that contains 4 ounces of Ethiopian and 12 ounces of Colombian,
and a **superior blend** that contains 8 ounces of Ethiopian and 8 ounces of Colombian.

Let x denote the number of packages of the economy blend and y the number of packages of the superior blend. Write a system of inequalities that describe the possible numbers of packages of each kind of blend.

Solution First, we need to use consistent units.

How many ounces in a pound?

Amount of Ethiopian in one bag of Econ blend:

Amount of Colombian in one bag of Econ blend:

Amount of Ethiopian in one bag of Premium blend:

Amount of Colombian in one bag of Premium blend:

Write system of inequalities: