Chapter 5: Probability

Section 5.2: The Addition Rule and Complement Rule

COMPOUND EVENTS

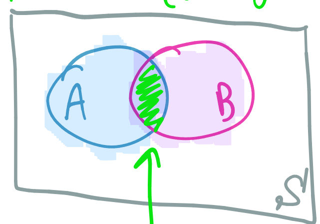
Def A **compound event** is any event combining two or more simple events. → one outcome

— mathematician's "OR"

NOTATION: $P(A \text{ or } B)$ denotes the probability that event A occurs or event B occurs (or both.)

	FORMAL ADDITION RULE
Symbolic	$\boxed{P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)}$
Meaning	*The probability of event A or event B is the sum of each event's probability of occurring individually, minus the probability of both events occurring simultaneously.*
Note	$P(A \text{ and } B)$ denotes the probability that A and B both occur at the same time (only one event).

intersection
remove: to avoid double counting it



A and B

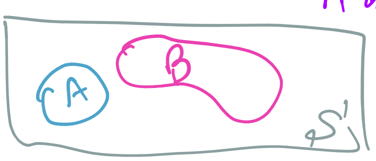
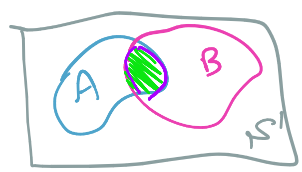
die
Ex $S = \{1, 2, \ldots, 6\}$ $A = \{\text{even}\}$ $B = \{\text{less than } 4\}$ $A \text{ and } B = \{2, 4\}$

DISJOINT / MUTUALLY EXCLUSIVE EVENTS

Def Two events are **disjoint** (or **mutually exclusive**) if they cannot occur at the same time.

Note: If events A and B are disjoint / mutually exclusive $\Rightarrow$ $P(A \text{ and } B) = 0$.

Add Rule for Disjoint Events: If A and B are disjoint, then $\boxed{P(A \text{ or } B) = P(A) + P(B)}$

If NO…	If YES…
Disjoint Events have *No Overlap* A and B = nothing 	**Not Disjoint** Events *have Overlap* A and B = have something 

Ex: Determine whether the following are disjoint events.

(a) **A** = A coin landing on Heads **B** = A coin landing on Tails A → H, B → T disjoint $S = \{H, T\}$	(b) **A** = {1, 2, 3, 4} **B** = {2, 3, 5, 6, 7} A and B = {2, 3} not disjoint
(c) **A** = person plays soccer **B** = person plays baseball not disjoint there examples of kids who play both sports	(d) **A** = Roll an even number on a 6-sided fair die. **B** = Roll an odd number on a 6-sided fair die. A = {2, 4, 6} B = {1, 3, 5} A and B nothing disjoint

Ex: Let $P(E) = 0.11$, $P(F) = 0.78$, $P(G) = 0.56$, $P(F \text{ and } G) = 0.4$, and events E and F are disjoint.

(a) Find $P(F \text{ or } G)$

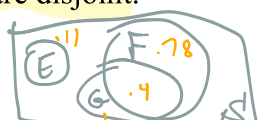
$= P(F) + P(G) - P(F \text{ and } G)$

$= 0.78 + 0.56 - 0.4 = \boxed{0.94}$

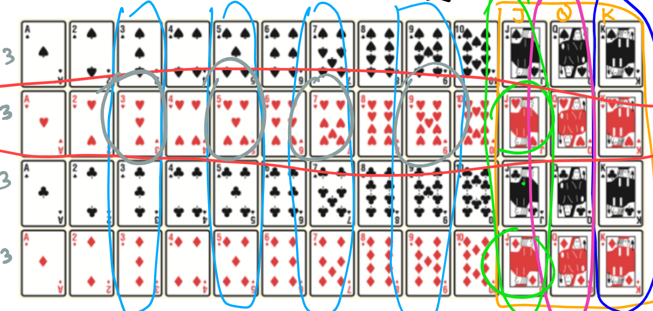
(b) Find $P(E \text{ or } F)$

$= P(E) + P(F) - \underbrace{P(E \text{ and } F)}_{=0}$

$= P(E) + P(F) = 0.11 + 0.78 = \boxed{0.89}$



Ex: Suppose that a single card is selected from a standard 52-card deck, such as shown below.



(a) What is the probability that the card drawn is a king?

$$P(K) = \frac{\#K}{\#S'} = \frac{4}{52} = \boxed{0.077}$$

(b) What is the probability that the card drawn is a king or a queen?

K or Q – disjoint!

$$P(K \text{ or } Q) = P(K) + P(Q) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{0.154}$$

(c) What is the probability that the card drawn is a Jack or a red?

not disjoint

$$P(J \text{ or } R) = P(J) + P(R) - P(J \text{ and } R)$$
$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52} = \boxed{0.538}$$

(d) What is the probability that the card is an odd number or a heart?

$$P(\text{odd or } H) = P(\text{odd}) + P(H) - P(\text{odd } \& \text{ } H)$$
$$= \frac{4 \cdot 4}{52} + \frac{13}{52} - \frac{4}{52}$$
$$= \frac{16+13-4}{52} = \frac{25}{52} = \boxed{0.481}$$

Ex: A study of 1,000 recently deceased people is summarized in the following table.

3 approaches: Empirical

marginal distribution

Cause of Death

	Cancer	Heart Disease	Other	
Smokers	100	180	120	Σ = 400
Non-smokers	100	120	380	Σ = 600
	Σ = 200	Σ = 300	Σ = 500	1000

Fill out the marginal distributions in the table, then find the probability of randomly selecting:

(a) someone who died of cancer.

C

$$P(C) = \frac{\#C}{\#S'} = \frac{200}{1000} = \boxed{0.2}$$

(b) someone who did not die of cancer.

$$P(\text{not } C) = P(HD) + P(\text{Other})$$
$$= \frac{300}{1000} + \frac{500}{1000} = 0.3 + 0.5 = \boxed{0.8}$$

(c) someone who died of heart disease and cancer.

$$P(HD \text{ and } C) = \boxed{0}$$

can't be both (in this table)

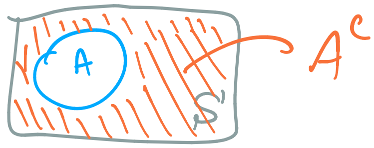
(d) someone who died of heart disease or cancer.

$$P(HD \text{ or } C) = P(HD) + P(C) \quad \text{(b/c disjoint)}$$
$$= \frac{300}{1000} + \frac{200}{1000}$$
$$= 0.3 + 0.2 = \boxed{0.5}$$

(e) someone who smoked or died of heart disease.

$$P(S \text{ or } HD) = P(S) + P(HD) - P(S \text{ and } HD)$$
$$= \frac{400}{1000} + \frac{300}{1000} - \frac{180}{1000} = \frac{520}{1000} = \boxed{0.52}$$

Complement of an Event



Def The **complement** (denoted A^c or $\overline{A}$) of event A, consists of all outcomes in which event A doesn't occur. That is, A^c is all the outcomes in S that are **not in** A.

Ex: Suppose we flip a fair coin and roll a fair die at the same time record the outcomes. Use set notation and probability notation in your answers.

$\# S = 12$

(a) Using set notation, write the sample space.

(simple) $\boxed{S = \{ H1, H2, H3, \underbrace{H4, H5, H6}_{E}, T1, T2, T3, T4, T5, T6 \}}$ $\emptyset$

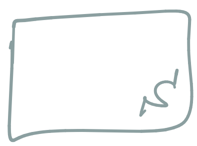
$S = \{ (H,1), (H,2), (H,3), \text{etc} \}$ (more abstract)

(b) Let E denote the event of flipping a heads and a number greater than three. Express the complement of E, E^c, using set notation.

$\boxed{E^c = \{ H1, H2, H3, \underbrace{T1, T2, \ldots, T6}_{6} \}}$ $\qquad \# E^c = 9$

(c) Find $P(E^c)$.

$$P(E^c) = \frac{\# E^c}{\# S} = \frac{9}{12} = \boxed{0.75}$$



Empty Set & More Set Theory

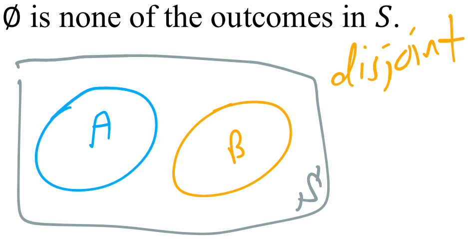
Def The **empty set** (denoted { } or ∅) is the set with no outcomes. That is, ∅ is none of the outcomes in S.

Some notes:

A and $B = \emptyset$

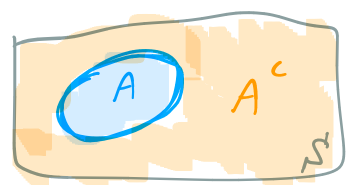
1. If A and B are disjoint, then ~~$A \cap B$~~ =
2. ~~$P(A \cap A^c)$~~ = $P(A \text{ and } A^c) = \emptyset$ (disjoint)
3. ~~$P(A \cup A^c)$~~ = $P(A \text{ or } A^c) = P(S) = 1$

Hint: draw Venn Diagrams



Theorem Complement Rule

$$\boxed{P(A^c) = 1 - P(A)}$$



Add Rule / Disjoint

Proof By (3) above: $1 = P(S) = P(A \text{ or } A^c) \stackrel{\downarrow}{=} P(A) + P(A^c)$

$1 = P(A) + P(A^c) \xrightarrow{\text{solve}} P(A^c) = 1 - P(A).$ $\square$