MATH 5B – Calculus II Spring 2019

Exam 2

Study Guide **Dr. Jorge Basilio**

§9.1–§9.4, §6.8, §7.1-7.4 Pasadena City College gbasilio@pasadena.edu

General Exam Info

Exams are a way for you to show me what you have learned (and please show all your steps so I can see this!) and to give you a sense of accomplishment! They are meant to be challenging and not just homework problems with the numbers changed. I really want to prepare you for university level math classes–so some exams may be longer or more challenging than others. Remember that I do grade fairly and my goal is to push you to succeed and excel in this class.

- Attendance required for all exams–I do NOT drop the lowest exam score.
- **"Make-up Exams"** are given only in extreme cases and at instructor's discretion; a student is allowed at most one make-up exam. (Documentation must be provided for the instructor to even consider a make up exam. This means you would need a doctor's note, etc.) A "Make-Up Exam" means you will be allowed to replace the missing score with the percentage you earn on the final exam. Please contact your instructor as soon as possible should there be a problem.
- **Your student ID is required for all exams.**
- During the exams–you will be required to leave your backpack and all non-test items at the front of the room, including cell phones and smart watches. Only your pencil/eraser and calculator will be allowed during the exam, and there will be a calculator check. Should you need to leave during the exam please ask for permission first before leaving and leave your cell phone with me. Not doing these things could result in a 0 on your exam.
- Once the exam is graded and returned, any problem you would like me to revisit must be brought to my attention by the next class session.
- Always keep your exams!

Exam 1 Date & Time		
• Exam 2	Thursday, April 11	4:00 pm – 5:20 pm

ALSO, after the exam:

- As the test is only 80 minutes long, we will have a 10 minutes break, then continue with new material.
- I take attendance at the end of class on test days.

Exam 2 Specific Info

- Material covered: Anything from Exam 1 PLUS §9.1, 9.2, 9.3, 9.4, 6.8, 7.1, 7.2, 7.3, 7.4
- You will need a calculator (only scientific allowed)–you won't be able to use your phone.
- Almost all questions have multiple parts
- Be prepared for the following types of questions: True-False-Sometimes, Multiple-Choice, and Free-response
- Be prepared to prove certain results (see below for more details)
- No cheat sheet is allowed on Exam 2, but I will provide a formula sheet (see below).
- Be prepared for questions with multiple parts that build on each other

Section 9.1

- Know the definitions of a **differential equation (DE)** and what it **solution** is.
- Know what the terms: **general solution**, **particular solution**, and **initial conditions**.
- Be able to verify whether or not a given function is a solution to a specified DE.
- Memorize/Know: the solutions to the **Natural growth/decay equation** from Chapter 6: $DE: \dfrac{dP}{dt} = kP$ have solutions $\boxed{P(t) = Ce^{kt}}$. It has a unique solutions if initial conditions $P_0 = P(t_0)$ are specified. In this case, $C = P_0$.
- Memorize/Know: the solutions to the **Newton's Law of Cooling** from Chapter 6: $DE: \dfrac{dT}{dt} = k(T - T_S)$ have solutions $\boxed{T(t) = T_S + Ce^{kt}}$. It has a unique solutions if initial conditions $T_0 = T(t_0)$ are specified.
- Memorize/Know: the solutions to the DEs of the simple form: $\dfrac{dy}{dx} = f(x)$ have solutions $\boxed{y(x) = \int f(x)dx}$. Don't forget the $+C$. It has a unique solutions if initial conditions $y_0 = y(x_0)$ are specified.
- Be able to find general and particular solutions to DEs of the above form (previous bullet point)
- Know the **Logistic Equation**: $\dfrac{dP}{dt} = kP(1 - \dfrac{P}{M})$. Know what **carrying capacity** M means and understand the qualitative behavior of the DE.
- Know what it means to "describe the **qualitative behavior** of a DE". Be able to use the DE to select the correct solution by using the graph. (See Activity 4 from the class notes).

Section 9.2

- Know what a **slope field (or direction field)** is.
- Given a DE in the form: $\dfrac{dy}{dx} = F(x, y)$ be able to recognize the correct corresponding slope field. Also, be able to sketch a particular solution given the graph of a slope field.
- Memorize/Know **Euler's Method**:
  To **approximate** a solution to $y' = F(x, y)$ using **step size** h, follow the following algorithm:
  - Given the initial conditions: $P_0 = (x_0, y_0)$ (x_0 and $y_0 = y(x_0)$).
  - To generate the next point $P_n = (x_n, y_n)$ (for $n = 1, 2, 3, \ldots$) follow:
    * $x_n = x_{n-1} + h = x_0 + nh$
    * $y_n = y_{n-1} + F(x_{n-1}, y_{n-1})h$
- Be prepared to perform Euler's method for 5 steps using only a scientific calculator.
- Optional - be able to compare the **error** of your approximation if given the exact solution $y(x)$.

Section 9.3

- Know what a **separable DE** is.
- Be able to determine whether or not a DE separable.
- Be able to solve many separable DEs using the technique **separation of variables (SoV)**. (Practice this!)

Section 9.4

- Be able to prove that the only solutions to the Natural Growth/Decay equation are of the above form using SoV.
- Memorize/Know: the solutions to the logistic equation, $\dfrac{dP}{dt} = kP\left(1 - \dfrac{P}{M}\right)$, for a constant $k \neq 0$, with initial conditions $P_0 = P(0)$ and carrying capacity M, are of the form: $\boxed{P(t) = \dfrac{M}{1 + Ae^{-kt}}}$, where $A = \frac{M - P_0}{P_0}$.
- Be able to use the solutions to the Logistic Eq to predict the population at a later date. See Activity 3 from the class notes.
- You do not need to know the proof of the Logistic Equation.

Chapter 9 Practice Problems

Page 631: # 12, 14

Page 637–638: # 1, 3-6, 23

Page 645: # 1, 3, 5, 7, 11, 13, 14

Page 657–658: # 5, 6, 7

Page 674: True-False Quiz: # 1, 2, 3

Page 674–675: Exercises: # 1.a, 3, 7,

Section 6.8

- Know what **indeterminate forms** are.
- Be able to identify the different indeterminate forms:
  - Fractions: $\frac{0}{0}$, $\quad \frac{\infty}{\infty}$
  - Products: $0 \cdot \infty$
  - Differences: $\infty - \infty$
  - Powers: 0^0, $\quad \infty^0$, $\quad 1^\infty$
- Know the conditions under which **L'Hôpital's Rule**, $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$, may be used.
- Practice a lot of problems with L'Hôpital's Rule! Be able to evaluate limits corresponding to each of the above indeterminate forms.
- Know the tricks needed to evaluate limits corresponding to each of the above indeterminate forms.
  - Fractions: Case: $\frac{0}{0}$, $\quad \frac{\infty}{\infty}$: classic L'Hop
  - Case: Products: $0 \cdot \infty$: re-arrange $f(x) \cdot g(x)$ into $f(x)/(1/g(x))$ or $g(x)/(1/f(x))$ to produce previous case
  - Case: Differences: $\infty - \infty$: write as a single fraction (likely, using common denominator trick)
  - Case: Powers: 0^0, $\quad \infty^0$, $\quad 1^\infty$: need to use the $e^{\ln(f(x)^{g(x)}} = e^{g(x)\ln f(x)}$ trick.
- Be able to use L'Hôpital's Rule to assist in curve sketching (important Calc 1 topic). See Example 1 in the notes. **Note:** Although we didn't go over this in class, I think it's important and would like you to be ready for questions of this type.
- Know what **rates of growth** means and how to use L'Hôpital's Rule to determine which function grows faster than another.

Section 6.8 Practice Problems

Page 504: True-False Quiz: # 18, 19

Page 506: Exercises: 63–77 odd, 70, 72, 78, 81

Section 7.1

- Know the tools in the **integration toolbox**.
- Memorize/Know: the formulas for ADRs given on page 5 of the notes.
- Know how to use **Integration by Parts (IBPs)** with and without limits of integration: $\boxed{\int u\,dv = uv - \int v\,du}$
- PRACTICE A LOT OF IBPs!!
- Know how to use the **I-trick** (or "$2I$ trick" in the notes)
- Study the examples from Activity 5 "some harder IBPs" from the notes and be prepared for **one hard** IBP problem.
- You do not need to know the reduction formulas and I will not test you on their use.

Section 7.2

- Memorize/Know the **flow chart for evaluating** $\int \sin^n(x)\cos^m(x)dx$ and the techniques for evaluating the different cases:
  - Brief summary is two cases: even and odd (for more details see flow chart)
  - even: use **double-angle formulas** and u-sub
  - odd: use "split one trick", **Pythagorean identity**, and u-sub
- Practice a lot of these!
- You do not need to know the **product-to-sum formulas**. I will provide them on a formula sheet. You do need to know how to solve problems using them. See Activity 5 from the notes.

Section 7.3

- Know when to use **trigonometric integrals**.
- Be able to recognize which substitution goes with which type of integrals.
  - Integrals with $\sqrt{a^2 - x^2}$ use the substitution $x = a\sin(\theta)$
  - Integrals with $\sqrt{a^2 + x^2}$ use the substitution $x = a\tan(\theta)$
  - Integrals with $\sqrt{x^2 - a^2}$ use the substitution $x = a\sec(\theta)$
  - Pay attention to the intervals for which the values work
- Know how to go back to the original variables after integration
- Know to draw the correct corresponding triangle using the substitution.
- Practice a lot of trig substitution problems!!
- Be able to compute the area of circles and ellipses by setting up an integral and evaluating it using trig sub.
- Be able to compute volumes using the disk, washer, shell, or slicing method (from Calc 1) combined with trig sub
- Optional - setting up a hard volume with trig sub (like the volume of a doughnut) might show up but only as a possible extra credit problem.

Section 7.4

- Know what a **proper** rational function
- Know how to use the **method of partial fractions** to evaluate integral:
  - Case I: $Q(x) = (a_1x - b_1)(a_2x - b_2)\cdots(a_kx - b_k)$ is product of **distinct** linear factors in denominator. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \cdots + \frac{A_k}{a_kx + b_k}$$
  - Case II: $Q(x) = (ax + b)^k$ repeated roots of Q. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{A_k}{(ax + b)^k}$$
  - Case III: $Q(x) = (ax^2 + bx + c)^k$ repeated irreducible. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}$$
  - Case IV: Assume $\deg(P) \geq \deg(Q)$. Then

    Use **long division** first to write $\dfrac{P(x)}{Q(x)} = S(x) + \dfrac{R(x)}{Q(x)}$

    where $S(x)$ is a polynomial and $R(x)$ is the remainder with $\deg(R) < \deg(Q)$.

    Then $\dfrac{R(x)}{Q(x)}$ is proper and use Case I, II, or III.
- Be able to do easy and medium level problems with the method of partial fractions. I will not put very hard problems on the test (like Activity 3 (b) from the notes).

Chapter 7 Practice Problems

Page 531: # 4, 5, 12

Page 577: True-False Quiz: # 1–4,

Page 577–578: Exercises: 3, 4, 5, 6, 7, 9, 10, 11, 14, 15, 16, 17, 24, 26, 32, 33

Formula Sheet

- $\boxed{\int u\,dv = uv - \int v\,du}$
- $\boxed{\cos^2(x) + \sin^2(x) = 1}$ $\boxed{1 + \tan^2(x) = \sec^2(x)}$
- $\boxed{\cos^2(x) = \frac{1}{2}(1 + \cos(2x))}$ $\boxed{\sin^2(x) = \frac{1}{2}(1 - \cos(2x))}$
- $\boxed{\sin(2x) = 2\sin(x)\cos(x)}$
- $\boxed{\sin(A)\cos(B) = \frac{1}{2}(\sin(A-B) + \sin(A+B))}$ $\boxed{\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))}$

  $\boxed{\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B))}$
- $\boxed{x = a\sin(\theta)}$ $\boxed{x = a\tan(\theta)}$ $\boxed{x = a\sec(\theta)}$
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  - Case I: $Q(x) = (a_1x - b_1)(a_2x - b_2)\cdots(a_kx - b_k)$ is product of **distinct** linear factors in denominator. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \cdots + \frac{A_k}{a_kx + b_k}$$
  - Case II: $Q(x) = (ax + b)^k$ repeated roots of Q. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{A_k}{(ax + b)^k}$$
  - Case III: $Q(x) = (ax^2 + bx + c)^k$ repeated irreducible. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}$$
  - Case IV: Assume $\deg(P) \geq \deg(Q)$. Then

    Use **long division** first to write $\frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$

    where $S(x)$ is a polynomial and $R(x)$ is the remainder with $\deg(R) < \deg(Q)$.

    Then $\frac{R(x)}{Q(x)}$ is proper and use Case I, II, or III.
- Disk Method: $\boxed{V = \int \pi[R(x)]^2\,dx}$
- Washer Method: $\boxed{V = \int \left(\pi\,[R(x)]^2 - \pi\,[r(x)]^2\right)\,dx}$
- Shell Method: $\boxed{V = \int 2\pi x f(x)\,dx}$