Chapter 9: Inferences from Two Samples (Two Parameters)

Section 9.1: Two Proportions Stat 50

Introduction Scenario:

Think about whether you plan to vote in the next presidential election in 2020. Let's say I split the responses into two groups: Females and Males

Decide Notation

p_1 = prop. of F will vote

p_2 = prop. of M will vote

Notice: There are **TWO** proportions here: $p_1 = \frac{12}{12} = 1.00$ (F)

$p_2 = \frac{3}{5} = 0.60$ (M)

	#	Yes		
M	5	2	5	3
F	5	5	12	12

QUESTION: Is there a statistically significant difference?

We don't care what the proportions actually are, we care about whether they're the SAME or not.

** In this section (9.1), we will only deal with independent samples with QUALITATIVE variables. ("categorical")

The Logic

Note p_1, p_2 population parameters; $\hat{p}_1$, $\hat{p}_2$ are sample parameters

If $p_1 = p_2$ then $p_1 - p_2 =$ 0.

So we will first collect sample proportion (SRS) and find $\hat{p}_1$ and $\hat{p}_2$ • $\hat{p}_1 = \frac{x_1}{n_1}$ • $\hat{p}_2 = \frac{x_2}{n_2}$

Then we will see if $\hat{p}_1 - \hat{p}_2$ is anywhere close to 0.

Recall the Logic: Say $H_0: p_1 = p_2$ and $H_1: p_1 < p_2$ (left). If we get… "a little or a lot"

• $\hat{p}_1 = \hat{p}_2$ fail to reject H_0 • $\hat{p}_1 > \hat{p}_2$ Fail to Reject H_0

• $\hat{p}_1 < \hat{p}_2$ by "a little" fail to reject H_0 • $\hat{p}_1 < \hat{p}_2$ by "a lot" then we REJECT H_0

Steps for Hypothesis Test Regarding the Difference between p_1 and p_2 (TWO PROPORTIONS)

Step 0

Pre-Step: *Check Requirements*

• The samples are simple random and independent (probabilities don't effect each other)

• At least five success $n\hat{p} \geq 5$ and five failures $n\hat{q} \geq 5$ for each sample.

Step 1: *Hypotheses*

Two Tailed	Left Tailed	Right Tailed
$H_0: p_1 = p_2$ $H_1: p_1 \neq p_2$	$H_0: p_1 = p_2$ $H_1: p_1 < p_2$	$H_0: p_1 = p_2$ $H_1: p_1 > p_2$

Step 2: *Level of Significance* $\alpha = P(\text{Type I Error})$

If it's not given, then use $\alpha = 0.05$. Choice depends on seriousness of making Type I error.

Step 3: *Test Statistic*

$$z^* = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\left(\frac{\bar{p}\bar{q}}{n_1}\right) + \left(\frac{\bar{p}\bar{q}}{n_2}\right)}}$$ where the point estimate is $\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$ "p bar" total proportion (combined)

$\hat{p} = \hat{p}_1 - \hat{p}_2$

note $p_1 - p_2 = 0$ by H_0

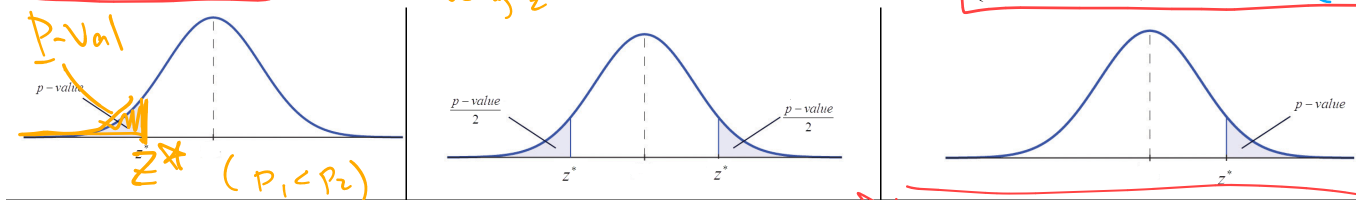
$\bar{q} = 1 - \bar{p}$ (complement)

• P low, null go
• P high, null fly

Step 4: *Find a Critical Value or P-Value*

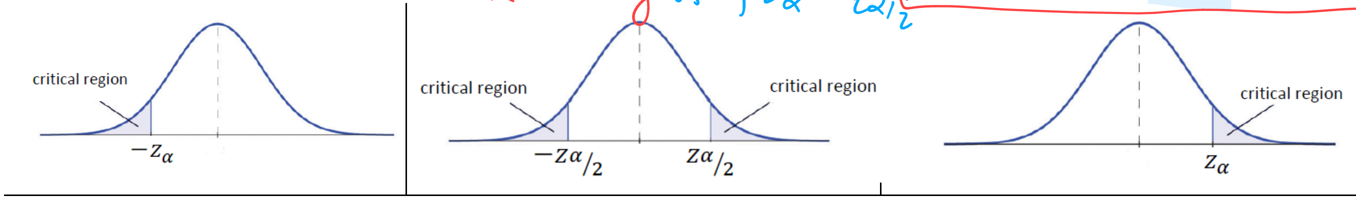
P-VALUE METHOD $P = P(\text{Crit Reg})$ using z^* **DECISION**

$\begin{cases} \text{Reject } H_0 \sim \text{ if } P\text{-value} \le \alpha \\ \text{Fail to Reject } H_0 \sim \text{ if } P\text{-value} > \alpha \end{cases}$



CRITICAL VALUE METHOD Critical Reg using z_α or $z_{\alpha/2}$ **DECISION**

$\begin{cases} \text{Reject } H_0 \sim \text{ if } z^* \text{ lies in the critical region} \\ \text{Fail to Reject } H_0 \sim \text{ if } z^* \text{ doesn't lie in the critical region} \end{cases}$



Step 5: Write a *CONCLUSION* either rejecting or failing to reject H_0

Ex: An insurance company is concerned that men are more likely to speed than women. In a sample of 500 (n_1) randomly selected women, 27 (x_1) have been ticketed for speeding in the last year. In a sample of 250 (n_2) randomly selected men, 26 (x_2) have been ticketed for speeding in the last year. Use a 0.05 (α) significance level to test the insurance company's claim that the percentage of women ticketed for speeding is less than the percentage of men. ← p_2

↳ proportion ← p_1

p_1: proportion of W speed.
p_2: proportion of M speed.

Null and Alternative Hypothesis

$$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 < p_2 \end{cases} \quad (\text{Left Tailed Test})$$

Step 0 Check Req
① SRS ✓ ② independent ✓
③ ✓ $n_1 p_1 \ge 5$: $(500)(27/500) = 27$ & $n_2 p_2 \ge 5$: $(250)(26/250) = 26$ ✓
✓ $n_1 q_1 \ge 5$: $(500)\left(\frac{473}{500}\right) = 473$ & $n_2 q_2 \ge 5$: $(250)\left(\frac{224}{250}\right) = 224$ ✓

Step 2 $\boxed{\alpha = 0.05}$

Test Statistic

$$z^* = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$

$\hat{p}_1 = \frac{x_1}{n_1} = \frac{27}{500}$

$\hat{p}_1 = 0.054$

$\hat{p}_2 = \frac{x_2}{n_2} = \frac{26}{250}$

$\hat{p}_2 = 0.104$

$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2}$

$\bar{p} = \frac{53}{750}$

$\bar{p} = 0.071$

$\bar{q} = 0.929$

$$z^* = \frac{(0.054 - 0.104) - (0)}{\sqrt{\frac{(0.071)(0.929)}{500} + \frac{(0.071)(0.929)}{250}}} = -2.51337\ldots$$

$\boxed{z^* = -2.51}$

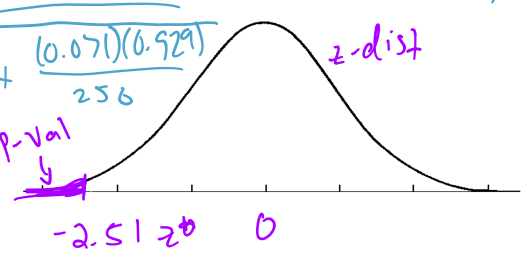
P-value/Critical Region

$P(\text{Crit Reg})$

$= P(z < -2.51)$

$= 0.5 - \text{normalcdf}(-2.51, 0, 0, 1) = 0.00604$

$\boxed{P = 0.00604}$



Decision about Null Hypothesis

$\alpha = 0.05$
$P = 0.00604$
$P < \alpha \rightarrow$ P low, null go
$\boxed{\text{Reject } H_0}$

Conclusion (M→E)

"There is sufficient statistical evidence to support the claim that the percentage of women ticked for speeding last year is less than the percentage of men ticket for speeding."

How to choose critical region

P-Value Method:

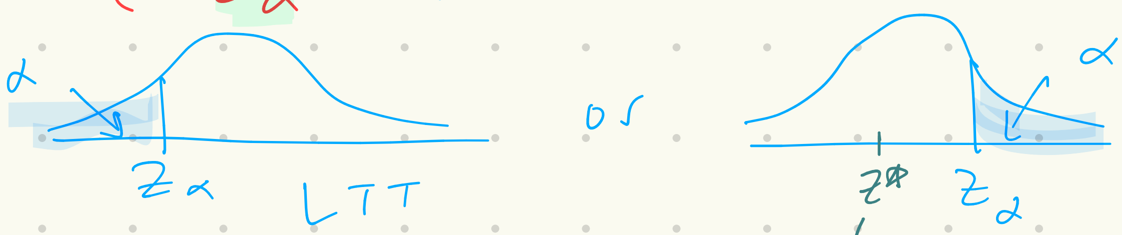
Critical region is determined by using Tailed Test & z^* as cut-offs

Critical Value Method

Decision is made z^* inside CR (R H0)
outside CR (FTR H0)

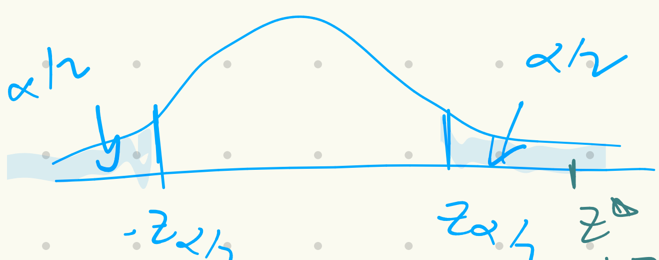
Critical region is determined by using z_α & Tailed Test:

→ use z_α when one-tailed test!



↳ outside CR, FTR H_0

⇒ use $z_{\alpha/2}$ when two-tailed test!



z^* inside CR
↳ so Reject H_0

Ex: In clinical trials of the anti-inflammatory drug Inflaminex, adult and adolescent allergy patients were randomly divided into two groups. Some patients received 500mcg of Inflaminex, while some patients received a placebo. Of the 2103 n_1 patients who received Inflaminex, 520 x_1 reported bloody noses as a side effect. Of the 1671 patients who received the placebo, 368 reported bloody noses as a side effect. Is there significant evidence to conclude that the proportion of Inflaminex users who experienced bloody noses as a side effect is greater than the proportion of the placebo group at the $\alpha = 0.01$ level of significance?

USE CALCULATOR !

Null and Alternative Hypothesis

$$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 > p_2 \end{cases}$$ (Right Tailed)

- p_1: proportion of patients taking drug w/ bloody noses
- p_2: proportion of patients taking placebo w/ bloody noses

Test Statistic

(still do this part)

$\boxed{z^* = 1.95}$ (Calculator)

From Calc

$x_1 = 520$ $x_2 = 368$

$n_1 = 2103$ $n_2 = 1671$

$\hat{p}_1 = 0.247$ $\hat{p}_2 = 0.220$

$\bar{p} = 0.235$

$\bar{q} = 0.765$

$$z^* = \frac{(0.247 - 0.22)}{\sqrt{\frac{0.235 * 0.765}{2103} + \frac{0.235 * 0.765}{1671}}} = 1.943\ldots$$

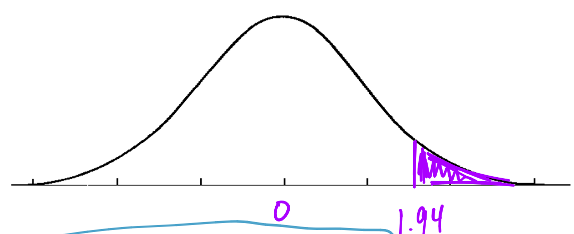
$\boxed{z^* = 1.94}$

P-value/Critical Region

$P = 0.5 - \text{normalcdf}(0, 1.94, 0, 1)$

$\boxed{P = 0.0262}$

$\boxed{P = 0.0256}$ ← using $z^* = 1.95$



Decision about Null Hypothesis

$\alpha = 0.01$

$P = 0.026$

$P > \alpha \rightarrow$ P high, null fly $\boxed{\text{Fail to Reject } H_0}$

Conclusion

" There is not enough statistical evidence to support the claim that the proportion of Imflamex users who get bloody noses as a side effect is greater than the proportion of placebo group. "

GRAPHING CALCULATOR (TI-83 OR 84)

Instructions: (a) STAT ⇒ TESTS ⇒ 2-PropZTest

(b) Enter
$$\begin{cases} x_1 \,/\, x_2 = \text{number of success in sample \#1 / \#2} \\ n_1 \,/\, n_2 = \text{size of sample \#1 / \#2} \\ p_1 \sim \text{alternative hypothesis} \end{cases}$$

CONFIDENCE INTERVAL FOR THE DIFFERENCE OF TWO POPULATION PROPORTIONS

Alternative Forms: $\left(\hat{p}_1 - \hat{p}_2\right) - E < \underset{\text{difference}}{\overset{\text{pop. prop}}{p_1 - p_2}} < \left(\hat{p}_1 - \hat{p}_2\right) + E$ or $\left(\hat{p}_1 - \hat{p}_2\right) \pm E$

where the margin of error is given by $\boxed{E = z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}_1 \hat{q}_1}{n_1} + \dfrac{\hat{p}_2 \hat{q}_2}{n_2}}}$

Compare w/ one proportion $E = z_{\alpha/2} \cdot \sqrt{\dfrac{\hat{p}\hat{q}}{n}}$

GRAPHING CALCULATOR (TI-83 OR 84)

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ 2-PropZInt

interval

Note you can sometimes have negatives since we looking at "differences".

(b) Enter
- x_1 / x_2 = number of successes in sample #1 / #2
- n_1 / n_2 = size of sample #1 / #2
- $C-level$ = confidence level

Ex: A study was conducted to test the effectiveness of a sweetener called xylitol in preventing ear infections in preschool children. In a randomized experiment, 159 (n_2) preschool children took five daily doses of xylitol, and 46 (x_2) of these children got an ear infection during the three months of the study. Meanwhile, 165 (n_1) children took five daily doses of placebo syrup, and 68 (x_1) of these children got an ear infection during the study. Construct a 90% confidence interval for the difference in the proportion of children that got ear infections for the control group (p_1) and the xylitol group (p_2).

$p_1 - p_2$

- p_1: prop kid w/ ear infection of control gp.
- p_2: prop kids w/ ear infection of xylitol gp.

Find the point estimate (difference between sample proportions)

Point Est: $\hat{p}_1 - \hat{p}_2 = 0.412 - 0.289 = \boxed{0.123}$

Determine critical value $z_{\alpha/2}$

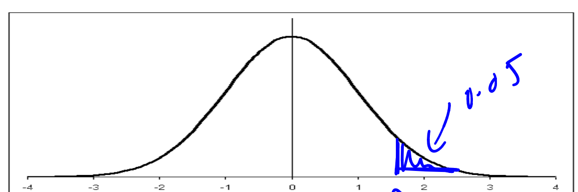
$CL = 0.90$

$\alpha = 1 - CL = 0.1$

$\alpha/2 = 0.05$

$\boxed{z_{0.05} = 1.64}$

$z_{\alpha/2} = \text{invNorm}(0.05, 0, 1, \text{Right})$



$z_{0.05} = 1.64$

Find margin of error

$E = \dfrac{(\hat{p}+E) - (\hat{p}-E)}{2} = \dfrac{E+E}{2} = E$

$E = \dfrac{0.20925 - 0.03638}{2} = 0.086435$

$\boxed{E = 0.086}$

Construct confidence interval CI: $(\hat{p} - E, \hat{p} + E)$

CI: $(0.03638, 0.20925)$ (from calc)

$\hat{p}-E$ $\hat{p}$ $\hat{p}+E$

E E

$\boxed{CI: (0.036, 0.209)}$

Does it appear that the sweetener is effective at reducing ear infections?

"It does appear that xylitol is effective at reducing ear infections."

B/c prop of control gets more ear infections b/c $\hat{p}$ is positive & big.

Step 1

Hypothesis Tests — Two Proportions: p_1 p_2 (pop prop)

$$\begin{cases} H_0: p_1 = p_2 \\ H_A: p_1 < p_2 \text{ or } p_1 \neq p_2 \text{ or } p_1 > p_2 \end{cases}$$

(Left TT) (Two TT) (Right TT)

Step 0 Check Requirements

① SRS ② independent ③ $n_1\hat{p}_1 \geq 5$ & $n_2\hat{p}_2 \geq 5$

$n_1\hat{q}_1 \geq 5$ $n_2\hat{q}_2 \geq 5$

Step 2 Level of Significance (α)

(if not given, choose $\alpha = 0.05$)

Step 3 Test Statistic

$$\boxed{z^* = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\dfrac{\bar{p}\bar{q}}{n_1} + \dfrac{\bar{p}\bar{q}}{n_2}}}}$$

$\bar{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$ "total proportion"

$\bar{q} = 1 - \bar{p}$

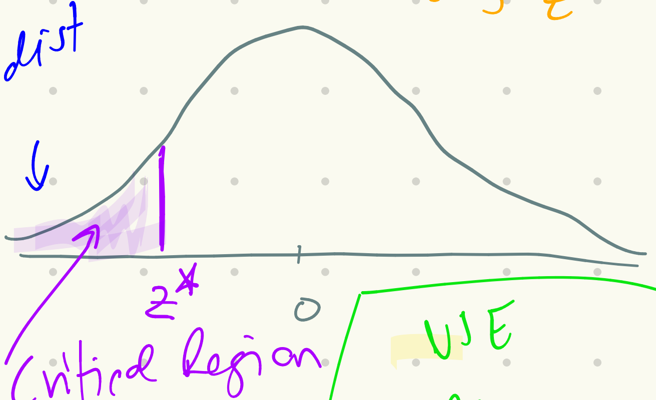
Step 4 P-Value Method or Critical Value Method

P-Value Method

Compute P-Value

P-Value = P(Critical Region using z^*)

z-dist



USE NORMALCDF

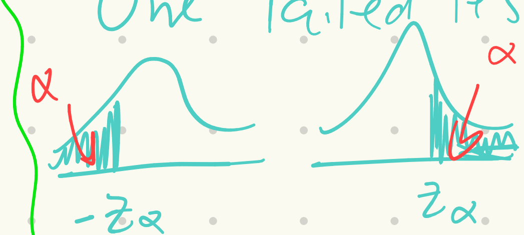
Critical Value Method

Compute Critical Region

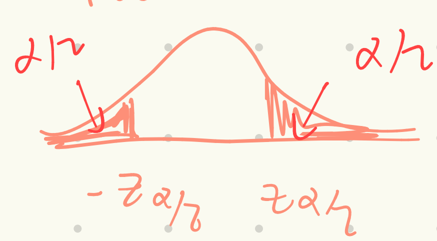
USING z_α or $z_{\alpha/2}$

How to Pick z_α or $z_{\alpha/2}$?

One Tailed Test



Two Tailed Test



USE invNORM

Step 4 Decision

P-Value M

- if $P < \alpha$ (P low)
  then REJECT H_0 (null go)

- if $P \geq \alpha$ (P high)
  then FAIL TO REJECT H_0
  (null fly)

Critical Value M

Decision

- if z^* is INSIDE Critical Region
  then REJECT H_0

- if z^* is OUTSIDE Critical Region
  then Fail to Reject H_0

Step 5 Conclusion

"There is/isnt enough statistical evidence to support claim"
(alt. Hyp)

Steps for a Hypothesis Test When Applied to testing μ_1 and μ_2 (Two Means)

Pre-Step: *Check Requirements*

- The samples are independent and randomly obtained
- The values of the population standard deviation σ_1 and σ_2 are not known and unequal.
- Populations are normally distributed **OR** ($n_1 >$ 30 **and** $n_2 >$ 30)

Step 1: *Hypotheses*

$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$	$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$

Step 2: *Level of Significance* α

If it's not given, then use 0.05. Choice depends on seriousness of making Type I error.

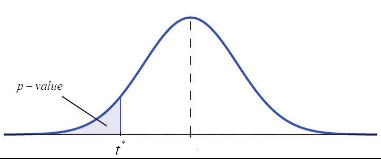
Step 3: *Test Statistic*

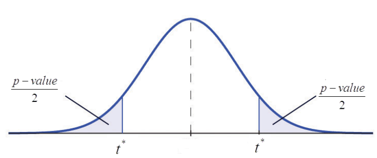
$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

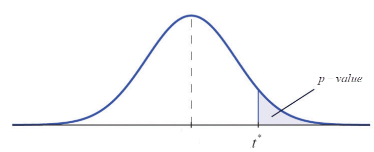
Step 4: *Find a Critical Value or P-Value*

P-VALUE METHOD **DECISION**

$\begin{cases} \text{Reject } H_0 \sim \text{ if } P\text{-value} \le \alpha \\ \text{Fail to Reject } H_0 \sim \text{ if } P\text{-value} > \alpha \end{cases}$

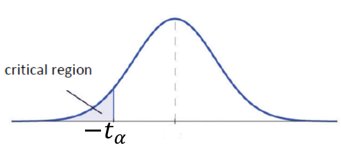


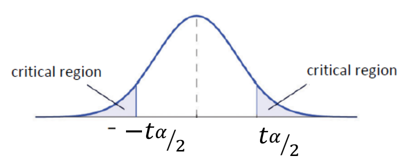


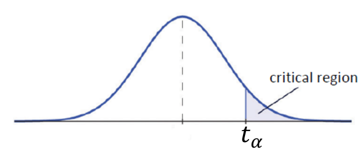


CRITICAL VALUE METHOD **DECISION**

$\begin{cases} \text{Reject } H_0 \sim \text{ if } t^* \text{ lies in the critical region} \\ \text{Fail to Reject } H_0 \sim \text{ if } t^* \text{ doesn't lie in the critical region} \end{cases}$







Step 5: Write a *CONCLUSION* either rejecting or failing to reject H_0

GRAPHING CALCULATOR (TI-83 OR 84) INSTRUCTIONS

Instructions: (a) $\boxed{\text{STAT} \Rightarrow \text{TESTS} \Rightarrow \text{2-SampTTest}}$

(b) Enter
$$\begin{cases} \bar{x}_1 \,/\, \bar{x}_2 = \text{mean of sample \#1 / \#2} \\ s_1 \,/\, s_2 = \text{standard deviation of sample \#1 / \#2} \\ n_1 \,/\, n_2 = \text{size of sample \#1 / \#2} \\ \mu_1 \sim \text{alternative hypothesis } (\textit{not pooled}) \end{cases}$$

Ex: A study of zinc-deficient mothers was conducted to determine whether zinc supplements during pregnancy results in babies with increased weights at birth. 294 expectant mothers were given a zinc supplement, and the mean birth weight was 3214 grams with a standard deviation of 669 g. There were 286 expectant mothers who were given a placebo, and the mean weight was 3088 g with a standard deviation of 728 grams. Using a 0.01 significance level, is there sufficient evidence to support the claim that a zinc supplement does result in increased birth weights?

μ_1 n_1 μ_2 σ_1 n_2 σ_2

μ_1 = mean babies weight from mom's w/ supplement

μ_2 = mean " " from placebo group

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_A: \mu_1 > \mu_2 \end{cases}$$

Check Req

(1) SRS ✓ (2) independent (3) size ✓

zinc group	placebo group
$\bar{x}_1 = 3214$ g	$\bar{x}_2 = 3088$ g
$s_1 = 669$ g	$s_2 = 728$ g
$n_1 = 294$	$n_2 = 286$

Test Statistic

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s^2}{n_1} + \frac{s^2}{n_2}}}$$

$$t^* = \frac{(3214 - 3088) - (0)}{\sqrt{\frac{669^2}{294} + \frac{728^2}{286}}} = 2.17 \qquad \boxed{t^* = 2.17}$$

P-value/Critical Region

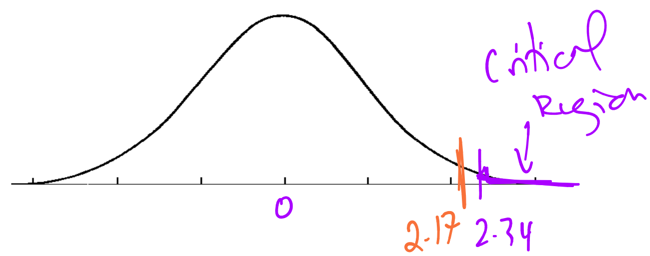
df = 286 - 1

Use smaller one

$\alpha = 0.01$

Right Tailed Test

$t_\alpha = \text{invT}(1 - 0.01, 285) = \boxed{2.34}$



Decision about Null Hypothesis

↳ t^* is outside Critical Region → $\boxed{\text{Fail to Reject } H_0}$

Conclusion

(M→E) "There is insufficient evidence to support the claim that a zinc supplement does increase birth weights."

Check calc 2 means T

$\boxed{\text{2 SampTTest}}$

$t^* = 2.17$

P-Val $P = 0.015$

$P > \alpha \rightarrow$ FTR H_0

same decision & conclusion!

n_1 n_2

Ex: A professor at a large community college wanted to determine whether there is a difference in the means of final exam scores between students who were allowed to text in class and those who weren't. She believed that the mean of the final exam scores for the texting class would be lower than that of the non-texting class. Was the professor correct? She randomly selected 30 final exam scores from each group, and they are listed below.

67.6	41.2	85.3	55.9	82.4	91.2	73.5	94.1	64.7	64.7
70.6	38.2	61.8	88.2	70.6	58.8	91.2	73.5	82.4	35.5
94.1	88.2	64.7	55.9	88.2	97.1	85.3	61.8	79.4	79.4

Texting class n_1

class

→ compute $\bar{x}_1, s_1$

77.9	95.3	81.2	74.1	98.8	88.2	85.9	92.9	87.1	88.2
69.4	57.6	69.4	67.1	97.6	85.9	88.2	91.8	78.8	71.8
98.8	61.2	92.9	90.6	97.6	100	95.3	83.5	92.9	89.4

Non-texting class n_2

class

→ compute $\bar{x}_2, s_2$

$\boxed{\text{2-SampTTest}}$

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_A: \mu_1 < \mu_2 \end{cases}$$ (Left Tailed)

$\bar{x}_1 = 72.85$

$s_1 = 16.92$

$\bar{x}_2 = 84.98$

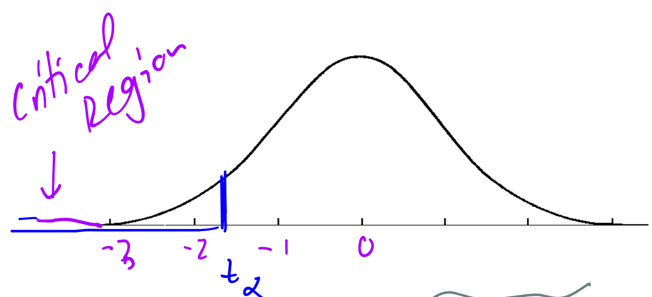
$s_2 = 11.71$

Test Statistic

$t^* = -3.23$

P-value/Critical Region

$P = 0.00108$



$t_\alpha = invT(0.05, 29)$
$= -1.70$

Decision about Null Hypothesis

$\alpha = 0.05$

$P < \alpha \rightarrow$ Reject H_0

Conclusion

"There is enough evidence that texting makes you dumb."

CONFIDENCE INTERVAL FOR THE DIFFERENCE OF TWO POPULATION MEANS

Alternative Forms: $\left(\bar{x}_1 - \bar{x}_2\right) - E < \mu_1 - \mu_2 < \left(\bar{x}_1 - \bar{x}_2\right) + E$ or $\left(\bar{x}_1 - \bar{x}_2\right) \pm E$

CI : $(\bar{\bar{x}} - E, \bar{\bar{x}} + E)$ $\quad \bar{\bar{x}} = \bar{x}_1 - \bar{x}_2$

where the margin of error is given by $\boxed{E = t_{\alpha/2} \cdot \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

GRAPHING CALCULATOR (TI-83 OR 84) INSTRUCTIONS

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ 2-SampTInt

(b) Enter
$$\begin{cases} \bar{x}_1 \;/\; \bar{x}_2 = \text{mean of sample \#1 / \#2} \\ s_1 \;/\; s_2 = \text{standard deviation of sample \#1 / \#2} \\ n_1 \;/\; n_2 = \text{size of sample \#1 / \#2} \\ C-level = \text{confidence level } (\textit{not pooled}) \end{cases}$$

Ex: The Gallup Organization wanted to investigate the time that American men and women spend hanging out with their friends. A random sample of 700 men surveyed spent a mean time of 10 hrs per week with their friends with a standard deviation of 1.9 hours. On the other hand, 740 women surveyed spent a mean time of 7.5 hours with a standard deviation of 1.6 hours. Construct a 95% confidence interval estimate for the difference between the corresponding population means.

s_1 $\quad$ n_1 $\quad$ n_2 $\quad$ $\bar{x}_1$ $\quad$ $\bar{x}_2$ $\quad$ s_2

Men
$\bar{x}_1 = 10$ hrs/wk
$s_1 = 1.9$ hrs

Women
$\bar{x}_2 = 7.5$ hr/wk
$s_2 = 1.6$ hr

Find the point estimate (difference between sample means)

Sample diff of means: ($\bar{\bar{x}}$) $\quad \bar{x}_1 - \bar{x}_2 = 10 - 7.5 = 2.5$

$\boxed{\bar{\bar{x}} = 2.5 \text{ hrs/wk}}$

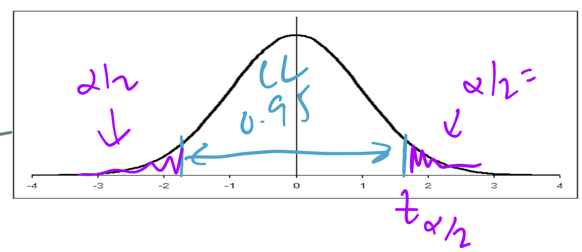
Determine critical value $t_{\alpha/2}$

Calc : 2sampT Int

CL = 0.95
$\alpha = 0.05$
$\alpha/2 = 0.025$

$t_{\alpha/2} = \text{invT}(\underbrace{1-0.025}_{1-\alpha/2}, \underbrace{699}_{df})$

$\boxed{t_{\alpha/2} = 1.96}$

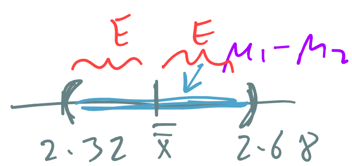


Find margin of error

$E = 2.68 - 2.5 = 0.18$ $\quad \boxed{E = 0.18}$

Construct confidence interval

$\boxed{CI : (2.32, 2.68)}$



$\mu_1 - \mu_2$ is b/w 2.32 & 2.68

Does it appear that there is a difference between men and women?

(M→E) yes, there does appear to be a difference!
We can predict with 95% confidence that men spend between 2.32 & 2.68 hours per week more w/ their friends than women!!

Section 9.3: Two Dependent Samples (Matched Pairs)

Def Two sets of observations are *paired* if each observation in one set has a special correspondence or connection with exactly one observation in the other set.

Ex: State if the samples are dependent or independent and if the variable is qualitative or quantitative.

1) Among competing acne medications, does one perform better than the other? To answer this question, researchers applied Medication A to one part of the subject's face and Medication B to a different part of the subject's face to determine the proportion of subjects whose acne cleared up for each medication. The part of the face that received Medication A was randomly determined.

Sample 1 = Med A on one part of face

Sample 2 = Med B on another part of face

Dependent Sample (same person)

Variable = whether ACNE clears up or not (Qualitative)

2) A researcher wishes to determine the effects of alcohol on people's reaction time. She randomly divides 100 people 21 years or older into two groups. Group 1 is asked to drink 3 ounces of alcohol, while group 2 drinks a placebo. Thirty minutes later they measure their reaction time.

Sample 1 = 21 yr or older drink 3 oz (Gp 1)

Sample 2 = 21 y. or older drink placebo

Independent Samples

Variable = reaction time of person (Quantitative) (& continuous)

3) A statistician wants to compare the treatment of female and male actors based on their ages. She looks at the ages of the females and males who won best actor/~~actress~~ in the last five years of the Oscars.

Sample 1 = female actors who won oscar in last 5 yr.

Sample 2 = male actors who won Oscar in last 5 yrs

Independent Sample

Variable = ages of person when won Oscar (Quantitative) (& interval LOM)

Turn Two Samples into One Sample:

Consider an experiment where a researcher throws a stick towards someone and first asks them to catch it with their dominant hand then again with their non-dominant hand. The times below showed how long it took several individuals to react to the toss.

X one variable Y second variable d

Student	Dominant Hand, X_i	Nondominant Hand, Y_i	Difference, d_i
1	0.177	0.179	0.177 − 0.179 = −0.002
2	0.210	0.202	0.210 − 0.202 = 0.008
3	0.186	0.208	−0.022
4	0.189	0.184	0.005
5	0.198	0.215	−0.017
6	0.194	0.193	0.001
7	0.160	0.194	−0.034
8	0.163	0.160	0.003
9	0.166	0.209	−0.043
10	0.152	0.164	−0.012
11	0.190	0.210	−0.020
12	0.172	0.197	−0.025
			$\sum d_i = -0.158$

These are **Dependent Samples** because: same person

Create one **New Sample of *Differences*** then run a Matched Pair Test (dependent variables).

Symbols:

True mean of the differences = μ_d

Sample standard deviation of the differences = s_d

Sample size = n $(= n_1 = n_2)$

$\boxed{d = X - Y}$ $\boxed{\mu_d = \frac{\sum d}{n}}$

$\bar{d}$ = sample difference

Recall the logic: Say $H_0: \mu_d = 0$ and $H_1: \mu_d < 0$. If we get…

- $\bar{d} = 0$ we ___FTR___ H_0
- $\bar{d} > 0$ we ___FTR___ H_0
- $\bar{d} < 0$ by "a little", we ___FTR___ H_0
- $\bar{d} < 0$ by "a lot", we ___REJECT___ H_0

Steps for Hypothesis Test when Applied to testing μ_d

Pre-Step: *Check Requirements*

- Samples are dependent and simple random
- Differences are **normal** or $\boldsymbol{n > 30}$

Step 1: *Hypotheses*

H_0: $\mu_d = 0$

H_1: $\mu_d < 0$ or $\mu_d > 0$ or $\mu_d \neq 0$

Step 2: *Level of Significance*

α (0.05 if not given)

Step 3: *Test Statistic* ***(note: it can be negative!)***
Find a z-score, t-value, X^2 value or F-value

$O^* = \dfrac{\text{sample} - \text{parameter}}{\text{sampling st. dev.}}$

Compare w/ one mean

$t^* = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$

$$\boxed{t^* = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}}$$

Compare w/ two indep. means

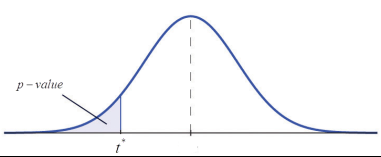
$t^* = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

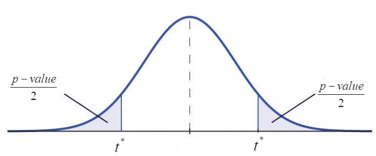
Step 4: *Find a Critical Value or P-Value*

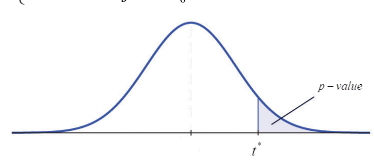
P-VALUE METHOD (t^*)

DECISION

Reject H_0 ~ if *P-value* $\leq \alpha$
Fail to Reject H_0 ~ if *P-value* $> \alpha$



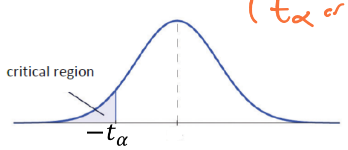


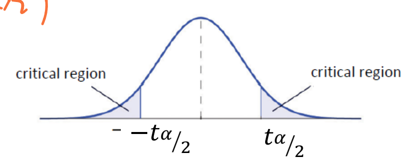


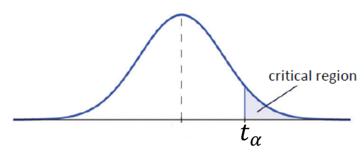
CRITICAL VALUE METHOD (t_α or $t_{\alpha/2}$)

DECISION

Reject H_0 ~ if t^* lies in the critical region
Fail to Reject H_0 ~ if t^* doesn't lie in the critical region







Step 5: Write a *CONCLUSION* either rejecting or failing to reject H_0

GRAPHING CALCULATOR (TI-83 OR 84)

huh? ok

Warning TTest is for Matched Pairs

Instructions: (a) $\boxed{\text{STAT} \Rightarrow \text{TESTS} \Rightarrow \text{T-Test}}$

(b) Enter

- μ_0 = population mean stated in H_0 (μ_d)
- s = sample standard deviation (s_d)
- $\bar{x}$ = sample mean $(\bar{d})$
- n = sample size
- μ ~ alternative hypothesis

Ex: It is a commonly held belief that Crossovers are safer than small cars. If a Crossover and small car are in a collision, does the Crossover sustain less damage (as suggested by the cost of repair)? Consumer Reports crashed Crossovers into small cars, with the Crossover moving 15 miles per hour and the front of the Crossover crashing into the rear of the small car. The data is normally distributed. Below are the repair costs:

	Crossover into Car	Small Car Damage (x_1)	Crossover Damage (x_2)	Difference (d) $d = x_1 - x_2$
1	Lexus RX-350 into Honda Insight	1274	1721	-447
2	Nissan Pathfinder into Hyundai Elantra	2327	1434	893
3	Toyota RAV4 into Kia Forte	3223	850	2373
4	Jeep Cherokee into Kia Niro Hybrid	2058	2329	-271
5	Ford Explorer into Toyota Camry	3095	1415	1680
6	Honda CR-V into Ford Focus	3386	1470	1916
7	Chevrolet Equinox into Nissan Sentra	4560	2884	1676

compute $\bar{d}$ (mean of sample diff) 1-VAR STATS

Dependent b/c they crash into each other (linked)

$\sum d =$

Do the sample data suggest that Crossovers are safer? Use the level of significance $\alpha = 0.01$.

(same $\mu_d = 0$, cross repairs less $\mu_d > 0$)

x_2 - repair cost for Crossovers

x_1 - repair cost for Small

$d = x_1 - x_2$

μ_d = average diff.

$d > 0$: when $x_1 > x_2$ (repair small car bigger than repair cost crossover)

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu_d = 0 \\ H_A: \mu_d > 0 \end{cases}$$ (Right Tailed Test)

$\bar{d} = 1117.1$

$s_d = 1100.6$

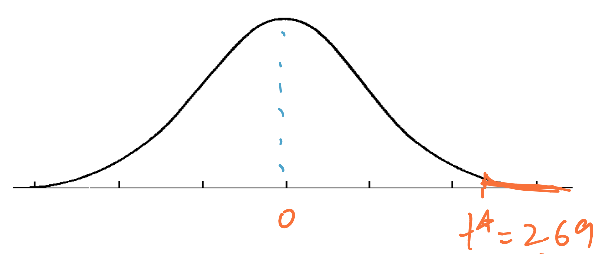
$n = 7$

Test Statistic

$$t^* = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{(1117.1 - 0)}{\left(\frac{1100.7}{\sqrt{7}}\right)} = \boxed{2.69}$$

P-value/Critical Region

$$P = P(t > 2.69)$$
$$= 0.5 - t\,cdf(\overset{a}{0}, \overset{b}{2.69}, \overset{df}{6})$$
$$\boxed{P = 0.0180}$$



Decision about Null Hypothesis

$\alpha = 0.01$

$P = 0.018$

$P > \alpha \longrightarrow$ P high, null fly

$\boxed{\text{Fail to Reject } H_0}$

Conclusion

"There isn't enough evidence to support the claim that the Crossovers sustain less true mean damage costs than small cars while crashing into each other (dependent) while moving at 15 mph."

Confidence Interval for the Mean Difference from Dependent Samples

Alternative Forms: $\bar{d} - E < \mu_d < \bar{d} + E$ or $\bar{d} \pm E$

CI: $(\bar{d} - E, \bar{d} + E)$

Calculator use $\boxed{\text{T Interval}}$

Same as CI for one mean (Ch 8.2) but use instead $\bar{d}$ & S_d

where the margin of error is given by $\boxed{E = t_{\alpha/2} \cdot \dfrac{s_d}{\sqrt{n}}}$

Ex: A company claims that its 12-week special exercise program significantly reduces weight. A random sample of eight persons was selected, and the following table gives the weights (in lbs) of those eight persons before and after the program.

• Enter

dependent

	Weight (in pounds)								
Before x_1	180	195	177	221	208	199	148	230	L_1
After x_2	185	187	171	214	208	194	150	227	L_2

$L_3 = d$
$= x_1 - x_2$

1-Var Stats (L_3)
$\bar{d} = 2.75$
$S_d = 4.65$

Construct a 90% confidence interval for the mean before-after differences.

Find point estimate (sample mean)

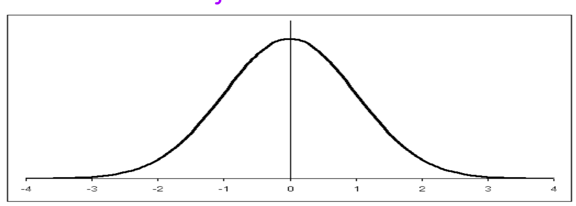
point estimate (sample) : $\boxed{\bar{d} = 2.8 \text{ lbs/person}}$

$d = x_1 - x_2$
before − after
is weight loss: $d > 0$
no weight loss: $d < 0$ (weight gain)

Determine critical value $t_{\alpha/2}$

$t_{\alpha/2} = \text{invT}(\underbrace{1-0.05}_{1-\alpha/2}, \underbrace{7}_{df})$

$\boxed{t_{\alpha/2} = 1.89}$



CL = 0.9
$\alpha = 0.1$
$\alpha/2 = 0.05$

Find margin of error

$E = t_{\alpha/2} \cdot \dfrac{S_d}{\sqrt{n}}$

$E = 1.89 * \dfrac{4.65}{\sqrt{8}} = 3.107\ldots$ $\boxed{E = 3.1 \text{ lbs/person}}$

Construct confidence interval

CI: $(\bar{d} - E, \bar{d} + E)$

$\boxed{\text{CI: } (-0.3, 5.9)}$

Does it appear that the weight loss program is effective?

- We would predict w/ 90% confidence either no weight loss or slight weight loss (5.9 lbs).
- (Opinion: 6 lbs over 12 weeks is not significant)