Section 10.4 Systems of Nonlinear Equations

Objectives
- Substitution Method for Nonlinear SOE
- Elimination Method for Nonlinear SOE
- Graphing Method for Nonlinear SOE
- Number of solutions of Nonlinear SOE

• Nonlinear SOEs

Defn 1

A **system of nonlinear equations** is a set of equations that involve the same variables and at least one variable has either an exponent greater than two or is composed with a transcendental function.
The **soluton set** to a nonlinear SOE is the set of all points that solve each equation simultaneously.

↳ Ex: $\{(-1,2)\}$ means $x=-1$, $y=2$ 3 variables $x=1, y=2, z=3$ write $\{(1,2,3)\}$.

Reminder: we write a solution set in the following way:

How to solve Nonlinear SOEs There are three methods for solving Nonlinear SOEs:
1. Substitution Method
2. Elimination Method
3. Graphing Method

Note Please review §10.1 and 10.2 if you don't remember how to solve linear equations using these methods.

Remarks
- It's important to know all 3 methods since usually one method is easier than the others.
- Moreover, method 3 (graphing method) allows us to solve nonlinear SOEs that are impossible to give exact answers to by estimating their intersection points using technology such as Desmos.

How to spot "no solutions" How can you spot there are no solutions without access to a graph?
- By arriving at **contradictions** (like $0=3$ or $2=1$ etc).
  For example, $\begin{cases} x+y &= 1 \\ x+y &= 0 \end{cases}$
  ↑ not true statements

substitution

① $x = 1-y$

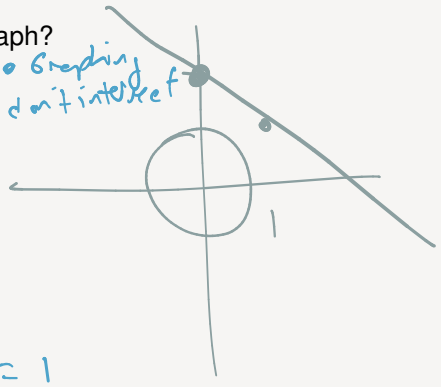
sub into ②: $(1-y)+y=0$

$1=0$

contradiction!

absurdité

- By getting complex solutions.
  For example, $\begin{cases} x^2+y^2 &= 1 \\ x+y &= 2 \end{cases}$



Use substitution:

② $y = 2-x$

sub into ①: $x^2 + (2-x)^2 = 1$

$x^2 + 4 - 4x + x^2 = 1$

$2x^2 - 4x + 3 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{4 \pm \sqrt{16-24}}{4} = \frac{4 \pm \sqrt{-8}}{4}$$

imaginary #

} 2 complex sol

• Substitution Method

Substitution Method Steps to solve a SOE using the substitution method:

1. Solve for one variable
2. Substitute
3. Back-substitute
4. Check your answers

Ex 1 Substutuion Method Solve the following nonlinear SOE using the substitution method:

$$\begin{cases} x^2 + y^2 = 100 \\ x - 2y = -10 \end{cases}$$

① $x - 2y = -10$

$x = 2y - 10$

② $(2y-10)^2 + y^2 = 100$

$4y^2 - 20y + 100 + y^2 = 100$

$5y^2 - 20y = 0$

$5y(y-4) = 0$

$y = 0$ or $y = 4$

③ Back substitute $x = 2y - 10$

$y = 0$: $x = 2 \cdot 0 - 10 = -10$ $(-10, 0)$ (x, y)

$y = 4$: $x = 2 \cdot 4 - 10 = -2$ $(-2, 4)$ (x, y)

solution set $\{(-10,0), (-2,4)\}$

~~$\{(0,-10), (4,-2)\}$~~ wrong way!

• Elimination Method

Elimination Method Steps to solve a SOE using the elimination method:

1. Combine the equations to eliminate one of the variables
2. Solve the remaining equation of one variable
3. Back-substitute
4. Check your answers

Ex 2 Elimination Method Solve the following nonlinear SOE using the elimination method:

$$\begin{cases} x^2 + y^2 = 16 \\ x - y^2 = -4 \end{cases}$$

① Combine ① + ②:

$x^2 + y^2 = 16$

$+ \quad x - y^2 = -4$

$x^2 + x = 12$

② $x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4$ or $x = 3$

③ plug into $x - y^2 = -4$

$x = -4$	$x = 3$
$-4 - y^2 = -4$	$3 - y^2 = -4$
$-y^2 = 0$	$7 - y^2 = 0$
$y = \pm\sqrt{0} = 0$	$y^2 = 7$
	$y = \pm\sqrt{7}$

solution set

$\{(-4, 0), (3, \sqrt{7}), (3, -\sqrt{7})\}$

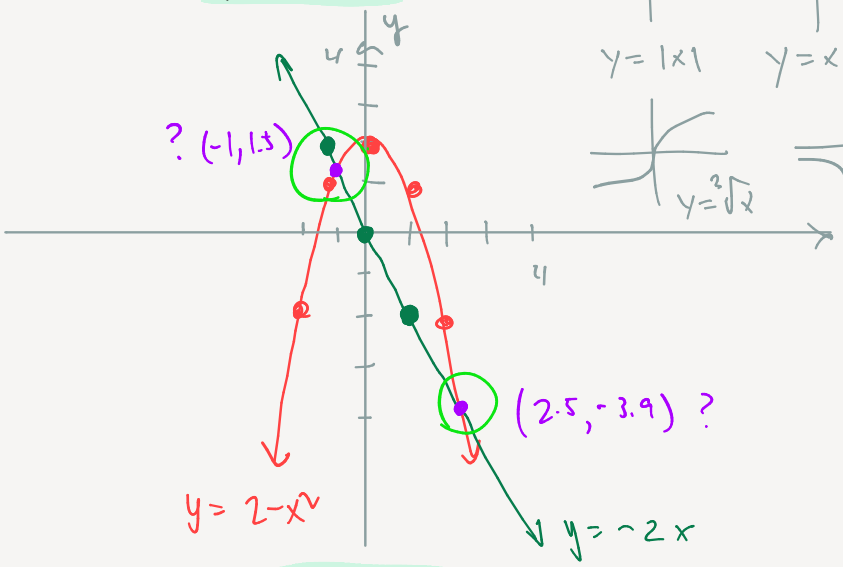
• Graphing Method

Graphing Method Steps to solve a SOE using the graphing method:

1. Sketch the graph of each equation on the same coordinate plane
2. Find, or approximate, the points where they intersect
3. Check your answers

Ex 3 Graphing Method Solve the following nonlinear SOE using the graphing method:

$$\begin{cases} 2x + y &= 0 \\ x^2 + y &= 2 \end{cases}$$



① $2x + y = 0$

$y = -2x$ "y =" notation

② $x^2 + y = 2$

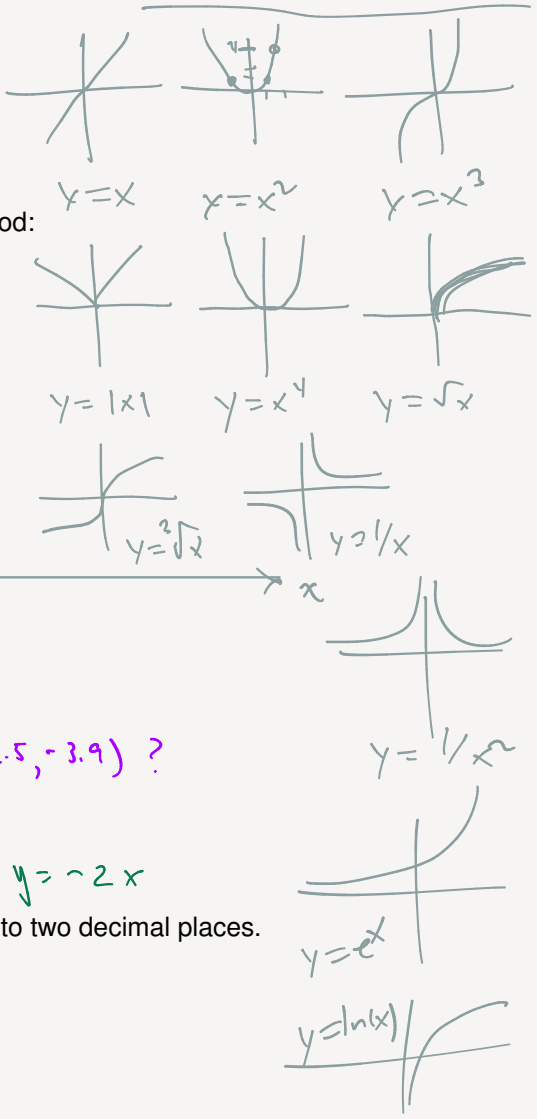
$y = 2 - x^2$

Solve algebraically: $-2x = 2 - x^2$

$x^2 - 2x - 2 = 0$

doesn't factor!

10 Memorable Functions

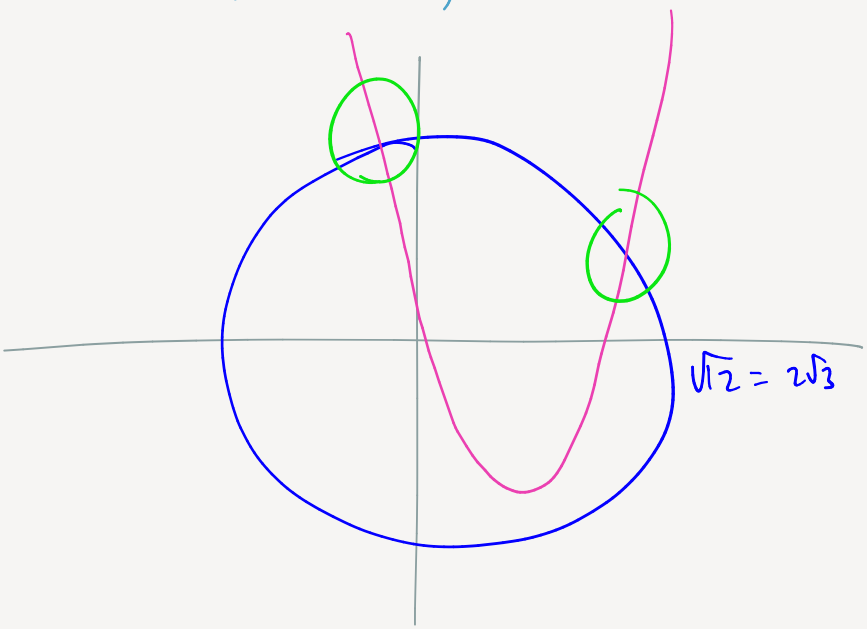


Ex 4 Graphing Method Solve the following nonlinear SOE using Desmos, rounded to two decimal places.

$$\begin{cases} y = 2x^2 - 5x \\ x^2 + y^2 = 12 \end{cases}$$

You don't need to find the exact solutions.

$\{(-0.56, 3.42), (2.85, 1.97)\}$



• Number of Solutions to Nonlinear SOEs

Recall that with linear equations we only have **three possibilities** for the number of solutions:

Number of Solutions of LINEAR SOEs When we study linear SOEs in two variables, there are only 3 possibilities for the solution set.

Case 1: There's only one solution. ..... solution set is a single point
Case 2: There's Infinitely many solutions. ..... solution set is an entire line
Case 3: No solutions ..... solution set is empty; parallel lines

The reason is because "linear equations" do not bend or curve, they are straight or flat.

Ex $\begin{cases} x+y=1 \\ 2x+2y=2 \end{cases}$ solution set $\{(x,y) \mid x \in \mathbb{R},\ x+y=1\}$

another way: $\{(x, 1-x) \mid x \in \mathbb{R}\}$

HOWEVER, **NONLINEAR** equations **CAN** bend or curve!

This means there are all sorts of possibilities for the number of solutions.

Number of Solutions of NONlinear SOEs There can be **ANY** number of solutions to systems of **NON**linear equations!

Ex 5 Graphing Method You can check using Desmos that the following nonlinear SOE

$$\begin{cases} 10\sin(x) - 3y &= 2 \\ x^4 + y^5 - x^3y^3 &= 1 \end{cases}$$

has 26 solutions.

Ex 6 Graphing Method You can check using Desmos that the following nonlinear SOE

$$\begin{cases} y = x\cos(x) \\ x = e^y \end{cases}$$

has infinitely many solutions without being the same equations (unlike two linear equations).