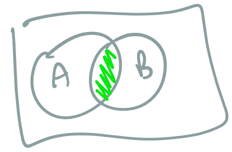
Chapter 5: Probability

Section 5.3: Independence and Multiplicative Rule



sum rule

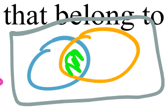
Compound Events

Previously: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Def A **compound event** is any event combining two or more simple events.

Notation: $P(A \text{ and } B)$ can denote two different things:

- It is the outcomes that belong to both A and B, that is in the intersection of both. Use this when making "one selection"



A and B overlap / intersection

- It also is used when making "two selections": it is the outcomes in A in the 1_{st} trial followed by the outcomes in B in the 2_{nd} trial

No venn Diagram! $P(A \text{ and } B) = P(A \text{ first selection and } B \text{ second selection})$

Independence vs. Dependence

Def Two events are **independent** if the occurrence of one event does not affect the probability of the occurrence of the other event. Ex: flipping coin repeatedly, dice rolls, …

Def If two events are not independent, they are said to be **dependent**. (next section)

Ex: **Independent events? Or Dependent events? Why or why not?**

(a) $A =$ You find a parking spot $B =$ First week of school dependent ;	(b) $C =$ You pass your class $D =$ Your mom is a good cook independent

Multiplication Rule for Independent Events	
Symbolic	$P(A \text{ and } B) = P(A) * P(B)$
Meaning	*When two events are independent, the probability of event A followed by event B is found by multiplying the probability of event A by the probability of event B.*

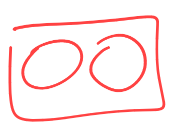
Explanation of Formula

A and B

$S = \{HH, HT, TH, TT\}$ $\#S = 4$

Let S be the sample space of flipping a coin twice. Let A be the event of H on first flip and B the be even of H on second flip. Then P(H on 1_{st} and H on 2_{nd}) = $P(A \text{ and } B) = \frac{\# A \text{ and } B}{\# S} = \frac{1}{4}$. Now, $P(A) = \frac{2}{4} = \frac{1}{2}$ and $P(B) = \frac{2}{4} = \frac{1}{2}$ so, $P(A)P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ This example show why the formula works. We will prove a more general version in the next section.

Disjoint Events vs Independent Events



- When making one selection, these do mean the same thing.
- When making two or more selections, these do NOT mean the same thing.

next section

Ex: Consider rolling a single dice. Let A = "roll an even" and B = "roll an odd". We know these are disjoint events and $P(A) = 0.5$ and $P(B) = 0.5$. If we roll a dice twice and we know A has occurred then the probability of B occurring changes from 0.5 to 0!

FORMULAS

1. When making **two selections**: if two events are independent, then $\boxed{P(\overset{1st}{A} \text{ and } \overset{2nd}{B}) = P(A) * P(B)}$

2. When making **one selection**: if two events are independent or dependent, then $\boxed{P(A \text{ and } B) = \frac{\# A \text{ and } B}{\# S}}$

Ex: About 13% of the population is left-handed.

(a) Explain why selecting multiple people from population may be considered independent.

population is huge! so randomly selecting people is independent for probabilities.

(b) If two people are randomly selected, what is the probability that both are left-handed?

$P(\text{one LH}) = 0.13$ convert 13% to a decimal b/c probabilities are between 0 & 1.

view as independent

$P(1^{st} \text{ LH and } 2^{nd} \text{ LH}) = P(LH) * P(LH) = 0.13 * 0.13 = (0.13)^2 = \boxed{0.0169}$

(c) If five people are randomly selected, what is the probability that all five are left-handed?

$P(\overset{1st}{LH} \text{ and } \overset{2nd}{LH} \text{ and } \overset{3rd}{LH} \text{ and } \overset{4th}{LH} \text{ and } \overset{5th}{LH})$

independent

$= P(LH) * P(LH) * P(LH) * P(LH) * P(LH) = (0.13)^5$

$= 3.71 \text{ E} -5 = 3.71 \times 10^{-5} = \boxed{0.0000371}$ Rounding Rule (3 sig figs)

GTQ

Ex: A bag contains an assortment of Jolly Rancher candies. Specifically, there are 5 apple, 8 watermelon, 10 cherry, and 15 grape flavored candies. You get to randomly select three candies *with replacement*.

↳ put candies back after each selection.

↳ independent events

$5 + 8 + 10 + 15 = 38$ candies

(a) Find the probability of picking **three** grape Jolly Ranchers.

$P(\overset{1st}{G} \text{ and } \overset{2nd}{G} \text{ and } \overset{3rd}{G}) = P(G) * P(G) * P(G)$

$= \frac{15}{38} * \frac{15}{38} * \frac{15}{38}$

$= \left(\frac{15}{38}\right)^3 = \boxed{0.0615}$

(b) Find the probability of **not** getting any apple Jolly Ranchers.

$P(\underset{A^c}{\text{not } A} \text{ and } \underset{A^c}{\text{not } A} \text{ and } \underset{A^c}{\text{not } A}) = P(\text{not } A)^3 = \left(\frac{33}{38}\right)^3 = \boxed{0.655}$

another way: $= [1 - P(A)]^3 = \left[1 - \frac{5}{38}\right]^3$

COMPLEMENTS: THE PROBABILITY OF "AT LEAST ONE"

We use some basic logic to come up with a simple way to compute the probability of events with "at least one …". If we let A = { none of … } then the complement of A, A^c, means not A. But, negative nothing is the same thing as "at least one" must be true. Formally, we use the complement rule: $P(A^c) = 1 - P(A)$

Really useful! Efficient

Note: complement Rule $P(A^c) = 1 - P(A) \Rightarrow \boxed{P(\text{at least one}) = 1 - P(\text{none})}$

Ex: Let's say that for the next seven days, the probability of rain is 5%. Assume the chance of rain each day is independent. What is the probability that it rains ***at least one day*** over the next seven days.

Ridiculously LONG way…

P(at least one day of rain)

= **P(** {exactly 1 day R} or {exactly 2 day R} or … or {exactly 7 day R} **)**

= **P**(1 R) + **P**(2 R) + … + **P**(7 R)

lots of possibilites

$= [P(RNNNNNN) + P(NRNNNNN)$ + $P(NNRNNNN) + \dots + P(NNNNNNR)]$

Rain on first day, rain second day, rain on third

$= [P(RRNNNNN) + P(NRRNNNN) + \dots]$ + …

more than 7 ways

The Sane Way…

What's the ***complement*** *of at least one day of rain?*

$P(\text{at least one } R) = 1 - P(\text{no R on 7 days})$

$= 1 - P(NNNNNNN)$

and and and ….

$= 1 - \underbrace{P(N)\cdot P(N)\dots P(N)}_{7\text{ times}}$

$= 1 - P(N)^7$

$= 1 - 0.95^7 = \boxed{0.302}$

30.2%

GTQ

Ex: A bag contains an assortment of Jolly Rancher candies. Specifically, there are 5 apple, 8 watermelon, 10 cherry, and 15 grape flavored candies. You get to randomly select three candies ~~without~~ with *replacement*. Find the probability of getting **at least one** watermelon.

with → independent

38 candies
8 watermelon
30 no watermelon

3 selections

$P(\text{at least one } W) = 1 - P(\text{no } W)$

$= 1 - \left(\frac{30}{38}\right)^3$

$= \boxed{0.508}$

one "threat detected" $P(T) = 0.92$

one "no threat detected" $P(T^c) = 1 - 0.92 = 0.08$

Ex: A satellite defense system has five independent satellites that each have a 0.92 chance of detecting a missile threat.

(a) What's the probability that **at least** one satellite does detect a missile threat?

5 select

$P(\text{at least one } T) = 1 - P(T^c)$

$= 1 - 0.08^5$

$= 0.9999\dots = \boxed{1.00}$

(b) What's the probability that **at least** one satellite does not detect a missile threat?

5 select

$P(\text{at least one } T^c) = 1 - P(T^{c^c})$

$= 1 - P(T)$

$= 1 - 0.92^5$

$= \boxed{0.341}$