Section 10.3: Hypothesis Test for a Population Mean

MR. T IS MEAN
• CI • HT

GOAL: Make a decision about μ based on $\bar{x}$ using probability (sampling distribution).

HYPOTHESIS TESTING: CLAIM ABOUT A MEAN (σ NOT KNOWN)

Requirements

1. The sample observations are a simple random sample.
2. The value of the population standard deviation σ is not known. population (but we do know sample st. dev s)
3. Either or both of the given conditions are satisfied: The population is normally distributed or $n > 30$

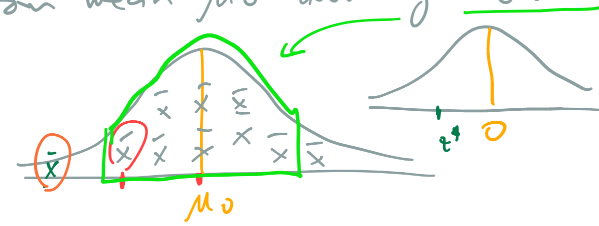
MUST CHECK

New Notation: "past (population) mean." μ_0

Ex1: Recall the logic behind a hypothesis test: Say $H_0: \mu = 10$ and $H_A: \mu < 10$ (now; μ_0 past mean; alternative hyp.)

If we take a sample and find the point estimate… ($\bar{x}$)

- $\bar{x} = 10$ then we Fail to Reject H_0
- $\bar{x} < 10$ by "a little", then we Fail to Reject H_0
- $\bar{x} > 10$ then we Fail to Reject H_0
- $\bar{x} < 10$ by "a lot", then we REJECT H_0
  - ie support H_A

Steps for *a* Hypothesis Test When Applied to Testing Population Mean μ (σ unknown)	
Step 0: *Check Requirements* • It is a valid SRS sample • The requirements are met to use the needed distribution:	• σ unknown • distribution is normal OR $n > 30$ (or both)
Step 1: *Hypotheses* now, past: H_0: μ = μ_0 H_A: $\mu < \mu_0$ or $\mu \neq \mu_0$ or $\mu > \mu_0$	***Step 2:*** *Level of Significance* $\alpha = P(\text{Type I Error})$ If not given, we ASSUME $\alpha = 0.05$
Step 3: *Test Statistic* (Find a ~~z-score~~, t-value or ~~χ^2 — value~~) $t^* = \dfrac{\bar{x} - \mu_0}{\sigma_{\mu_0}} = \boxed{t^* = \dfrac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}}$ • how many standard deviations away from mean μ_0 assuming H_0 is TRUE 	
Step 4: *Find a critical value or P-value to check using either the Critical Value Method OR P-value Method.*	
Step 5: *Make a **decision** AND draw a **conclusion***	

↳ "support H_A" or "do not support H_A"

Null and Alternative Hypothesis

Left-tailed	Two-tailed	Right-tailed
$H_0: p = p_0$	$H_0: p = p_0$	$H_0: p = p_0$
$H_A: p < p_0$	$H_A: p \neq p_0$	$H_A: p > p_0$

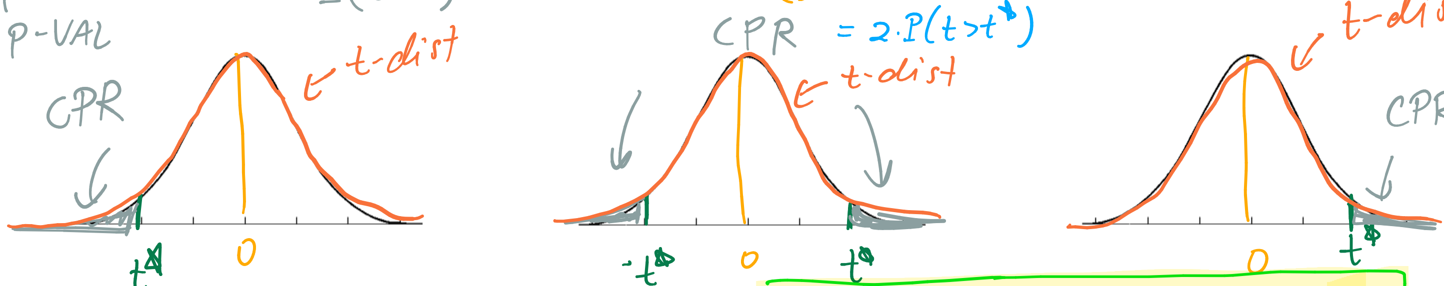
P-Value Method

Def **P-Value**: *probability that a sample is as extreme as our test statistic or more extreme assuming* H_0 *is true.*

Key: *use test statistic* t^* *to draw the* **critical region (CPR)** *and compute probability of it.*

P-VAL $\rightarrow$ $P = P(t < t^*) = P(CPR)$ $\qquad$ $P = P(t < -t^* \text{ or } t > t^*)$ $\qquad$ $P = P(t > t^*)$

CPR $= 2 \cdot P(t > t^*)$



HOW TO COMPUTE PROBABILITY? $\quad$ USE NEW $\quad$ Probability = tcdf(a b df)

n−1

Make a Decision: $\begin{cases} \text{If } P \leq \alpha, \text{then we "Reject } H_0\text{"} \\ \text{If } P > \alpha, \text{then we "Fail to Reject } H_0\text{"} \end{cases}$

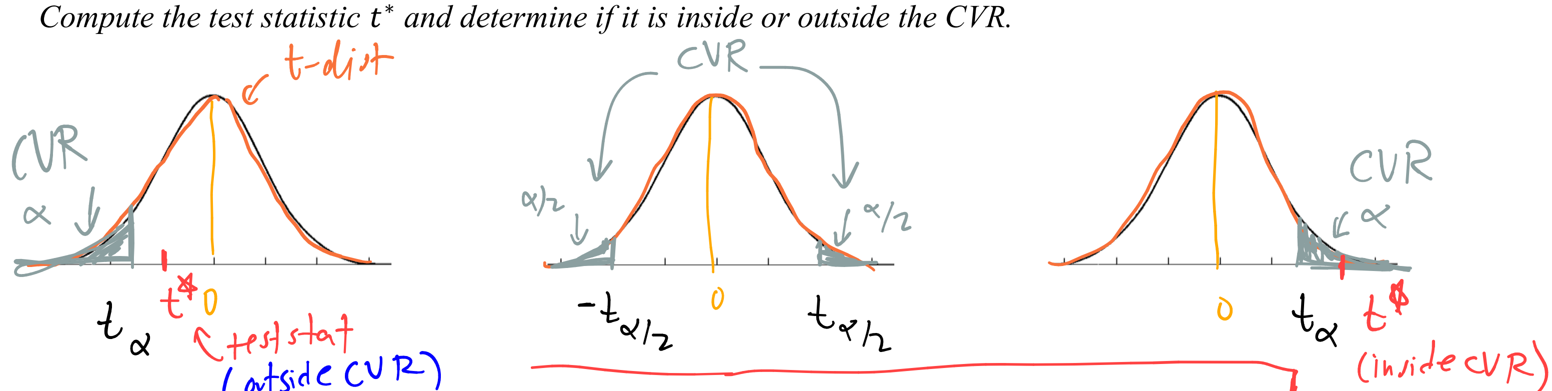
If P low, Null GO!
If P high, Null fly!

Critical Value Method

one tailed test $\quad$ two tailed test

Key: *Use the level of significance,* α*, to compute the critical value* t_α *or* $t_{\alpha/2}$ *(depending on one or two-tailed test) which determines the* **critical region (CVR)**.

Compute the test statistic t^* *and determine if it is inside or outside the CVR.*



Make a Decision: $\begin{cases} \text{If } t^* \text{ is INSIDE the CVR, then we "}\textbf{Reject } \mathbf{H_0}\text{"} \\ \text{If } t^* \text{ is OUTSIDE the CVR, then we "}\textbf{Fail to Reject } \mathbf{H_0}\text{"} \end{cases}$

$t_\alpha = \text{invT}(\alpha \text{ or } \alpha/2, df)$

past

$n = 40 \quad df = n - 1 = 39$

$\bar{x}$

Ex2: Kaiser Foundation hospital claims that the mean waiting time for patients to be seen in the emergency room is 20 minutes. A random sample of 40 patients produced a mean waiting time of 18.5 minutes and a standard deviation of 4.0 minutes. Use a 0.10 significance level to $\boxed{\text{test the claim}}$ that the mean waiting time is equal to 20 minutes. Use the __P-VALUE__ method.

s $\quad \alpha \quad$ HT

↳ changed $\Rightarrow \neq$

Check requirements

(1) SRS ✓ (2) σ unknown ✓ (3) normal or $n > 30$ ✓ (likely)

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu = 20 \text{ min} & (\mu_0 = 20 \text{ min}) \\ H_A: \mu \neq 20 \text{ min} & (\text{Two Tailed Test}) \end{cases}$$

Test Statistic

$$t^* = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{18.5 - 20}{\left(\frac{4.0}{\sqrt{40}}\right)} = \boxed{-2.37}$$

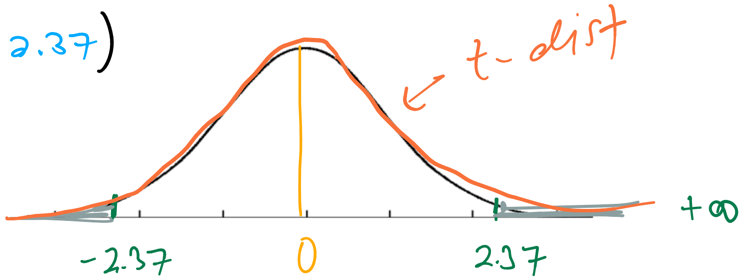
P-value/Critical Region

$$P(CPR) = P(t < -2.37 \text{ or } t > 2.37)$$

$$= 2 \cdot P(t > 2.37)$$

$$= 2 \cdot tcdf(\underset{a}{2.37}, \underset{b}{1E99}, \underset{df}{39})$$

$$\boxed{PVal = 0.0228}$$



Decision about Null Hypothesis

- $P = 0.0228$ $\quad \alpha > P \longrightarrow$ P is low, Null must go! $\boxed{\text{Reject } H_0}$
- $\alpha = 0.1$

Conclusion

"There is enough statistical evidence to support the claim that the mean waiting time is different than 20 minutes long at Kaiser Hospitals."

Identify the Type I error

$\alpha = P(\text{Type I Error})$ $\quad$ Type I Error = $\underbrace{\text{Reject } H_0}_{\mu \neq 20} \mid \underbrace{H_0 \text{ is T}}_{\mu = 20 \text{ min}}$

↳ conclude μ is not 20 when in reality it is 20 min.

Identify the Type II error

$\beta = P(\text{Type II Error})$ $\quad$ Type II Error = $\underbrace{\text{Fail to Reject } H_0}_{\mu = 20} \mid \underbrace{H_0 \text{ is F}}_{\mu \neq 20}$

↳ conclude μ is 20 when in reality is not 20 min.

conclude \ reality	H_0 T	H_0 F
FTRHo		II
RHo	I	

Ex3: According to the Bureau of Labor Statistics, the mean amount of money spent by a household on alcohol in the US is \$565 per year. A church group wants to check this claim and took a random sample of 45 households and found that mean amount spent on alcohol per year was \$520 with a standard deviation of \$167. Test the church group's claim that the mean amount of money spent on alcohol per year is less than \$565. Use the <u>Critical Value</u> method.

past $\bar{x}$ n s HT

parameter? mean

Check requirements

① SRS ✓ ② Indep. ✓ ③ normally dist OR $n>30$? ✓
(likely to ~ not given)

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu = \$565/yr \quad (\mu_0) \\ H_A: \mu < \$565/yr \quad \text{(Left-Tailed Test)} \end{cases}$$

Test Statistic

$n=45$
$\bar{x}=520$
$s=167$

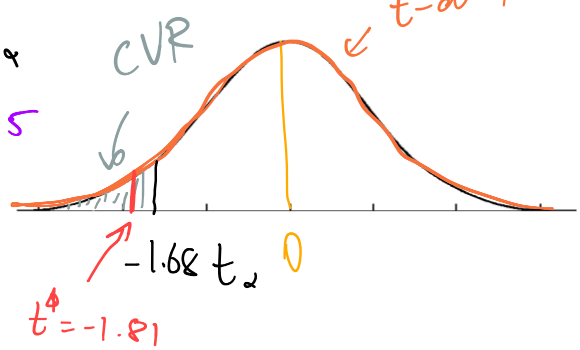
$$t^* = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}} = \frac{520-565}{\frac{167}{\sqrt{45}}} = \boxed{-1.81}$$

~~P-value~~/Critical Region

CVR Left-tailed test → Use t_α

Level of Significance? $\alpha = ?$ Use $\alpha = 0.05$

$$t_\alpha = t_{0.05} = \text{invT}(\underset{\alpha}{0.05}, \underset{df}{44}) = \boxed{-1.68}$$



Decision about Null Hypothesis

t^* is INSIDE CVR ⟶ Decision: Reject H_0!

Conclusion

" There is enough statistical evidence to support the claim that the mean amount of money spent on alcohol per year is less than \$565. "

GRAPHING CALCULATOR (TI-83 OR 84)

Instructions: (a) STAT ⇒ TESTS ⇒ T-Test → ~~Data~~ / Stats

calc tells you "t" is test statistic (not critical t_α)

$\alpha = 0.05$
$P = 0.0387$
$P < \alpha$ "P low, null go"
↳ Reject H_0

(b) Enter
μ_0 = population mean stated in H_0 (past)
s = sample standard deviation
$\bar{x}$ = sample mean
n = sample size
μ ◻ alternative hypothesis

tells us test stat & p-val

p-value
$P = 0.0387$

Ex4: The Instagram handle @getfollowers claims that they can increase the number of followers someone has on Instagram. In March 2015 the mean number of followers for a US teen was 150, so a random sample of 12 US teens with 150 followers was taken. The following is the number of followers, which is normally distributed, these US teens had after they paid @getfollowers for their help. Using a 0.01 level of significance, determine whether @getfollowers is effective at increasing the number of Instagram followers.

μ_0

n

Use the P-Val method.

Parameter: mean $\bar{x} = 158.9$ $s = 14.6$ $n = 12$

Use 1-VAR STATS

160	200	152	150	145	151	162	158	156	149	154	170

→ use to find $\bar{x}$ & s

Check requirements

① SRS ✓ ② Indep ✓ ③ normal ✓ or ~~n≥30~~?

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu = 150 \text{ followers } (\mu_0) \\ H_A: \mu > 150 \text{ followers (Right Tailed Test)} \end{cases}$$

Test Statistic

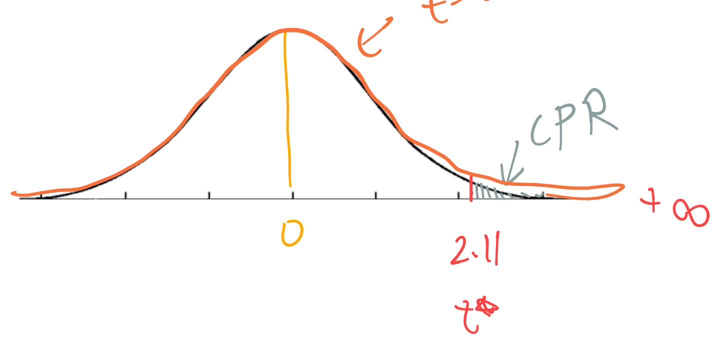
$$t^* = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{158.9 - 150}{\frac{14.6}{\sqrt{12}}} = \boxed{2.11}$$

P-value/Critical Region

P-Val: $P = P(\text{CPR})$

$P = \text{tcdf}(\overset{a}{2.11}, \overset{b}{1E99}, \overset{df}{11})$

$$\boxed{P = 0.0293}$$



Decision about Null Hypothesis

$\alpha = 0.01$

$P = 0.0293$

$P > \alpha \longrightarrow$ "P is high, Null fly" → "Fail to Reject H_0"

Conclusion

"There is NOT enough statistical evidence to support the claim that @getfollowers is effective at increasing the number of Insta followers among US Teens."

Logic of Hypothesis Tests (Cover this at the end of the chapter)

Mean

We now explain how Hypothesis Tests work. It starts with a sampling distribution. Recall that we have a past claim, μ_0. We want to know if it has changed so we find a new sample, $\bar{x}$, of size n. Then we know what the sampling distribution should look like assuming the null hypothesis, $\mu = \mu_0$, is true. In Chapter 8 we learned that the sampling distribution is a normal distribution with $\mu_{\mu_0} = \mu_0$ and $\sigma_{\mu_0} = \frac{\sigma}{\sqrt{n}}$.

But if we don't know the population mean, then we likely don't know the population standard deviation as well. So we want to use the sample standard deviation, s, instead. To do this, we can't use the normal distribution but instead use the t-distribution.

The rest of the story is essentially the same as we discussed before.