SARAH • LAWRENCE • COLLEGE

HHW# 3 - Final FALL 2016

Math 3005. Calculus 1. Jorge Eduardo Basilio

Posted on Thurs 12/21

DIRECTIONS

A total of 250 points are possible on this assignment, broken down by:
Part 1: 40 points
Part 2: 150 points
Part 3: 60 points

To earn a passing score (C) on this exam, please solve enough problems in the corresponding parts to guarantee scores of at least:

Part 1: 30 points
Part 2: 50 points
Part 3: 20 points

To guarantee an A, please solve enough problems to earn at least 150 points.

- All work turned in must be NEAT, ORGANIZED, AND STAPLED. Handwriting should be neat and legible.
- Also, it must have a cover page with the following information: the Course Name, Section, Instructor's name (me!), Date Due, name of the assignment and your name all neatly and clearly marked. In addition, write the following statement and sign-your name under it: *"I swear to neither receive nor give any unauthorized assistance on this assignment."*
- It must be written on blank or lined 8.5" by 11" sized paper using only the front side of the page.
- It can be written in pen or pencil. If written in pen, mistakes must be neatly crossed out, or erased with white-out.
- Be sure to label each problem carefully and staple work sequentially.
- Be sure to show all work on every problem.
- The level of "polish" should be very high, just like it would be when turning in an essay in an English class. You should work out solutions on scratch paper before you write your final draft. When applicable, write in complete sentences.
- This assignment is open book and open notes (from seminar, RPs, or previous HHWs), however, you are to work entirely on your own (or you may ask me questions). Consulting other people, other textbooks or the internet is strictly forbidden.

DUE TUESDAY, DECEMBER 20, 5:00 PM

Please turn in in digitally to your Google drive. If you scan it, please convert it to a single PDF file in the correct order. Thanks!

PART I: Definitions, Theorems, and Facts

1. [5] If a differentiable function $f(x)$ satisfies $f(0) = f(5)$, then which of the following must be true?

(A) There exists c in $(0, 5)$ with $f'(c) < 0$

(B) There exists c in $(0, 5)$ with $f(c) = 0$

(C) There exists c in $(0, 5)$ with $f'(c) > 0$

(D) There exists c in $(0, 5)$ with $f'(c) = 0$

(E) There exists c in $(0, 5)$ with $f''(c) = 0$

2. [10] TRUE or FALSE (Please print the entire word in your answer):

________ (i) If f is continuous on $[a, b]$, then $\int_a^b f(x)\,dx$ is infinitely many functions

________ (ii) If a function is differentiable at a point, it is continuous there

________ (iii) The absolute maximum of a function on a closed interval never occurs at the endpoints

________ (iv) If a function changes from increasing to decreasing at $x = c$ then $f(c)$ is a relative maximum.

________ (v) $\int_1^e \frac{1}{t}\,dt = 1$

________ (vi) The Chain Rule says $\dfrac{df}{dx} = \dfrac{df}{du}\dfrac{du}{dx}$ when u is a function of x and $f(x) = f(u(x))$.

________ (vii) The identity $\ln(e^x) = x$ is true for all x

________ (viii) If the rate of change of the rate of change is increasing, then the function must be increasing.

________ (ix) The anti-derivative of acceleration is the position

________ (x) If $f''(x) > 0$ for all x, then f is concave upward for all x

3. [5] The formula for the n^{th}-lower sum, $S_L(n)$, for the area under the curve $y = f(x)$ is

(A) $\displaystyle\sum_{i=1}^{n} m_i \Delta x$

(B) $\displaystyle\sum_{i=1}^{n} \left(f(x_i) - f(x_{i-1})\right) \Delta x$

(C) $\displaystyle\sum_{i=1}^{n} f(m_i) \Delta x$

(D) $\displaystyle\sum_{i=1}^{n} f(x_i) \Delta x$

(E) $\displaystyle\sum_{i=1}^{n} f(x_{i-1}) \Delta x$

4. [5] Identify the correct statement of the Mean Value Theorem:

(A) You can always find a number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

(B) It is impossible to find a number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

(C) Let f be differentiable on the open intevral (a,b). Then there exists at least one number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

(D) Let f be a continuous on a closed interval $[a,b]$. Then there exists at least one number c in (a,b) such that $f(c) = \dfrac{f(b)-f(a)}{b-a}$

(E) Let f be a continuous on a closed interval $[a,b]$ and differentiable on the open intevral (a,b). Then there exists at least one number c in (a,b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

5. [5] Evaluate: $\dfrac{d}{dx}\left[\dfrac{1}{2\pi}\displaystyle\int_{10}^{x} \dfrac{\ln(\sqrt[3]{t}) + e^{t^5}}{\sqrt{11t^7 - 100}}\,dt\right] =$

6. [5] In order to compute $S_U(n)$, the upper sum approximation to the curve $y = x^3 + 2$ from $x = 0$ to $x = 2$, one must compute:

(A) $\displaystyle\sum_{i=1}^{n}\left(\left(\frac{2}{n}i\right)^3 + 2\right)\left(\frac{2}{n}\right)$

(B) $\displaystyle\sum_{i=1}^{n}\left(\left(\frac{2}{n}(i-1)\right)^3 + 2\right)$

(C) $\displaystyle\sum_{i=1}^{n}\left(\frac{i}{n}\right)^3\left(\frac{2}{n}\right)$

(D) $\displaystyle\sum_{i=1}^{n} i^3\left(\frac{2}{n}\right)$

(E) $\displaystyle\frac{8}{n^3}\sum_{i=1}^{n} i^3$

7. [5] For $f(x) = \cos(x)$, $0 \le x \le 1$, the expression

$$\frac{1}{5}\left(1 + \cos\left(\frac{1}{5}\right) + \cos\left(\frac{2}{5}\right) + \cos\left(\frac{3}{5}\right) + \cos\left(\frac{4}{5}\right)\right)$$

is equal to

(A) $S_U(5)$

(B) $S_L(5)$

(C) $S_U(1/5)$

(D) $\displaystyle\int_0^1 \cos(x)dx$

(E) $\displaystyle\int_0^1 \cos\left(\frac{x}{5}\right)dx$

PART II: Problem Solving

8. [20] Find the following derivatives:

(i) $\dfrac{d}{dx}\left(2x^7 - 11x^4 + 19\right)$

(ii) $\dfrac{d}{dx}(e + x^e + e^x)$.

(iii) $\dfrac{d}{dx}(\sin(3x)e^x)$

(iv) $\dfrac{d}{dx}(\ln(x^2 + x^4 + x^6 + 1))$

9. [10] Use logarithmic differentiation to find the derivative of $y = \ln\left(\dfrac{x^2+1}{x^3(x-1)^2}\right)$.

10. [15] Evaluate the following anti-derivatives:

(i) $\displaystyle\int (2x^3 - 24x^2 - 22x + 13.66)\,dx$

(ii) $\displaystyle\int 3x^2 \sin(x^3 + 1)\,dx$

(iii) $\displaystyle\int \frac{2\pi \cos(\ln(x))}{x}\,dx \quad (x > 0)$

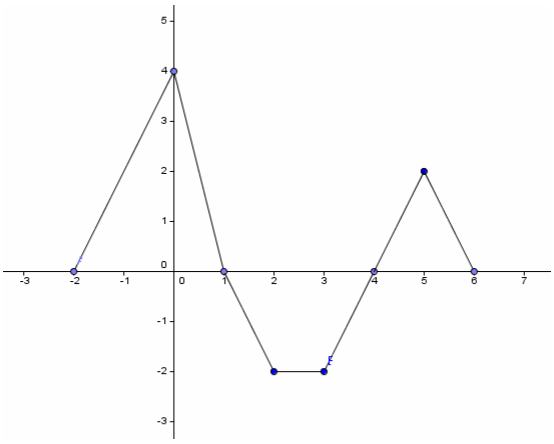
11. [15] Evaluate the following definite integrals:

(i) $\displaystyle\int_{-2}^{2} (x^2 + 1)\,dx$

(ii) $\displaystyle\int_{6}^{6} \frac{x^7}{\sqrt{e^{\sqrt{x}} + 9}}\,dx$

(iii) $\displaystyle\int_{2}^{10} 3\sqrt{x - 1}\,dx$

12. [5] The graph of $f(x)$ is given below. Find $\int_{-2}^{6} f(x)\,dx$.



13. [10] Solve the following ordinary differential equation:

$$\frac{x\,\frac{dy}{dx} - 8x}{\sqrt{x}} = 2$$

(i) the general solution

(ii) the particular solution when $y(1/4) = -4$

14. [10] Let $P(x) = 2(g(x))^3$. Given that $g(5) = -2$ and $g'(5) = 3$, find

(i) the derivative of P at $x = 5$.

(ii) the approximate value of $P(-1.9)$.

15. [10] Archimedes decides to build a rectangular box with square ends and no top. It is to be built to hold 800 cubic pygme (A pygme is a unit of measuring length used in ancient Greece and 1 pygme $\approx$ 14 inches). Assume that the material of the base costs 75 Drachmas per square pygme, and the material of the sides costs 25 Drachmas per square pygme (a Drachma is a silver coin (ie currency) used in ancient Greece). Find the most economical dimensions. Write your answer in a sentence ($M \to E$).

16. [10] A particle is moving with an acceleration given by $a(t) = 6t + 2$ (note that this is not a free-falling object), where t is measured in seconds and s (position) is measured in meters. If the initial position is given by $s_0 = s(0) = 3$ and the initial velocity is given by $v_0 = v(0) = 4$, then find the position function of the particle at time t seconds.

17. [10] Consider $h(x) = \dfrac{e^x + e^{-x}}{2}$ for $-\infty < x < \infty$. Find the interval(s) where h is decreasing.

(A) $(-\infty, \infty)$

(B) $(-1, 0)$

(C) $(0, +\infty)$

(D) $(0, 1)$

(E) $(-\infty, 0)$

18. [10] If $F(x) = \displaystyle\int_0^x (t^3 - 12t)\, dt$, write find the interval(s) where the graph of F is concave down.

(A) $(-\infty, \infty)$

(B) $(-\infty, 2) \cup (2, +\infty)$

(C) $(2, +\infty)$

(D) $(-2, 2)$

(E) Cannot be determined.

19. [10] Suppose that f and g are continuous everywhere, and $f(x) < 0$ for all x. Also, assume that they satisfy:

$$\int_1^5 f(x)\, dx = -4 \quad \text{and} \quad \int_1^5 g(x)\, dx = 0 \quad \text{and} \quad \int_3^5 f(x)\, dx = -1 \quad \text{and} \quad \int_3^5 g(x)\, dx = 7$$

Calculate

(i) $\displaystyle\int_5^1 (f(x) + 1)\, dx$

(ii) $\displaystyle\int_1^3 g(x)\, dx$

20. [15] Consider $f(x) = x^2 + 1$ on the interval [0,1].

(i) Find the upper estimate, $S_U(5)$, for the area under the curve $f(x)$ from $x = 0$ to $x = 1$. [Note that $n = 5$ here.]

(ii) Now, find the exact area. (hint: use the FTC II, not the definition)

(iii) How good is this approximation? The difference is within

(A) 1.0 (B) 0.5 (C) 0.1 (D) 0.01 (E) 0.001

PART III: Critical Thinking

21. [30] Suppose $f(x)$ and $g(x)$ are everywhere differentiable functions whose values are shown below at for some values of x.

x	0	1	2
$f(x)$	4	0	2
$f'(x)$	0	1	4
$g(x)$	0	3	1
$g'(x)$	3	5	1

Find:

(i) $\lim_{x\to 0} \dfrac{f(e^x)}{g(2+x)}$

(ii) $\lim_{x\to 1} \dfrac{f(x)+3}{|g(x)-3|}$

(iii) $h'(2)$, where $h(x) = x^2 f(x)$

(iv) $h'(1)$, where $h(x) = g(f(2x))$

(v) $\int_1^2 f'(x)\,dx$

(vi) $\int_1^2 e^{g(x)} g'(x)\,dx$

Below is the graph of a function f.

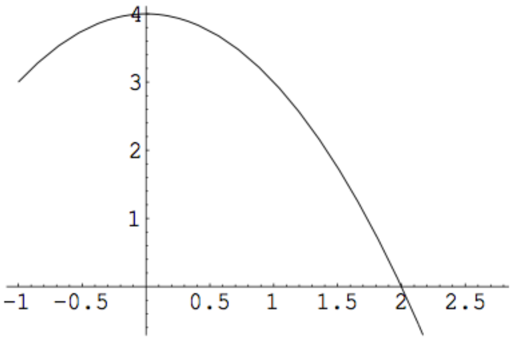


Figure: The graph of f.

Consider the function $g(x) = \int_0^x f(t)dt$.

22. [5] Then for $0 < x < 2$, the function $g(x)$ is

(A) increasing and concave down

(B) increasing and concave up

(C) decreasing and concave down

(D) decreasing and concave up

(E) not enough information given

Justify your answer.

23. [5] Using the same function $g(x)$, then

(A) $g(0) = 0$, $g'(0) = 0$, and $g'(2) = 1$

(B) $g(0) = 0$, $g'(0) = 1$, and $g'(2) = 0$

(C) $g(0) = 1$, $g'(0) = 0$, and $g'(2) = 1$

(D) $g(0) = 0$, $g'(0) = 4$, and $g'(2) = 0$

(E) not enough information given

Justify your answer.

24. [10] Peeling an orange changes its volume V. What does ΔV represent?

(A) the surface area of the orange

(B) the volume of the rind

(C) the volume of the "edible part" of the orange

(D) $-1\times$ (volume of the rind)

Explain your reasoning and include a sketch in your explanation.

25. [10] **True or False:** If $f(x)$ is continuous on a closed interval, then it is enough to look at the points where $f'(x) = 0$ in order to find its absolute maxima and minima.

Explain your reasoning and cite examples.

– HAPPY HOLIDAYS! –