MATH 5B - Single Variable Calculus II — Spring 2019

§11.9 Representation of Function as Power Series

In-class Activity 11.9

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Activity 1:

Find the **power series representation** of

$$f(x) = \frac{1}{1+x^7}$$

and include the **interval of convergence**.

Activity 2:

Find the **power series representation** of

$$f(x) = \frac{2x}{1+x}$$

and include the **interval of convergence**.

Activity 3:

Find the **power series representation** of

$$f(x) = \frac{x^2}{4 + 3x}$$

and include the **interval of convergence**.

Activity 4:

Find the **power series representation** of $f(x) = \frac{2}{(1+x)^2}$ using the PSR of $\frac{2x}{1+x}$ from Activity 2.

Theorem 1: PSR for Natural Log

The power series for the natural logarithm is

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} \qquad \text{for} \qquad |x| < 1$$

Activity 5:

Approximate $\ln(1.1)$ with an error less than 10^{-5}.

Activity 6:

(a) Find the **power series representation** of $\arctan(x) = \tan^{-1}(x)$.

(b) Use part (a), to derive Leibniz' formula given in the Chapter 11 intro:

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Activity 7:

Use the series in part (b) of Activity 5 to approximate π by using 5 terms.

Activity 8:

Approximate

$$\int_0^{0.1} \frac{1}{1+x^7}\,dx$$

with an error less than 10^{-10}.