

Quiz 5: 4.3, 5.1, 5.2

Dr. Jorge Basilio

NAME (PRINT): SOLUTIONS

SCORE: 26/26

SIGNATURE: _____

Directions

- YOU ARE ALLOWED TO USE A CALCULATOR ON THIS EXAM. (Ti83/Ti83+/Ti84/Ti84+/Ti84+CE-T, or scientific calculator)
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a box around your final answer where applicable.

26 Quiz (X points)

Problem 1: 12 pts

(a) (2 pts) What are the two requirements for a **probability distribution**?

$$(1) \sum P(x) = 1 \quad (2) 0 \leq P(x) \leq 1 \quad \text{for each } x$$

(b) A couple plans to have three children. Let X be the random variable corresponding to the number of girls the couple will have.(i) (2 pt) Using set notation, write the **sample space** of the "experiment" of having 3 kids. (Hint: use the G or B notation. So one outcome is GBG, for example)

$$S = \{ \underbrace{GGG}_{x=3}, \underbrace{GGB, GBG, BGG}_{x=2}, \underbrace{GGB, BBG, BGB}_{x=1}, \underbrace{BBB}_{x=0} \} \quad (2^3=8)$$

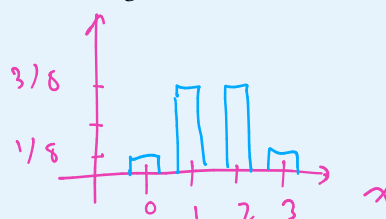
(ii) (1 pt) Give all the **values** x can take.

$$x = 0, 1, 2, 3 \quad X = \# \text{ girls in 3 kids}$$

(iii) (3 pts) Find the **probability distribution** of X given as a table of values.

x	$P(x)$
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x	$P(x)$
0	$1/8$
1	$3/8$
2	$3/8$
3	$1/8$

(iv) (2 pts) Find the **mean** number of girls the couple can have.

$$\mu = \sum x \cdot P(x) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} + 3 \cdot \frac{1}{8} = \frac{12}{8} = 1.5$$

$$\boxed{\mu = 1.5 \text{ girls}}$$

(v) (2 pts) How many girls can the couple **expect** to have?

↳ mean = expectation

"We expect 1.5 girls out of 3 kids"

Problem 2: 10 pts

$$P(x) = n \cdot x \cdot p^x \cdot q^{n-x}$$

(a) (1 pt) State the four requirements for a **binomial probability distribution**.

- Two
- (1) Fixed # of trials (n)
 - (2) trials are independent
 - (3) Each outcome has 2 categories (S or F)
 - (4) Prob of "success" is constant (p)

(b) (1 pt) Give the **calculator syntax** that computes the binomial probability distribution: $P(x) = \text{binompdf}(n, p, x)$

$P(x)$ = prob. that exactly x successes out of n .

(c) We survey 5 PCC students and ask "Do you want to transfer to a four-year college?" Assume that 85% of PCC students want to transfer to a four-year college.

(i) (2 pts) What is the **probability** that all 5 students want to transfer?

$$P(5) = \text{binompdf}(5, 0.85, 5)$$

$$n=5, p=0.85, x=5$$

$$= 0.444$$

(ii) (2 pts) What is the **probability** that 3 or 4 students want to transfer?

$$P(3 \text{ or } 4) = P(3) + P(4)$$

$$= \text{binompdf}(5, 0.85, 3) + \text{binompdf}(5, 0.85, 4) = 0.530$$

(iii) (2 pts) What is the **probability** that at least one student wants to transfer?

$$\begin{aligned} P(x \geq 1) &= 1 - P(0) \\ &= 1 - \text{binompdf}(5, 0.85, 0) \\ &= 0.9999... = 1.00 \end{aligned}$$

(iv) (2 pts) What is the **probability** that no more than 3 student want to transfer?

$$\begin{aligned} P(x \leq 3) &= \text{binomcdf}(5, 0.85, 3) \\ &= 0.165 \end{aligned}$$

(d) According to the U.S. Office of Adolescent Health, nearly 90% of adult smokers in America started smoking before turning 18 years old.

(i) (2 pts) If 1000 adult smokers are randomly selected, how many would we **expect** to have started smoking before turning 18 years old? ($M \rightarrow E$)

$$\mu = n \cdot p = 1000 \cdot 0.9 = 900 \text{ people started smoking before 18yo}$$

$$n=1000 \quad p=0.9$$

(ii) (2 pts) Would it be **unusual** (significantly low or high) to observe 800 smokers who started smoking before turning 18 years old in a sample of 1000 adult smokers?

unusual $\rightarrow \text{Prob} \leq 0.05$

$$\rightarrow x \leq \bar{x} - 2\sigma \text{ (sig low)}$$

$$x \geq \bar{x} + 2\sigma \text{ (sig high)}$$

$$\bullet x = 900 + 2(9.5) = 919$$

$$\bullet x = 900 - 2(9.5) = 881$$

$$\bullet x = 800 < 881$$

"yes, it would be unusual for 800 smokers to start by 18yo."

$$\sigma = \sqrt{n \cdot p \cdot q} \quad (q = 1 - p)$$

$$\sigma = \sqrt{1000 \cdot 0.9 \cdot 0.1} = 9.5$$