Review 7.2 & 7.3 Date Thurs 3/19

Section 7.2

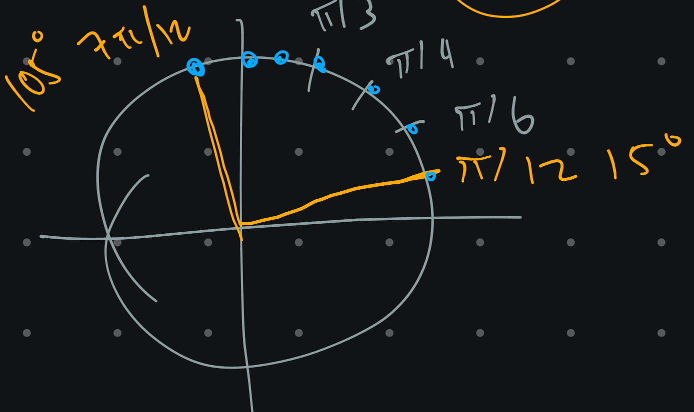
Add/Subtract

M $\sin(s \pm t) = \sin(s)\cos(t) \pm \cos(s)\sin(t)$

M $\cos(s \pm t) = \cos(s)\cos(t) \mp \sin(s)\sin(t)$

M $\tan(s \pm t) = \dfrac{\tan(s) \pm \tan(t)}{1 \mp \tan(s)\tan(t)}$

Ex 1 $\sec\left(\frac{7\pi}{12}\right) = \sec\left(\cancel{\frac{4\pi + 3\pi}{12}}\right)$



$= \sec\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right)$

$= \sec\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \dfrac{1}{\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)}$

$= \dfrac{1}{\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)}$

$= \dfrac{1}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}$

$= \dfrac{1}{\frac{\sqrt{2} - \sqrt{6}}{4}}$

$= \boxed{\dfrac{4}{\sqrt{2} - \sqrt{6}}}$

Ex2 Prove: $\cos(x-\pi) = -\cos(x)$.

use cosine even

Proof $LHS = \cos(x-\pi) = \cos\big((-)(\pi - x)\big)$ (alg)

$= \cos(\pi - x)$ (cosine even)

$= \cos\left(\left(\frac{\pi}{2}\right) + \left(\frac{\pi}{2} - x\right)\right)$ (alg)

$= \cos\left(\frac{\pi}{2}\right)\cdot\cos\left(\frac{\pi}{2}-x\right) - \sin\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}-x\right)$ (Add Formula)

($\cos\left(\frac{\pi}{2}\right) = 0$, $\sin\left(\frac{\pi}{2}\right) = 1$)

$= -\sin\left(\frac{\pi}{2} - x\right)$

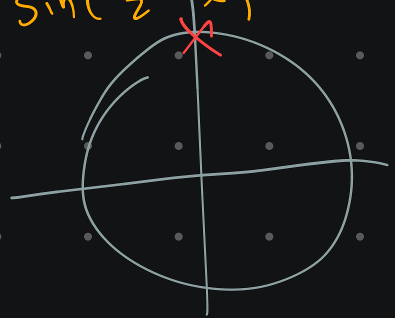
$= -\cos(x)$ (cofunction ID)

$= RHS$. $\square$

- **Cofunction ID**

$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$

$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$



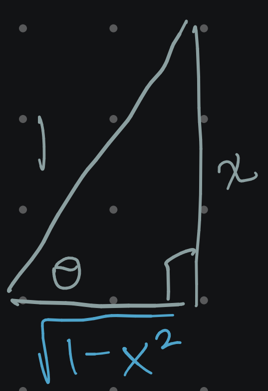
★GTQ

Ex 3 Re-write in terms of x & y : $\cos\big(\underbrace{\sin^{-1}(x)}_{\theta} - \underbrace{\tan^{-1}(y)}_{\phi}\big)$

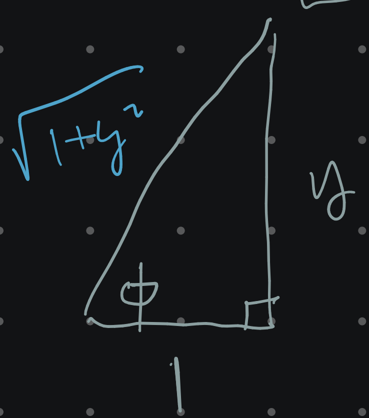
key#1

$\boxed{\theta = \sin^{-1}(x)}$ $\longleftrightarrow$ $\sin\theta = \boxed{\frac{x}{1}}$

$\boxed{\phi = \tan^{-1}(y)}$ $\longleftrightarrow$ $\tan\phi = \boxed{\frac{y}{1}}$



$?^2 + x^2 = 1$



$1^2 + y^2 = ?^2$

$= \cos(\theta - \phi)$

$= \cos\theta\cos\phi + \sin\theta\sin\phi$

$= \left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{1}{\sqrt{1+y^2}}\right) + \left(\frac{x}{1}\right)\left(\frac{y}{\sqrt{1+y^2}}\right)$

$= \boxed{\frac{\sqrt{1-x^2} + xy}{\sqrt{1+y^2}}}$

7.3

- Double Angle
- Lowering Powers
- Half-Angle
- ~~Product to Sum & Sum to Product~~

☆ Double-Angle ☆

M $\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$ M

$= 1 - 2\sin^2(x)$ M

$= 2\cos^2(x) - 1$ M

M $\tan(2x) = \dfrac{2\tan(x)}{1-\tan^2(x)}$

Lowering Powers ☆

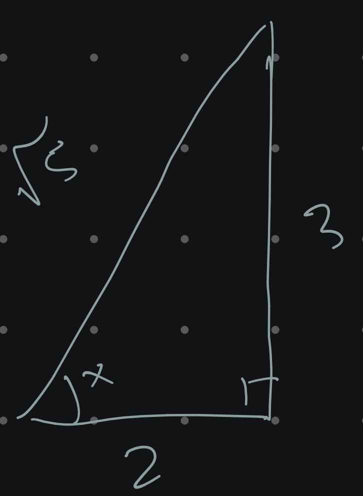
$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

Half-Angle

$$\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

Ex1 Find $\sin(2x)$ & $\cos(2x)$ Given that $\cot(x) = \frac{2}{3}$ & $\sin(x) > 0$



$$\cot(x) = \frac{2}{3} = \frac{\cos(x)}{\sin(x)}$$

$$\sin(2x) = 2\sin(x)\cos(x) = 2\left(\frac{3}{\sqrt5}\right)\left(\frac{2}{\sqrt5}\right) = \boxed{\frac{12}{5}}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = \left(\frac{2}{\sqrt5}\right)^2 - \left(\frac{3}{\sqrt5}\right)^2 = \frac{4}{5} - \frac{9}{5} = \frac{-5}{5} = \boxed{-1}$$

GTQ!

Ex Lower the powers of: $\sin^4(x)\cos^2(x)$.

Sol $\sin^4(x)\cos^2(x) = \left(\sin^2(x)\right)^2\left(\frac{1+\cos(2x)}{2}\right)$

$$= \left(\frac{1-\cos(2x)}{2}\right)^2\left(\frac{1+\cos(2x)}{2}\right)$$

$(a-b)^2$

$a^2 - 2ab + b^2$

$$= \frac{1}{8}\left(1 - 2\cos(2x) + \cos^2(2x)\right)\left(1+\cos(2x)\right)$$

$$= \frac{1}{8}\left(1 - 2\cos(2x) + \left(\frac{1+\cos(4x)}{2}\right)\right)\left(1+\cos(2x)\right)$$

$$= \frac{1}{16}\left(2 - 4\cos(2x) + 1 + \cos(4x)\right)\left(1+\cos(2x)\right)$$

$$= \boxed{\frac{1}{16}\left(3 - 4\cos(2x) + \cos(4x)\right)\left(1+\cos(2x)\right)}$$

leave in factored form :)

Ex Simplify as one term: a) $2\sin(7x)\cos(7x) = \sin(14x)$.

b) $\cos^2(31^\circ) - \sin^2(31^\circ) = \cos(62^\circ)$

GTQ

Ex Write in terms of x: $\tan(2\underbrace{\cos^{-1}(x)}_{=\theta})$

$\theta = \cos^{-1}(x)$

$\cos\theta = \boxed{\frac{x}{1}}$

$$= \tan(2\theta)$$

$$= \frac{2\tan\theta}{1-\tan^2\theta}$$

$$= \frac{2\left(\frac{\sqrt{1-x^2}}{x}\right)}{1-\left(\frac{\sqrt{1-x^2}}{x}\right)^2}$$

all algebra!

$$= \frac{\frac{2\sqrt{1-x^2}}{x}}{\frac{x^2}{x^2}\cdot 1-\left(\frac{1-x^2}{x^2}\right)}$$

$$= \frac{\frac{2\sqrt{1-x^2}}{x}}{\frac{x^2-(1-x^2)}{x^2}}$$

x^2-1+x^2

$2x^2-1$

$$= \frac{2\sqrt{1-x^2}}{\cancel{x}}\cdot\frac{x^{\cancel{2}\,1}}{2x^2-1}$$

$$= \boxed{\frac{2x\sqrt{1-x^2}}{2x^2-1}}$$