MATH 460: Probability & Statistics   September 2018

Chapter 4: Probability Distribution Functions

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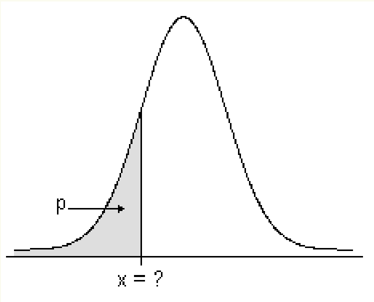
Chapter 4: Probability Distribution Functions

Inverse Normal Distribution

Definition 1: Inverse-Normal-Distribution

This is an informal name to the process of working backwards from a known (or given) probability to find an x-value.

Given probability p of the area under the curve of the PDF to the left of the value x_p, we want to find what the value x_p is. It really helps to draw a picture to help explain what we are doing:



Essentially, we solving for x in the equation: $P(X \leq x) = p$.

USING CALCULATOR TI83: invNorm(p,μ,σ)

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- Given probability p of the area under the curve of the PDF to the left of the value x_p, we can find what the value x_p is by using the "inverse normal distribution:"

$$\text{invNorm}(p, \mu, \sigma) = x_p \tag{1}$$

Activity 1: Inverse-Normal-Distribution

Find the 90th percentile for a normal distribution with a mean of 70 and a standard deviation of 4.5.

Activity 2: Inverse-Normal-Distribution

The time it takes employees to get to work from home (in minutes) is normally distributed with a mean of 30 minutes, and a standard deviation of 5 minutes. Find:

(a) the percentage of employees that take between 28 and 37 minutes to get to work (Hint: this is not an inverse problem)

(b) The number of minutes the longest it would take the bottom employee in the bottom 5% of the data to get to work. (Hint: this is an inverse problem)

Activity 3: Inverse-Normal-Distribution

An average light bulb manufactured in a factory lasts 280 days with a standard deviation of 45 days. Assume that bulb life is normally distributed.

(a) What is the probability that an Acme light bulb will last at most 360 days? (Hint: this is not an inverse problem)

(b) What bulb life separates the bottom 12%? (Hint: this is an inverse problem)

Poisson Distribution

Definition 2: Poisson-Distribution

For a Poisson distribution, we make the following assumptions:

- X is a discrete random variable, i.e. it takes values 0,1,2, ... So X = the number of successes of a Poisson random variable.
- the number of successes in two disjoint time intervals is independent
- the probability of success during a small time interval is proportional to the entire length of the time interval

Then, if μ is the mean of the number of successes in the given time interval, the probability that X is successful x times is given by

$$P(X = x) = \frac{e^{-\mu} \cdot \mu^x}{x!} \tag{2}$$

Fact: A Poisson distribution has $\sigma = \sqrt{\mu}$.

USING CALCULATOR TI83: poissonpdf(μ,x)

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- Given mean μ and the number of successes x, the probability can be calculated by the "poisson distribution:"

$$\text{poissonpdf}(\mu, x) = P(X = x) \tag{3}$$

Activity 4: Poisson-Distribution

A Life Insurance (LI) salesman sells on average 3 LI policies per week. Assuming a Poisson Distribution, calculate the probability that in a given week she will sell:

(a) some policies

(b) 2 or more but less than 5 policies

(c) Assuming a five day workweek, what is the probability that in a given day, she will sell a policy?