Math 5B - Calculus II | Spring 2019

Exam 4

§11.6–11.11

Dr. Jorge Basilio

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gbasilio@pasadena.edu

Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

NAME (PRINT): Solutions SIGNATURE: ______________________

Directions

- YOU ARE ALLOWED TO USE ONLY A SCIENTIFIC CALCULATOR ON THIS EXAM.
- You have 90 minutes to complete this exam.
- The exam totals **130 points**. There is also one extra-credit problems, worth 5 points (so 135 points are possible).
- There are 7 problems, many of them with multiple parts.
- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you your writing utensils.
- Cell phones must be turned off and put away in with your items in the front of the classroom.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E).
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a $\boxed{\text{box around your final answer}}$ where applicable.
- Give $\boxed{\text{UNITS}}$ where applicable.
- **PLEASE CHECK YOUR WORK!!!**
- If you need extra space, there is extra space on the back of the cover page and clearly indicate that you are continuing your work there in the original location.
- If you finish early, you may take a break but you must come back to class by 5:30 and we will have class.
- I will take attendance at the end of class

Score	Grade

This page is intentionally blank. It may be used for scratch paper. If you wish for me to grade your work on this page, please (i) label the problem you are working on, (ii) box your answer, (iii) indicate in the original problem's location that you will continue your work on this page.

Problem 4(b): Need to use Fact $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = e$.

- $\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$

  def of "e" from handout!

- $\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^{n+1} = \lim_{u\to\infty}\left(\frac{u}{u+1}\right)^u = \lim_{u\to\infty}\left(\frac{u+1}{u}\right)^{-u} = \lim_{n\to\infty}\left[\left(\frac{u+1}{u}\right)^u\right]^{-1} = \Big[\underbrace{\lim_{u\to\infty}\left(1+\frac{1}{u}\right)^u}_{=e}\Big]^{-1} = e^{-1} = \frac{1}{e}$

  $u = n+1$ so $u\to\infty$ as $n\to\infty$

  trick! make it look like "e"

so $\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^{n+1}\left(\frac{n+1}{n}\right)^n = \left(\frac{1}{e}\right)(e) = 1$!

Problem 1: 12 pts (2 pts each)

Fill-in the blank:

(a) The exact value of $\sum_{n=0}^{\infty} \frac{(-1)^n 8^{2n+1}}{(2n+1)!}$ equals sin(8)

$\sum_{0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = \sin(x) \quad x = 8 \quad (x \in \mathbb{R})$

(b) The exact value of $\sum_{n=0}^{\infty} \frac{(-1)^n 0.1^{n+1}}{n+1}$ equals ln(1.1)

$\sum_{0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \ln(x+1) \quad x = 0.1 \quad (\text{since } |x| < 1)$

(c) The exact value of $\sum_{n=0}^{\infty} \frac{(-1)^n 8^{2n+1}}{2n+1}$ equals arctan(8)

$\sum_{0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \arctan(x) \quad x = 8$

TRUE or **FALSE** (please spell out/write the entire word for credit). *(No work needed)*

(a) FALSE Suppose that the series $\sum c_n x^n$ has radius of convergence 12 and the series $\sum d_n x^n$ has a radius of convergence 13. Then the radius of convergence of $\sum (c_n + d_n)x^n$ is 13.

(b) TRUE $e = 1 + 1 + \frac{1}{2} + \frac{1}{6} + \cdots + \frac{1}{n!} + \cdots$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + 1 + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{n!}, \quad w/\ x = 1$

(c) TRUE The sereis $\sum_{n=5}^{\infty} n^{-\cosh(\ln(5))}$ is convergent.

$p = \cosh(\ln(5)) = \frac{e^{\ln 5} + e^{-\ln 5}}{2} = \frac{5 + \frac{1}{5}}{2} \geq 2.5 > 1$

Problem 2: 12 pts (4 pts each)

(a) Find the first <u>five derivatives</u> of $f(x) = \sinh(x)$ at $x = 0$.

$\sinh(0) = \frac{e^0 - e^0}{2} = 0, \quad \cosh(0) = \frac{e^0 + e^0}{2} = 1.$

$f'(x) = \cosh(x) \longrightarrow f'(0) = 1$

$f''(x) = \sinh(x) \longrightarrow f''(0) = 0$

$f'''(x) = \cosh(x) \longrightarrow f'''(0) = 1$

$f^{(4)}(x) = \sinh(x) \longrightarrow f^{(4)}(0) = 0$

$f^{(5)}(x) = \cosh(x) \longrightarrow f^{(5)}(0) = 1$

(b) Find the **Taylor Polynomial** $T_5(x)$ for $f(x)$ at $x = 0$.

$T_5(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5$

$= 0 + \frac{1}{1}x + 0 + \frac{1}{6}x^3 + 0 + \frac{1}{5!}x^5$

$\boxed{T_5(x) = x + \frac{x^3}{6} + \frac{x^5}{60}}$

(c) Find the **Taylor series** for $f(x)$ at $x = 0$.

notice pattern continues as in part(a). even powers $= 0$, odd powers only & coefficient are $\frac{f^{(n)}(0)}{n!} = \frac{1}{n!}$

$\boxed{T(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}}$

Recall AC $\Rightarrow \sum |a_n| < \infty$ (hence $\sum a_n < \infty$ too)

CC $\Rightarrow \sum a_n < \infty$ BUT $\sum |a_n| = \infty$

Problem 3: 36 pts (6 pts each)

Determine whether the series **absolutely converges or conditionally converges:**
For each problem, state clearly which tests your are using and which series you are comparing with.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n^3}{n^4+1}$ conditionally converges

- $\sum \frac{(-1)^n n^3}{n^4+1}$ converges by AST b/c if $b_n = \frac{n^3}{n^4+1}$ then

① $b_n > 0$ ✓ ② $\lim_{n\to\infty}\left(\frac{n^3}{n^4+1}\right) = 0$ ✓ ③ $\{b_n\}$ decreasing: let $f(x) = \frac{x^3}{x^4+1}$

Then $f'(x) = \frac{(3x^2)(x^4+1) - x^3(4x^3)}{(x^4+1)^2}$

$= \frac{-x^6 + 3x^2}{(x^4+1)^2} < 0$ ✓ when $x \in (3^{1/4}, \infty)$

- $\sum \left|\frac{(-1)^n n^3}{n^4+1}\right| = \sum \frac{n^3}{n^4+1} \to D$ by Squint w/ $b_n = 1/n$.

↳ D by Harmonic series

$L = \lim \left|\frac{\frac{n^3}{n^4+1}}{\frac{1}{n}}\right| = \lim_{n\to\infty} \frac{n^4}{n^4+1} = 1$. So by Limit comparison.

since $0 < L = 1 < \infty$, both series diverge!

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{5^n}$ absolutely converges

- $\sum \left|\frac{(-1)^n 2^n}{5^n}\right| = \sum \left(\frac{2}{5}\right)^n \to$ converges by geometric series w/ $r = 2/5 < 1$

- since $\sum |a_n|$ converges, $\sum a_n$ converges absolutely.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n^3+1}}$ absolutely converges

- $\sum \left|\frac{(-1)^n}{\sqrt{n^3+1}}\right| = \sum \frac{1}{\sqrt{n^3+1}} \to$ converges, "SQUINT" compare w/ $\sum \frac{1}{n^{3/2}}$ (which C by $p = 3/2 > 1$)

- $L = \lim_{n\to\infty}\left|\frac{\frac{1}{\sqrt{n^3+1}}}{\frac{1}{n^{3/2}}}\right| = \lim_{n\to\infty}\left(\frac{\sqrt{n^3}}{\sqrt{n^3+1}}\right) = \lim_{n\to\infty}\sqrt{\frac{n^3}{n^3+1}} = \sqrt{\lim_{n\to\infty}\left(\frac{n^3}{n^3+1}\right)} = 1$

$\to$ so by LCT, $0 < L = 1 < \infty$ we have both converge.

- Since $\sum |a_n|$ converges, so does $\sum a_n$. So our series converges absolutely.

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3}$ absolutely converges

- $\sum \left|\frac{(-1)^n}{n^3}\right| = \sum \frac{1}{n^3} \to$ converges by p-series test w/ $p = 3 > 1$

since $\sum |a_n|$ converges, $\sum a_n$ converges absolutely.

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{2^n}$ diverges

- $\sum \frac{(-1)^n 5^n}{2^n} = \sum \left(-\frac{5}{2}\right)^n \longrightarrow$ diverges by geometric series w/ $r = -5/2$ since $|r| = \left|-\frac{5}{2}\right| > 1$

(f) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos(n)}{n!}$ absolutely converges

- $\sum \left|\frac{\cos(n)}{n!}\right| \leq \sum \frac{1}{n!} \to$ converges by Ratio Test:

since $-1 \leq \cos(x) \leq 1$

Use Ratio Test:

$L = \lim_{n\to\infty}\left|\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}}\right| = \lim_{n\to\infty}\left(\frac{\cancel{n!}}{(n+1)\cancel{n!}}\right) = \lim_{n\to\infty}\left(\frac{1}{n+1}\right) = 0$

Since $L = 0 < 1$, Root test $\Rightarrow$ converges absolutely

Problem 4: 20 pts (10 pts each)

Find the **interval of convergence** and the **radius of convergence** for the following power series:
Don't forget to check the endpoints!

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{\sqrt{n}}(x+1)^n$

ROC • ratio test

$$L = \lim_{n\to\infty}\left|\frac{\frac{2^{n+1}(x+1)^{n+1}}{\sqrt{n+1}}}{\frac{2^n(x+1)^n}{\sqrt{n}}}\right| = \lim_{n\to\infty}\left|\frac{2^{n+1}}{2^n}\frac{\sqrt{n}}{\sqrt{n+1}}\frac{(x+1)^{n+1}}{(x+1)^n}\right|$$

$$= \lim_{n\to\infty}\left|2\sqrt{\frac{n}{n+1}}(x+1)\right| = 2|x+1|\cdot\lim_{n\to\infty}\sqrt{\frac{n}{n+1}} = 2|x+1|$$

• By Ratio test: converges if $L<1$:

$2|x+1|<1 \to |x+1|<\frac{1}{2}$ so $\boxed{R=\frac{1}{2}}$

• IOC: $|x+1|<\frac{1}{2}$

$-\frac{1}{2}<x+1<\frac{1}{2}$

$-1 \quad -1 \quad -1$

$-\frac{3}{2}<x<-\frac{1}{2}$

Check endpoints:

• $x=-\frac{1}{2}$: $\sum \frac{2^n}{\sqrt{n}}\left(\frac{1}{2}\right)^n = \sum \frac{1}{\sqrt{n}} \to D$ by p-series $p=1/2$

• $x=-\frac{3}{2}$: $\sum \frac{2^n}{\sqrt{n}}\left(-\frac{1}{2}\right)^n = \sum \frac{(-1)^n}{\sqrt{n}} \to C$

by Alt series test since $\{\frac{1}{\sqrt{n}}\}$ decreasing: $\frac{1}{\sqrt{n+1}}<\frac{1}{\sqrt{n}}$.

so $\boxed{\text{IOC: } \left[-\frac{3}{2},-\frac{1}{2}\right)}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-n)^n}{4^n(n+1)^n}(2x-1)^n$

• Use Ratio test:

$$L = \lim_{n\to\infty}\left|\frac{\frac{(-(n+1))^{n+1}(2x-1)^{n+1}}{4^{n+1}(n+2)^{n+1}}}{\frac{(-n)^n(2x-1)^n}{4^n(n+1)^n}}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(-1)^{n+1}}(n+1)^{n+1}(2x-1)^{n+1}\cdot\cancel{4^n}\cdot(n+1)^n}{\cancel{(-1)^n}n^n\cancel{(2x-1)^n}4^{n+1}(n+2)^{n+1}}\right|$$

$$= \lim_{n\to\infty}\left|\left(\frac{n+1}{n+2}\right)^{n+1}\cdot\left(\frac{n+1}{n}\right)^n\frac{(2x-1)}{4}\right| = \frac{|2x-1|}{4}\lim_{n\to\infty}\underbrace{\left(\frac{n+1}{n+2}\right)^{n+1}}_{\to 1/e}\underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e} = \frac{|2x-1|}{4}$$

(see scratch work - front)

so by Ratio test converges when $L=\frac{|2x-1|}{4}<1$ so $|2x-1|<4$

$\to -4<2x-1<4$

$+1 \quad +1 \quad +1$

$-3<2x<5$

$-\frac{3}{2}<x<\frac{5}{2}$

$\boxed{\text{IOC is } \left(-\frac{3}{2},\frac{5}{2}\right)}$

ROC is

$R = \frac{\frac{5}{2}-\left(-\frac{3}{2}\right)}{2} = \frac{\frac{8}{2}}{2} = 2$

$\boxed{\text{ROC is } R=2}$

Check endpoints

• $x=-\frac{3}{2}$: $\sum \frac{(-n)^n(-4)^n}{4^n(n+1)^n} = \sum \frac{\cancel{(-1)^n}n^n\cancel{(-1)^n}\cancel{4^n}}{\cancel{4^n}(n+1)^n}$

$= \sum\left(\frac{n}{n+1}\right)^n \to D$ by Test for divergence

since $\left(\frac{n}{n+1}\right)^n \to e \neq 0$

• $x=\frac{5}{2}$: $\sum \frac{(-n)^n 4^n}{4^n(n+1)^n}$

$= \sum(-1)^n\left(\frac{n}{n+1}\right)^n \to D$

by Test for Divergence

$\lim(-1)^n\left(\frac{n}{n+1}\right)^n = \text{DNE}$ (oscillates b/w $\pm e$)

Problem 5: 6 pts

Evaluate the limit using your knowledge of power/Taylor series (no credit will be given if L'Hôpital's Rule is used):

$$\lim_{x\to 0}\left(\frac{\sin(x^3)-x^3}{x^9}\right) =$$

• We know:

$$\sin(x) = \sum_0^\infty \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

• substituting x^3:

$$\sin(x^3) = \sum_0^\infty \frac{(-1)^n(x^3)^{2n+1}}{(2n+1)!}$$

$$= \frac{(x^3)^1}{1!} + \frac{(-1)(x^3)^3}{3!} + \frac{(-1)^2(x^3)^5}{5!} + \cdots$$

$$= x^3 - \frac{x^9}{6} + \frac{x^{15}}{5!} + \cdots$$

$$= \lim_{x\to 0}\left[\frac{\left(\cancel{x^3} - x^9/6 + x^{15}/5! + \cdots\right) - \cancel{x^3}}{x^9}\right]$$

$$= \lim_{x\to 0}\left[\frac{-\frac{\cancel{x^9}}{6} + \frac{x^{15}}{5!} + \cdots}{\cancel{x^9}}\right]$$

$$= \lim_{x\to 0}\left[-\frac{1}{6} + \underbrace{\frac{x^6}{5!} + \text{higher terms}}_{=0 \text{ when } x=0}\right]$$

$$= \boxed{-\frac{1}{6}}$$

Problem 6: 24 pts (6 pts each)

Test the series for convergence or divergence.
Classify as absolute or conditional. If convergence is conditional, show how absolute convergence was ruled out.
Clearly state which test your are using and verify the test applies to the given series.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^3}$ absolutely converges

- series is positive terms, so if converges then it's AC.
- use integral test: with $f(x) = \frac{1}{x(\ln(x))^3}$

$$\int_2^{\infty} \frac{1}{x(\ln(x))^3}\,dx = \int_{\ln(2)}^{\infty} \frac{1}{u^3}\,du = \left.\frac{u^{-2}}{-2}\right|_{\ln(2)}^{\infty} = \left.\frac{-1}{2u^2}\right|_{\ln(2)}^{\infty}$$

$u = \ln(x)$ $\quad x = 2 \to u = \ln(2)$

$du = \frac{1}{x}dx$ $\quad x = \infty \to u = \ln(\infty) = \infty$

$$= \frac{-1}{\infty} + \frac{1}{2(\ln(2))^2} = \frac{1}{2(\ln(2))^2}$$ finite! converges!

so converges absolutely.

Check conditions for integral test: ① $f(x) = \frac{1}{x\ln(x)^3} > 0$ ② f cont on ③ $f'(x) = \frac{0 - 1[(1)(\ln(x))^3 - x\,3[\ln(x)^2 \cdot \frac{1}{x}]]}{[x(\ln(x))^3]^2}$

$= \frac{-\ln(x)^3 - \ln(x)^2}{[x(\ln(x))^3]} < 0$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{e^n}{n^2}$ diverges

Use test for divergence:

$$\lim_{n\to\infty}\left(\frac{e^n}{n^2}\right) \overset{\text{L'Hôp}}{=} \lim_{n\to\infty}\left(\frac{e^n}{2n}\right) \overset{\text{L'Hôp}}{=} \lim_{n\to\infty}\left(\frac{e^n}{2}\right) = \infty \neq 0$$

$\frac{\infty}{\infty}$ $\quad\frac{\infty}{\infty}$

(c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!}$ absolutely convergent

- Use Ratio Test:

$$L = \lim_{n\to\infty}\left|\frac{\frac{\cancel{(-1)^{n+1}}\,\pi^{2(n+1)}}{(2(n+1))!}}{\frac{\cancel{(-1)^n}\,\pi^{2n}}{(2n)!}}\right| = \lim_{n\to\infty} \frac{\pi^{2n+2}\,(2n)!}{\cancel{\pi^{2n}}\,(2n+2)!}$$

$$= \lim_{n\to\infty} \frac{\pi^2\,(2n)!}{(2n+2)(2n+1)(2n)!} = \lim_{n\to\infty}\left(\frac{\pi^2}{(2n+2)(2n+1)}\right) = 0$$

- Since $L = 0 < 1$, series converges absolutely by Ratio Test.

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{5+e^n}$ absolutely convergent

- positive terms, so if converges, then AC
- use Basic Comparison Test:

$$\sum_0^{\infty} \frac{1}{5+e^n} \leq \sum \frac{1}{e^n} = \sum \left(\frac{1}{e}\right)^n \to$$ converges by geometric series $r = \frac{1}{e} < 1$.

Problem 7: 20 pts

(6 pts) (a) Find the anti-derivative: $\int e^{-x^2}\,dx$.

- Use $e^x = \sum_0^\infty \frac{x^n}{n!} \rightarrow e^{-x^2} = \sum_0^\infty \frac{(-x^2)^n}{n!} = \sum_0^\infty \frac{(-1)^n x^{2n}}{n!}$

- integrating using Just like Poly th:

$$\int e^{-x^2}dx = \int\left[\sum \frac{(-1)^n x^{2n}}{n!}\right]dx = \sum_0^\infty \frac{(-1)^n}{n!}\left[\int x^{2n}dx\right] = \sum_0^\infty \frac{(-1)^n}{n!}\frac{x^{2n+1}}{2n+1} + C$$

$$= \boxed{\sum_{n=0}^\infty \frac{(-1)^n}{n!}\frac{x^{2n+1}}{2n+1} + C}$$

(4 pts) (a) Use part (a) to find $\int_0^1 e^{-x^2}\,dx$ expressed as a series.

$$\int_0^1 e^{-x^2}dx = \left[\sum_{n=0}^\infty \frac{(-1)^n}{n!}\frac{x^{2n+1}}{2n+1} + C\right]_{x=0}^{x=1} = \left[\sum_{n=0}^\infty \frac{(-1)^n}{n!}\frac{1^{2n+1}}{2n+1} + \cancel{C}\right] - \cancel{\left[\sum_{n=0}^\infty \frac{(-1)^n}{n!}\frac{0^{2n+1}}{2n+1} + C\right]}$$

$$= \boxed{\sum_{n=0}^\infty \frac{(-1)^n}{n!(2n+1)}}$$

(6 pts) (a) Use the Alternating Series Test (include in your answer verification that the conditions are met) to find the number of terms needed in part (b) so that the error is less that $\frac{1}{100}$.

AST Remainder Thm: if $\sum a_n$ is alt series, $|R_n| \le b_{n+1}$. Let $b_n = \frac{1}{n!(2n+1)}$.

Need $b_{n+1} < \frac{1}{100}$:

$$\frac{1}{(n+1)!(2(n+1)+1)} < \frac{1}{100}$$

$$100 < (n+1)!(2n+3)$$

plug-in values of n:

$n=1$ $100 \overset{?}{<} 2!(5) = 10$ No

$n=2$ $100 \overset{?}{<} 3!7 = 42$ No

$n=3$ $100 \overset{?}{<} 4!9 = 216$ ✓ yes!

so $n=3$ means we need

$\boxed{\text{need 4 terms (including 0)}}$

check conditions for AST

$\sum_0^\infty \frac{(-1)^n}{n!(2n+1)}$ $+ - + - + -$ ✓

let $b_n = \frac{1}{n!(2n+1)}$.

Need
1. $b_n > 0$ ✓
2. $\lim b_n = 0$ ✓
3. $\{b_n\}$ decreasing:

$b_{n+1} = \frac{1}{(n+1)!(2n+3)} < \frac{1}{n!(2n+3)} < \frac{1}{n!(2n+1)} = b_n$ ✓

(4 pts) (a) Estimate $\int_0^1 e^{-x^2}\,dx$ so that the error is less that $\frac{1}{100}$. Give both the fractional and decimal (*rounded to the nearest thousandths*) answers.

$$\int_0^1 e^{-x^2}dx = \sum_{n=0}^\infty \frac{(-1)^n}{n!(2n+1)} \approx \sum_{n=0}^3 \frac{(-1)^n}{n!(2n+1)} = \frac{(-1)^0}{0!\,1} + \frac{(-1)^1}{1!(3)} + \frac{(-1)^2}{2!(5)} + \frac{(-1)^3}{3!(7)}$$

$$= 1 - \frac{1}{3} + \frac{1}{10} - \frac{1}{42}$$

$$= \frac{2}{3} + \frac{42-10}{420} = \frac{2}{3} + \frac{32}{420}$$

$$= \frac{2}{3} + \frac{8}{105} = \frac{70+8}{105} = \frac{78}{105} = \frac{26}{35}$$

$\frac{16}{210} = \frac{8}{105}$

$$\boxed{\int_0^1 e^{-x^2}dx \approx \frac{26}{35} \approx 0.743}$$

Problem 8: Extra-credit: 5 pts

You may attempt this only if every questions on the exam has an attempted solution. Otherwise it will not be graded.

Use power series $y(x) = \sum_{n=0}^{\infty} c_n x^n$ to solve the differential equation: $y' - x^2 y = 0$. Give both the general solution and the particular solution when $y(0) = 1$.

Post Exam Survey

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

1. When taking the exam I felt
   (a) Rushed. I wanted more time.
   (b) Relaxed. I had enough time.
   (c) Amazed. I had tons of extra time.
2. The week before the test I did all my homework on time: YES NO
3. The week before the test, in addition to the homework I followed a study plan. YES NO
   (a) I think this helped: YES NO
4. The day before the test I spend ____________ hours studying and reviewing.
   (a) I think that was enough time: YES NO
5. The night before the test:
   (a) I stayed up very late cramming for the test
   (b) I stayed up very late, but I wasn't doing math
   (c) I didn't need to cram because I was prepared
   (d) I got a good night's sleep so my brain would function well.
6. I think I got the following grade on this test: ____________
7. Strategies that worked well for me were (please elaborate):
8. Next time I will do an even better job preparing for the test by: