

MT 460 Midterm*Please show ALL work and BOX answers!!*

Name: _____

1. Answer the following counting questions.

- a. Out of ten people, in how many ways can President, Vice President and Secretary be selected?

- b. How many permutations of the letters in the word "STAT" can you have?

- c. Among 6 cars, 3 will be selected to go to a convention. In how many ways can these cars be selected?

2. Answer the following probability questions.

- a. What is the probability of drawing an Ace or a Clubs card from a well shuffled 52 card deck?

- b. You roll two dice. What is the probability that the sum is a 3 or 5?

- c. Out of 325 screws, 15 are defective, if you select 30 from the batch, what is the probability of getting exactly 4 defective screws?

3. A couple plans to have 6 kids. What is the probability that there will be at least three girls? Assume probability of 0.5 to have a boy and the events are independent. Use Binomial PDF.

4. A single ball is drawn at random from a box containing 12 orange balls, 10 black balls, and 8 blue balls.
 - a. Determine the probability that it is blue.

 - b. Three balls are drawn successively. Find the probability that they are drawn in the order orange, black, and blue if the ball is not replaced.

5. Suppose that a coin is tossed three times so that the sample space is $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$. Let x represent the # of heads that can come up. Create a table for its Probability Distribution Function (PDF) and draw its histogram. Test if the sum of probability is 1.

| x | $P(x)$ |
|-----|--------|
| | |
| | |
| | |
| | |

Sum

6. Find the probability distribution of girls in a family of 6 children, assuming fair probability ($p = 0.5$). Note: This is a binomial distribution.

| x | $P(x)$ |
|-----|--------|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |

Sum

- a. What is the probability that the couple will have at most 4 girls?
- b. Find the mean and standard deviation of the binomial pdf. You can use the shortcut formulas.
- $$E(X) = \mu = \sum_x x \cdot P(x) \quad Var(X) = \sigma^2 = \sum_x (x - \mu)^2 P(x) \quad Std(X) = \sqrt{Var(X)}$$

7. Suppose a game is to be played with a single die, assumed fair, where the player wins the dollar amount of the number rolled if it is even and losses the amount if it is odd. Find mean (expectation) of x , which denotes payout for this game.

| x | $P(x)$ |
|-------|--------|
| -\$2 | 1/6 |
| \$4 | 1/6 |
| -\$6 | 1/6 |
| \$8 | 1/6 |
| -\$10 | 1/6 |
| \$12 | 1/6 |

Sum

8. Acme Realty reports that it sells 75 homes in 25 days. What is the probability that exactly 2 homes will be sold tomorrow? Use Poisson distribution.

9. Daily travel times (in minutes) for an airport are normally distributed with mean of 130 and a standard deviation of 17.
 - a. What is the probability that a random flight has travel time less than 105 minutes?

 - b. What is the probability that a random flight has travel time more than 170?

10. An average light bulb manufactured in a factory lasts 280 days with a standard deviation of 45 days. Assume that bulb life is normally distributed.
 - a. What is the probability that an Acme light bulb will last at most 360 days?

 - b. What bulb life separates the bottom 12%? This is an inverse norm problem.