Jan 16

Chapter 5: Discrete Probability Distributions

Section 5.1: Probability Distributions

Capital X

Notation

X = name of RV

x = value X can take

(RV) **Random Variable** (X): is a numerical measure of the outcome of a probability "experiment", so its value is determined by chance.

Discrete Random Variable: either finite or countable number of values.

gaps!

0 1 2 3 4 ....

go forever

Continuous Random Variable: has (measurable) infinitely many values.

0 1 2 3 4

no gaps!

Ex: Identify the random variable and sample space.

(a) Coin toss for Heads

X = # of Heads in 10 coin flips,

the sample space x = 3 (eg.) ($x = 0, 1, \ldots, 10$)

(simple event) one outcome

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(b) An experiment that measures the time between arrivals of cars at a drive-through. continuous

X = time b/w car arrivals,

the sample space t $t > 0$.

0 → t

PROBABILITY DISTRIBUTIONS

Def A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a graph, table, or formula. (x) (X)

REQUIREMENTS

1. $\sum P(x) = 1$ (where x assumes all possible values.)
2. $0 \le P(x) \le 1$ (for every individual value of x.)

Ex: Are the following a probability distribution? If not, state why.

valid prob. dist.

A.

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	0.01

2 ✓
1 ?

$\Sigma P(x) = 1$?

NO $\Sigma = 0.97$

$\Sigma = 0.97$

B. Not PD

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	−0.01

BAD

C.

x	$P(x)$
0	0.16
1	0.18
2	0.22
3	0.10
4	0.30
5	0.04

$\Sigma = 1.0$

2 ✓
1 ? ✓

yes, satisfies both conditions!

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Ex: A couple plans to have four children. Let x be the number of boys the couple will have. Find the probability distribution for the number of boys.

X = # of boys out of 4 kids.

$x = 0, 1, 2, 3, 4$

$S = \{GGGG, BGGG, \ldots\}$

$\#S = 16$

GGGG − $x=0$

BGGG, GBGG, GGBG, GGGB } $x=1$

BBGG, GGBB, GBBG, BGGB, BGBG, GBGB } $x=2$

BBBG, BBGB, BGBB, GBBB } $x=3$

BBBB $x=4$

Probability Dist.

x	$P(x)$
0	1/16 = 0.0625
1	4/16 = 0.25
2	6/16 = 0.375
3	4/16 = 0.25
4	1/16 = 0.0625

Prob = # outcomes / # sample space

Formulas are used when given a Prob. Dist.

MEAN VALUE

FORMULA: $\boxed{\mu = \Sigma[x \cdot P(x)]}$

Note: The Greek letter μ is read "mu"

VARIANCE

FORMULA: $\boxed{\sigma^2 = \Sigma\left[(x-\mu)^2 \cdot P(x)\right]}$ or $\sigma^2 = \Sigma\left[x^2 \cdot P(x)\right] - \mu^2$

STANDARD DEVIATION

FORMULA: $\boxed{\sigma = \sqrt{\Sigma\left[x^2 \cdot P(x)\right] - \mu^2}}$

Note: The Greek letter σ is read "sigma"

Round-Off Rule: Carry one more decimal place than is used for the random variable. → Stats Rule for Rounding

Ex: In a study of brand recognition, random groups of four people are interviewed. Let x be the number of people who recognize Jeff Bezos when shown a picture. The following probability distribution gives the likelihood of the random variable. (RV) X = # of people who recognize Jeff Bezos out of 4

values of RV X

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	0.01	0 * 0.01 = 0	0^2 * 0.01 = 0
1	0.10	1 * 0.10 = 0.1	1^2 * 0.10 = 0.10
2	0.24	2 * 0.24 = 0.48	2^2 * 0.24 = 0.96
3	0.30	3 * 0.30 = 0.90	3^2 * 0.30 = 2.7
4	0.35	4 * 0.35 = 1.4	4^2 * 0.35 = 5.6
	$\Sigma P(x) = 1$	$\underbrace{\Sigma[x \cdot P(x)]}_{\mu} = 2.88$	$\Sigma[x^2 \cdot P(x)] = 9.36$

(a) What is the probability that more than two people will recognize a picture of Jeff Bezos?

$x = 3, 4$ "more than 2"

$P(x=3 \text{ or } x=4) = P(x=3) + P(x=4) - \underbrace{P(x=3 \text{ and } x=4)}_{=0} = 0.3 + 0.35 = \boxed{0.65}$

(b) What is the probability that at most three people will recognize a picture of Jeff Bezos?

$x = \boxed{0\ 1\ 2\ 3}\ 4$ "inequality" $x \leq 3$

Use the "1 minus trick" $P(\bar{A}) = 1 - P(A)$

$P(x \leq 3) = P(x \neq 4) = 1 - P(x=4) = 1 - 0.35 = \boxed{0.65}$

(c) Find the mean number of people who recognize a picture of Jeff Bezos.

$\mu = \Sigma[x \cdot P(x)] = 2.88 = \boxed{2.9 \text{ people recognize J. Bezos out of 4}}$

rounding rule use RV

(d) Find the standard deviation of the given probability distribution.

$\sigma = \sqrt{\underbrace{\Sigma[x^2 \cdot P(x)]}_{9.36} - \mu^2} = \sqrt{9.36 - (2.88)^2} = 1.032279\ldots\ldots$

$\boxed{\sigma = 1.0 \text{ people recognize Bezos out of 4}}$

EXPECTED VALUE

Def The **expected value** (denoted E) of a discrete random variable represents the mean value of the outcomes.

FORMULA: $\boxed{E = \sum [x \cdot P(x)]}$ units of E are same as units of x (or X)

Assume: if win get $1 back ("bet")

Ex: There is a game in Vegas where you can win $4 or $9 but it costs $1 to play the game. The probability of winning $4 is 0.3 and the probability of winning $9 is 0.1. Find the expected value for this game.

x	$P(x)$
-1	0.6
4	0.3
9	0.1

$\sum P(x) = 1$

$0.6 = 1 - 0.3 - 0.1$

- X = value of winning or losing from playing game (once).
- x = -$1, $+4, $9
- $E = \sum[x \cdot P(x)] = (-1)*0.6 + (4)*(0.3) + (9)*0.1$

$\boxed{E = \$1.5}$

Interpretation of μ: Over the long run (if we play this game MANY times) we **expect** the mean profit to be $1.50.

→ • if E > 0, keep playing • if E < 0, don't turn play • if E = 0, up to you!

Ex: When someone buys a life insurance policy, that policy will pay out a sum of money to a benefactor upon the death of the policyholder. Suppose a 25-year-old male buys a $150,000 1-year term life insurance policy for $250. The probability that the male will not survive the year is 0.0013.

The experiment has two possible outcomes: survive or die. Let the random variable X represent the money lost or gained by the life insurance company for the 25-year-old male after many years. What is the expected value for the company?

X = money lost or gained by insurance Co.

x =

	x	$P(x)$
survive	$250	0.9987
die	-$149,750	0.0013

P(die) = 0.0013

P(Survive) = 1 − P(die) = 1 − 0.0013 = .9987

"1 minus trick"

$E = \sum[x \cdot P(x)] = (250)*(0.9987) + (-149750)*(0.0013) = \boxed{\$55}$

Over 1,000 policies, how much should they expect to make?

Insurance Co. expects to make only $55 per policy.

- 1000 * 0.0013 = 1.3
- $1.3(-149750) + 998.7(250) = \frac{55000 \text{ (total made)}}{1000 \text{ policies}}$

Ex: Find the expected value of the random variable. Round to the three decimal places.

A contractor is considering a sale that promises a profit of $29,000 with a probability of 0.7 or a loss (due to bad weather, strikes, and such) of $3,000 with a probability of 0.3. What is their expected profit?

A. $19,400
B. $21,200
C. $20,300
D. $22,400
E. $26,000