§11.1 Sequences

In-class Activity 11.1

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Activity 1:

List the first 5 terms of the sequence:

(a) $\{a_n\}_{n=1}^{\infty}$ where $a_n = \frac{1}{n}$.

(b) $\left\{\dfrac{n}{n+1}\right\}_{n=1}^{\infty}$

(c) $\left\{(-1)^n \dfrac{n}{2^n}\right\}_{n=1}^{\infty}$

Activity 2:

Find the general term of the sequence determined by the terms of the sums:

(a) Leibniz: $\dfrac{\pi}{4} = 1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots$

(b) Euler: $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{4} + \dfrac{1}{9} + \dfrac{1}{16} + \cdots$

(c) Euler: $e = 1 + \dfrac{1}{2} + \dfrac{1}{6} + \dfrac{1}{24} + \cdots$

Use Sage to visualize the graph of the sequences by plotting the points: (n, a_n)

Activity 3:

Determine whether the sequences converge or diverge. If they converge, determine their limit.

(a) $\left\{\frac{(-1)^n}{2n}\right\}_{n=1}^{\infty}$

(b) $\{(-1)^n\}_{n=0}^{\infty}$

(c) $\{\cos(\pi n)\}_{n=0}^{\infty}$

(d) $\left\{\cos\left(\frac{\pi}{2}+\pi n\right)\right\}_{n=0}^{\infty}$

Activity 4:

Evaluate the limits of sequences:

(a) $\lim_{n\to\infty}\left(\frac{2n^2+n+1}{n^2+1}\right)$

(b) $\lim_{n\to\infty}\left(\frac{2n+1}{e^n-11}\right)$

Activity 5:

Let $a_1 = 2$ and $a_{n+1} = \frac{1}{2}(a_n + 6)$ for $n \geq 2$. Sequences defined in this way are called reccurrence relations.

(a) Compute the first 8 terms of the sequence

(b) Based on part (a) value do you predict that $\{a_n\}_{n=1}^{\infty}$ converges to?

(c) How can you prove your prediction correct?

Activity 6:

(a) Show $\left\{\dfrac{2}{n+3}\right\}_{n=1}^{\infty}$ is decreasing.

(b) Use the ID test to show that $\left\{\dfrac{2n}{n^2+1}\right\}_{n=1}^{\infty}$ is decreasing.

Activity 7:

Verify that $a_n = \sqrt{n+1} - \sqrt{n}$ is decreasing and bounded below.
Does $\lim_{n\to\infty} a_n$ exist?