§11.7: Strategies for Testing Series

Ch 11: Infinite Sequences and Series
Math 5B: Calculus II

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Outline

1. Guiding Questions
2. Introduction
3. Testing Series Toolbox

Guiding Questions for §11.7

Guiding Question(s)

1. What are strategies for Testing Series?

Introduction

- Similar to integration (where we studies many techniques), we have many techniques for determining whether a series converges or diverges.
- In this section, we'll develop a testing series toolbox.

Testing Series Toolbox

Memorize all the series tests! Pay close attention to the conditions needed.

- Tool #1 Try Test for Divergence First
- Tool #2 Is it a geometric series or *p*-series? Is it a alternating series?
  - Keep in mind the algebraic rules you can apply to convergent series. Try apply them to get a Geometric series into the correct form.
- Tool #3 Does the "Squint Test" give you a series you know to C or D (*p*-series, geometric series, or alternating series)? If YES, then use the Limit Comparison Test (or the Ratio Test)
- Tool #4 Is $\sqrt[n]{|a_n|}$ easy to analyze? Try the Root Test.
- Tool #5 Is $\left|\frac{a_{n+1}}{a_n}\right|$ easy to analyze? Try the Ratio Test.
- Tool #6 Does the series have positive terms?
  - Are there easy comparison? Try the Comparison Test.
  - Is there a positive, decreasing $f(x)$ with $f(n) = a_n$ and $\int f(x)dx$ doable? Try the Integral Test.

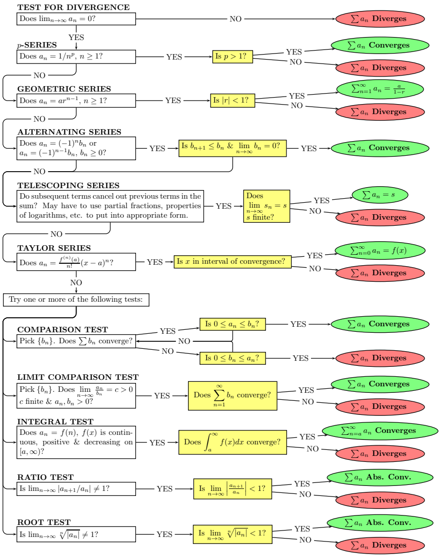
Testing Series Toolbox

Remarks

- **Only use tests you're allowed to use!**
  There's no point in trying to use a test if the series doesn't fit the necessary hypotheses.
- **Remember: some tests can be inconclusive!**
  For example, Ratio and Root Tests are inconclusive when $L = 1$.
- **Re-write the series terms**. You can use algebra rules to simplify series.

Testing Series Toolbox

Series Convergence/Divergence Flow Chart



Problems 1-38 from Stewart's *Calculus*, page 784

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+n}$
4. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1}\frac{n-1}{n^2+n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{2^{3n}}$
6. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$
7. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$
8. $\displaystyle\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$
9. $\displaystyle\sum_{k=1}^{\infty} k^2 e^{-k}$
10. $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$
11. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n\ln(n)}$
12. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+25}$
13. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$
14. $\displaystyle\sum_{n=1}^{\infty} \sin(n)$
15. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{2\cdot 5\cdot 8\cdot\dots\cdot(3n+2)}$
16. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$
17. $\displaystyle\sum_{n=1}^{\infty} (-1)^n 2^{1/n}$
18. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$
19. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$
20. $\displaystyle\sum_{k=1}^{\infty} \frac{k+5}{5^k}$
21. $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$
22. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$
23. $\displaystyle\sum_{n=1}^{\infty} \tan(1/n)$
24. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2+4n}$
25. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$
26. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$
27. $\displaystyle\sum_{k=1}^{\infty} \frac{k\ln(k)}{(k+1)^3}$
28. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$
29. $\displaystyle\sum_{n=1}^{\infty} \frac{\tan^{-1}(n)}{n\sqrt{n}}$
30. $\displaystyle\sum_{j=1}^{\infty} (-1)^j \frac{\sqrt{j}}{j+5}$
31. $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$
32. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$
33. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$
34. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+n\cos^2(n)}$
35. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$
36. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln(n))^{\ln(n)}}$
37. $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{2}-1)^n$
38. $\displaystyle\sum_{n=1}^{\infty} (\sqrt[n]{2}-1)$