Stat 50 - Elementary Statistics — Winter 2020

Exam 2

Ch 4, 5

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Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

NAME (PRINT): Solutions _____ SIGNATURE: _____

Directions

- YOU ARE ALLOWED TO USE A CALCULATOR ON THIS EXAM. (Ti83/Ti83+/Ti84/Ti84+/Ti84+CE-T, or scientific calculator)
- You have 80 minutes to complete this exam.
- The exam totals **110 points** but will be graded out of 100 only. (So it is possible to get 110% on this exam ☺).
- There are 9 problems, many of them with multiple parts.
- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you any writing utensils.
- Cell phones must be turned off and put away in with your items in the front of the classroom.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a box around your final answer where applicable.
- **PLEASE INCLUDE UNITS** where applicable
- **PLEASE CHECK YOUR WORK!!!**
- If you need extra space, there is extra space on the back of the cover page and clearly indicate that you are continuing your work there in the original location.
- Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E).

Score	Grade

This page is intentionally blank. It may be used for scratch paper. If you wish for me to grade your work on this page, please (i) label the problem you are working on, (ii) box your answer, (iii) indicate in the original problem's location that you will continue your work on this page.

Problem 1: 16 pts

Answer each question for the following table. For probability, state your answer as **un-simplified** fractions AND as decimals (following our rounding rule).

	Low Obesity	Average Obesity	High Obesity	Total
Hypertension	24	33	46	103
No Hypertension	109	101	87	297
Total	133	134	133	400

Let **T** = Hypertension, **N** = No Hypertension, **L** = Low Obesity, **A** = Average Obesity, **H** = High Obesity

(2 pt) (a) Find $P(T)$:

$P(T) = \frac{103}{400} = 0.258$

careful: don't confuse with π "hypertension"

(2 pt) (b) Find $P(A \text{ and } N)$:

$P(A \text{ and } N) = \frac{101}{400} = 0.253$

(average obesity, No hypertension)

(2 pt) (c) Find $P(H \text{ or } N)$:

$P(H \text{ or } N) = P(H) + P(N) - P(H \text{ and } N) = \frac{133}{400} + \frac{297}{400} - \frac{87}{400} = \frac{343}{400} = 0.858$

(2 pt) (d) Find the probability that a person has hypertension given that the person is of Average Obesity.

(T, A)

$P(T|A) = \frac{P(T \text{ and } A)}{P(A)} = \frac{33/400}{134/400} = \frac{33}{134} = 0.246$

(Hypertension, given, Average Obesity)

(2 pt) (e) Are the events "Hypertension" and "Low Obesity" independent? Explain why or why not.

(T, L)

The events T and L are not independent because they influence each other's probabilities. Moreover, notice these two events share 24 people in common. Independent events must be disjoint.

(2 pt) (f) Are the events "No Hypertension" and "Average Obesity" mutually exclusive? Explain why or why not.

(N, A)

The events N and A are not mutually exclusive because they are not disjoint. Both events share 101 people in common.

(2 pt) (g) Two adults are randomly selected without replacement, what is the probability both are of Average Obesity?

(A)

$P(A \text{ and } A) = P(A) * P(A) = \frac{134}{400} * \frac{133}{399} = 0.112$

(1st select., 2nd sele., 1st, 2nd)

(technically $P(A|A)$)

(2 pt) (h) Three adults are randomly selected with replacement, what is the probability at least one has Hypertension?

(T)

$P(\text{at least one } T) = 1 - P(\text{none } T)$

$= 1 - P(\bar{T} \text{ and } \bar{T} \text{ and } \bar{T})$ (1st, 2nd, 3rd)

$= 1 - \left(\frac{297}{400}\right)^3 = 0.591$

- with replacement: so put back so $P(\bar{T})$ are same & we mult. rule
- $P(T) = 103/400$
- $P(\bar{T}) = 297/400$
- Note $\bar{T} = N$

Problem 2: 4 pts (1 pts each)

Fill in the blanks:

(a) 0 $\leq P(A) \leq$ 1

(b) $P(A) = 0$ means event A is an impossible event.

(c) $P(A) = 1$ means event A is a certain event.

Multiple-choice. Select the correct answer:

(d) The shaded area in the Venn Diagram represents

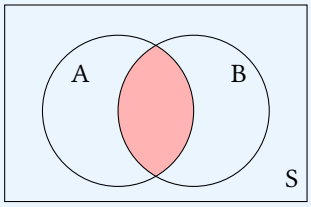
(A) $A|B$

B (B) A and B

(C) A or B

(D) $B|A$

(E) the entire sample space S

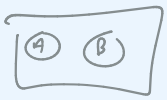


Problem 3: 6 pts (1 pts each)

TRUE or **FALSE** (please spell out/write the entire word for credit).

(Hint: you might find it helpful to use Venn Diagrams to help you arrive at an answer)

(a) TRUE If A and B are mutually exclusive then $P(A \text{ and } B) = 0$.



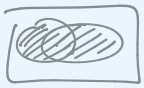
(b) FALSE If A and B are mutually exclusive then $P(A \text{ or } B) = 1$. see ↗

(c) FALSE If $P(A) > P(B)$ then event ~~A~~ B is more likely than event ~~B~~ A.

(d) TRUE If our population is 40,000 students, then a sample of 35 can be treated as being independent even if we sample without replacement. 5% Rule for cumbersome calculations $\frac{35}{40{,}000} = 0.000875$ way less 5%

(e) TRUE If $P(A) = 0.00081$ then A is an unusual event.
↳ is less than 0.05

(f) FALSE We always have $P(A \text{ or } B) \geq P(A \text{ and } B)$.



↖ always smaller.

Problem 4: 6 pts

(2 pt) (a) Consider the following table of values: What does p have to be to make the following a **probability distribution**? Show your mathematical reasoning.

x	$P(x)$
-12	0.222
20	0.235
23	0.324
35	p

Need ① $\sum p(x) = 1$ ② $p(x) \geq 0$ ↳ $0 \leq p(x) \leq 1$

$\sum p(x) = 1$

$0.222 + 0.235 + 0.324 + p = 1$

$0.781 + p = 1$

$p = 1 - 0.781 = 0.219.$

$p = 0.219$

(4 pt) (b) Find the **mean** and the **standard deviation** of the probability distribution. Show your mathematical reasoning.

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	0.33	$0 \cdot 0.33 = 0$	$0^2 \cdot 0.33 = 0$
1	0.30	$1 \cdot 0.3 = 0.3$	$1^2 \cdot 0.3 = 0.3$
2	0.25	$2 \cdot 0.25 = 0.5$	$2^2 \cdot 0.25 = 1$
3	0.12	$3 \cdot 0.12 = 0.36$	$3^2 \cdot 0.12 = 1.08$
		$\sum x \cdot P(x) = 1.16$	$\sum x^2 \cdot P(x) = 2.38$

mean $\mu = \sum x \cdot P(x) = 1.16 =$ 1.2 units

Note rounding rule uses x-values. Note no unit given.

standard deviation $\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$

$= \sqrt{2.38 - (1.16)^2}$

$= 1.01705\ldots =$ 1.0 units

Problem 5: 26 pts

Let X be a random variable that counts the number of Heads can may occur out of three coin flips (assume that the coin is fair).

(2 pt) (a) Using set notation, write the **sample space** of the experiment of flipping 3 coins. *(Hint: use the H or T notation. So one outcome is HHT, for example)*

$S = \{$ TTT, HTT, THT, TTH, HHT, HTH, THH, HHH $\}$

($x=0$: 0 Heads; $x=1$: 1 Heads; $x=2$: 2 Heads; $x=3$: 3 Heads)

Note $\#S = 8$

Sample space is set of all possible outcomes.

(2 pt) (b) What does x represent? What are all the values can it be?

x = # of Heads out of 3 coin flips

x = values X can take.

$x = 0, 1, 2, 3$

(4 pt) (c) Is X a **binomial probability distribution**? Explain why or why not.

yes, X is a binomial prob. dist. because it satisfies all four conditions:

(1) $n = 3$ (fixed # trials) (2) each trial is independent. (3) only two outcomes S (Heads) F (tail) (4) probability of success is constant ($p = 0.5$)

(4 pt) (c) Give the **probability distribution** associated to X.

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	$1/8 = 0.125$	0	0
1	$3/8 = 0.375$	0.375	0.375
2	$3/8 = 0.375$	0.75	1.5
3	$1/8 = 0.125$	0.375	1.125
		$\Sigma = 1.5$	$\Sigma = 3$

(2 pt) (d) What is the probability of getting **at most** two Heads?

x: 0 1 2 | 3 "at most 2"

$P(x \le 2) = 0.125 + 0.375 + 0.375 = 0.875$

[another way: $P(x \le 2) =$ binomcdf$(3, 0.5, 2) = 0.875$]

(2 pt) (e) What is the probability of getting **at least** two Heads?

0 | 1 2 3 "at least 2"

$P(x \ge 2) = P(x=2 \text{ or } x=3) = 0.375 + 0.125 = 0.5$

[another way: $P(x \ge 2) = 1 - P(x \le 1) = 1 -$ binomcdf$(3, 0.5, 1) = 0.5$]

(2 pt) (f) What is the **mean**?

$\mu = \sum x \cdot P(x)$

$\mu = 1.5$ Heads in 3 flips

Note units of μ same as units of X or x

Note μ is "average" or "expectation"

(2 pt) (g) What is the **standard deviation**?

$\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$

$\sigma = \sqrt{3 - (1.5)^2}$

$= 0.8660\ldots$

$\sigma = 0.9$ Heads in 3 flips

Problem 6: 6 pts

Fidelity life insurance will sell a $100,000 one-year term life insurance policy to 30 year-olds males for a premium of $161. Based on U.S. Department of Health and Human Services, it has been determined that there is a 99.86% chance that a randomly selected 30 year old male survives the year.

$p = 0.9986$ $q = 1 - p = 1 - 0.9986 = 0.0014$

Find the **expected value** to the company (that is, from the company's point of view) of this policy for a 30 year old male. Include in your answer the corresponding probability distribution. Also, please write your final answer in a complete sentence $(M \to E)$.

	x	$P(x)$
lives:	\$161	0.9986
dies:	\$ −99,839	0.0014
		$\Sigma = 1$ ✓

- X = the amount gained (in \$) or lost for the insurance co. for one policy for one year for a 30 y.o. male client.
- $E(X) = \sum x \cdot P(x)$

  $= (161)(0.9986) + (-99839)(0.0014)$

  $= \$21.00$

  $E = \$21.00$
- $(M \to E)$

  "The insurance company expects to gain \$21.00 over many similar policies (30 y.o males)."

Problem 7: 16 pts

Assume we have a standard deck of 52 cards (see below). Use proper probability notation and rules when writing your answer.

(2 pt) (a) When one card is drawn, what is $P(\text{Black and 4})$?

drawing one card so use intersection of cards: 4 club & 4 spade.

$P(\text{Black and 4}) = \frac{2}{52} = 0.038$

(2 pt) (b) When one card is drawn, what is $P(\text{Spade or Heart})$?

$P(\text{Spade or Heart}) = P(\text{Spade}) + P(\text{Heart}) - P(\text{Spade and Heart}) = \frac{13}{52} + \frac{13}{52} - \frac{0}{52}$

$= \frac{26}{52} = 0.5$

(2 pt) (c) When one card is drawn, what is $P(\overline{\text{Face}})$?

$P(\overline{\text{Face}}) = P(\text{not Face}) = 1 - P(\text{Face}) = 1 - \frac{12}{52} = \frac{52}{52} - \frac{12}{52} = \frac{40}{52} = 0.769$

(2 pt) (d) When two cards are drawn with replacement, find $P(\text{Two Queens})$?

$P(Q \text{ and } Q) = P(Q) * P(Q) = \frac{4}{52} * \frac{4}{52} = 0.00592$

2 cards use multiplication rule

Note use 3 sig fig rounding rule

(2 pt) (e) When two cards are drawn with replacement, find $P(1^{st}$ is a Black card and 2^{nd} is a Face card)?

$P(B \text{ and Face}) = P(B) * P(\text{Face}) = \frac{26}{52} * \frac{12}{52} = 0.115$

(2 pt) (f) When two cards are drawn **without** replacement, find $P(\text{Two Red Face cards})$?

$P(RF \text{ and } RF) = P(RF) * P(RF) = \frac{6}{52} * \frac{5}{51} = 0.0113$

(2 pt) (g) When two cards are drawn **without** replacement, find $P(2^{nd}$ card is Heart $|\ 1^{st}$ card is Black)?

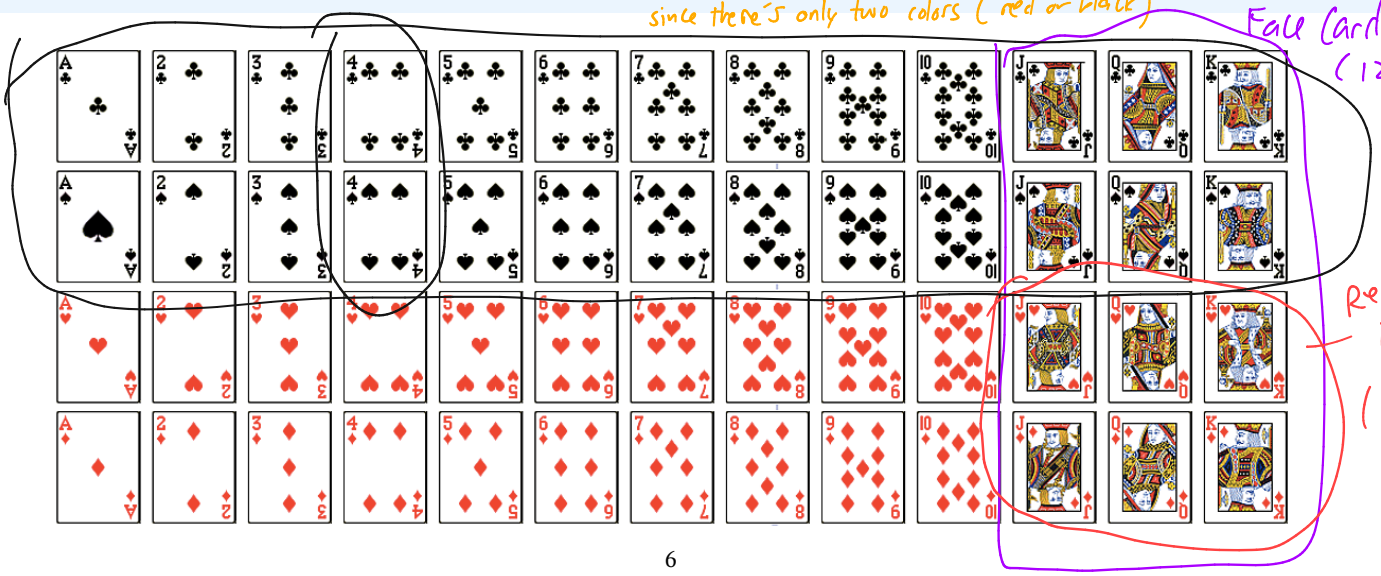
$P(H|B) = P(B \text{ and } H) = P(B) * P(H) = \frac{26}{52} * \frac{13}{51} = 0.127$

Note since Hearts & Black cards are disjoint, P(H) stays at 13 in numerator, but denominator drops to 51 b/c drawing without replacement.

(2 pt) (h) When two cards are drawn **without** replacement, find $P(\text{Two different colors})$?

$P(\text{one color and other color}) = P(\text{one color}) * P(\text{other color}) = \frac{26}{52} * \frac{26}{51} = 0.255$

since there's only two colors (red or black)



Problem 8: 12 pts

A fair, **12-sided die** is rolled. Let **A** = { **odd** numbers }, **B** = { **even** numbers }, and **C** = { numbers **greater than** 7 }.

(3 pt) (a) Using set notation, express the **sample space**.

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ $\#S = 12$

(3 pt) (b) Using set notation, express the event A and calculate $P(A)$.

$A = \{1, 3, 5, 7, 9, 11\}$ $P(A) = \frac{\#\text{ outcomes in } A}{\#\text{ outcomes in } S} = \frac{6}{12} = 0.5$

(3 pt) (c) Using set notation, express the event $\overline{C}$ and calculate $P(\overline{C})$.

$C = \{8, 9, 10, 11, 12\}$ $\overline{C} = \text{not } C = \{1, 2, 3, 4, 5, 6, 7\}$ $P(\overline{C}) = \frac{\#\text{ in } \overline{C}}{\#\text{ in } S} = \frac{7}{12} = 0.583$

(3 pt) (d) Using set notation, express the event $A|\overline{C}$ and calculate $P(A|\overline{C})$.

$A|\overline{C}$ means A and $\overline{C}$

$A|\overline{C} = \{1, 3, 5, 7\}$ $P(A|\overline{C}) = \frac{P(A \text{ and } \overline{C})}{P(\overline{C})} = \frac{4/12}{7/12} = \frac{4}{7} = 0.571$

Problem 9: 18 pts

Individual plays on a slot machine are independent. The probability of winning on any play is 0.18.

- X = # of wins in slot machine out of n trials
- notice this is a binomial dist.
  1. fixed trials ($n = 3$ or 15)
  2. independent
  3. bi → win $P(\text{win}) = 0.18$, → lose $P(\text{lose}) = 0.82$
  4. probability win constant. $p = 0.18$

(3 pt) (a) What is the probability of winning three times in a row?

binomial $n = 3$, $p = 0.18$, $x = 3$

$P(x = 3) = \text{binompdf}(3, 0.18, 3) = 0.00583$

this is a very low probability of winning all 3 times! 0.583% less than 1%

For parts (b)-(e), suppose the slot machine is played 15 times in a row. $n = 15$

(3 pt) (b) What is the probability of winning **exactly** 10 times?

exactly 10

$P(x = 10) = \text{binompdf}(15, 0.18, 10) = 3.98 \times 10^{-5} = 0.0000398$

(3 pt (c) What is the probability of winning **at least** 10 times?

at least 10

0 1 ... 9 | 10 11 ... 15

$P(x \geq 10) = 1 - P(x \leq 9) = 1 - \text{binomcdf}(15, 0.18, 9) = 4.40 \times 10^{-5} = 0.0000440$

(3 pt (d) What is the probability of winning **less than** 8 times?

less than 8

0 1 ... 7 | 8 9 ... 15

$P(x < 8) = P(x \leq 7) = \text{binomcdf}(15, 0.18, 7) = 0.998$

(3 pt (e) Calculate the **mean** and **standard deviation**.

Since X is a binomial prob. dist., we can use $\mu = n \cdot p$ and $\sigma = \sqrt{n \cdot p \cdot q}$

$n = 15$, $p = 0.18$, $q = 1 - p = 0.82$

- $\mu = n \cdot p = 15 * 0.18 = 2.7$ wins in 15 plays
- $\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{15 * 0.18 * 0.82} = 1.48795\ldots = 1.5$

$\sigma = 1.5$ wins in 15 plays

Note rounding use x = # of wins

(3 pt (f) Are 6 wins out of 15 **significantly high**? Justify your answer with statistical reasoning and give your answer in a complete sentence $(M \to E)$.

2 ways to answer this:

Method 1: range rule of thumb

Method 2: use Probability < 0.05

Method 1

- $x = 6$ wins
- Sig High: $\mu + 2\sigma$
- $\mu + 2\sigma = 2.7 + 2(1.5) = 5.7$
- $x = 6 > 5.7$ → yes, sig high

Method 2

- $P(x = 6) = \text{binompdf}(15, 0.18, 6) = 0.0285$
- Since $0.0285 < 0.05$, 6 wins is sig high

$(M \to E)$

"Six wins is significantly high because it is more than 2 standard deviations above the mean, or the probability is 0.0285 which is unusual."

Formula Sheet

- $P(A \textbf{ or } B) = P(A) + P(B) - P(A \textbf{ and } B)$
- $P(A \textbf{ and } B) = P(A) * P(B)$
- $P(A \textbf{ and } B) = P(A) * P(B|A)$
- $P(B|A) = \dfrac{P(A \textbf{ and } B)}{P(A)}$
- $P(\bar{A}) = 1 - P(A)$

- $\mu = \sum[x \cdot P(x)]$
- $\sigma^2 = \sum(x - \mu)^2 \cdot P(x)]$
- $\sigma = \sqrt{\sum[x^2 \cdot P(x)] - \mu^2}$
- $E(X) = \sum[x \cdot P(x)]$

Post Exam Survey

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

1. When taking the exam I felt
   (a) Rushed. I wanted more time.
   (b) Relaxed. I had enough time.
   (c) Amazed. I had tons of extra time.
2. The week before the test I did all my homework on time: YES NO
3. The week before the test, in addition to the homework I followed a study plan. YES NO
   (a) I think this helped: YES NO
4. The day before the test I spend _____ hours studying and reviewing.
   (a) I think that was enough time: YES NO
5. The night before the test:
   (a) I stayed up very late cramming for the test
   (b) I stayed up very late, but I wasn't doing math
   (c) I didn't need to cram because I was prepared
   (d) I got a good night's sleep so my brain would function well.
6. I think I got the following grade on this test: _____
7. Strategies that worked well for me were (please elaborate):

8. Next time I will do an even better job preparing for the test by: