SCIT — Southern California Institute of Technology

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Chapter 4: Probability Distribution Functions

Binomial Distribution

Activity 1: Binomial-Distribution-Probability

For each part, please label the n, X, and p in addition to your work and answer. Leave answers as decimals and round to three decimal places.

Find the probability that in tossing a fair coin three times, there will appear

(a) three heads

(b) two tails and a head

(c) at least one head

(d) not more than one tail

Activity 2: Binomial-Distribution-Probability

For each part, please label the n, X, and p in addition to your work and answer. Give answers as percentages and round to one decimal place.

Find the probability that in five tosses of a fair die, a 3 will appear

(a) twice

(b) at most once

(c) at least two times

Activity 3: Probability

A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversized or undersized. What is the probability that a batch of 10 pistons will contain

(a) no more than 2 rejects?

(b) at least 2 rejects?

Visualizing a Binomial Distribution

Activity 4: Visualizing-Binomial-Distribution

Let X be the number of heads that turn up after flipping a coin five times. Then $n = 5$ and x can be $0, 1, 2, 3, 4, 5$. We can calculate the probability of zero heads turning up with $P(X = 0)$, one head turning up with $P(X = 1)$, etc. Using our calculator check that:

$$P(X = 0) = \frac{1}{32}, P(X = 1) = \frac{5}{32}, P(X = 2) = \frac{10}{32}, P(X = 3) = \frac{10}{32}, P(X = 4) = \frac{5}{32}, P(X = 5) = \frac{1}{32}$$

(a) Plot a histogram for the random variable X probability distribution.

To do this, on the horizontal axis scale from $x = 0, 1, 2, 3, 4, 5$ and the vertical axis scale from 0 to $10/32$ with $1/32$ intervals.

(b) Describe any interesting features from your histogram.

(c) Sketch what you think the histogram would look like for the same random variable X but the number of trails is $n = 100$.

Activity 5: Visualizing-Binomial-Distribution

Let our experiment be shooting free-throws. Assume that the probability of making a freethrow is 70% and that these are independent events. Let X be the number of made in taking six shots.

(a) Use your calculator to find $P(X = x)$ for $x = 0, 1, 2, 3, 4, 5, 6$.

(b) Plot a bar graph for the random variable X probability distribution.

(c) Describe any interesting features from your histogram. Is it positively skewed, negatively skewed, or symmetric?

Activity 6: Probability

Hospital records show that of the patients suffering from a certain disease, 75% die of it.

(a) What is the probability that of 6 randomly selected patients, 4 will recover? Give your answer as a percent rounded to the nearest tenth.

(b) Plot the probability distribution function.

(c) Is the histogram positively skewed, negatively skewed, or symmetric?

Normal Distribution

Activity 7: Normal-Distribution

Weight (in grams) of bags of sugar from a factory are normally distributed, with a mean of 1000g, and standard deviation of 13g. Find the following.

(a) The probability that a randomly selected bad of sugar weighs in between 974g and 1000g. Do this with a calculator.

(b) The percentage of bags whose weight is above 1026g. Do this with a calculator.

Activity 8: Normal-Distribution

The time it takes employees to get to work from home (in minutes) is normally distributed with a mean of 30 minutes, and a standard deviation of 5 minutes. Find the following.

(a) The percentage of employees that take between 20 and 40 minutes to get to work. Do this without a calculator.

(b) The percentage of employees that take between 28 and 37 minutes to get to work. Do this with a calculator.

Activity 9: Convert-z-values

Convert each of the following between x and z values.

(a) $x = 35$ where $\mu = 40$, $\sigma = 2$

(b) $x = 130$ where $\mu = 100$, $\sigma = 12$

(c) $z = -0.57$ where $\mu = 14$, $\sigma = 1.5$

Activity 10: Standard-Normal-Distribution

Find the area under the standard normal curve between

(a) $z = 0$ and $z = 1.2$

(b) $z = -0.68$ and $z = 0$

(c) $z = -0.46$ and $z = 2.21$

(d) to the right of $z = -1.28$

Activity 11: Standard-Normal-Distribution

The mean weight of 500 male students at a certain college is 151 lb and the standard deviation is 15 lb. Assuming that the weights are normally distributed, find without using a calculator how many students weigh

(a) between 120 and 155 lb

(b) more than 185 lb.

Activity 12: Inverse-Normal-Distribution

Find the 90th percentile for a normal distribution with a mean of 70 and a standard deviation of 4.5.

Activity 13: Inverse-Normal-Distribution

The time it takes employees to get to work from home (in minutes) is normally distributed with a mean of 30 minutes, and a standard deviation of 5 minutes. Find:

(a) the percentage of employees that take between 28 and 37 minutes to get to work (Hint: this is not an inverse problem)

(b) The number of minutes the longest it would take the bottom employee in the bottom 5% of the data to get to work. (Hint: this is an inverse problem)

Activity 14: Inverse-Normal-Distribution

An average light bulb manufactured in a factory lasts 280 days with a standard deviation of 45 days. Assume that bulb life is normally distributed.

(a) What is the probability that an Acme light bulb will last at most 360 days? (Hint: this is not an inverse problem)

(b) What bulb life separates the bottom 12%? (Hint: this is an inverse problem)