

Jan 16

Chapter 5: Discrete Probability Distributions

Section 5.1: Probability Distributions

Capital X

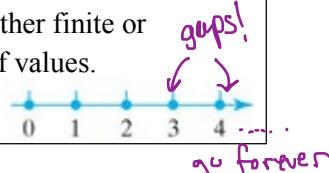
(RV)

Random Variable (X) is a numerical measure of the outcome of a probability "experiment", so its value is determined by chance.

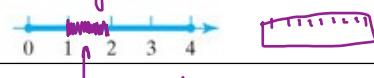
Notation

$\underline{X} = \text{name of RV}$
 $x = \text{value } X \text{ can take}$

Discrete Random Variable: either finite or countable number of values.



Continuous Random Variable: has (measurable) infinitely many values.



Ex: Identify the random variable and sample space.

(a) Coin toss for Heads

$X = \# \text{ of Heads in 10 coin flips}$

the sample space $x = 3$ (e.g.) $(x=0, 1, \dots, 10)$
 (single event) \uparrow one outcome $H T T H H T T T T$

PROBABILITY DISTRIBUTIONS

Def A **probability distribution** is a description that gives the probability for each value of the random variable. It is often expressed in the format of a graph, table, or formula.

REQUIREMENTS

1. $\sum P(x) = 1$

(where x assumes all possible values.)

2. $0 \leq P(x) \leq 1$

(for every individual value of x .)

Ex: Are the following a probability distribution? If not, state why.

2 ✓
1 ?

$\sum P(x) = 1?$
No $\sum = 0.97$

A.	x	$P(x)$
	0	0.16
	1	0.18
	2	0.22
	3	0.10
	4	0.30
	5	0.01

$\Sigma = 0.97$

B.	x	$P(x)$
	0	0.16
	1	0.18
	2	0.22
	3	0.10
	4	0.30
	5	-0.01

NOT PD

C.	x	$P(x)$
	0	0.16
	1	0.18
	2	0.22
	3	0.10
	4	0.30
	5	0.04

2 ✓
1 ? ✓

yes, satisfies both conditions!

$\Sigma = 1.0$

Probability Dist.

x	$P(x)$
0	$1/16 = 0.0625$
1	$4/16 = 0.25$
2	$6/16 = 0.375$
3	$4/16 = 0.25$
4	$1/16 = 0.0625$

$P_{\text{prob}} = \frac{\# \text{ of cases}}{\# \text{ sample space}}$

Jan 17 Ex: A couple plans to have four children. Let x be the number of boys the couple will have. Find the probability distribution for the number of boys.

$X = \# \text{ of boys out of 4 kids.}$
 $x = 0, 1, 2, 3, 4$

$S = \{GGGG, BGGG, GBGG, GGBG, GGGG\}$
 $\#S = 16$

$\{x=0\} = \{GGGG\}$
 $\{x=1\} = \{BGGG, GG-BB, GBBG, GB-GG\}$
 $\{x=2\} = \{BBG-G, BB-BB, BGBB, GB-BB\}$
 $\{x=3\} = \{BBB-G, BBB-B, BGBB, GB-BB\}$
 $\{x=4\} = \{BBBB\}$

Formulas are used when given a Prob. Dist.

MEAN VALUE

FORMULA:

$$\mu = \sum [x \cdot P(x)]$$

Note: The Greek letter μ is read "mu"

VARIANCE

FORMULA:

$$\sigma^2 = \sum [(x - \mu)^2 \cdot P(x)]$$

$$\text{or } \sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2$$

STANDARD DEVIATION

FORMULA:

$$\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$$

Note: The Greek letter σ is read "sigma"

Round-Off Rule: Carry one more decimal place than is used for the random variable. → Starts Rule for Rounding

Ex: In a study of brand recognition, random groups of four people are interviewed. Let x be the number of people who recognize Jeff Bezos when shown a picture. The following probability distribution gives the likelihood of the random variable. (RV) $X = \# \text{ of people who recognize Jeff Bezos out of 4}$

values of RV X

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	0.01	$0 * 0.01 = 0$	$0^2 * 0.01 = 0$
1	0.10	$1 * 0.10 = 0.1$	$1^2 * 0.10 = 0.10$
2	0.24	$2 * 0.24 = 0.48$	$2^2 * 0.24 = 0.96$
3	0.30	$3 * 0.30 = 0.90$	$3^2 * 0.30 = 2.7$
4	0.35	$4 * 0.35 = 1.4$	$4^2 * 0.35 = 5.6$
	$\sum P(x) = 1$	$\sum [x \cdot P(x)] = 2.88$	$\sum [x^2 \cdot P(x)] = 9.36$

(a) What is the probability that more than two people will recognize a picture of Jeff Bezos?

$x = 3, 4$ "more than 2"

$$P(X=3 \text{ or } X=4) = P(X=3) + P(X=4) - P(X=3 \text{ and } X=4) = 0.3 + 0.35 = 0.65$$

(b) What is the probability that at most three people will recognize a picture of Jeff Bezos?

$$x = 0, 1, 2, 3, 4 \rightsquigarrow \text{"inequality"} \quad x \leq 3$$

Use the "1 minus trick" $P(\bar{A}) = 1 - P(A)$

$$P(X \leq 3) = P(X=4) = 1 - P(X=4) = 1 - 0.35 = 0.65$$

(c) Find the mean number of people who recognize a picture of Jeff Bezos.

$$\mu = \sum [x \cdot P(x)] = 2.88 = \boxed{2.9 \text{ people recognize J. Bezos out of 4}}$$

↑
rounding rule use RV

(d) Find the standard deviation of the given probability distribution.

$$\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2} = \sqrt{9.36 - (2.88)^2} = 1.032279 \dots$$

↑
 9.36

$$\sigma = 1.0 \text{ people recognize Bezos out of 4}$$

EXPECTED VALUE

Def The **expected value** (denoted E) of a discrete random variable represents the mean value of the outcomes.

FORMULA:

$$E = \sum [x \cdot P(x)]$$

units of E are same as units of x (or X)

Assume: if win get \$1 back ("bet")

Ex: There is a game in Vegas where you can win \$4 or \$9 but it costs \$1 to play the game. The probability of winning \$4 is 0.3 and the probability of winning \$9 is 0.1. Find the expected value for this game.

x	$P(x)$
-1	0.6
4	0.3
9	0.1

$$\sum P(x) = 1$$

- X = value of winning or losing from playing game (once).
- $x = -1, 4, 9$
- $E = \sum [x \cdot P(x)] = (-1) * 0.6 + (4) * (0.3) + (9) * 0.1$

$$E = \$1.5$$

$$0.6 = 1 - 0.3 - 0.1$$

Interpretation of μ : Over the long run (if we play this game MANY times) we expect the mean profit to be \$1.50.

↳ if $E > 0$, keep playing • if $E < 0$, don't even play • if $E = 0$, up to you!

Ex: When someone buys a life insurance policy, that policy will pay out a sum of money to a benefactor upon the death of the policyholder. Suppose a 25-year-old male buys a \$150,000 1-year term life insurance policy for \$250. The probability that the male will not survive the year is 0.0013.

The experiment has two possible outcomes: survive or die. Let the random variable X represent the money lost or gained by the life insurance company for the 25-year-old male after many years. What is the expected value for the company?

x	$P(x)$
survive \$250	0.9987
die -\$149,750	0.0013

X = money lost or gained by insurance co.
 $x = \begin{cases} \$250 \\ -\$149,750 \end{cases}$

$$P(\text{die}) = 0.0013$$

$$P(\text{Survive}) = 1 - P(\text{die}) = 1 - 0.0013 = 0.9987$$

"minustick"

$$E = \sum [x \cdot P(x)] = (250) * (0.9987)$$

$$+ (-149,750) * (0.0013)$$

$$= 1.3$$

$$= \$55$$

$$= 1.3(-149,750) + 0.9987(250) = 55000 \text{ (total worth)} \\ 1000 \text{ policies}$$

Over 1,000 policies, how much should they expect to make?

Insurance Co. expects to make only
\$55 per policy.

Ex: Find the expected value of the random variable. Round to the three decimal places.

A contractor is considering a sale that promises a profit of \$29,000 with a probability of 0.7 or a loss (due to bad weather, strikes, and such) of \$3,000 with a probability of 0.3. What is their expected profit?

- A. \$19,400
- B. \$21,200
- C. \$20,300
- D. \$22,400
- E. \$26,000