HT
CI

Chapter 11: Inferences on Two Samples

Section 11.3: Inference about Two Population Means: Independent Samples

Steps for a Hypothesis Test When Applied to testing $\boldsymbol{\mu_1}$ and $\boldsymbol{\mu_2}$ (Independent)

Step 0: *Check Requirements*

- The samples are independent and randomly obtained
- The values of the population standard deviation σ_1 and σ_2 are not known and unequal. ✓
- Populations are (normally distributed) **OR** ($n_1 >$ 30 **and** $n_2 >$ 30)

Step 1: *Hypotheses*

$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 < \mu_2$ Left Tailed Test	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 \neq \mu_2$ Two Tailed Test	$H_0: \mu_1 = \mu_2$ $H_A: \mu_1 > \mu_2$ Right Tailed Test

Step 2: *Level of Significance* α

If it's not given, then use 0.05. Choice depends on seriousness of making Type I error.

Step 3: *Test Statistic*

compare w/: 1 Mean

$$t^* = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

$$\boxed{t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}}$$

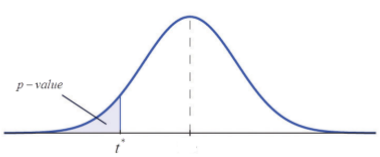
- Parameter: difference of independent means
- $\mu_1 - \mu_2$. $H_0: \mu_1 - \mu_2 = 0$
- $n_1, n_2, \bar{x}_1, \bar{x}_2$

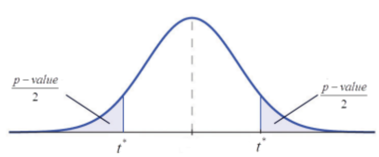
Step 4: *Find a Critical Value or P-Value*

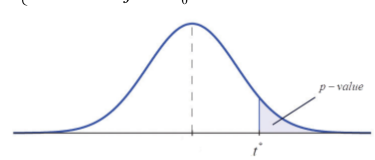
P-VALUE METHOD

DECISION

Reject H_0 □ if *P-value* $\leq \alpha$
Fail to Reject H_0 □ if *P-value* $> \alpha$



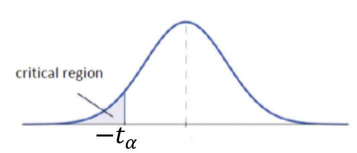


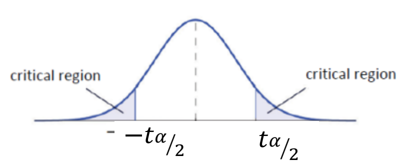


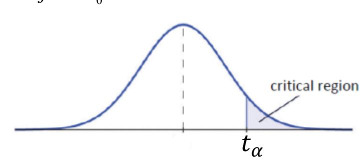
CRITICAL VALUE METHOD

DECISION

Reject H_0 □ if t^* lies in the critical region
Fail to Reject H_0 □ if t^* doesn't lie in the critical region







Step 5: State the *DECISION* and write a *CONCLUSION* either rejecting or failing to reject H_0

GRAPHING CALCULATOR (TI-83 OR 84) INSTRUCTIONS

Instructions: (a) STAT ⇒ TESTS ⇒ $\boxed{\text{2-SampTTest}}$

(b) Enter
- $\bar{x}_1$ / $\bar{x}_2$ = mean of sample #1 / #2
- s_1 / s_2 = standard deviation of sample #1 / #2
- n_1 / n_2 = size of sample #1 / #2
- μ_1 □ alternative hypothesis (*not pooled*)

μ_2 - mean weight babies w/o Zinc Supp.

μ_1 - mean weight of babies from mothers with Zinc supplement

Ex 1: A study of zinc-deficient mothers was conducted to determine whether zinc supplements during pregnancy results in babies with increased weights at birth. 294 expectant mothers were given a zinc supplement, and the mean birth weight was 3214 grams with a standard deviation of 669 g. There were 286 expectant mothers who were given a placebo, and the mean weight was 3088 g with a standard deviation of 728 grams. Using a 0.01 significance level, is there sufficient evidence to support the claim that a zinc supplement does result in increased birth weights? | HT

2 independent means : Group 1 - zinc Group 2 - Placebo

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_A: \mu_1 > \mu_2 \end{cases}$$ (Right-Tailed Test)

$\boxed{\text{Calc}}$ 2 SampTTest

Test Statistic

$$\boxed{t^* = 2.17}$$

$n_1 = 294$

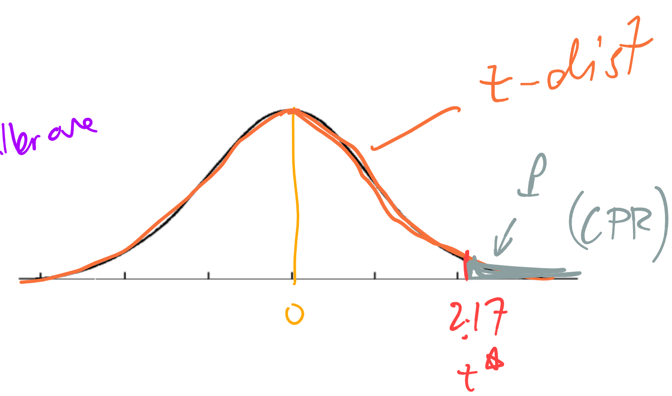
$n_2 = 286$

P-value/Critical Region

$$\boxed{P = 0.0153}$$

df - smaller one

$P(t > t^*) = \text{tcdf}(2.17, 1E99, 285)$



Decision about Null Hypothesis

$\alpha = 0.01$

$P = 0.0153$

$P > \alpha \longrightarrow$ "P is high, Null will fly"

$\boxed{\text{FTR } H_0}$

Conclusion

"There is not enough statistical evidence to support the claim that a zinc supplement does result in increased weights for newborn babies."

GTQ!

HT 2 groups Independent

Ex 2: A professor at a large community college wanted to determine whether there is a difference in the means of final exam scores between students who were allowed to text in class and those who weren't. She believed that the mean of the final exam scores for the texting class would be lower than that of the non-texting class but her students didn't think so. Were the students correct? The professor randomly selected 30 final exam scores from each group, and they are listed below.

Use 1-VAR-STATS

67.6	41.2	85.3	55.9	82.4	91.2	73.5	94.1	64.7	64.7
70.6	38.2	61.8	88.2	70.6	58.8	91.2	73.5	82.4	35.5
94.1	88.2	64.7	55.9	88.2	97.1	85.3	61.8	79.4	79.4

Texting class μ_1

$\bar{x}_1 = 72.85$

$s_1 = 16.92$

77.9	95.3	81.2	74.1	98.8	88.2	85.9	92.9	87.1	88.2
69.4	57.6	69.4	67.1	97.6	85.9	88.2	91.8	78.8	71.8
98.8	61.2	92.9	90.6	97.6	100	95.3	83.5	92.9	89.4

Non-texting class μ_2

$\bar{x}_2 = 84.98$

$s_2 = 11.72$

Null and Alternative Hypothesis

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_A: \mu_1 < \mu_2 \end{cases}$$

(Left-Tailed Test)

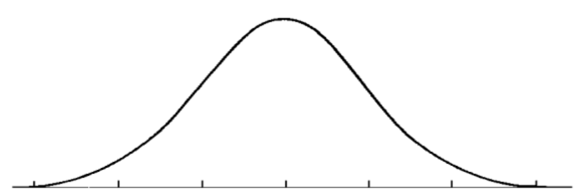
2 Samp T Test

Test Statistic

$\boxed{t^* = -3.23}$

P-value/Critical Region

$\boxed{P = 0.00108}$



Decision about Null Hypothesis

α not given

use $\alpha = 0.05$

$P = 0.00108$

$\alpha = 0.05$

$P < \alpha \longrightarrow$ Reject H_0

"P low, Null go"

Conclusion

population parameter: $\mu_1 - \mu_2$

CONFIDENCE INTERVAL FOR THE DIFFERENCE OF TWO POPULATION MEANS

Alternative Forms: $(\bar{x}_1 - \bar{x}_2) - E < \mu_1 - \mu_2 < (\bar{x}_1 - \bar{x}_2) + E$ or $\boxed{(\bar{x}_1 - \bar{x}_2) \pm E}$ or

where the margin of error is given by $\boxed{E = t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

Point Estimate: $\bar{x}_1 - \bar{x}_2$

compare w/ $E = t_{\alpha/2} \cdot \sqrt{\frac{s}{n}}$

GRAPHING CALCULATOR (TI-83 OR 84) INSTRUCTIONS

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ $\boxed{\text{2-SampTInt}}$

(b) Enter
- $\bar{x}_1 \,/\, \bar{x}_2$ = mean of sample #1 / #2
- $s_1 \,/\, s_2$ = standard deviation of sample #1 / #2
- $n_1 \,/\, n_2$ = size of sample #1 / #2
- $C-level$ = confidence level (*not pooled*)

→ Independent

Ex 3: The Gallup Organization wanted to investigate the time that American men and women spend hanging out with their friends. A random sample of 700 men surveyed spent a mean time of 10 hrs per week with their friends with a standard deviation of 1.9 hours. On the other hand, 740 women surveyed spent a mean time of 7.5 hours with a standard deviation of 1.6 hours. Construct a 95% confidence interval estimate for the difference between the corresponding population means.

↳ CI

↳ parameter, $\mu_1 - \mu_2$ (men, women)

Two Samples: Independent

Find the point estimate (difference between sample means)

$\bar{x}_1 - \bar{x}_2 = 10 - 7.5 = \boxed{2.5 \text{ hrs/wk}}$

Determine critical value $t_{\alpha/2}$

$CL = 0.95$

$\alpha = 1 - CL = 0.05$

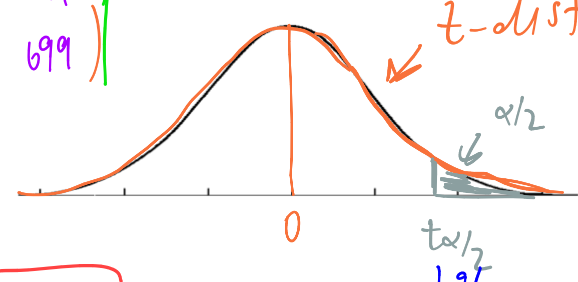
$\alpha/2 = 0.025$

$t_{\alpha/2} = t_{0.025} = |\text{invT}(0.025, \underset{df}{699})|$

$t_{0.025} = 1.96$

InvT is Mean

t-dist



Find margin of error

$E = t_{\alpha/2} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (1.96)\sqrt{\frac{(1.9)^2}{700} + \frac{(1.6)^2}{740}} = \boxed{0.18}$

Construct confidence interval

Calculator (2.32, 2.68)

$(\bar{x}_1 - \bar{x}_2) \pm E$: 2.5 ± 0.18

$\boxed{\text{CI: } (2.32, 2.68)}$ units hrs/wk.

Estimating $2.32 < \mu_1 - \mu_2 < 2.68$

Does it appear that there is a difference between men and women?

"Yes, it appears that men spend on average, 2.32 hours per week more than women with friends."