MATH 110B - Precalculus II | Spring 2020

Exam 1

§6.1–6.5, 7.1

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March_9

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Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

NAME (PRINT): Solutions SIGNATURE: ____________________

Directions

- YOU ARE ALLOWED TO USE ONLY A SCIENTIFIC CALCULATOR ON THIS EXAM.
- You have 85 minutes to complete this exam.
- The exam totals **100 points** with 5 points of extra-credit possible.
- There are 7 problems (plus one extra-credit problem at the end), many of them with multiple parts.
- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you your writing utensils.
- Cell phones must be turned off and put away in with your items in the front of the classroom.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E). For True/False questions, you must spell out the entire word "true" or "false" in your answer.
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- A problem which requires a **proof** means you must provide a general proof in complete sentences. Do not use logical short-hand in proofs.
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a $\boxed{\text{box around your final answer}}$ where applicable.
- **PLEASE CHECK YOUR WORK!!!**

Score	Grade

This page is intentionally blank. It may be used for scratch paper. If you wish for me to grade your work on this page, please (i) label the problem you are working on, (ii) box your answer, (iii) indicate in the original problem's location that you will continue your work on this page.

Problem 1: 20 pts (2 pts each)

Fill-in the blank: *(No work needed)*

(a) The **inverse property** $\tan\left(\tan^{-1}(x)\right) = x$ is true for $x \in$ $(-\infty, \infty)$ or $\mathbb{R}$

(b) The **domain** of $\tan(x)$ is all $x \in \mathbb{R} : x \neq \frac{\pi}{2} + \pi k,\ k \in \mathbb{Z}$ ($\mathbb{Z}$ shorthand for integers)

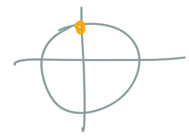
Can also give: $\ldots \cup \left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \ldots$ ← must be w/ dots...

(c) The **asymptotes** of $y = \sec(x)$ on the principal branch occur at $x = \frac{\pi}{2},\ x = \frac{3\pi}{2}$ $1/\cos(x)$ $\cos(x) \neq 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

(d) Give one of the **Pythagorean identities**: $\sin^2(t) + \cos^2(t) = 1$ (2 others possible)

(e) The **terminal point** corresponding to the $t \in \mathbb{R}$ when $t = \frac{402}{4}\pi$ is $(0, 1)$



$\frac{402}{4}\pi = \frac{400}{4}\pi + \frac{2\pi}{4} = 100\pi + \frac{\pi}{2} = 2\pi(50) + \frac{\pi}{2}$

TRUE or **FALSE** (please spell out/write the entire word for credit). *(No work needed)*

(a) TRUE The function $\csc(x)$ is **periodic** with period 2π because $\csc(x + 2\pi) = \dfrac{1}{\sin(x + 2\pi)} = \dfrac{1}{\sin(x)} = \csc(x)$.

(b) FALSE The function $\sec(x)$ is **even** since $\sec(-x) = \cancel{-}\sec(x)$.

(c) FALSE The equation $\sin(x) + \cos(x) = 1$ is an identity. @ $x = \frac{\pi}{6}$: $\sin(x) + \cos(x) = \frac{1}{2} + \frac{\sqrt{3}}{2} \neq 1$

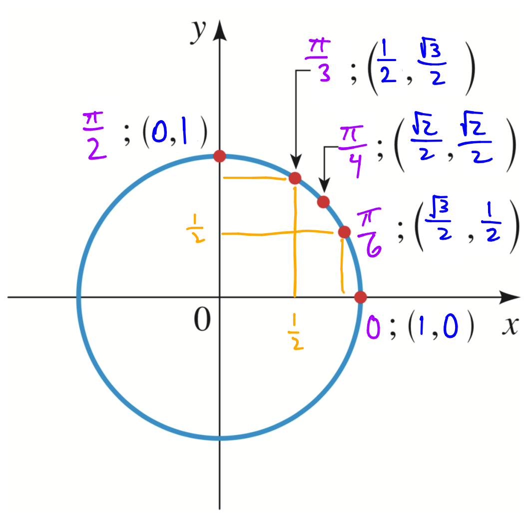
(d) FALSE The graph of $y = 10\cos\left(\pi x + \frac{\pi}{7}\right)$ has **amplitude** 10, **period** 2, and is the graph of $y = \cos(x)$ **shifted horizontally** to the ~~right~~ left by $\frac{1}{7}$.

$\frac{2\pi}{\pi} = 2$

(e) TRUE The domain of the **arccosine** function, $\cos^{-1}(x)$, is $[-1, 1]$.

Problem 2: 15 points - 1 pt each blank

Fill out **Quadrant I** of the unit circle. Include the terminal points (x, y) corresponding to the real numbers $t \in [0, \pi/2]$ discussed in class. Please put the t to the left of the semicolon (;).



Do not need to provide the corresponding angle in degrees, only in radians (i.e. a real number).

Problem 3: 6 pts

Which of the statements below are true **identities**? *If it is an identity, provide a proof; if it is not an identity give a reason why.*

(a) $\sin^4(t) - \cos^4(t) = \sin^2(t) - \cos^2(t)$

$\boxed{\text{Identity}}$

factor: $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$

Proof $LHS = \sin^4(t) - \cos^4(t)$

$= (\sin^2(t) - \cos^2(t)) \cdot \underbrace{(\sin^2(t) + \cos^2(t))}_{=1 \text{ Pythagorean ID}}$

$= \sin^2(t) - \cos^2(t)$

$= RHS. \quad \square$

(b) $\sin(2x) = 2\sin(x)$

$\boxed{\text{Not an identity; an equation}}$

Check: $x = \frac{\pi}{2}$

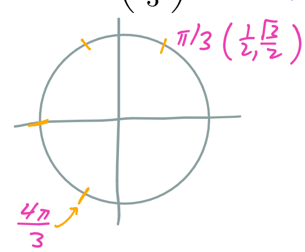
- $LHS = \sin\left(2\left(\frac{\pi}{2}\right)\right) = \sin(\pi) = 0$
- $RHS = 2 \cdot \sin\left(\frac{\pi}{2}\right) = 2 \cdot 1 = 2$

not equal!

Problem 4: 24 pts

Evaluate the following trigonometric functions. Given an exact value.

(a) $\tan\left(\frac{4\pi}{3}\right) = \frac{y}{x} = \frac{\sqrt{3}/\cancel{2}}{1/\cancel{2}} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$

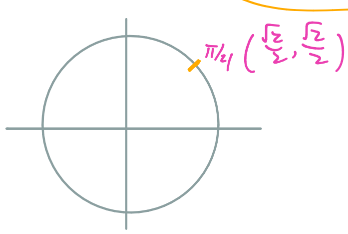


(b) $\tan^{-1}(1) = t = \boxed{\pi/4}$

$t = \tan^{-1}(1)$ "reverse cancel"

$\tan(t) = 1 = \frac{\sqrt{2}/2}{\sqrt{2}/2}$

$t = \pi/4$



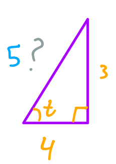
(c) $\sin\left(\underbrace{\tan^{-1}\left(\frac{3}{4}\right)}_{t = \tan^{-1}(3/4)}\right) = \sin(t) = \boxed{3/5}$

use triangle!

soh cah toa

$\longleftrightarrow \tan(t) = 3/4 = opp/adj$

use triangle approach



$3^2 + 4^2 = ?^2$

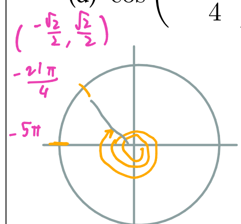
$9 + 16 = ?^2$

$25 = ?^2$

$\boxed{5 = ?}$

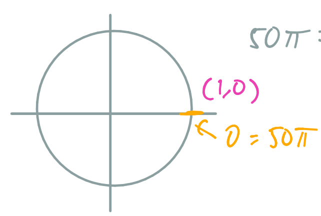
(d) $\cos\left(-\frac{21\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

$\frac{-21\pi}{4} = \frac{-20\pi - \pi}{4} = -5\pi - \frac{\pi}{4}$



(e) $\csc(50\pi) = \frac{1}{\sin(50\pi)} = \frac{1}{0}$ $\boxed{\text{DNE or undefined}}$

$50\pi = 2\pi \cdot 25$

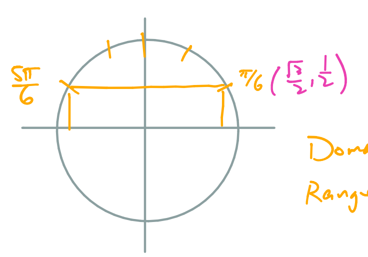


(f) $\cos^{-1}\left(\cos\left(\frac{17\pi}{6}\right)\right) = \boxed{\frac{5\pi}{6}}$

$\frac{17}{6}\pi = \frac{12}{6}\pi + \frac{5\pi}{6} = 2\pi + \frac{5\pi}{6}$

Domain $[-1, 1]$

Range $[0, \pi]$



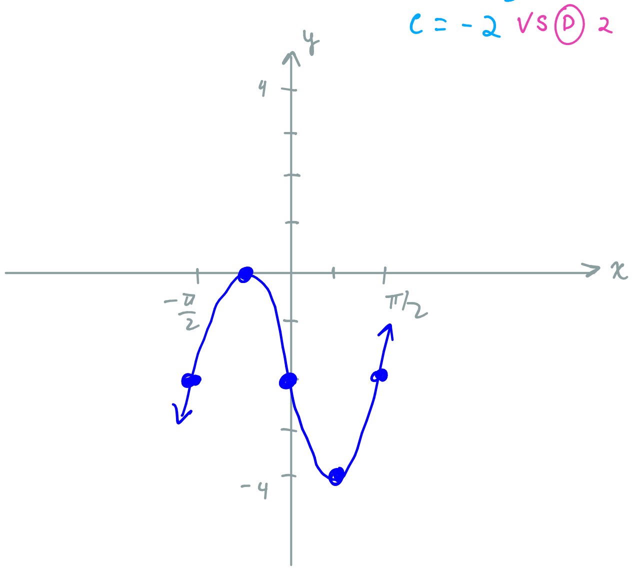
Problem 5: 24 pts

Graph the following trigonometric functions. Include at least one full period. Be sure to label and scale your axes appropriately.

(a) $y = 2\sin(2x + \pi) - 2$

$y = 2\sin\left(2\left(x - \left(-\frac{\pi}{2}\right)\right)\right) - 2$

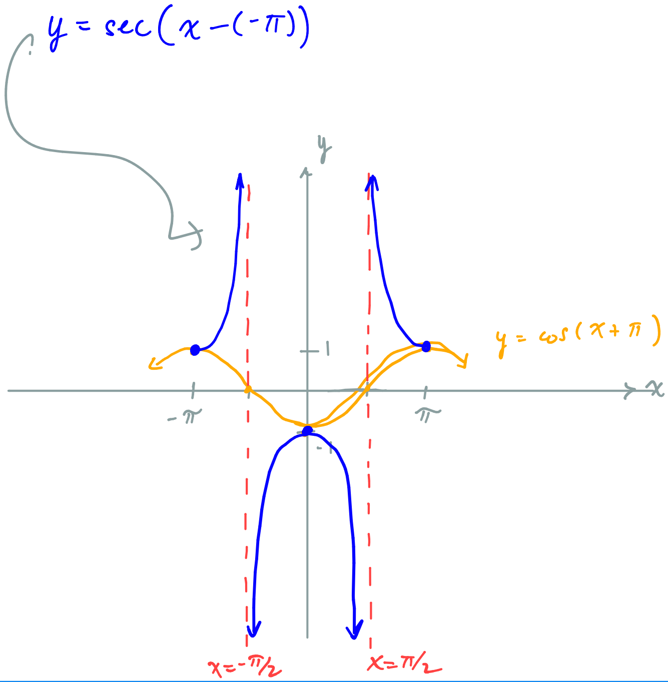
$a = 2$ amplitude
$k = 2$ Period: $\frac{2\pi}{2} = \pi$
$b = -\pi/2$ HS L $\pi/2$
$c = -2$ VS D 2



(b) $y = \sec(x + \pi)$

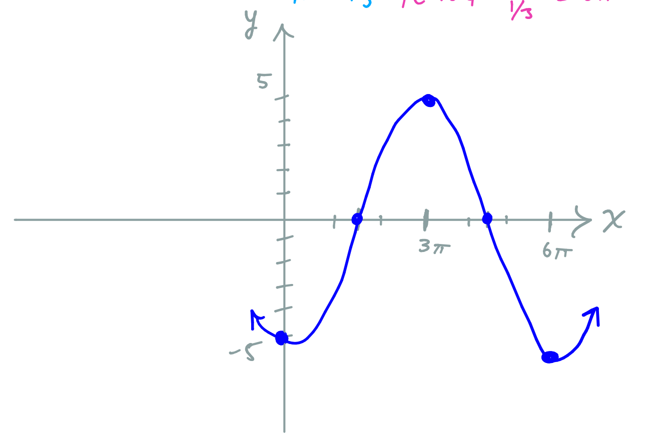
$b = -\pi$ HS L π

$y = \sec(x - (-\pi))$



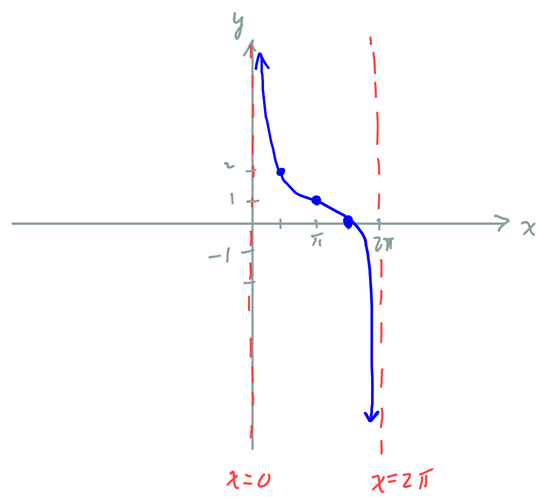
(c) $y = -5\cos\left(\frac{x}{3}\right)$

$|a| = 5$ amplitude
$a = -5$ reflection accross x-axis
$k = 1/3$ Period: $\frac{2\pi}{1/3} = 6\pi$



(d) $y = 1 + \cot\left(\frac{1}{2}x\right)$

$k = 1/2$: Period $\frac{\pi}{1/2} = 2\pi$
$c = 1$: VS U 1



Problem 6: 6 pts

Prove the following identity: $\tan^2(t) = \dfrac{\cos^2(t) + \tan^2(t) - 1}{\sin^2(t)}$.

Show all steps. Prove it as discussed in class. That is, start with one side and show, after a series of valid steps, that you get the other side. Include a square (□) to indicate the end of your proof.

Proof:

$$RHS = \frac{\cos^2(t) + \tan^2(t) - 1}{\sin^2(t)}$$

(use Pythag)

$$= \frac{\cos^2(t) + \tan^2(t) - (\sin^2(t) + \cos^2(t))}{\sin^2(t)}$$

$$= \frac{\cancel{\cos^2(t)} + \tan^2(t) - \sin^2(t) - \cancel{\cos^2(t)}}{\sin^2(t)}$$

("Petal" trick)

$$= \frac{\tan^2(t) - \sin^2(t)}{\sin^2(t)}$$

$$= \frac{\tan^2(t)}{\sin^2(t)} - 1$$

$$= \frac{\frac{\sin^2(t)}{\cos^2(t)}}{\frac{\sin^2(t)}{1}} - 1$$

$$= \frac{\cancel{\sin^2(t)}}{\cos^2(t)} \cdot \frac{1}{\cancel{\sin^2(t)}} - 1$$

$$= \sec^2(t) - 1$$

(use Pythag: $\tan^2(t) + 1 = \sec^2(t)$
$\tan^2(t) = \sec^2(t) - 1$)

$$= \tan^2(t)$$

$$= LHS. \quad \square$$

(Q.E.D.)

use either/or

Another way:

$$RHS = \frac{\cos^2(t)}{\sin^2(t)} + \frac{\tan^2(t)}{\sin^2(t)} - \frac{1}{\sin^2(t)}$$

$$= \cot^2(t) + \frac{\cancel{\sin^2(t)}}{\cos^2(t)} \cdot \frac{1}{\cancel{\sin^2(t)}} - \csc^2(t)$$

$1 + \cot^2(t) = \csc^2(t)$

$$= \cot^2(t) + \sec^2(t) - \csc^2(t)$$

$$= (\cancel{\csc^2(t)} - 1) + \sec^2(t) - \cancel{\csc^2(t)}$$

$$= \sec^2(t) - 1 = \tan^2(t) = LHS \quad \square$$

Problem 7: 5 pts

Make the **trigonometric substitution** $x = 2\sec(\theta)$ into the expression $\dfrac{\sqrt{x^2-4}}{x}$ and simplify completely. Assume that $0 < \theta < \pi/2$.

$$\frac{\sqrt{x^2-4}}{x} = \frac{\sqrt{(2\sec\theta)^2 - 4}}{2\sec\theta} = \frac{\sqrt{4\sec^2\theta - 4}}{2\sec\theta} = \frac{\sqrt{4(\sec^2\theta - 1)}}{2\sec\theta} = \frac{\sqrt{4}\sqrt{\tan^2\theta}}{2\sec\theta} = \frac{\cancel{2}\cdot\tan\theta}{\cancel{2}\cdot\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

use $\tan^2\theta + 1 = \sec^2\theta$
$\Rightarrow \sec^2\theta - 1 = \tan^2\theta$

$$= \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1} = \boxed{\sin\theta},$$

Problem 8: Extra-credit (5 points)

You may attempt these only if every questions on the exam has an attempted solution. Otherwise they will not be graded.

(a) (2 pts) Sketch the function $y = x^2 \cdot \cos(x)$.

(b) (3 pts) Prove: $-x^2 \leq x^2 \cdot \cos(x) \leq x^2$ for all $x \in \mathbb{R}$.

Post Exam Survey

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

1. When taking the exam I felt
   (a) Rushed. I wanted more time.
   (b) Relaxed. I had enough time.
   (c) Amazed. I had tons of extra time.
2. The week before the test I did all my homework on time: YES NO
3. The week before the test, in addition to the homework I followed a study plan. YES NO
   (a) I think this helped: YES NO
4. The day before the test I spend ________ hours studying and reviewing.
   (a) I think that was enough time: YES NO
5. The night before the test:
   (a) I stayed up very late cramming for the test
   (b) I stayed up very late, but I wasn't doing math
   (c) I didn't need to cram because I was prepared
   (d) I got a good night's sleep so my brain would function well.
6. I think I got the following grade on this test: ________
7. Strategies that worked well for me were (please elaborate):
8. Next time I will do an even better job preparing for the test by:

please answer these!