MATH 110B - Precalculus II — Fall 2019

Exam 2

Ch 7

Dr. Jorge Basilio

jbasilio@glendale.edu

Oct_4

Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

NAME (PRINT): _____ SIGNATURE: _____

Directions

- YOU ARE ALLOWED TO USE ONLY A SCIENTIFIC CALCULATOR ON THIS EXAM.
- You have 85 minutes to complete this exam.
- The exam totals **102 points** with 5 points of extra-credit possible.
- There are 5 problems (plus one extra-credit problem at the end), many of them with multiple parts.
- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you your writing utensils.
- Cell phones must be turned off and put away in with your items in the front of the classroom.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E). For True/False questions, you must spell out the entire word "true" or "false" in your answer.
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- A problem which requires a **proof** means you must provide a general proof in complete sentences. Do not use logical short-hand in proofs.
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a box around your final answer where applicable.
- **PLEASE CHECK YOUR WORK!!!**

Score	Grade

This page is intentionally blank. It may be used for scratch paper. If you wish for me to grade your work on this page, please (i) label the problem you are working on, (ii) box your answer, (iii) indicate in the original problem's location that you will continue your work on this page.

Problem 1: 20 pts (2 pts each)

Fill-in the blank: *(No work needed)*

(a) The **double-angle formula** for sine is _____ *(please give entire ID)*

(b) An **identity** is true for _____ values of the variable.

(c) If we know the value of $\cos(x)$ and the quadrant in which $x/2$ lies, then we can find the value of $\sin\left(\frac{x}{2}\right)$ by using the _____ Formula.

(d) The **basic trigonometric equation** $\cos(x) = 5$ has _____ solutions.

(e) The solutions to the **basic trigonometric equation** $\cos(x) = \frac{1}{5}$ in the interval $(-\pi, \pi]$ *rounded to the nearest thousandths* are _____.

TRUE or **FALSE** (please spell out/write the entire word for credit). *(No work needed)*

(a) _____ $\tan^{-1}(-1) = -\frac{\pi}{4}$.

(b) _____ $\cos^{-1}\left(\frac{2\pi}{3}\right) = -\frac{1}{2} + 2\pi k,\ k \in \mathbb{Z}$.

(c) _____ If θ is in Quadrant III, then $\sqrt{1-\cos^2(\theta)} = -\sin(\theta)$.

(d) _____ The function $f(x) = \sin(x) + \cos(x) + \sec(x) + \csc(x)$ has **period** 2π.

(e) _____ The solutions to $\left(\sin(x) + \frac{3}{2}\right)\left(\cos(x) + \frac{1}{2}\right) = 0$ in the interval $[0, 2\pi)$ are $x = \frac{\pi}{3}, \frac{4\pi}{3}$.

Problem 2: 10 points

Which of the following are **identities**? Select <u>all</u> that apply. *(No work needed)*

(A) $1 + \cot^2(x) = \csc^2(x)$.

(B) $\cos(-x) = -\cos(x)$.

(C) $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$.

(D) $\cos(2x) = \cos^2(x) - \sin^2(x)$.

(E) $\cos(x-y) = \cos(x)\cos(y) - \sin(x)\sin(y)$.

(F) $\tan(2x) = \dfrac{\tan(2x)}{1-\tan^2(x)}$.

(G) $\csc\left(\dfrac{\pi}{2} - x\right) = \cos(x)$.

(H) $\csc(x) = \sin^{-1}(x)$.

(I) $\cos^2(x) = \dfrac{1-\cos(2x)}{2}$.

(J) $\tan^2(x) = \dfrac{1-\cos(2x)}{1+\cos(2x)}$.

Problem 3: 36 pts

Evaluate the following trigonometric functions. Please give **exact values!** You must show work/formulas used to receive full credit.

(a) $\cos(75^\circ)$

(b) $\sin\left(\pi + \tan^{-1}\left(\frac{3}{7}\right)\right)$

(c) $\cos(95^\circ)\cos(50^\circ) + \sin(95^\circ)\sin(50^\circ)$

(d) $\sin\left(-\frac{\pi}{12}\right)$

(e) $\sin\left(\sin^{-1}\left(\frac{3}{5}\right) - \cos^{-1}\left(\frac{1}{2}\right)\right)$

(f) Find the exact value of $\sin(2x)$ given that $\sec(x) = 3/2$, $\csc(y) = 3$ and x and y are both in Quadrant I.

Problem 4: 30 pts

Find all solutions to the following **trigonometric equations**. Give **exact values!**

(a) $2\sin(x) - \sqrt{2} = 0$

(b) $\frac{1}{2}\sec(x) - 1 = 0$

Find all solutions in the interval $[0, 2\pi)$ to the following **trigonometric equations**. Give **exact values!**

(c) $\sin(x) = \cos(2x)$

(d) $\tan(x)\cos(x) - \cos(x) = 0$

(e) At what **points** do the functions $f(x) = \sin(4x)$ and $g(x) = \sin(2x)$ intersect in the interval $-\frac{\pi}{2} < x \leq \frac{\pi}{2}$?

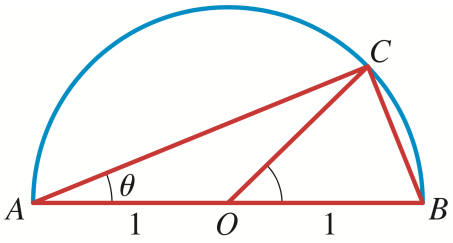
Problem 5: 6 pts

Prove the following identity: $\cos(2x) = \cos^4(x) - \sin^4(x)$.

Show all steps. Prove it as discussed in class. That is, start with one side and show, after a series of valid steps, that you get the other side.

Problem 6: Extra-credit (5 points)

Use the figure to provide a **geometric proof** of the Double-Angle Formula for sine.



Formula Sheet

- **Half-Angle Formulas**:

$$\boxed{\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}}}$$

$$\boxed{\cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}}$$

$$\boxed{\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin(u)}{1+\cos(u)}}$$

- **Product-to-Sum Formulas**:

$$\boxed{\sin(u)\cos(v) = \frac{1}{2}\left[\sin(u+v) + \sin(u-v)\right]}$$

$$\boxed{\sin(u)\cos(v) = \frac{1}{2}\left[\sin(u+v) - \sin(u-v)\right]}$$

$$\boxed{\cos(u)\cos(v) = \frac{1}{2}\left[\cos(u+v) + \cos(u-v)\right]}$$

$$\boxed{\sin(u)\sin(v) = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]}$$

- **Sum-to-Product Formulas**:

$$\boxed{\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$$\boxed{\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}$$

$$\boxed{\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$$\boxed{\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}$$

- **Pythagorean Formulas**:
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-
-
- **Addition and Subtraction Formulas**:
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- **Double-Angle Formulas**:
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- **Formulas for Lowering Powers**:
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Post Exam Survey

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

1. When taking the exam I felt
   (a) Rushed. I wanted more time.
   (b) Relaxed. I had enough time.
   (c) Amazed. I had tons of extra time.
2. The week before the test I did all my homework on time: YES NO
3. The week before the test, in addition to the homework I followed a study plan. YES NO
   (a) I think this helped: YES NO
4. The day before the test I spend _____ hours studying and reviewing.
   (a) I think that was enough time: YES NO
5. The night before the test:
   (a) I stayed up very late cramming for the test
   (b) I stayed up very late, but I wasn't doing math
   (c) I didn't need to cram because I was prepared
   (d) I got a good night's sleep so my brain would function well.
6. I think I got the following grade on this test: _____
7. Strategies that worked well for me were (please elaborate):
8. Next time I will do an even better job preparing for the test by: