

§9.3: Separable Equations

Ch 9: Differential Equations Math 5B: Calculus II

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Class #8 Notes

March 19, 2019
Spring 2019

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Guiding Questions for §9.3

Guiding Question(s)

- 1 What are **separable equations** and how can solve them?

Separable Equations - Introduction

Differential equations can be very hard to solve. So far, we've looked at:

- Solutions to a few DE:
 - Natural Growth/Decay: $\frac{dy}{dx} = ky \rightarrow y = Ce^{kx}$
 - Newton's Law of Cooling: $\frac{dT}{dt} = k(T - T_S) \rightarrow T(t) = T_S + Ce^{kt}$
 - Using anti-differentiation: $\frac{dy}{dx} = f(x) \rightarrow y(x) = \int f(x) dx$
- The qualitative behavior of a DE. That is, how the DE can tell us about the shape of the solutions.
- Slope Fields to help us visualize solutions
- Euler's method to approximate solutions (numerical algorithm)

Separable Equations - Introduction

- Let's look again at how we solved DEs of the form: $\frac{dy}{dx} = f(x)$.

$$\begin{aligned}\frac{dy}{dx} dx &= f(x) \\ \int \frac{dy}{dx} dx &= \int f(x) dx \\ y(x) &= \int f(x) dx\end{aligned}$$

- So can we generalize this simple trick to solve more complicated DEs?
- Yes, and we've given them a name: **separable equations**.

Recall we looked at DE of the form, $\frac{dy}{dx} = F(x, y)$, but didn't try to solve them and instead looked at their slope fields. This gave us a general idea about the solution curves.

Definition 1: Separable Equations

- The **separable equation** is a DE of the form where $F(x, y) = f(x)g(y)$, or

$$\frac{dy}{dx} = f(x)g(y)$$

- That is, the right-hand side **is a product** of a function in x only and a function in y only.

Examples

The following DEs are all separable equations:

(a) $\frac{dy}{dx} = x^2 y^2$

(d) $\frac{dy}{dx} = y$

(b) $\frac{dy}{dx} = \frac{x^2}{y^2}$

(e) $y' = \frac{6x^2}{2y + \cos(y)}$

(c) $y' + xe^y = 0$

(f) $\theta e^{t^2} \frac{d\theta}{dt} = t \sec(\theta)$

The following are NOT separable equations:

(g) $\frac{dy}{dx} = x^2 + y^2$

(h) $\frac{dy}{dx} = (x + y)^2$

How to solve Separable EQs

- The name comes from the fact that we can “separate the derivative” $\frac{dy}{dx}$ into two pieces dy and dx and because the right-hand side is a product $f(x)g(y)$ we can re-arrange the DE by “separating each variable on one side of the equal sign.”
- Here’s the general strategy to solving these DEs:
 - $\frac{dy}{dx} = f(x)g(y)$
 - $\frac{dy}{g(y)} = f(x)dx$ (“separate the x and y ”)
 - $\int \frac{dy}{g(y)} = \int f(x) dx$
 - After integrating both sides, we try to solve for y (or just leave it as an equation)
- We call this technique “separation of variables”.

Activity 1:

Find the general solutions of the following separable equations:

(a) $y \frac{dy}{dx} - x = 0$

(b) $y' = -ty$

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Activity 2:

(a) Solve the IVP:

$$y' = \frac{xy^3}{1+x^2}; \quad y(0) = -1$$

(b) Discuss the maximal interval where the solution exists (called an **interval of validity**).

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