Chapter 11: Goodness-of-Fit and Contingency Tables

Section 11.1: Goodness-of-Fit   Stat 50

Def   A **goodness-of-fit test** is used to test the hypothesis that an observed frequency distribution fits (or conforms to) some claimed distribution.

The M&M company claims that the distribution of plain M&M candies in a bag is 23% blue, 23% orange, 15% green, 12% red, 15% yellow, and 12% brown. Even though this is their claim, do you think this represents the true proportions of color distribution in all of the M&M bags? ***How would we check?***

H_0: $p_{Blue} = 0.23$, $p_O = 0.23$, $p_G = 0.15$, $p_R = 0.12$, $p_Y = 0.15$, $p_{Brown} = 0.12$

H_1: At least one proportion is not equal to stated proportions

What to Compare and How to Compare It

Expected Counts (E_i)	Observed Counts (O_i)
The number in each category we would expect to see if H_0 is true * Two ways of calculating the expected counts: 1. If the expected counts are EQUAL, then $E = \boxed{\frac{n}{k}}$ ← total observed ← categories 2. If the expected counts are not equal, then calculate using $E_i = \mu_i =$ $n_i p_i$ where $i = 1, 2 \ldots k$ (binomial dist)	Observe how many in your sample are in each category. **This information will be given.*

*If the observation and experiment counts are "close", then fail to reject H_0

* If the observation and experiment counts "far apart" (BIG DIFFERENCE), then REJECT H_0

Ex: Finding the expected counts.

roll dice: prob(roll 1) = 1/6 = prob.(roll 2)

(a) A single die is rolled 45 times with the following results. Assuming that the die is fair and all outcomes are equally likely, find the expected frequency E for each empty cell.

same prob so same exp!

Outcome	1	2	3	4	5	6
Observed O_i	13	6	12	9	3	2
Expected E_i	7.5	7.5	7.5	7.5	7.5	7.5

$E = \frac{n}{k} = \frac{45}{6}$ # times dice rolled / # categories $= 7.5$ $E = n \cdot p = 45 \times \left(\frac{1}{6}\right)$

(b) Jon works as an usher at a theatre. The theatre has 1000 seats that are accessed through five entrances. Each guest should use the entrance that's marked on their ticket. Entrances A and B should each have 30% of the guests using these entrances. Entrance C should have 20% of the guests using its entrance. Entrances D and E should each have 10% of the guests using these entrances. Find the expected frequency for each E for each entrance.

Entrance	A	B	C	D	E
Observed O_i	398	202	205	87	108
Expected E_i	300	300	200	100	100

Note $\sum E_i = n$
sum of expectations should equal total observed n

$E = n \cdot p$
$= 1000 * 0.3$
$= 300$

$E = np$
$= 1000 * 0.2$
$= 200$

$E = 1000 * 0.1$

Steps for Hypothesis Test for Goodness-of-Fit

What to Find…	***Check Requirements***
● Number of categories, k ● Expected Counts, E_i	● The data has to be randomly selected. (SRS) ● The sample data consist of frequency counts for each of the different categories. ↳ $O_i > 0$ ● For each category, the expected frequency is at least 5. $E_i \geq 5$ ($E = np$)

Step 1: *Hypotheses*

Dice

$H_0: \; p_1 = p_2 = \ldots = p_k$ (all outcomes are equally likely)

$H_1:$ at least one of the probabilities is different from the others (proportions)

M&M

$H_0: \; p_1 = \#, \; p_2 = \# \ldots p_k = \#$

$H_1:$ at least one of the probabilities is different from the claimed distribution (proportions)

Step 2: *Level of Significance*

α

(if not given, assume $\alpha = 0.05$)

Step 3: *Test Statistic*

χ "chi" ("kai")

$$\boxed{\chi_0^2 = \sum \frac{(O-E)^2}{E}}$$

• χ^2-distribution (new distribution)

• to find probabilities: $\chi^2cdf(a, b, df)$

$\boxed{df = k-1}$ k not n! k = # categories

ALWAYS __RIGHT__ **-TAILED TEST!**

Step 4: *Find a Critical Value or P-Value to check using either the Critical Value or P-value method.*

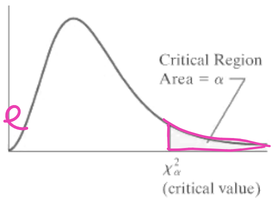
Step 5: *Make a decision and draw a conclusion.*

CRITICAL REGION METHOD

* Table A-4

$df = k - 1$

χ_0^2 is inside/outside



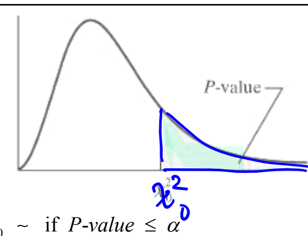
look up in a table

χ_α^2

$\begin{cases} \textit{Reject } H_0 \sim \text{ if } \chi^{2*} \text{ lies in the critical region} \\ \textit{Fail to Reject } H_0 \sim \text{ if } \chi^{2*} \text{ doesn't lie in the critical region} \end{cases}$

P-VALUE METHOD

$df = k - 1$

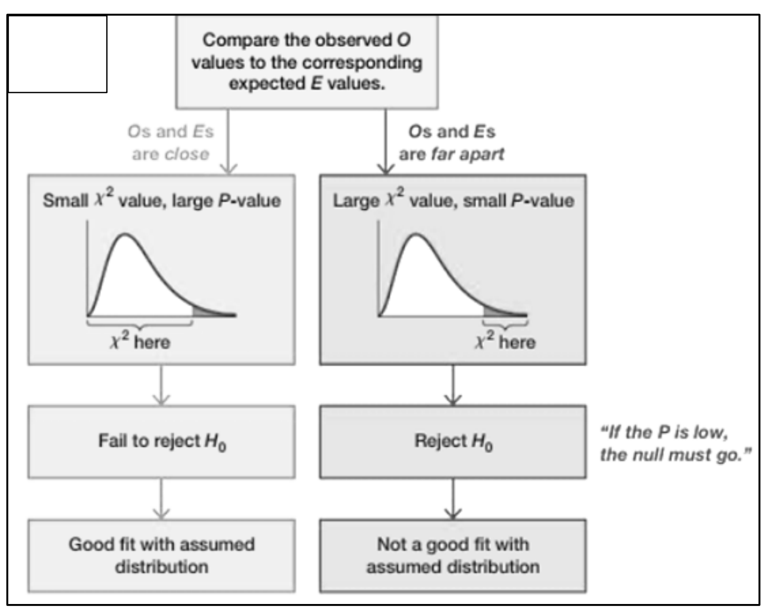


$\begin{cases} \textit{Reject } H_0 \sim \text{ if } \textit{P-value} \leq \alpha \\ \textit{Fail to Reject } H_0 \sim \text{ if } \textit{P-value} > \alpha \end{cases}$

GRAPHING CALCULATOR (TI-83 OR 84)

Instructions:

STAT $\Rightarrow$ TESTS $\Rightarrow$ D: $\boxed{\chi^2 GOF\text{-Test}}$



Ex: The M&M company claims that the distribution of plain M&M candies in a bag is 23% blue, 23% orange, 15% green, 12% red, 15% yellow, and 12% brown. Suppose we took a simple random sample of 400 M&Ms from the populations of all M&Ms. The results are shown below:

	COLOR	Blue	Orange	Green	Red	Yellow	Brown
O_i	**FREQUENCY**	53	66	38	96	88	59
E_i	**EXPECTED**	92	92	60	48	60	48

$E_i = n \cdot p_i$ 400 * 0.23

On Calc: L1 = Observed(O_i), L2 = Expected(E_i), L3 = $(O-E)^2/E$, L3 = $(L1-L2)^2/L2$

sum(L3) = · LIST(2nd Stat) · math · sum(5.)

Find E_i =

Is the proportion of each color different than the claim of the M&M's manufacturer?

Null and Alternative Hypothesis

H_0: $p_{Blue} = 0.23$, $p_{Orange} = 0.23$, $p_{Green} = 0.15$, $p_{red} = 0.12$, $p_{yellow} = 0.15$, $p_{Brown} = 0.12$

H_A: At least one proportion differs from a claimed proportion.

Test Statistic

$$\chi_0^2 = \sum \frac{(O-E)^2}{E} = 95.5$$

Check w/ Calc: $\boxed{\chi^2 \text{ GOF-Test}}$ → also $\chi_0^2 = 95.5$

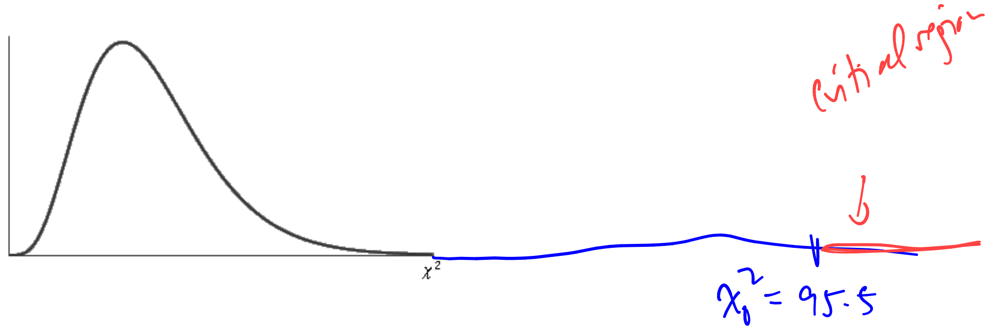
P-value/Critical Region

p = 4.61 E-19

$p = 4.61 \times 10^{-19}$

$p = 0.\underbrace{000\ldots\ldots}_{}0461$

$\boxed{p = 0^+}$ 18 zeros! TINY!



Decision about Null Hypothesis

$\alpha = 0.05$

$p = 0^+ < \alpha$ → Plow, null go $\boxed{\text{Reject } H_0}$

Conclusion

"There is enough statistical evidence to support the claim that at least one proportion of M&M colors is different than the M&M Factory's claimed proportion."

Ex: A company sells their products exclusively by mail. The company's management wants to find out if the number of orders received at the company's office on each of the five days of the week is the same. The company took a random sample of 400 orders received during a four-week period. The following table lists the frequency distribution for these orders by the day of the week.

	Monday	Tuesday	Wednesday	Thursday	Friday
(L1) O_i Number of Orders	92	71	65	83	89
(L2) E_i Expected Number	80	80	80	80	80

Test the claim that the orders are evenly distributed over the five days of the week. Use $\alpha = .025$

Null and Alternative Hypothesis

$H_0: p_1 = p_2 = p_3 = p_4 = p_5$

H_A: At least one proportion of orders is different than the other days.

key: X_1 – RV # of orders on Monday
X_2 – RV # of orders on Tuesday
etc...

Check Req
(1) $5 \geq 5$ ✓
(2) $O_i > 0$ ✓
(3) $E_i \geq 5$ ✓

Test Statistic

$\boxed{\chi_0^2 = 6.75}$

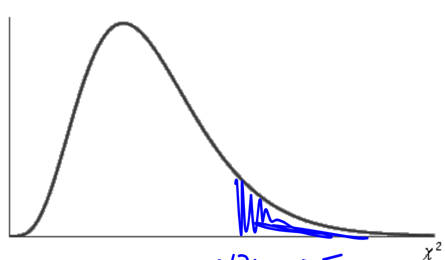
$\boxed{\chi^2 \text{ GOF Test}}$

P-value/Critical Region

$\alpha = 0.025$

$p = 0.1497...$

$\boxed{p = 0.150}$



$\chi_0^2 = 6.75$

Decision about Null Hypothesis

$p > \alpha$

p high, null fly → Fail to Reject H_0

Conclusion

"There is not enough statistical evidence to support that the true proportion of orders the company recieves differs on each day of the week."