Section 5.2: Binomial Probability Distributions

BINOMIAL PROBABILITY DISTRIBUTION

Def A **binomial probability distribution** results from a procedure that meets the given requirements.

1. The procedure has a *fixed number of trials*.
2. The trials must be *independent*.
   (i.e. the outcome of an individual trial does not affect the probabilities in other trials.)
3. Each trial must have all outcomes classified into *two categories*.
   (often referred to as success and failure)
4. The probability of a success remains *constant* in all trials.

NOTATION

Symbol	*Represents*
n	fixed number of trials
p	probability of success
q	probability of failure
x	specific number of success in n trials
$P(x)$	probability of getting exactly x successes among the n trials

$\to p = 0.19$ $\to$ "success"

Ex: According to Wikipedia, 19% of Mexican residents are vegetarians. If we randomly survey 20 Mexican residents, what's the probability 3 of the Mexicans are vegetarians? Fill in the values given the information presented.

Two Possible Outcomes: vegeterian (success) or not vegeterian (failure)

$n =$ 20 $p =$ 0.19 $q =$ 0.81 $x =$ 3

$(q = 1 - p)$

TWO METHODS TO FIND PROBABILITY OF A SPECIFIC VALUE

1. FORMULA: $P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

2. Use Graphing Calculator (TI-83 or 84)

   Instructions: (a) $2^{nd} \Rightarrow$ VARS $\Rightarrow$ DISTR

   (b) binompdf (n, p, x)

Graphing Calculator for ${}_nC_x$
math $\Rightarrow$ PROB $\Rightarrow$ 3: ${}_nC_x$

Ex: Use the problem from above (regarding vegetarianism in Mexico) to set up and evaluate using both methods.

Binomial?: ① n=5 ✓ ② independent ✓ ③ "bi" ✓ ④ yes ✓

yes / no ; yes, set p = prob. "yes". p = 0.2

Ex: We survey 5 PCC students and ask "Is this your first year here?" Assume that 20% of all PCC students are in their first year.

(a) What is the probability that all 5 students are new?

$n=5$, $p=0.2$, $x=5$

$P(5) = \text{binompdf}(5, 0.2, 5)$
$= 3.2E{-4}$
$= 3.2\times10^{-4} = \boxed{0.0032}$

(b) What is the probability that 2 or 3 will be new?

$P(2 \text{ or } 3) = P(2) + P(3)$
$= \text{binompdf}(5, 0.2, 2)$
$+ \text{binompdf}(5, 0.2, 3)$
$= \boxed{0.256}$

(c) What is the probability that at least one will be new?

$x = 0, \boxed{1, 2, 3, 4, 5}$

P(at least one) = 1 − P(none)
= 1 − P(X = 0)
= 1 − binompdf(5, 0.2, 0)
= $\boxed{0.672}$

*use "3 significant rule rounding"

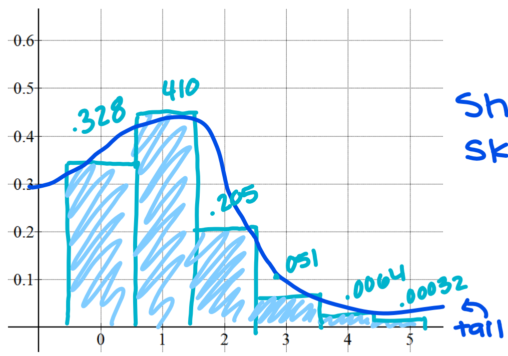
(d) Create a probability distribution table for this exercise.

x	$P(x)$
0	0.328
1	0.410
2	0.205
3	0.051
4	0.0064
5	0.00032

(use "2nd enter" trick)

· $\Sigma P(x) = 1 \approx 1$
· $0 \le P(x) \le 1$ ✓

(e)



Shape: skewed right

*If we surveyed 200 students, and I ask the probability that between 113 and 187 students are new, how would you find your answer?

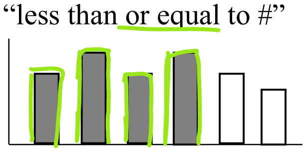
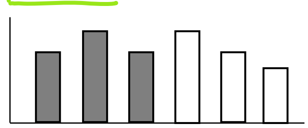
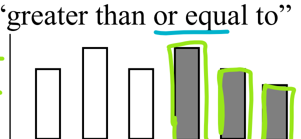
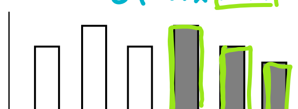
have data for 200, p = 0.2, x is between 113 & 187 P(113 or 114 . . or 187)

Finding a Cumulative Probability

Use Graphing Calculator (TI-83 or 84)

Instructions: (a) 2nd ⇒ VARS ⇒ DISTR

(b) $P(x \le \#) = \text{binomcdf}\,(n,\ p,\ \#)$

"no more than" or "at most #" or "less than or equal to #"	"fewer than #" or "less than #"	*Calculator
0,1,. . x . n  0 1 x	0 1 . . x . . n  0 1 x	0 1 2 . . x . . n $P(X \le x) = \text{binomcdf}(n, p, x)$
"at least" or "no less than" or "greater than or equal to"	"more than" or "greater than"	*Calculator
0 1 . . x . . n  x n	0 1 . . x . . n  x n	0 1 . . . x . . . n $P(X \ge x) = 1 - \text{binomcdf}(n, p, x-1)$

binomal → make
↘ miss

make FT = success

Ex: A basketball player makes 75% of the free throws he tries. If the player attempts 10 free throws in a game, find the probability that:

→ .75   → n=10

(a) the player will make at most six free throws. → ≤6

$\boxed{0\ 1\ 2\ 3\ 4\ 5\ 6}\ 7\ 8\ 9\ 10$

$P(X \le 6) = \text{binomcdf}(10, 0.75, 6)$
$= \boxed{0.224}$

(b) the player will make at least eight free throws. → ≥8

$0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ \boxed{8\ 9\ 10}$

$P(X \ge 8) = 1 - \text{binomcdf}(10, 0.75, 7) = \boxed{0.526}$
$= 1 - P(X \le 7)$

Ex: According to 2017 Washington Post article, approximately 53% of all U.S. households are wireless-only households (no landline). In a random sample of 20 households, what is the probability that…

· X - # of wireless only
· p = 0.53
· → wireless
 → not wireless

(a) fewer than 6 are wireless only?

$\boxed{0\ 1\ 2\ 3\ 4\ 5}\ 6\ 7 \ldots\ 20$

$P(X < 6) = P(X \le 5) = \text{binomcdf}(20, 0.53, 5)$
$= \boxed{0.0105}$

* 3 sig fig *

(c) more than 13 are wireless only?

$0\ 1 \ldots\ 12\ 13\ \boxed{14 \ldots\ 20}$

$P(X > 13) = P(X \ge 14)$
$= 1 - P(X \le 13) = 1 - \text{binomcdf}(20, 0.53, 13)$
$= \boxed{0.0958}$

Using Mean and Standard Deviation for Critical Thinking

μ *Mean Value*

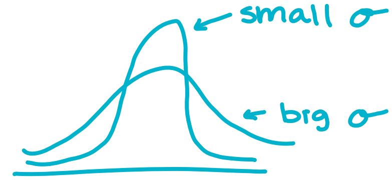
Formula: $\boxed{\mu = np}$ ← rounding (stats law of rounding)
↳ 1 more than data.

σ^2 *Variance*

Formula: $\boxed{\sigma^2 = npq}$ ← Just tenths

σ *Standard Deviation*

Formula: $\boxed{\sigma = \sqrt{npq}}$



Round-Off Rule: Round to the nearest tenth.

Ex: According to the U.S. Office of Adolescent Health, nearly 90% of adult smokers in America started smoking before turning 18 years old. → n

→ 0.9 = p

Prob smokers started before 18 y/o.

(a) If 300 adult smokers are randomly selected, how many would we expect to have started smoking before turning 18 years old?

expect → Expectation : $E(X) = \sum x \cdot P(x) \overset{\text{also}}{=} \text{mean} = \mu = n \cdot p = 300 \cdot 0.9 = \boxed{270 \text{ people}}$

(full sentence) (M → E) — "we expect 270 people to have started smoking before turning 18 y/o among 300 adult smokers."

math to english

(b) Would it be unusual (significantly high/low) to observe 240 smokers who started smoking before turning 18 years old in a random sample of 300 adult smokers? What may this suggest about the population that was observed?

$\mu - 2\sigma \le$ not significant $\le \mu + 2\sigma$

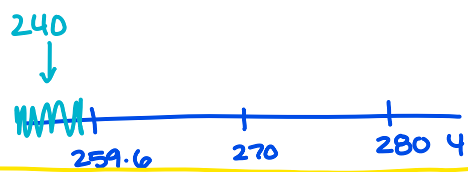
sig. low
scores $\le \mu - 2\sigma$

sig. high
scores $\ge \mu + 2\sigma$

$\mu + 2\sigma = 270 + 2(5.2) = 280.4$
$\mu - 2\sigma = 270 - 2(5.2) = 259.6$

Need · μ & σ

$\mu = 270$ ✓ $\sigma = \sqrt{300 \cdot 0.90(0.1)} = 5.2$



Yes, it would be unusual to observe 240 smokers who started by 18 y/o.