MATH 110B - Precalculus II Spring 2020

Exam 2

Ch 7 **Dr. Jorge Basilio**

April_1 **jbasilio@glendale.edu**

Solutions // Solutions //

Directions

1. Please hand-write the statement provided below in quotes; print your name; and sign your name below it that acknowledges the **honor code**:

   "On my honor, by printing and signing my name, I vow to neither receive nor give any unauthorized assistance on this examination. I understand what my professor has deemed appropriate and inappropriate for this test and vow to follow these rules."

2. The exam is written to last 1 hour and 25 minutes from 9:10 am to 10:35 am, however, you have until 11:05 am to submit this exam without penalty.

3. How to submit: upload a **single PDF file** of your solutions to Canvas no later than 11:05 am to avoid penalties.

4. You will need to be logged in to class via Zoom. You will also need to have your camera and microphone both functioning and turned ON for the duration of the test. Ask family members, roommates, to not disrupt you during the test. Please disable any virtual backgrounds.

5. Write your solutions to the exam on one side of the page (front side only, do not write on the back of the page).

6. Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown unless told otherwise. *If in doubt, ask for clarification.* Correct answers with little to no work will receive no points. Students might be randomly selected to have a 1-1 conference where you are asked to defend your work and explain to me all your steps on certain questions or problems that are similar to a test question.

7. **Penalties for late submissions:** Exams received between 11:06 and 11:15 am will have 10 points deducted from their score. Exams received between 11:16 and 11:25 am, I will deduct an additional point from your score for each additional minute the exam is late. Exams received after 11:26 am will not be graded and be given a score of 0.

8. **Allowed Materials**
   - Scientific calculator with no graphing features.
   - Blank pieces of paper to write your solutions. Writing utensils, erasers, etc
   - Physical copy of the textbook. No e-books. No solutions manuals.
   - Class notes (you can have your hand-written class notes)

9. **Materials NOT Allowed**
   - Do not use your cell phone (for any reason: do not send or receive texts or calls or use the internet, etc)
   - Do not use digital or printed out notes: the slides, the study guides, etc (only your textbook and hand-written class notes)
   - Do not consult your HW or ICAs
   - Do not give or receive any outside help (no getting help from a family member, friend, or any person either in person, via chat, message board, text message or any form of communication–again–you will be on camera the entire time so I will be looking for suspicious behavior)
   - Do not use your computer to look up anything using the internet (don't google; don't consult homework help websites, etc)

Continued on the next page...

Directions Continued:

10. The exam totals **102 points** but will be scored out of 100.
11. There are 6 problems (plus one extra-credit problem at the end), many of them with multiple parts.
12. Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
13. Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E). For True/False questions, you must spell out the entire word "true" or "false" in your answer.
14. Leave answers in exact form (as simplified as possible), unless told otherwise.
15. Put a $\boxed{\text{box around your final answer}}$ where applicable.
16. **PLEASE CHECK YOUR WORK!!!**
17. GOOD LUCK!!!!

Problem 1: 20 pts (2 pts each)

Note 2 other ID apply $\cos(2x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$

Fill-in the blank: *(No work needed)* You do not need to copy the problem.

(a) The **double-angle formula** for cosine is $\cos(2x) = \cos^2(x) - \sin^2(x)$ *(please give entire ID)*

(b) An **identity** is true for all/every values of the variable.

(c) If we know the value of $\cos(x)$ and the quadrant in which $x/2$ lies, then we can find the value of $\cos\left(\frac{x}{2}\right)$ by using which formula? half-angle formula: $\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}$.

(d) The **basic trigonometric equation** $\sin(x) = -3$ has NO solutions.

$-1 \le \sin(x) \le 1$

(e) The solutions to the **basic trigonometric equation** $\sin(x) = -\frac{3}{5}$ in the interval $(-\pi, \pi]$ *rounded to the nearest hundredths* are $x = -0.64$ & $x = 2.50$.

$x = \sin^{-1}(-3/5) = -0.64$

$x = -\pi - (-0.64) = 2.50$

TRUE or **FALSE** (please spell out/write the entire word for credit). *(No work needed)* You do not need to copy the problem.

(a) FALSE $\tan^{-1}(1) = \frac{5\pi}{4}$. $-\frac{\pi}{2} \le \tan^{-1}(x) \le \frac{\pi}{2}$

(b) FALSE $\sin^{-1}\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2} + 2\pi k, k \in \mathbb{Z}$. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{2}$ backwards!

(c) FALSE If θ is in Quadrant IV, then $\sqrt{1 - \sin^2(\theta)} = + \cos(\theta)$. cos > 0

(d) FALSE The function $f(x) = 2\sin(x) - 3\cos(x) + 4\sec(x) - 5\csc(x)$ has **period** 2π.

(e) TRUE The solutions to $\left(\sin(x) - \frac{1}{2}\right)\left(\cos(x) + \frac{5}{2}\right) = 0$ in the interval $[0, 2\pi)$ are $x = \frac{\pi}{6}, \frac{5\pi}{6}$.

$x = \frac{\pi}{6}, 5\pi/6$ $\sin(x) = 1/2$ $\cos(x) = -5/2$ no sol

Problem 2: 10 points

Which of the following are **identities**? Select all that apply by writing "Yes an ID" or "Not an ID" on your paper. For parts that are identities, you must also list the name. Grading: point will be awarded for correct answers and deducted for incorrect answers. *(No work needed)*. You do not need to copy the problem.

(A) $\sin(-x) = -\sin(x)$. odd — (A) Yes an ID

(B) $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$. — (B) Yes an ID

(C) $\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$. — (C) Not an ID

(D) $\sin^2(x) = \frac{1 + \cos(2x)}{2}$. — (D) Not an ID

(E) $\cos(2x) = \cos^2(x) + \sin^2(x)$. — (E) Not an ID

(F) $\sec(x) = \cos^{-1}(x)$. trick Q! $\cos^{-1}(x) = \arccos(x) \neq \frac{1}{\cos(x)}$ inverse of cosine — (F) Not an ID

(G) $\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$. — (G) Yes an ID

(H) $1 + \cot^2(x) = \csc^2(x)$. — (H) Yes an ID

(I) $\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1 - \cos(u)}{2}}$. — (I) Yes an ID

(J) $\csc\left(\frac{\pi}{2} + x\right) = \cos(x)$. $\csc\left(\frac{\pi}{2} + x\right) = \sec(-x) = \sec(x) \neq \cos(x)$ — (J) Not an ID

READ CAREFULLY!

~~Problem 3: 36 pts~~

Evaluate the following trigonometric functions. You must show work/formulas used to receive full credit. Please give **exact values!** *Correct answers without any supporting work will receive zero points!*

(a) $\sin(105°)$

(b) $\cos\left(\frac{7\pi}{12}\right)$

(c) $\cos\left(\tan^{-1}\left(\frac{6}{7}\right) - \pi\right)$

(d) $\sin\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)\right)$

(e) $\frac{1}{2}\cos\left(\frac{\pi}{12}\right) + \frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{12}\right)$

(f) Find the exact value of $\sin(2x)$ given that $\sec(x) = 7/3$, $\csc(y) = 3$ and x and y are both in Quadrant I.

Problem 4: 24 pts

Give **exact values!** *Correct answers without any supporting work will receive zero points!*

- Find all solutions to the following **trigonometric equations**.

(a) $2\cos(x) + \sqrt{2} = 0$

(b) $\cot(\theta) + 1 = 0$

- Find all solutions in the interval $[0, 2\pi)$ to the following **trigonometric equations**. Give **exact values!**

(c) $\cos(2x) = 3\sin(x) - 1$

(d) $4\sin(x)\cos(x) + 2\sin(x) - 2\cos(x) - 1 = 0$

Problem 5: 6 pts

At what **points** do the functions $f(x) = \sin(4x)$ and $g(x) = \sin(2x)$ intersect in the interval $[0, \pi)$?

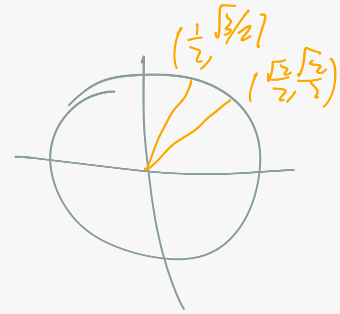
Problem 6: 6 pts

3 Typo!

Prove the following identity: $\frac{\cancel{2} + \cos(4x)}{4} = \cos^4(x) + \sin^4(x)$.

Show all steps. Prove it as discussed in class. That is, start with one side and show, after a series of valid steps, that you get the other side. Also, write the word "Proof:" and the beginning and end with a little square (□).
Hint: Lowering powers formulas

Problem 3



(a) $\sin(105^\circ) = \sin(45^\circ + 60^\circ) = \sin(\overset{\pi/4}{45^\circ})\cos(\overset{\pi/3}{60^\circ}) + \cos(\overset{\pi/4}{45^\circ})\sin(\overset{\pi/3}{60^\circ})$

$45^\circ + 60^\circ$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

(b) $\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

$\frac{7\pi}{12} = \frac{4\pi + 3\pi}{12} = \frac{\cancel{4}\pi}{\cancel{12}} + \frac{\cancel{3}\pi}{\cancel{12}}$

$= \frac{\pi}{3} + \frac{\pi}{4}$

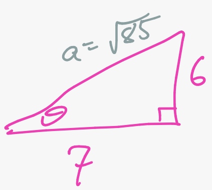
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

(c) $\cos\left(\underbrace{\tan^{-1}(6/7)}_{=\theta} - \pi\right) = \cos(\theta - \pi) = \cos(\theta)\underbrace{\cos(\pi)}_{=-1} + \cancel{\sin(\theta)}\underbrace{\sin(\pi)}_{=0}$

let $\theta = \tan^{-1}(6/7)$

$\tan\theta = 6/7$



$6^2 + 7^2 = a^2$

$85 = a^2$

$a = \sqrt{85}$

$$= -\cos\theta$$

$$= \boxed{-\frac{7}{\sqrt{85}}}$$

(d) $\sin\left(\underbrace{\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)}_{=\alpha} + \underbrace{\sin^{-1}\left(\frac{1}{2}\right)}_{=\beta}\right) = \sin(\alpha+\beta) = \sin\left(\frac{\pi}{6}+\frac{\pi}{6}\right) = \sin\left(\frac{2\pi}{6}\right)$

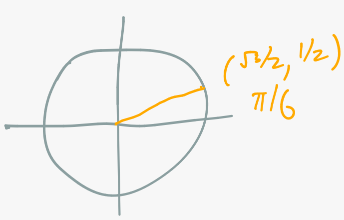
$\cos\alpha = \sqrt{3}/2$

$\alpha = \pi/6$

$\sin\beta = 1/2$

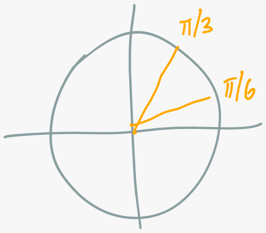
$\beta = \pi/6$

$$= \sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$$



Problem 3

(e) $\frac{1}{2}\cos\left(\frac{\pi}{12}\right) + \frac{\sqrt{3}}{2}\sin\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{12}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{12}\right)$



can choose either!

$= \cos\left(\frac{\pi}{3} - \frac{\pi}{12}\right)$ Use Cosine TRIG ID

$= \cos\left(\frac{4\pi - \pi}{12}\right) = \cos\left(\frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{4}\right) = \boxed{\frac{\sqrt{2}}{2}}$

(f) (f) Find the exact value of $\sin(2x)$ given that $\sec(x) = 7/3$, $\csc(y) = 3$ and x and y are both in Quadrant I.

trap! Don't need y!

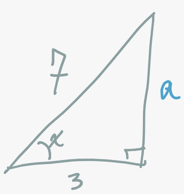
$\sin(2x) = 2\sin(x)\cos(x) = 2\left(\frac{a}{7}\right)\left(\frac{3}{7}\right)$

Use x only!

$= 2\left(\frac{2\sqrt{10}}{7}\right)\left(\frac{3}{7}\right) = \boxed{\frac{12\sqrt{10}}{49}}$

$\sec(x) = 7/3$

$\cos(x) = 3/7$



$3^2 + a^2 = 7^2$

$a^2 = 49 - 9$

$a = \sqrt{40} = 2\sqrt{10}$

Problem 4

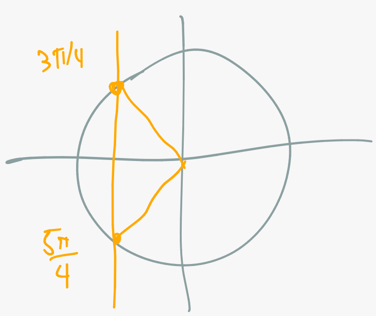
(Note parts a, b find all solutions)

(a) Solve: $2\cos(x) + \sqrt{2} = 0$ find all solutions

$\quad - \sqrt{2} \quad - \sqrt{2}$

$\frac{2\cos(x)}{2} = \frac{-\sqrt{2}}{2}$

$\cos(x) = \frac{-\sqrt{2}}{2}$



The solutions are:

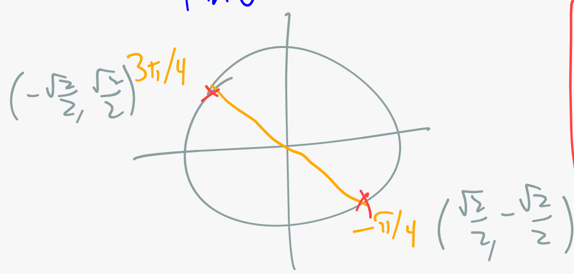
$x = \frac{3\pi}{4} + 2\pi k,$

$x = \frac{5\pi}{4} + 2\pi k,$

$k \in \mathbb{Z}$

Problem 4

(b) Solve: $\cot(\theta)+1=0 \Rightarrow \frac{1}{\tan\theta}=-1 \Rightarrow \tan\theta=-1$



$$\boxed{\theta=\frac{-\pi}{4}+2\pi k,\quad \theta=\frac{3\pi}{4}+2\pi k,\quad k\in\mathbb{Z}}$$

all solutions

(Note parts c,d find solutions in $[0,2\pi)$ only)

(c) $\cos(2x) = 3\sin(x) - 1$

(d) $4\sin(x)\cos(x) + 2\sin(x) - 2\cos(x) - 1 = 0$

(c) $\underbrace{\cos(2x)}_{\Rightarrow} = 3\sin(x)-1$

$$1-2\sin^2(x) = 3\sin(x)-1$$
$$-1+2\sin^2(x) \qquad\qquad -1+2\sin^2(x)$$

$$0 = 2\sin^2(x)+3\sin(x)-2$$

u-sub: let $u=\sin(x)$

$$0 = 2u^2+3u-2 \quad \text{(Factor)}$$
$$= (2u-1)(u+2)$$

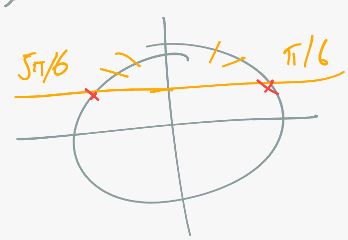
ZFP: $2u-1=0$ or $u+2=0$

$2\sin(x)-1=0$

$\sin(x)=1/2$ or $\sin(x)=-2$

No solutions! ☺

$\sin(x)=1/2$



$x\in[0,2\pi)$ only:

$$\boxed{x=\frac{\pi}{6},\ \frac{5\pi}{6}}$$

Problem 4

Factor by grouping

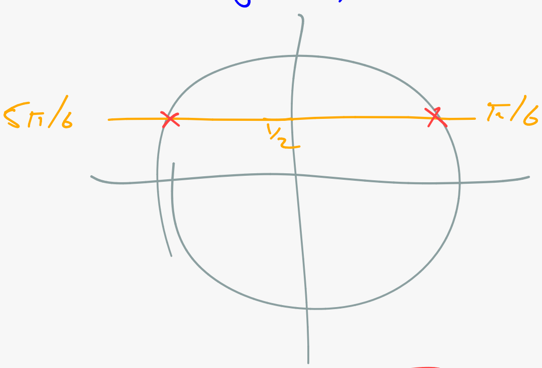
(d) $4\sin(x)\cos(x) + 2\sin(x) - 2\cos(x) - 1 = 0$

$$2\sin(x)\left(2\cos(x)+1\right) - \left(2\cos(x)+1\right) = 0$$

$$\left(2\sin(x)-1\right)\left(2\cos(x)+1\right) = 0$$

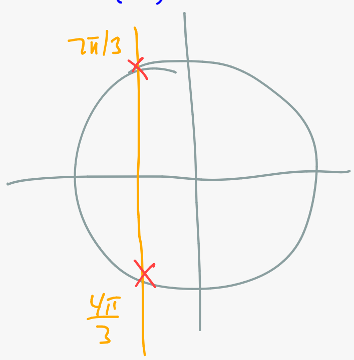
ZFP: $2\sin(x) - 1 = 0$ or $2\cos(x) + 1 = 0$

$\sin(x) = 1/2$



$$\boxed{x = \frac{\pi}{6}, 5\pi/6}$$

$\cos(x) = -1/2$



$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$$

Problem 5

Problem 5: 6 pts

At what **points** do the functions $f(x) = \sin(4x)$ and $g(x) = \sin(2x)$ intersect in the interval $[0, \pi)$?

points intersect when $f(x) = g(x)$

$$\iff \sin(4x) = \sin(2x)$$

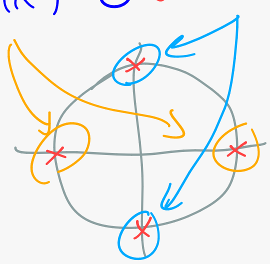
$$2\sin(2x)\cos(2x) = \sin(2x)$$

- when $\sin(2x) = 0$

$\sin(2x) = 0$

$2\sin(x)\cos(x) = 0$

$\sin(x) = 0$ or $\cos(x) = 0$



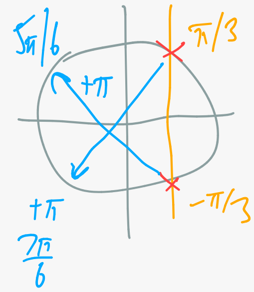
$x = 0, \pi$ ($\sin(x) = 0$)

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ ($\cos(x) = 0$)

- when $\sin(2x) \neq 0 \to$ divide by $\sin(2x)$

$2\cos(2x) = 1$

$\cos(2x) = 1/2$



$$\begin{cases} 2x = \frac{\pi}{3} + 2\pi k \\ 2x = -\frac{\pi}{3} + 2\pi k \end{cases}$$

$$\begin{cases} x = \frac{\pi}{6} + \pi k \\ x = -\frac{\pi}{6} + \pi k \end{cases}$$

$\hookrightarrow$ inside $[0, 2\pi)$

$x = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{5\pi}{6}$

- Now find points: $(x, f(x))$ or $(x, g(x))$

$g(x) = \sin(2x)$

$g(x) = 0$ @ $x = 0, \pi$

$g(x) = 0$ @ $x = \frac{\pi}{2}, \frac{3\pi}{2}$

- $x = \frac{\pi}{6}$: $g\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{3}\right) = \sqrt{3}/2$

$x = \frac{7\pi}{6}$: $g\left(\frac{7\pi}{6}\right) = \sin\left(\frac{7\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \sqrt{3}/2$

$x = 5\pi/6$: $g(5\pi/6) = \sin(5\pi/3) = -\sqrt{3}/2$

$\boxed{(0,0), (\pi, 0), \left(\frac{\pi}{2}, 0\right), \left(\frac{3\pi}{2}, 0\right)}$

$\boxed{\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right), \left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2}\right), \left(\frac{7\pi}{6}, \frac{\sqrt{3}}{2}\right)}$

Problem 6: 6 pts

typo!

Prove the following identity: $\dfrac{3 + \cos(4x)}{4} = \cos^4(x) + \sin^4(x)$.

Show all steps. Prove it as discussed in class. That is, start with one side and show, after a series of valid steps, that you get the other side. Also, write the word "Proof:" and the beginning and end with a little square (☐).
Hint: Lowering powers formulas

Proof RHS $= \cos^4(x) + \sin^4(x)$

$$= \left(\cos^2(x)\right)^2 + \left(\sin^2(x)\right)^2$$

$$= \left(\frac{1+\cos(2x)}{2}\right)^2 + \left(\frac{1-\cos(2x)}{2}\right)^2$$

$$= \frac{1}{4}\left(1+\cos(2x)\right)^2 + \frac{1}{4}\left(1-\cos(2x)\right)^2$$

$$= \frac{1}{4}\left[1^2 + 2\cos(2x) + \cos^2(2x)\right] + \frac{1}{4}\left[1^2 - 2\cos(2x) + \cos^2(2x)\right]$$

$$= \frac{1}{4}\left[1 \cancel{+ 2\cos(2x)} \boxed{+\cos^2(2x)} + 1 \cancel{- 2\cos(2x)} \boxed{+\cos^2(2x)}\right]$$

$$= \frac{1}{4}\left[2 + 2\cos^2(2x)\right]$$

$$= \frac{1}{4}\left[2 + \cancel{2}\left(\frac{1+\cos(4x)}{\cancel{2}}\right)\right]$$

$$= \frac{1}{4}\left[2 + 1 + \cos(4x)\right]$$

$$= \frac{3+\cos(4x)}{4} = \text{LHS}. \qquad \square$$

(or Q.E.D.)