Chapter 5: Probability

Section 5.4: Conditional Probability and General Multiplication Rule

RECALL: $P(A \text{ and } B)$ can denote two different things:

- It is the outcomes that belong to both A and B, that is in the intersection of both. Use this when making "one selection." Studied this in 5.3.
- It also is used when making "two selections": it is the outcomes in A in the 1st trial followed by the outcomes in B in the 2nd trial. We study this now.

FORMAL MULTIPLICATION RULE	
Symbolic	$\boxed{P(A \text{ and } B) = P(A) * P(B \mid A)}$
Meaning	*The probability of event A followed by event B is found by multiplying the probability of event A by the probability of event B.*
Note	$P(B \mid A)$ denotes the conditional probability of event B occurring after it is assumed that event A has already occurred.

INDEPENDENCE VS. DEPENDENCE

Def Two events are **independent** if the occurrence of one event *does not affect* the probability of the occurrence of the other event.

Note: *If events A and B are independent* $\Rightarrow$ $P(B \mid A) = P(B)$. ☆

If events A and B are independent $\Rightarrow$ $P(A \mid B) = P(A)$. ☆

ie probability of B is unaffected by A

Def If two events are not independent, they are said to be **dependent**.

SUMMARY: Two selections

$P(A \text{ and } B)$ $\nearrow$ $= P(A) \cdot P(B \mid A)$ (if A and B are dependent)

$P(A \text{ and } B)$ $\searrow$ $= P(A) \cdot P(B)$ (if A and B are independent)

GTQ

Ex 1: A bag contains an assortment of Jolly Rancher candies. Specifically, there are 5 apple, 8 watermelon, 10 cherry, and 15 grape flavored candies. You get to randomly **select three** candies *without replacement.*

ie once select candie it doesn't go back to bag !! eat !!

Dependent Events

Conditional Prob.

(a) Find the probability of picking three grape Jolly Ranchers.

Dependent Events use conditional

1st 2nd 3rd

$P(G \& G \& G) = P(G) * P(G) * P(G)$

$= \frac{15}{38} * \frac{14}{37} * \frac{13}{36} = \boxed{0.0539}$

(b) Find the probability of **not** getting any apple Jolly Ranchers.

5 apple, 38 candies

1st 2nd 3rd

$P(A^c \& A^c \& A^c) = P(A^c) \cdot P(A^c) \cdot P(A^c)$

$= \frac{33}{38} * \frac{32}{37} * \frac{31}{36} = \boxed{0.597}$

NOTE Compare this example with the example in section 5.3 that has "with replacement".

Ex2: In the table is the highest level of education information for 50 applicants for a job.

total 50

Bachelor's Degree	35
Master's Degree	15

independent / not listing twice

(a) If two of these fifty applicants names are chosen at random, *without replacement*, then what is the probability that the 1st selected has a Bachelor's degree and the 2nd has a Master's degree?

$$P(\overset{1st}{BD} \text{ and } \overset{2nd}{MD}) = P(BD) * P(MD)$$
$$= \frac{35}{50} * \frac{15}{49} = \frac{3}{14} = \boxed{0.214}$$

not very different

(b) What would the probability in (a) be if *replacement was allowed*?

$$P(\overset{1st}{BD} \text{ and } \overset{2nd}{MD}) = \frac{35}{50} * \frac{15}{50} = \frac{21}{100} = \boxed{0.21}$$

The 5% Guideline for Cumbersome Calculations (View Dependent as Independent)

If a sample size is no more than 5% of the size of the population, treat the selections as being independent.

Ex3: A quality control analyst randomly selects 3 different car ignition systems from a manufacturing process that has just produced 200 systems, including 5 that are defective. → Prob of defective

$$P(D) = \frac{5}{200} = 0.025$$

(a) What is the probability that all 3 ignition systems are good?

Dependent

$$P(\overset{1st}{G} \& \overset{2nd}{G} \& \overset{3rd}{G}) = \underset{1st}{\overset{P(G)}{\frac{195}{200}}} * \underset{2nd}{\overset{P(G)}{\frac{194}{199}}} * \underset{3rd}{\overset{P(G)}{\frac{193}{198}}} = \boxed{0.926}$$

(b) Use the 5% guideline for treating the events as independent, and find the probability that all 3 ignition systems are good.

view indep.

$$P(\overset{1st}{G} \& \overset{2nd}{G} \& \overset{3rd}{G}) = \frac{195}{200} * \frac{195}{200} * \frac{195}{200} = \left(\frac{195}{200}\right)^3$$
$$= 0.975^3 = \boxed{0.927}$$

View as Independent
3/200 = 0.015
= 1.5%

Conditional Probability

Def A **conditional probability** of an event is a probability obtained with the additional information that some other event has already occurred.

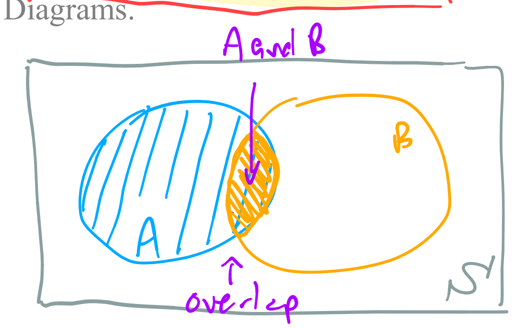
Notation: $P(B \mid A)$ denotes the conditional probability that event B occurs, given that event A has already occurred.

Same as: General Mult. Rule
$P(A \text{ and } B) = P(A) * P(B \mid A)$

Formula: $$\boxed{P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}}$$

Explanation: Use Venn Diagrams.

$$P(E) = \frac{\#E}{\#S}$$



Short-cut

$$\boxed{P(B \mid A) = \frac{\# A \text{ and } B}{\# A}} \cdot \frac{\frac{1}{\#S}}{\frac{1}{\#S}}$$

$$= \frac{\#A \text{ and } B / \#S}{\#A / \#S} = \frac{P(A \text{ and } B)}{P(A)}$$

explanation

4/2

Ex4: Let A =Today is your birthday and B =Your birthday is in this month. (April)

(a) Are events A and B dependent?

Yes!

(b) $P(A) = 1/365 = 0.00274$

(c) $P(A|B)$

$= P(\text{BD is } 4/2 \mid \text{April})$

$= \frac{1}{30} = 0.0333$

(d) $P(B|A) = P(\text{BD is April} \mid \text{BD is } 4/2) = 1.00$

6TQ

Ex5: The following table gives the mortality data for passengers of the Titanic.

Empirical Approach to Probability

	Men M	*Women* W	*Children* C	
S *Survived*	332	318	56	$\Sigma =$ 706
D *Died*	1360	104	53	$\Sigma =$ 1517
	$\Sigma =$ 1692	$\Sigma =$ 422	$\Sigma =$ 109	2223

Find the probability of randomly selecting:

(a) a passenger who died, given that the person was a man.

$P(D|M) = \frac{\# D \,\&\, M}{\# M} = \frac{1360}{1692} = \boxed{0.804}$

(b) a woman, given that the passenger survived.

$P(W|S) = \frac{\# W \text{ and } S}{\# S} = \frac{318}{706} = \boxed{0.450}$

(c) a survivor, given that the passenger was a child.

$P(S|C) = \frac{\# S \text{ and } C}{\# C} = \frac{56}{109} = \boxed{0.514}$

Note used shortcut

$P(S|C) = \frac{P(S \text{ and } C)}{P(C)} = \frac{\frac{56}{2223}}{\frac{109}{2223}}$

Ex6: The table to the right shows the status of 200 registered college students.

(a) What is the probability that a part time student is female?

	Part Time	Full Time	Total
Female	80	40	120
Male	60	20	80
Total	140	60	200

(b) What is the probability that a randomly selected student is part time, given that they are a male?

(c) What is the probability that **at least one** randomly selected female is a full time student when selecting three college students (*without replacement*)?