Math 5B - Calculus II Spring 2019

Exam 2

§9.1-9.4, 6.8, 7.1-7.4

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April_11

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Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

NAME (PRINT): Solutions SIGNATURE: ____________________

Directions

- YOU ARE ALLOWED TO USE ONLY A SCIENTIFIC CALCULATOR ON THIS EXAM.
- You have 90 minutes to complete this exam.
- The exam totals **110 points**, but it will be scored out of 100.
- There are 8 problems, many of them with multiple parts.
- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you your writing utensils.
- Cell phones must be turned off and put away in with your items in the front of the classroom.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E).
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a $\boxed{\text{box around your final answer}}$ where applicable.
- Give $\boxed{\text{UNITS}}$ where applicable.
- **PLEASE CHECK YOUR WORK!!!**
- If you need extra space, there is extra space on the back of the cover page and clearly indicate that you are continuing your work there in the original location.
- If you finish early, you may take a break but you must come back to class by 5:30 and we will have class.
- I will take attendance at the end of class

Score	Grade

This page is intentionally blank. It may be used for scratch paper. If you wish for me to grade your work on this page, please (i) label the problem you are working on, (ii) box your answer, (iii) indicate in the original problem's location that you will continue your work on this page.

$F(x,y)$ gives slope at a point $P(x,y)$

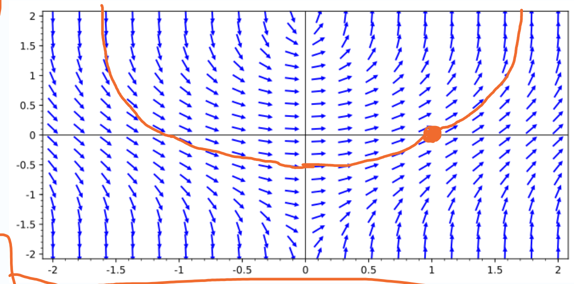
Problem 1: 14 pts

always positive

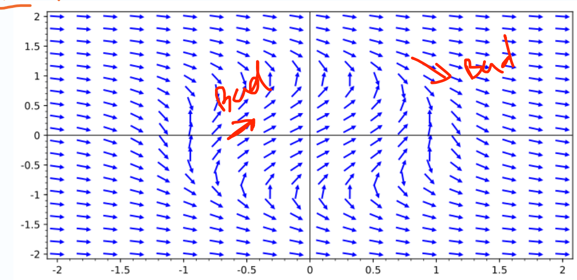
also notice $F(0,y)=0$ so all slopes on y-axis$(x=0)$ are horizontal

(4 pts) (a) Select the $\boxed{\text{slope field}}$ that corresponds to $y' = F(x,y)$ with $\boxed{F(x,y) = x \cdot e^{y^2}}$ and also sketch the solution passing through $(1,0)$.

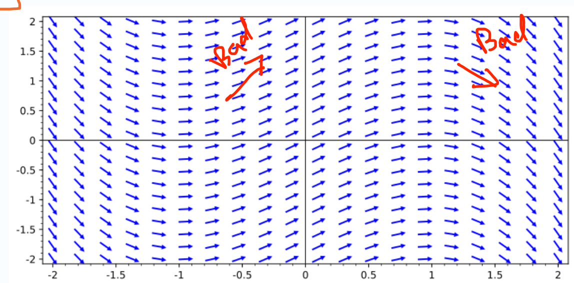
Key observation $F>0$ when $x>0$ & $F<0$ when $x<0$ → only A has this property



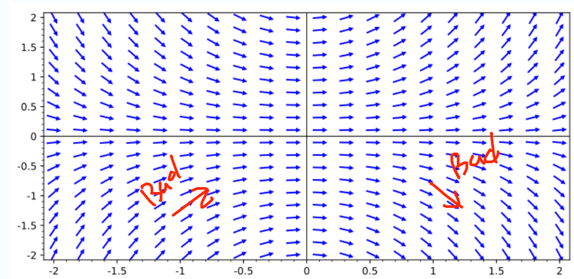
$\boxed{\text{(A)}}$



(C)



(B)



(D)

(10 pts) (b) A deer population grows logistically with growth constant $k = 0.6$ (units are $year^{-1}$) in a forrest with carrying capacity of $\boxed{2{,}000 \text{ deer.}}$

M

↳ gives time units as "years"

(i) Find the $\boxed{\text{population}}$ of deer after t years if the initial population is $1,200$ deer.

↳ Find $P(t)$

↳ $P_0 = 1200$

Givens
- logistic growth
  ↳ $P(t) = \dfrac{M}{1+Ae^{-kt}}$
- carrying capacity: $M = 2000$
- $k = 0.6$
- initial pop: $P_0 = 1200$

Find
$P(t)$

$A = \dfrac{M - P_0}{P_0}$

$= \dfrac{2000-1200}{1200}$

$= \dfrac{800}{1200} = \dfrac{2}{3}$

Thus, $\boxed{P(t) = \dfrac{2000}{1+\left(\frac{2}{3}\right)e^{-0.6t}}}$

or multiply by 3:

$\boxed{P(t) = \dfrac{6000}{3+2e^{-0.6t}}}$ (prefered) [4 pts]

(ii) How $\boxed{\text{long}}$ does it take for the deer population to reach 1,900? [Round to the nearest hundredth.]

↳ Find t

↳ 2 decimals

Given
- $P(t) = 1900$

Find
t

Solve for t: $P(t) = 1900$

$$\frac{6000}{3+2e^{-0.6t}} = 1900$$

$$\frac{6000}{1900} = 3 + 2e^{-0.6t}$$

$-3 \qquad -3$

$$\frac{\cancel{6000}\,60}{\cancel{1900}\,19} - 3 = 2e^{-0.6t}$$

$\div 2 \qquad \div 2$

$$\ln\left(\frac{1}{2}\left(-3+\frac{60}{19}\right)\right) = \ln\left(e^{-0.6t}\right)$$

$$\frac{-0.6t}{-0.6} = \frac{\ln\left(\frac{1}{2}\left(-3+\frac{60}{19}\right)\right)}{-0.6}$$

$$t = \frac{\ln\left(\frac{1}{2}\left(-3+\frac{60}{19}\right)\right)}{-0.6}$$

$t \approx 4.2316\ldots\ldots$

$\boxed{t = 4.23 \text{ years}}$ [6 pts]

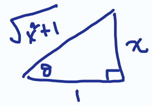
Problem 2: 12 pts

Find the general solutions to the following DEs:

(a) $\int \frac{dy}{dx}dx = \int \frac{1}{(x^2+1)^2}dx$

b/c of square can't use $\tan^{-1}$ instead use trig sub w/ $x = \tan\theta$

$y = \int \frac{1}{(x^2+1)^2}dx$ $\quad \begin{cases} x = \tan\theta \\ dx = \sec^2\theta \, d\theta \end{cases}$



$= \int \frac{1}{(\tan^2\theta+1)^2}(\sec^2\theta \, d\theta)$

$= \int \frac{\sec^2\theta}{(\sec^2\theta)^2}d\theta$

$= \int \frac{1}{\sec^2\theta}d\theta$

$= \int \cos^2\theta \, d\theta$ use formula: $\cos^2\theta = \frac{1+\cos(2\theta)}{2}$

$= \int \frac{1+\cos(2\theta)}{2}d\theta$

$= \frac{\theta}{2} + \frac{\sin(2\theta)}{4} + C$

$= \frac{\theta}{2} + \frac{2\sin(\theta)\cos\theta}{4} + C$ use $\sin(2\theta) = 2\sin\theta\cos\theta$

Go back to x variable:

$y(x) = \frac{\theta}{2} + \frac{\sin\theta\cos\theta}{2} + C$

$= \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{1}{\sqrt{x^2+1}}\right) + C$

$\boxed{y(x) = \frac{1}{2}\left(\tan^{-1}(x) + \frac{x}{x^2+1}\right) + C}$

(b) $xy' = \sqrt{1-y^2}$

$x\frac{dy}{dx} = \sqrt{1-y^2}$ (use SOV)

$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{1}{x}dx$ must know how to integrate these from memory!

$\sin^{-1}(y) = \ln|x| + C$

to solve for y, apply sin( ) to both sides:

$\sin(\sin^{-1}(y)) = \sin(\ln|x| + C)$

$\boxed{y(x) = \sin(\ln|x| + C)}$

Problem 3: 12 pts

Solve the following IVPs (i.e. find the particular solutions):

(a) $y' + 7y = 0$, IC: $y(0) = 150$

$y' = -7y$ ← this is law of natural decay

$y = Ce^{-7t}$ (or can use SOV)

apply IC: $y = 150$ when $t = 0$

$150 = Ce^0$

$150 = C$

$\boxed{y(t) = 150e^{-7t}}$

Solving using SOV:

$\frac{dy}{dt} = -7y$ (or $\frac{dy}{dx} = -7x$)

$\int \frac{dy}{y} = \int -7dt$

$e^{\ln|y|} = e^{-7t+C}$

$|y| = e^{-7t+C} = e^{-7t} \cdot \underbrace{e^C}_{\text{always } \oplus}$

$\to y = Ce^{-7t}$ (redefine e^C to be C)

(b) $\frac{dy}{dt} = \frac{t}{y - t^2y}$, IC: $y(0) = 4$

$\frac{dy}{dt} = \frac{t}{y(1-t^2)}$ (use SOV)

$\int y\,dy = \int \frac{t}{1-t^2}dt$ $\quad \begin{bmatrix} u = 1-t^2 \\ du = -2t\,dt \to t\,dt = -\frac{1}{2}du \end{bmatrix}$

$\frac{1}{2}y^2 = \int \frac{1}{u}\left(-\frac{1}{2}du\right)$

$\cancel{\frac{1}{2}}y^2 = -\cancel{\frac{1}{2}}\ln|u| + C$

$y^2 = -\ln|1-t^2| + C$

$y = \pm\sqrt{C - \ln|1-t^2|}$

IC $y(0) = 4$

so take $+\sqrt{\ }$

$4 = +\sqrt{C - \cancel{\ln|1-0^2|}}$ (=0)

$4 = \sqrt{C}$

$16 = C$

$\leadsto \boxed{y(t) = \sqrt{16 - \ln|1-t^2|} \text{ or } y(t) = \sqrt{16 - \ln|1-t^2|}}$

$F(x,y) = x^2 - y$

Euler's Method

$x_n = x_{n-1} + h$

$y_n = y_{n-1} + h \cdot F(x_{n-1}, y_{n-1})$

Problem 4: 14 pts

Consider the IVP: $\boxed{y' = x^2 - y}$ with $\boxed{y(3) = 1.}$ (x_0, y_0)

Use Euler's Method with step size $h = 0.2$ to approximate $y(4)$ where $y(x)$ is the exact solution. Show all your steps.

$h = 0.2$ $y(4)$ = requires 5 steps ($0.2 * 5 = 1$ to get from 3 to 4)

$\begin{cases} x_0 = 3 \\ y_0 = 1 \end{cases}$

$x_1 = 3.2$

$y_1 = y_0 + hF(x_0, y_0)$

$= 1 + 0.2F(3,1)$

$= 1 + 0.2[9-1]$

$= 2.6$

$\begin{cases} x_1 = 3.2 \\ y_1 = 2.6 \end{cases}$

$x_2 = 3.4$

$y_2 = y_1 + hF(x_1, y_1)$

$= 2.6 + 0.2 \cdot F(3.2, 2.6)$

$= 2.6 + 0.2[10.24 - 2.6]$

$= 4.128$

$\begin{cases} x_2 = 3.4 \\ y_2 = 4.128 \end{cases}$

$x_2 = 3.4$

$y_2 = 4.128$

$x_3 = 3.6$

$y_3 = y_2 + h \cdot F(x_2, y_2)$

$= 4.128 + 0.2 \cdot F(3.4, 4.128)$

$= 4.128 + 0.2 \cdot [3.4^2 - 4.128]$

$= 5.6144$

$\begin{cases} x_3 = 3.6 \\ y_3 = 5.6144 \end{cases}$

$x_4 = 3.8$

$y_4 = y_3 + h \cdot F(x_3, y_3)$

$= 5.6144 + 0.2 \cdot F(3.6, 5.6144)$

$= 5.6144 + 0.2 \cdot [3.6^2 - 5.6144]$

$= 7.08352$

$\begin{cases} x_4 = 3.8 \\ y_4 = 7.08352 \end{cases}$

$x_4 = 3.8$

$y_4 = 7.08352$

$x_5 = 4$

$y_5 = y_4 + h \cdot F(x_4, y_4)$

$= 7.08352 + 0.2 \cdot F(3.8, 7.08352)$

$= 7.08352 + 0.2 \cdot [3.8^2 - 7.08352]$

$= 8.554816$

$\begin{cases} x_5 = 4.0 \\ y_5 = 8.554816 \end{cases}$

$\boxed{y(4) = y_5 = 8.554816}$

$y(4)$ means $y(x)$ at $x = 4$

$y_5 \approx y(4)$

Problem 5: 12 pts

Evaluate the following limits. In your work, identify the type of indeterminate form(s).

(a) $\lim_{x\to\infty} \frac{x^5+1}{e^x} = \frac{\infty}{\infty}$

$\overset{L'H\hat{o}p}{=} \lim_{x\to\infty} \left(\frac{5x^4}{e^x} \right) = \frac{\infty}{\infty}$

$\overset{L'H\hat{o}p}{=} \lim_{x\to\infty} \left(\frac{20x^3}{e^x} \right) = \frac{\infty}{\infty}$

$\overset{L'H\hat{o}p}{=} \lim_{x\to\infty} \left(\frac{60x^2}{e^x} \right) = \frac{\infty}{\infty}$

$\overset{L'H\hat{o}p}{=} \lim_{x\to\infty} \left(\frac{120x}{e^x} \right) = \frac{\infty}{\infty}$

$\overset{L'H\hat{o}p}{=} \lim_{x\to\infty} \left(\frac{120}{e^x} \right) = \boxed{0}$

(b) $\lim_{x\to 0} \frac{\ln(1+x)}{\cos(x) + e^x - 2} = \frac{\ln(1)}{1+1-2} = \frac{0}{0}$

$\overset{L'H\hat{o}p}{=} \lim_{x\to 0} \left(\frac{\frac{1}{1+x}}{-\sin(x) + e^x} \right)$

$= \frac{\frac{1}{1+0}}{-\sin(0) + e^0}$

$= \frac{1}{1}$

$= \boxed{1}$

Problem 6: 24 pts

Evaluate the following integrals.

(a) $\int \sin^8(x)\cos^5(x)\,dx$ ← "at least one odd" case

$= \int \sin^8(x)(\cos^2(x))^2(\cos(x)\,dx)$ · separate out a cos(x) · get ready to replace $\cos^4(x)$

$= \int \sin^8(x)(1-\sin^2(x))^2(\cos(x)\,dx)$

$u = \sin(x)$
$du = \cos(x)\,dx$

$= \int u^8(1-u^2)^2\,du$

$= \int u^8(1-2u^2+u^4)\,du$

$= \int (u^8 - 2u^{10} + u^{12})\,du$

$= \frac{1}{9}u^9 - \frac{2}{11}u^{11} + \frac{1}{13}u^{13} + C = \boxed{\frac{1}{9}\sin^9(x) - \frac{2}{11}\sin^{11}(x) + \frac{1}{13}\sin^{13}(x) + C}$

go back to x!

Need partial Fractions

(b) $\int \frac{5x^2-7x+3}{x^2(x-3)}\,dx = \int\left(\frac{a}{x} + \frac{b}{x^2} + \frac{c}{x-3}\right)dx$

$\frac{5x^2-7x+3}{x^2(x-3)} = \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x-3}$

$5x^2-7x+3 = ax(x-3) + b(x-3) + cx^2$

$x=0$: $3 = b(-3) \to \boxed{b=-1}$

$x=3$: $45-21+3 = 9c$
$27 = 9c \to \boxed{c=3}$

$x=1$: $5-7+3 = a(-2)+b(-2)+c$
$1 = -2a-2b+c$
$1 = -2a+2+3$
$-4 = -2a \to \boxed{a=2}$

$= \int\left(\frac{2}{x} - \frac{1}{x^2} + \frac{3}{x-3}\right)dx$

$= 2\ln|x| + \frac{1}{x} + 3\ln|x-3| + C$

$= \boxed{\ln(x^2) + \frac{1}{x} + \ln|x-3|^3 + C}$

or

$= \boxed{\frac{1}{x} + \ln\left|x^2(x-3)^3\right| + C}$

IBPs

(c) $\int t^5\ln(t)\,dt \overset{IBP}{=} (\ln t)\left(\frac{t^6}{6}\right) - \int \frac{t^6}{6}\left(\frac{1}{t}dt\right)$

IBP

$u = \ln(t)$	$dv = t^5\,dt$
$du = \frac{1}{t}dt$	$v = \frac{t^6}{6}$

$= \frac{1}{6}t^6\ln(t) - \int \frac{t^5}{6}\,dt$

$= \boxed{\frac{1}{6}t^6\ln(t) - \frac{1}{36}t^6 + C}$.

IBPs & "Trick"

(d) $\underbrace{\int e^{3x}\sin(2x)\,dx}_{=I} \overset{IBP①}{=} \left(-\frac{\cos(2x)}{2}\right)(e^{3x}) - \int -\frac{\cos(2x)}{2}\cdot 3e^{3x}\,dx$

IBP①

$u = e^{3x}$	$dv = \sin(2x)\,dx$
$du = 3e^{3x}\,dx$	$v = -\frac{\cos(2x)}{2}$

$= -\frac{1}{2}\cos(2x)e^{3x} + \frac{3}{2}\int e^{3x}\cos(2x)\,dx$

IBP②

$u = e^{3x}$	$dv = \cos(2x)$
$du = 3e^{3x}dx$	$v = \frac{\sin(2x)}{2}$

IBP②

$= -\frac{1}{2}\cos(2x)e^{3x} + \frac{3}{2}\left[\frac{1}{2}\sin(2x)e^{3x} - \int \frac{1}{2}\sin(2x)\cdot 3e^{3x}\,dx\right]$

$= -\frac{1}{2}\cos(2x)e^{3x} + \frac{3}{4}\sin(2x)e^{3x} - \frac{9}{4}\underbrace{\int e^{3x}\sin(2x)\,dx}_{=I}$

$I = \frac{1}{2}e^{3x}\left(-\cos(2x) + \frac{3}{2}\sin(2x)\right) - \frac{9}{4}I$

$+\frac{9}{4}I \qquad\qquad +\frac{9}{4}I$

$\frac{13}{4}I = \frac{1}{2}e^{3x}\left(-\cos(2x) + \frac{3}{2}\sin(2x)\right)$

$\boxed{I = \frac{1}{13}e^{3x}\left(3\sin(2x) - 2\cos(2x)\right) + C}$

Problem 7: 12 pts

Evaluate the following limits. In your work, identify the type of indeterminate form(s).

(a) $\lim\limits_{x\to\infty} x^{2/x} = \infty^0$ need to use $e^{\ln(x)} = x$ "inverse prop"

$$= \lim_{x\to\infty} e^{\ln\left(x^{2/x}\right)} \quad \text{(inverse prop)}$$

$$= \lim_{x\to\infty} \left(e^{\frac{2}{x}\ln(x)}\right) \quad \text{(log prop)}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{2\ln x}{x}\right)} \ \frac{\infty}{\infty} \quad (e^x \text{ is continuous})$$

$$\overset{\text{L'Hôp}}{=} e^{\lim\limits_{x\to\infty}\left(\frac{2/x}{1}\right)}$$

$$= e^{\frac{2}{\infty}} = e^0 = \boxed{1}$$

(b) $\lim\limits_{x\to 1^+} \left(\dfrac{x}{x-1} - \dfrac{1}{\ln(x)}\right) = \frac{1}{+0} - \frac{1}{+0} = \infty - \infty$ use common denom trick

$$= \lim_{x\to 1^+}\left(\frac{x\cdot\ln(x) - (x-1)}{(x-1)\cdot\ln(x)}\right) = \frac{0}{0}$$

$$\overset{\text{L'Hôp}}{=} \lim_{x\to 1^+}\left(\frac{[1]\ln(x) + x\left[\frac{1}{x}\right] - 1}{[1]\ln(x) + (x-1)\left[\frac{1}{x}\right]}\right)$$

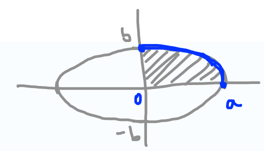
$$= \lim_{x\to 1^+}\left(\frac{\ln(x) + \cancel{1} - \cancel{1}}{\ln(x) + 1 - \frac{1}{x}}\right) = \frac{0}{0}$$

$$\overset{\text{L'Hôp}}{=} \lim_{x\to 1^+}\left(\frac{\frac{1}{x}}{\frac{1}{x} + 0 + \frac{1}{x^2}}\right) = \frac{1}{1+1} = \boxed{\frac{1}{2}}$$

Problem 8: 10 pts

→ don't forget units

Determine the $\boxed{\textbf{area}}$ of the ellipse



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where $a > b > 0$, by setting up and evaluating an appropriate integral. Show all work to receive credit.

EQ of ellipse as function:

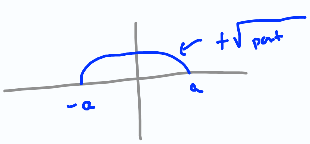
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$-\frac{x^2}{a^2} \qquad -\frac{x^2}{a^2}$$

$$b^2\left(\frac{y^2}{b^2}\right) = \left(1 - \frac{x^2}{a^2}\right)b^2$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right)$$

$$y = \pm\sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)}$$



$$\text{Area of Ellipse} = 4\int_0^a \sqrt{b^2\left(1 - \frac{x^2}{a^2}\right)}\, dx$$

$$= 4\int_0^a \sqrt{b^2}\sqrt{\frac{a^2 - x^2}{a^2}}\, dx$$

$$= 4b\int_0^a \frac{\sqrt{a^2 - x^2}}{a}\, dx$$

$$= \frac{4b}{a}\underbrace{\int_0^a \sqrt{a^2 - x^2}\, dx}_{\text{trig sub}}$$

trig sub: $x = a\sin\theta$, $dx = a\cos\theta\, d\theta$

Change limits too:
$x = 0$: $0 = a\sin\theta \to \theta = 0$
$x = a$: $a = a\sin\theta$, $1 = \sin\theta \to \theta = \pi/2$

$$= \frac{4b}{a}\left[\int_0^{\pi/2} \sqrt{a^2 - (a\sin\theta)^2}\,(a\cos\theta\, d\theta)\right]$$

$$= \frac{4b}{a}\int_0^{\pi/2} \underbrace{\sqrt{a^2(1 - \sin^2\theta)}}_{\sqrt{a^2\cos^2\theta} \,=\, a\cos\theta} a\cos\theta\, d\theta$$

$$= \frac{4b}{\cancel{a}}\int_0^{\pi/2} (a\cos\theta)(\cancel{a}\cos\theta)\, d\theta$$

$$= 4ab\int_0^{\pi/2} \underbrace{\cos^2\theta}_{\text{ID}}\, d\theta$$

$$= 4ab\int_0^{\pi/2} \frac{1}{2}\left(1 + \cos(2\theta)\right) d\theta$$

$$= 4ab\left[\frac{1}{2}\left(\theta + \frac{1}{2}\sin(2\theta)\right)\right]_0^{\pi/2}$$

$$= 4ab\left[\left(\frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{4}\sin(\pi)\right) - \left(\frac{1}{2}\cdot 0 + \frac{1}{4}\sin(0)\right)\right]$$

$$= \cancel{4}ab\left[\frac{\pi}{\cancel{4}}\right]$$

$$= \boxed{\pi ab \text{ square units}}$$

Formula Sheet

- $\boxed{\int u\,dv = uv - \int v\,du}$
- $\boxed{\int \tan(\theta)\,d\theta = \ln|\sec(\theta)| + C}$ $\boxed{\int \sec(\theta)\,d\theta = \ln|\sec(\theta) + \tan(\theta)| + C}$
- $\boxed{\cos^2(x) + \sin^2(x) = 1}$ $\boxed{1 + \tan^2(x) = \sec^2(x)}$
- $\boxed{\cos^2(x) = \frac{1}{2}(1 + \cos(2x))}$ $\boxed{\sin^2(x) = \frac{1}{2}(1 - \cos(2x))}$
- $\boxed{\sin(2x) = 2\sin(x)\cos(x)}$
- $\boxed{\sin(A)\cos(B) = \frac{1}{2}(\sin(A-B) + \sin(A+B))}$ $\boxed{\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))}$
  $\boxed{\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B))}$
- $\boxed{x = a\sin(\theta)}$ $\boxed{x = a\tan(\theta)}$ $\boxed{x = a\sec(\theta)}$
-
  - Case I: $Q(x) = (a_1x - b_1)(a_2x - b_2)\cdots(a_kx - b_k)$ is product of **distinct** linear factors in denominator. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \cdots + \frac{A_k}{a_kx + b_k}$$
  - Case II: $Q(x) = (ax + b)^k$ repeated roots of Q. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \cdots + \frac{A_k}{(ax + b)^k}$$
  - Case III: $Q(x) = (ax^2 + bx + c)^k$ repeated irreducible. Then
    $$\frac{P(x)}{Q(x)} = \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}$$
  - Case IV: Assume $\deg(P) \geq \deg(Q)$. Then

    Use **long division** first to write $\frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$

    where $S(x)$ is a polynomial and $R(x)$ is the remainder with $\deg(R) < \deg(Q)$.

    Then $\frac{R(x)}{Q(x)}$ is proper and use Case I, II, or III.

Post Exam Survey

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

1. When taking the exam I felt
   (a) Rushed. I wanted more time.
   (b) Relaxed. I had enough time.
   (c) Amazed. I had tons of extra time.
2. The week before the test I did all my homework on time: YES NO
3. The week before the test, in addition to the homework I followed a study plan. YES NO
   (a) I think this helped: YES NO
4. The day before the test I spend ________ hours studying and reviewing.
   (a) I think that was enough time: YES NO
5. The night before the test:
   (a) I stayed up very late cramming for the test
   (b) I stayed up very late, but I wasn't doing math
   (c) I didn't need to cram because I was prepared
   (d) I got a good night's sleep so my brain would function well.
6. I think I got the following grade on this test: ________
7. Strategies that worked well for me were (please elaborate):
8. Next time I will do an even better job preparing for the test by: