Chapter 11: Inferences on Two Samples

Section 11.2: Inference about Two Population Means: Dependent Samples (Matched-Pairs)

Def Two sets of observations are *paired* if each observation in one set has a special correspondence or connection with exactly one observation in the other set. Dependent Samples

Ex 1: State if the samples are dependent or independent and if the variable is qualitative or quantitative.

1) Among competing acne medications, does one perform better than the other? To answer this question, researchers applied Medication A to one part of the subject's face and Medication B to a different part of the subject's face to determine the proportion of subjects whose acne cleared up for each medication. The part of the face that received Medication A was randomly determined.

Sample 1 = apply Med A to one side of person's face

Sample 2 = apply med B to other side of person's face

Dependent! Matched Pair

Variable = whether or not Acne clears up on person's side of face (Qualitative)

2) A researcher wishes to determine the effects of alcohol on people's reaction time. She randomly divides 100 people 21 years or older into two groups. Group 1 is asked to drink 3 ounces of alcohol, while group 2 drinks a placebo. Thirty minutes later they measure their reaction time.

Sample 1 = Group 1 drinks alcohol

Independent

Sample 2 = Group 2 drinks placebo

Variable = Reaction time to alcohol (Quantitative) Time → continuous → Level of measurement: Ratio

3) A statistician wants to compare the treatment of female and male actors based on their ages. She looks at the ages of the females and males who won best actor/actress in the last five years of the Oscars.

Sample 1 = Male actors

Independent

Sample 2 = Female actors

Variable = Age at time won oscar (Quantitative) → time discrete → Level of measurement: Interval

TURN TWO SAMPLES INTO ONE SAMPLE:

x_1: reaction time dominant hand x_2: reaction time of NON-dominant hand

Consider an experiment where a researcher throws a stick towards someone and first asks them to catch it with their dominant hand then again with their non-dominant hand. The times below showed how long it took several individuals to react to the toss.

L3 = L2 − L1

Student	Dominant Hand, x_1 (L1)	Nondominant Hand, x_2 (L2)	Difference, d
1	0.177	0.179	0.177 − 0.179 = −0.002
2	0.210	0.202	0.210 − 0.202 = 0.008
3	0.186	0.208	−0.022
4	0.189	0.184	0.005
5	0.198	0.215	−0.017
6	0.194	0.193	0.001
7	0.160	0.194	−0.034
8	0.163	0.160	0.003
9	0.166	0.209	−0.043
10	0.152	0.164	−0.012
11	0.190	0.210	−0.020
12	0.172	0.197	−0.025
n			$\sum d_i = -0.158$

use 1-VAR stats

These are **Dependent Samples** because: its the same person!

Create one **New Sample of *Differences*** then run a Matched-Pairs Test.

True mean of the differences = $\mu_d = \mu_1 - \mu_2$

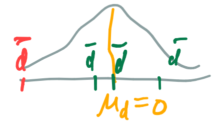
Sample size = $n = n_1 = n_2$

Sample differences = $d = x_1 - x_2$

Sample mean of the differences = $\bar{d}$

Sample standard deviation of the differences = s_d

$\mu_1 = \mu_2$ $\quad\quad$ $\mu_1 - \mu_2 < 0 \rightarrow \mu_1 < \mu_2$ $\quad$ μ_1 smaller than μ_2



Recall the logic: Say $H_0: \mu_d = 0$ and $H_1: \mu_d < 0$. If we get…

- $\bar{d} = 0$ we ___FTR___ H_0
- $\bar{d} > 0$ we ___FTR___ H_0
- $\bar{d} < 0$ by "a little", we ___FTR___ H_0
- $\bar{d} < 0$ by "a lot", we ___reject___ H_0

Explain in words: what each alternative hypothesis means: $\mu_d < 0$ or $\mu_d > 0$ or $\mu_d \neq 0$

$\mu_d < 0 \rightarrow \mu_1 - \mu_2 < 0 \rightarrow \mu_1 < \mu_2$: The mean of sample 1 is LESS THAN mean sample 2.

Steps for Hypothesis Test when Applied to testing μ_d

Pre-Step: *Check Requirements*	***Step 1:*** *Hypotheses*	***Step 2:*** *Level of Significance*
• Samples are dependent and simple random • Differences are **normal** or $\boldsymbol{n > 30}$	H_0: $\mu_d = 0$ H_A: $\mu_d < 0$ or $\mu_d > 0$ or $\mu_d \neq 0$	α ($\alpha = 0.05$ if not given)

Step 3: *Test Statistic* ***(note: it can be negative!)***
Find a z-score, t-value, X^2 value or F-value

Reminder μ_r T is MEAN (μ)

compare w/ one mean:

$t^* = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$ 10.3

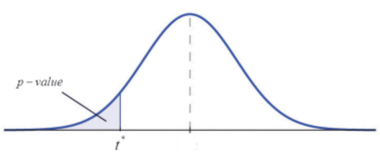
$$\boxed{t^* = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}}}$$

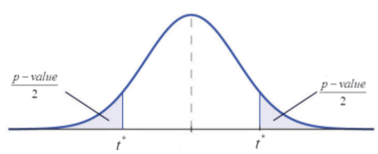
Step 4: *Find a Critical Value or P-Value*

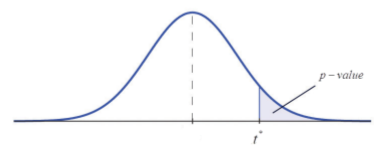
P-VALUE METHOD

DECISION

$\begin{cases} \text{Reject } H_0 & \text{if } P\text{-value} \leq \alpha \\ \text{Fail to Reject } H_0 & \text{if } P\text{-value} > \alpha \end{cases}$



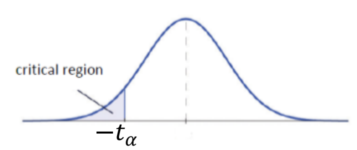


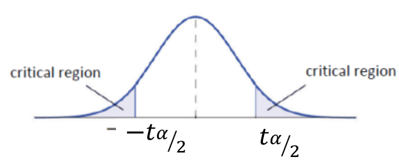


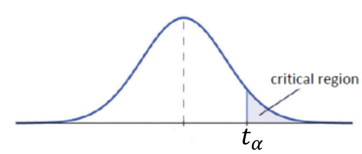
CRITICAL VALUE METHOD

DECISION

$\begin{cases} \text{Reject } H_0 & \text{if } t^* \text{ lies in the critical region} \\ \text{Fail to Reject } H_0 & \text{if } t^* \text{ doesn't lie in the critical region} \end{cases}$







Step 5: Make DECISION and write a *CONCLUSION* either rejecting or failing to reject H_0

GRAPHING CALCULATOR (TI-83 OR 84)

We actually use the same T-Test that we used in one independent sample in section 10.3. The difference (pun intended) now is that we use the differences of two means as a single mean.

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ $\boxed{\text{T-Test}}$ same!

HT for 1 mean

(b) Enter $\begin{cases} \mu_0 = \text{population mean stated in } H_0 \quad (\mu_d) \\ s = \text{sample standard deviation} \quad (s_d) \\ \bar{x} = \text{sample mean} \quad (\bar{d}) \\ n = \text{sample size} \\ \mu \ \square \ \text{alternative hypothesis} \end{cases}$

claim (HT)

Ex 2: It is a commonly held belief that Crossovers are safer than small cars. If a Crossover and small car are in a collision, does the Crossover sustain less damage (as suggested by the cost of repair)? Consumer Reports crashed Crossovers into small cars, with the Crossover moving 15 miles per hour and the front of the Crossover crashing into the rear of the small car. The data is normally distributed. Below are the repair costs: L3 = L2 − L1

df = 6

	Crossover into Car	Small Car Damage (L1, x_1)	Crossover Damage (L2, x_2)	Differences: d
1	Lexus RX-350 into Honda Insight	1274	1721	−447
2	Nissan Pathfinder into Hyundai Elantra	2327	1434	893
3	Toyota RAV4 into Kia Forte	3223	850	2373
4	Jeep Cherokee into Kia Niro Hybrid	2058	2329	−281
5	Ford Explorer into Toyota Camry	3095	1415	1680
6	Honda CR-V into Ford Focus	3386	1470	1916
7	Chevrolet Equinox into Nissan Sentra	4560	2884	1676

$\bar{d} = 1115.7$

$s_d = 1102.7$

HT

$\mu_2 < \mu_1 \rightarrow \mu_1 - \mu_2 > 0 \rightarrow \mu_d > 0$

Do the sample data suggest that Crossovers are safer? Use the level of significance $\alpha = 0.01$. Use PVM.

Check requirements

① Dependent? Yes, b/c are in same accident

② normal? ✓ ~~n ≥ 30~~

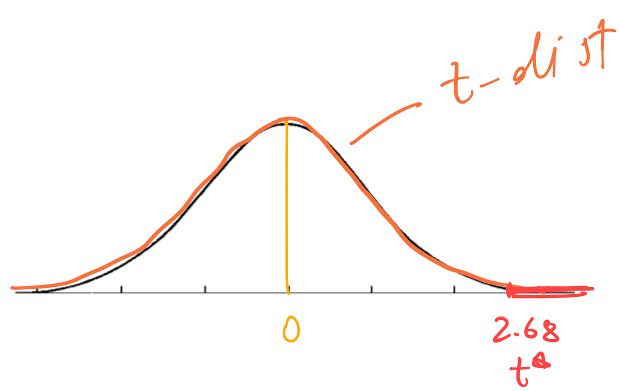
Null and Alternative Hypothesis

$\begin{cases} H_0: \mu_d = 0 \\ H_A: \mu_d > 0 \end{cases}$ (Right Tailed Test)

Test Statistic

$$t^* = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{1115.7 - 0}{\left(\frac{1102.7}{\sqrt{7}}\right)} = \boxed{2.68}$$

P-value



$P = P(t > t^*)$

$= \text{tcdf}(\overset{a}{2.68}, \overset{b}{1E99}, \overset{df}{6})$

$\boxed{P = 0.0183}$

Decision about Null Hypothesis

$P = 0.0183$ $\quad P > \alpha$

$\alpha = 0.01$

"P is high, Null w/ fly" ⟶ $\boxed{FTRH_0}$

Conclusion

"There isn't enough statistical evidence to conclude that crossovers are safer than compact cars as determined by the cost of repairs."

CONFIDENCE INTERVAL FOR THE MEAN DIFFERENCE FROM DEPENDENT SAMPLES

Compare w/ CI for one mean

Alternative Forms: $\bar{d} - E < \mu_d < \bar{d} + E$ or $\bar{d} \pm E$ or $\boxed{(\bar{d} - E, \bar{d} + E)}$

• $(\bar{x} - E, \bar{x} + E)$

where the **margin of error** is given by $\boxed{E = t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}}}$

• $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$

Calculator: 8. TInterval (means)

Ex 3: A company claims that its 12-week special exercise program significantly reduces weight. A random sample of eight persons was selected, and the following table gives the weights (in lbs) of those eight persons before and after the program.

$n = 8$ $df = 7$

Dependent

	Weight (in pounds)							
Before x_1	180	195	177	221	208	199	148	230
After x_2	185	187	171	214	208	194	150	227
$x_2 - x_1 = d$	5	-8	-6	-7	0	-5	2	-3

$\bar{d} = -2.8$

$s_d = 4.7$

Construct a 90% confidence interval for the mean before-after differences.

~~Find point estimate~~

$$\boxed{\bar{d} = -2.8 \text{ lb/12-week}}$$

Determine critical value $t_{\alpha/2}$

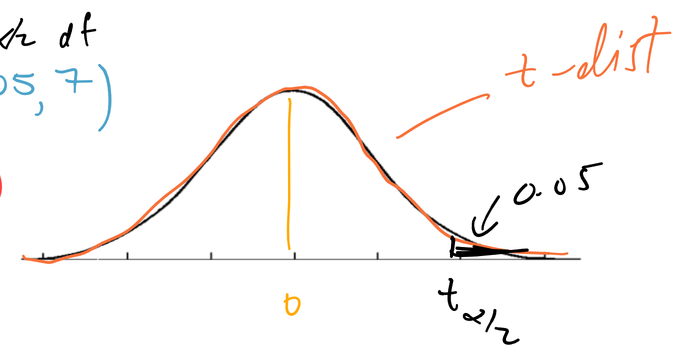
$CL = 0.9$

$\alpha = 1 - CL = 0.1$

$\alpha/2 = 0.05$

$t_{0.05} = \text{invT}(\underset{\alpha/2}{0.05}, \underset{df}{7})$

$\boxed{t_{0.05} \approx +1.89}$



Find margin of error

$$E = t_{\alpha/2} \cdot \frac{s_d}{\sqrt{n}} = 1.89 \cdot \frac{(4.7)}{\sqrt{8}} = \boxed{3.14}$$

Construct confidence interval

$\bar{d} \pm E$: -2.8 ± 3.14 : $\boxed{CI: (-5.9, 0.34)}$

Estimating μ_d : ≈ 0

Does it appear that the weight loss program is effective?

Estimating μ_d: mean of difference (after - before) of weight loss by all participants of the 12-week program

$-5.9 < \mu_d < 0.34$

≈ -6 ≈ 0

• marginally successful?

successful: $-10 < \mu_d < -5$

Before bigger!

Before x_2
After x_1

μ_d = before - after

$\mu_d > 0$

$\mu_d < 0$

CI μ_d $(\bar{d} - E, \bar{d} + E)$

$\bar{d} = +2.8$

$E = 3.14$

$2.8 \pm 3.14 = (-0.34, 5.9)$

$-0.34 < \mu_d < 5.9$