Section 10.2: Hypothesis Test for a $\boxed{\text{Population Proportion}}$

p - population
$\hat{p}$ - sample

GOAL: Make a decision about p based on $\hat{p}$ using probability (sampling dist).

STRUCTURE OF A HYPOTHESIS TEST

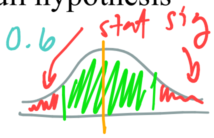
1) Make an assumption about reality	•
2) Look at a sample evidence	•
3) Determine whether it contradicts our assumption.	•

We won't be 100% certain, we will just be able to tell if sample data supports a statement or not.

BIG QUESTION: Is this STATISTICALLY SIGNIFICANT?

Def ***Statistically Significant*** When observed results are unlikely under the assumption that the null hypothesis is true and we reject the null hypothesis.

Ex $\begin{cases} H_0: p = 0.5 \\ H_A: p > 0.5 \end{cases}$ sample $\hat{p} = 0.57$ $\hat{p} = 0.6$ stat sig
$\hat{p} = 0.55$

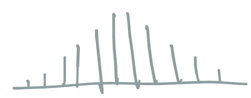


p_0 $\hat{p}$

HYPOTHESIS TESTING: CLAIM ABOUT A PROPORTION

Requirements

1. The sample observations are a simple random sample. SRS
2. The conditions for a BINOMIAL distribution are satisfied.
3. If $\boxed{n \cdot \hat{p} \cdot \hat{q} \geq 10}$, then the normal distribution can be used to approximate the distribution of sample proportions with mean μ = np and standard deviation σ= $\sqrt{npq}$ exactly as found when using the binomial distribution.



New Notation: "past proportion" $\boxed{p_0}$ "new" sample $\boxed{\hat{p}}$

$P(\text{success}) = p$
$P(\text{failure}) = q = 1 - p$

Steps for *a* Hypothesis Test When Applied to Testing Population Proportion *p*	
Step 0: *Check Requirements* • It is a valid SRS sample • The requirements are met to use the needed distribution.	BINOMIAL DIST 1 fixed trials (n) 2 independent events 3 "bi" S F 4 probability of events are constant.
Step 1: *Hypotheses* parameter "past" H_0: $p = p_0$ H_1: $p < p_0$ or $p \neq p_0$ or $p > p_0$	***Step 2:*** *Level of Significance* Convention: choose $\alpha = 0.05$ if LOS is not specified. $\alpha = P(\text{R}H_0 \mid H_0 T)$
Step 3: *Test Statistic* (Find a z-score, ~~t-value~~ or ~~χ^2-value~~) $z^* = \dfrac{\hat{p} - p_0}{\sigma_{p_0}}$ $\quad \boxed{z^* = \dfrac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}}}$ • p_0 = past prop ($q_0 = 1 - p_0$) • $\hat{p}$ = new sample prop • n = sample size Sampling	
Step 4: *Find a critical value or P-value to check using either the Critical Value Method OR P-value Method.*	
Step 5: *Make a **decision** AND draw a **conclusion***	

NULL AND ALTERNATIVE HYPOTHESIS

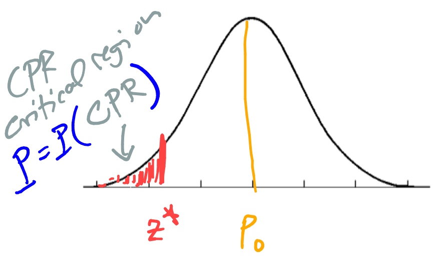
LEFT-TAILED	TWO-TAILED	RIGHT-TAILED
$H_0: \ p = p_0$	$H_0: \ p = p_0$	$H_0: \ p = p_0$
$H_1: \ p < p_0$	$H_1: \ p \neq p_0$	$H_1: \ p > p_0$

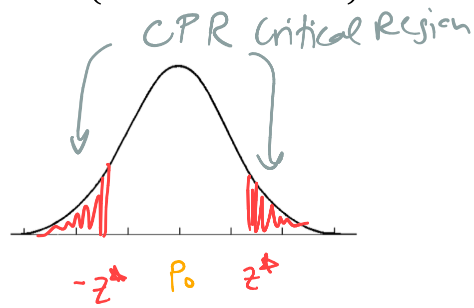
P-VALUE METHOD

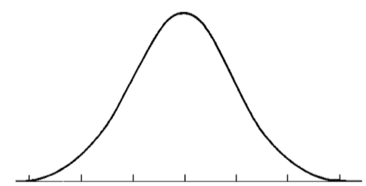
Def ***P-Value****: probability that a sample is as extreme as our test statistic or more extreme assuming H_0 is true.*

Key: *use test statistic z^* to draw the **critical region (CPR)** and compute probability of it.*

$P = P(z < z^*)$ = normalcdf(-1E99, z^*, 0, 1) $\quad P = P(z < -z^* \text{ or } z > z^*) \quad P = P(z > z^*)$







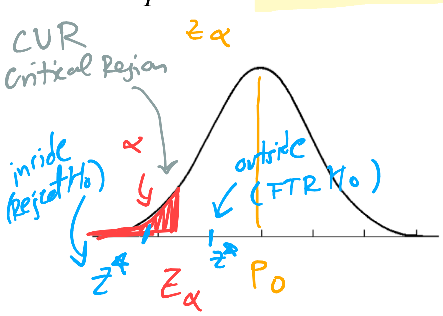
P-Val = P(CPR)

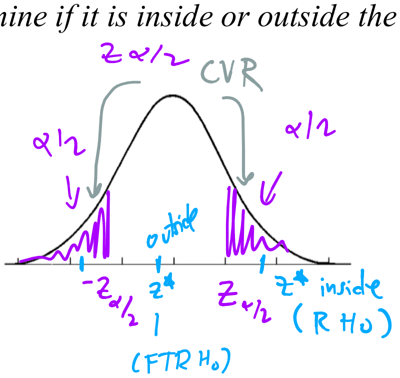
MAKE A DECISION: $\begin{cases} \textbf{If } P \le \alpha, \textbf{ then we } \text{"}\textbf{Reject } H_0\text{"} \\ \textbf{If } P > \alpha, \textbf{ then we } \text{"}\textbf{Fail to Reject } H_0\text{"} \end{cases}$

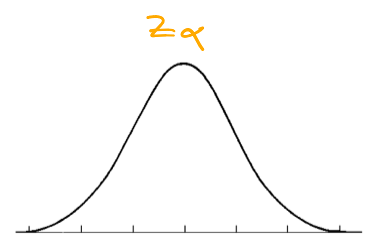
CRITICAL VALUE METHOD

Key: *Use the level of significance, α, to compute the critical value z_α or $z_{\alpha/2}$ (depending on one or two-tailed test) which determines the **critical region (CVR)**.*

Compute the test statistic z^* and determine if it is inside or outside the CVR.







z_α = invNorm(α, 0, 1, LEFT)

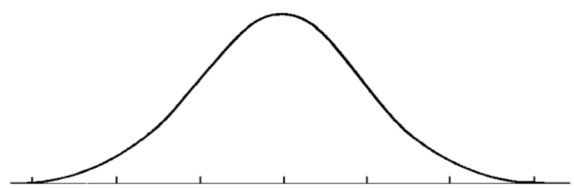
MAKE A DECISION: $\begin{cases} \text{If } z^* \text{ is INSIDE the CVR, then we "}\textbf{Reject } H_0\text{"} \\ \text{If } z^* \text{ is OUTSIDE the CVR, then we "}\textbf{Fail to Reject } H_0\text{"} \end{cases}$

Ex1: According to the Census Bureau, 8.8% of the U.S. population had no health insurance coverage in 2017. Suppose that in a recent random sample of 1200 Americans, 130 had no health insurance. Use a 0.02 significance level to test the claim that the current percentage of Americans who have no health insurance coverage is greater than 8.8%. Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

Identify the Type I error

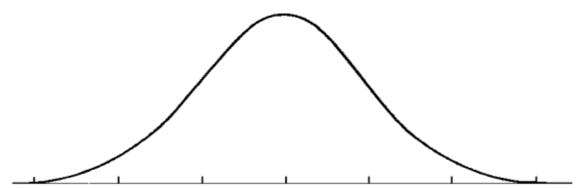
Identify the Type II error

Ex2: According to NPR, in 2016 32.1% of adults aged 18-34 lived at home with their parents. A sociologist recently randomly surveyed 500 people aged 18-34 and found that 143 of them did. At $\alpha = 0.05$, do we think the proportion has changed? Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

Graphing Calculator (TI-83 or 84)

Instructions: (a) STAT $\Rightarrow$ TESTS $\Rightarrow$ 1-PropZTest

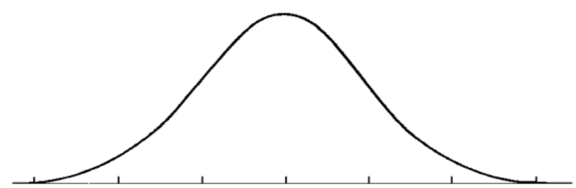
(b) Enter
$$\begin{cases} p_0 = \text{population proportion stated in } H_0 \\ x = \text{number of successes} \\ n = \text{number of trials} \\ prop \; \square \; \text{alternative hypothesis} \end{cases}$$

Ex3: In a 2016 Gallup poll, 34% of people said that it was morally acceptable to clone animals. In 2017, a survey found that 192 out of 600 randomly selected people believed that it was morally acceptable to clone animals. Use a 0.10 significance level to test the claim that less than 34% of all adults say that it is morally acceptable to clone animals. Use the ____________________ method.

Null and Alternative Hypothesis

Test Statistic

P-value/Critical Region



Decision about Null Hypothesis

Conclusion

Big Question: Is this ____________________________ ________________________________?

Def ***Statistically Significant*** When observed results are unlikely under the assumption that the null hypothesis is true and we reject the null hypothesis.

Logic of Hypothesis Tests (Cover this at the end of the chapter)

Classical Approach

We now explain how Hypothesis Tests work. It starts with a sampling distribution. Recall that we have a past claim, p_0. We want to know if it has changed so we find a new sample, $\hat{p}$, of size n. Then we know what the sampling distribution should look like assuming the null hypothesis, $p = p_0$, is true. In Chapter 8 we learned that the sampling distribution is a normal distribution with $\mu_{p_0} = p$ and $\sigma_{p_0} = \sqrt{p_0 \cdot q_0/n}$.

The key is we want to know whether our new sample, $\hat{p}$, could reasonably be determined by chance if H_0 is true.

If the null hypothesis is true, then we expect that our sample, $\hat{p}$, come from the "middle" of the sampling distribution. That means it should be, say, within 2 standard deviations from the mean (recall this corresponds to approximately 95% of the samples). To compute how many standard deviations away from the mean it is, we simply compute the z-score! We call this z-score the test statistic, z^*.

In symbols, this looks like this: $z^* = \frac{\hat{p}-p_0}{\sigma_{p_0}}$. We can use this number to make our decision.

If the test statistic is greater than 2 or less than -2, then this is saying that our sample is unlikely to have occurred by chance! So we reject the null hypothesis since have statistical evidence that our sample, $\hat{p}$, is unlikely to occur if the null hypothesis is true.

If the test statistics is between -2 and 2, then this is exactly what we would expect assuming the null hypothesis is true. That is, our sample is likely to have occurred if the null hypothesis is true.

P-value Approach

This is very similar to the classical approach. This time we compute the probability of obtaining a sample that is as extreme as $\hat{p}$, or more extreme. For example, if we are trying to do a Right-Tailed Test, then we compute the probability $P(z > \hat{p})$ using our new sample as the cut-off and the sampling distribution of p_0:

$$P(z > \hat{p}) = normalcdf\left(\hat{p}, 1\text{E}99, \mu_{p_0}, \sigma_{p_0}\right).$$

If P is small (less than 0.05), then it is unlikely that our sample, $\hat{p}$, was determined by chance! So we have statistical evidence that the null hypothesis is not true and we reject H_0. For example, if P=0.02, then 2 samples in 100 will give a sample proportion of $\hat{p}$ or higher.

If P is high (greater than 0.05), then this is not unusual assuming that the null hypothesis is true. We haven't proved the null hypothesis only found evidence that our sample is reasonable assuming it is true.

Confidence Interval Approach

There's actually one more way we can do Hypothesis Tests. We can use a confidence interval to make our decision. We construct a confidence interval using the new sample data, $\hat{p}$.

If the past claim, p_0, is outside the confidence interval then we "reject H_0."

If the past claim, p_0, is inside the confidence interval then we "fail to reject H_0."