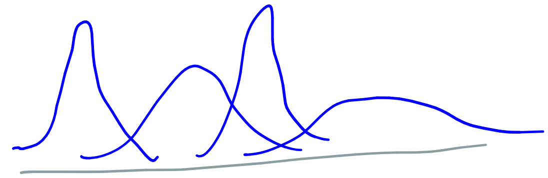
Chapter 7: The Normal Probability Distribution

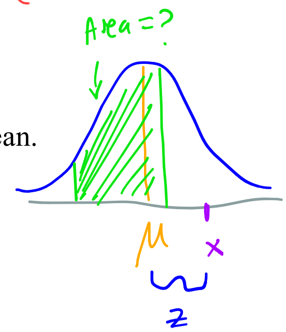
Section 7.2: Applications of the Normal Distribution



DESMOS: https://www.desmos.com/calculator/pooefgeuhi

Key Point: how many normal distribution curves are there? There's tons of them! (infinitely many)

Z SCORES



Def A z **score** is the number of standard deviations that a given value x is above or below the mean.

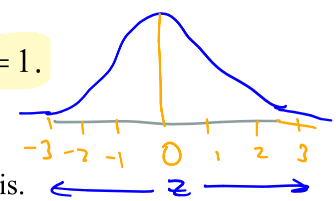
FORMULA: Sample: $z = \dfrac{x - \bar{x}}{s}$ Population: $z = \dfrac{x - \mu}{\sigma}$

Round-Off Rule: Round z scores to two decimal places.

STANDARD NORMAL DISTRIBUTION

Def The **standard normal distribution** is a normal probability distribution with $\mu = 0$ and $\sigma = 1$.

z-scores



IMPORTANT NOTES

1. The z-score is used on the horizontal axis. The x-values are also on the horizontal axis.
2. The **area** of the region under the curve **is equal to** the associated **probability** of occurrence.

TWO WAYS TO FIND AREA

1. ~~Use Table V in Appendix.~~

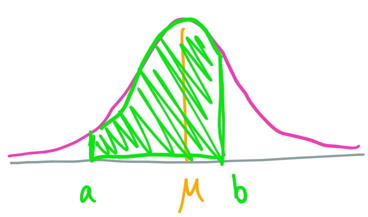
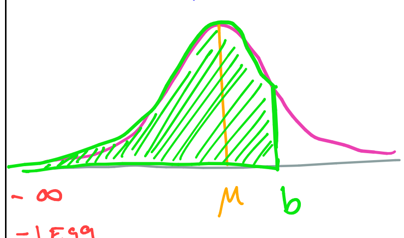
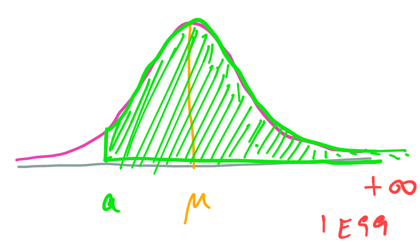
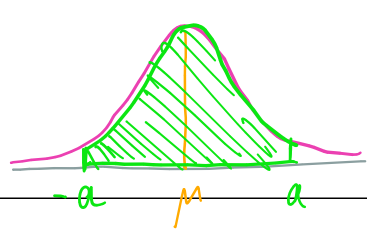
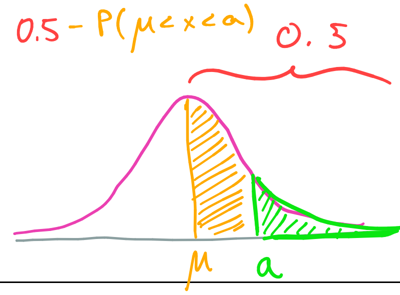
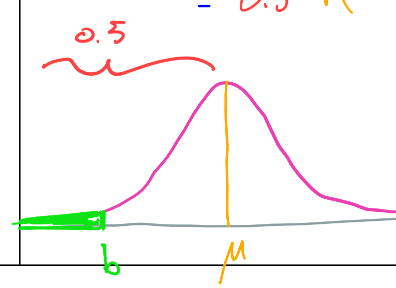
   ~~Look up the area under the curve that lies to the left of z-score (may first need to convert data to z-score).~~

2. **Use Graphing Calculator (TI-84 Plus)**

   (a) 2nd ⇒ VARS ⇒ DISTR

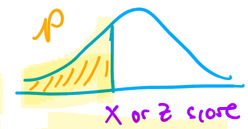
   (b) $\boxed{\textit{normalcdf}(\textit{lower}, \textit{upper}, \mu, \sigma)}$ ☆

PICTURES & NOTATION Sketch and shade the part of the normal curve with μ and σ. Also, provide the calculator instructions. Note: we use a = lower, b = upper. Note: Use -1e99 for $-\infty$ and 1e99 for $+\infty$.

$P(a < x < b)$	$P(x < b)$	$P(x > a)$
= normalcdf(a, b, μ, σ)	= normalcdf(−1E99, b, μ, σ)	= normalcdf(a, 1E99, μ, σ)
		
$P(-a < x < a)$	$P(x > a)$ and $a > \mu$	$P(x < b)$ and $b < \mu$
~~Use 1-Minus Trick~~ = normalcdf(−a, a, μ, σ)	Use 0.5-Minus Trick = 0.5 − P(μ < x < a)	Use 0.5-Minus Trick = 0.5 − P(b < x < μ)
		

Two Ways to Find Z-score/Quartile score (i.e. original x values)

OLDER CALC



USES LEFT

1. ~~Use Table V in Appendix.~~

   ~~Look up the z-score associated with the area that lies to left.~~

2. **Use Graphing Calculator (TI-84 Plus)**

   (a) 2nd $\Rightarrow$ VARS $\Rightarrow$ DISTR

   (b) invNorm(*area*, μ, σ, *TAIL*)

   $\overset{\|}{\text{prob}}$

   older calc don't have this

use z

$\mu = 0$ & $\sigma = 1$

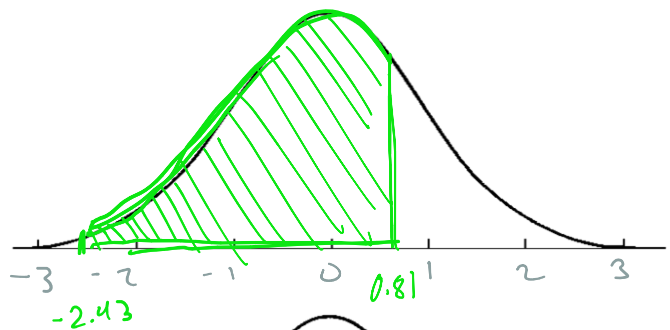
Ex1: Find the **probability** given the following **z-scores** for a standard normal distribution.

What is the mean? What is the standard deviation?

(a) $P(-2.43 < z < 0.81)$

$= \text{normalcdf}(\overset{a}{-2.43}, \overset{b}{0.81}, \overset{\mu}{0}, \overset{\sigma}{1})$

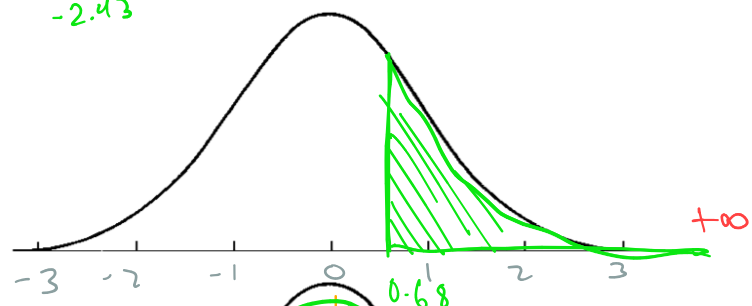
$= \boxed{0.783}$



(b) $P(z > 0.68)$

$= \text{normalcdf}(0.68, \overset{1E99}{+\infty}, 0, 1)$

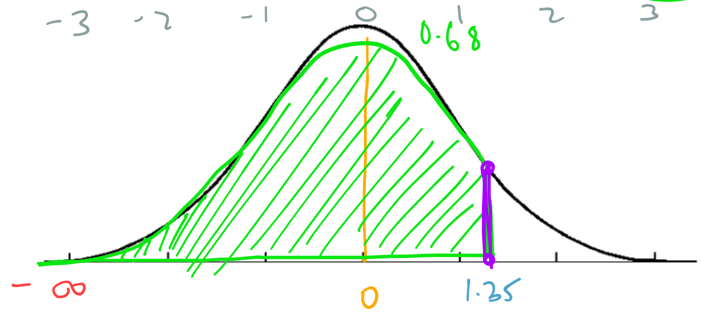
$= \boxed{0.248}$



(c) $P(z < 1.35)$ -1E99

$= \text{normalcdf}(-\infty, 1.35, 0, 1)$

$= \boxed{0.911}$



(d) Would $P(z \leq 1.35)$ differ from (c)?

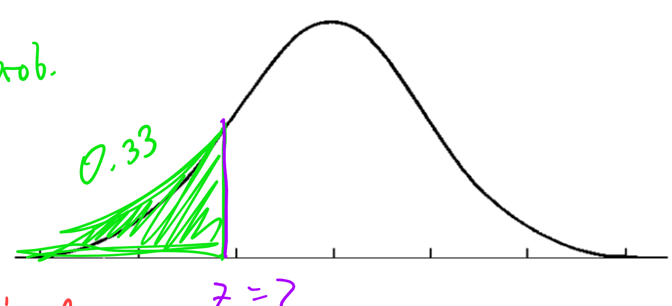
No! The same answer!

Ex2: Find the **z-score** associated with the 33th percentile.

percentage = prob.

$z = \text{invNorm}(\overset{p}{0.33}, \overset{\mu}{0}, \overset{\sigma}{1}, \overset{\text{TAIL}}{\text{LEFT}})$

$= \boxed{-0.44}$

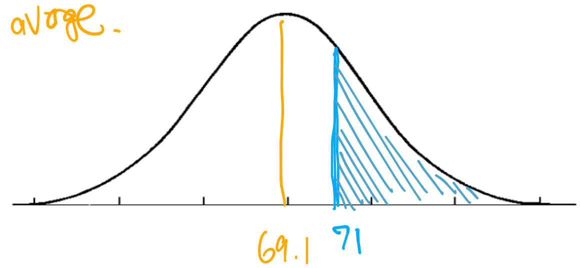


Recall Rounding Rule z-score is 2 decimal

Ex3: Consider your height in inches. Calculate the standardized value (z-score) for your height given that in the United States the average height for women is 63.7 inches with a standard deviation of 2.7 inches and for men is 69.1 inches with a standard deviation of 2.9 inches. *Would you be considered tall for your gender?*

$z_{height} = \dfrac{71-69.1}{2.9} = 0.66$ very slightly above avrage.

$x = 71$ inches



What is the probability that someone of your gender is taller than you?

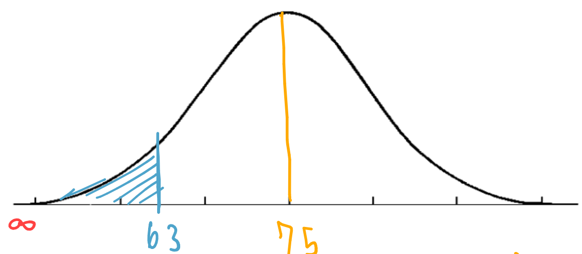
$P(x>71) = P(z>0.66)$

$= \text{normalcdf}(\overset{a}{71}, \overset{b}{1E99}, \overset{\mu}{69.1}, \overset{\sigma}{2.9})$

$= \boxed{0.256}$

Ex4: The average for the statistics exam was 75 and the standard deviation was 8. Andrey was told by the instructor that he scored 1.5 standard deviations below the mean, and the scores were normally distributed.
What was Andrey's exam score?

$x = \mu - 1.5\sigma = 75 - 1.5(8) = \boxed{63}$ Andrey's score.



What percentage of students did worse than Andrey?

$P(x<63) = \text{normalcdf}(\overset{a}{-1E99}, \overset{b}{63}, \overset{\mu}{75}, 8)$

$= 0.0668$ → write as percentage! $\boxed{6.68\%}$

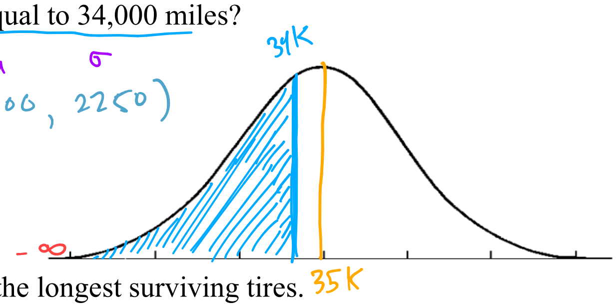
Only 7% of students did worse than Andrey on Test.

Ex5: The life spans of a brand of automobile tires are normally distributed with a mean life span of 35,000 miles and a standard deviation of 2250 miles. The life span of a randomly selected tire is 34,000 miles. Can you find the probability that a randomly selected automobile tire has a life-span less than or equal to 34,000 miles?

X = RV lifespan of tire

$P(x<34k) = \text{normalcdf}(\overset{a}{-1E99}, \overset{b}{34000}, \overset{\mu}{35000}, \overset{\sigma}{2250})$

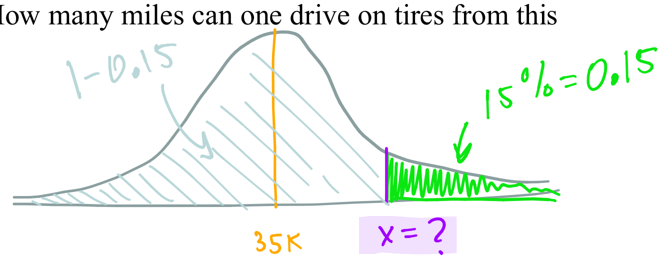
$= \boxed{0.328}$



What is the number of miles in the lifespan of tires in the top 15% of the longest surviving tires.
Consider tires in the top 15% of lifespans (i.e. the longest mileage). How many miles can one drive on tires from this range?

$X = \text{invNorm}(\overset{p}{0.15}, \overset{\mu}{35000}, \overset{\sigma}{2250}, \overset{\text{TAIL}}{\text{RIGHT}})$

$= \boxed{37{,}332 \text{ miles}}$



Folk w/o Tail on Calculator

$X = \text{invNorm}(\overset{1-p}{1-0.15}, \overset{\mu}{35000}, \overset{\sigma}{2250})$ "Left Tail"

Tires in top 15% will last at least 37,232 miles.

HOW TO CHOOSE BETWEEN normalcdf AND InvNorm:

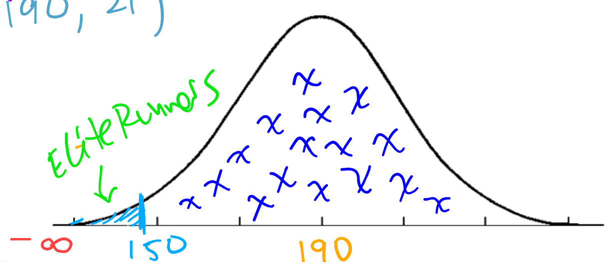
When to use **Normalcdf**?	When to use **InvNorm**?
To find the probability when the data values μ, σ, a, b are given.	When the probability (or area or percent) is given and we are trying to find the x-value or z-score.

Ex6: The completion times to run a road race are normally distributed with a mean of 190 minutes and a standard deviation of 21 minutes.

(a) What is the probability that a randomly selected runner will finish the race in less than 150 minutes?

$P(x<150) = \text{normalcdf}(-1E99, 150, 190, 21)$

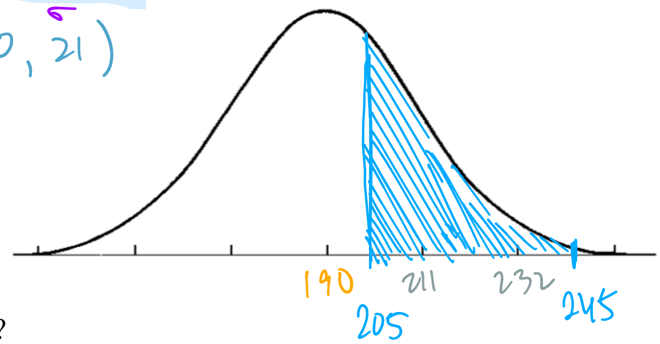
$= \boxed{0.0284}$



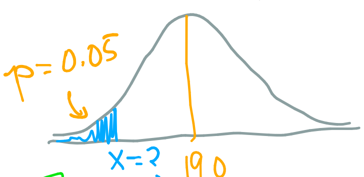
(b) What percentage of runners will finish the race between 205 and 245 minutes?

$P(205 < x < 245) = \text{normalcdf}(205, 245, 190, 21)$

$= \boxed{0.233}$



(c) In how many minutes do the fastest 5% of runners finish the race?



x = cut off for fastest 5%

$= \text{invNorm}(0.05, 190, 21)$

$= \boxed{155.5 \text{ min}}$

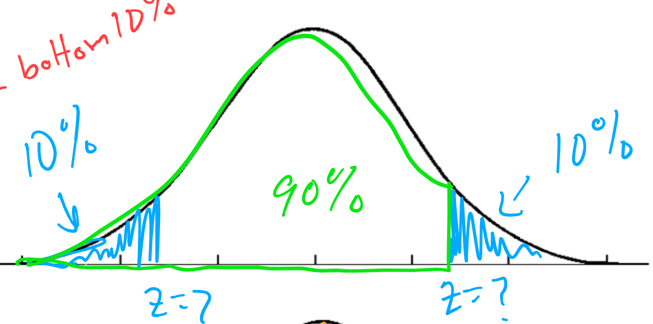
Elite 5% run race in 155.5 min or faster. (or less).

Ex7: Find the z-scores that separate the top 10% and bottom 10% of all values.

$z = \text{invNorm}(0.1, 0, 1) = \boxed{-1.28}$ — bottom 10%

top 10%: $\boxed{z = 1.28}$ $= \text{invNorm}(.9, 0, 1)$

OLD TI



CRITICAL VALUE

Def

Specific Notation: z_α is the **critical value** that denotes a z-score with an area of α to its RIGHT.

Ex8: Find $z_{0.05}$

$z_{0.05} = \text{invNorm}(0.05, 0, 1, \text{RIGHT})$

$= \boxed{1.64}$

OLD $z_{0.05} = \text{invNorm}(1 - 0.05, 0, 1)$