Stat 50 - Elementary Statistics — Spring 2019

Exam 2

Ch 4, 5

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April_4

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Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

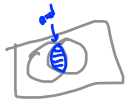
NAME (PRINT): Solutions _____ SIGNATURE: _____

Directions

- YOU ARE ALLOWED TO USE A CALCULATOR ON THIS EXAM. (Ti83/Ti83+/Ti84/Ti84+/Ti84+CE-T, or scientific calculator)
- You have 80 minutes to complete this exam.
- The exam totals **104 points** but will be graded out of 100 only. (So it is possible to get 104% on this exam ☺).
- There are ~~12~~ 9 problems, many of them with multiple parts.
- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you your writing utensils.
- Cell phones must be turned off and put away in with your items in the front of the classroom.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- Leave answers in exact form (as simplified as possible), unless told otherwise.
- Put a box around your final answer where applicable.
- **PLEASE INCLUDE UNITS** where applicable
- **PLEASE CHECK YOUR WORK!!!**
- If you need extra space, there is extra space on the back of the cover page and clearly indicate that you are continuing your work there in the original location.
- If you finish early, you may take a break but you must come back to class by 2:45 and we will have class.
- I will take attendance at the end of class

Score	Grade

More notes on using "and"



- there's two ways to use it: as the intersection of two sets when making "one selection"
  - Ex
    drawing one card
    that's a King and Spade

& when making "2 or more selections"
- Ex
  drawing two cards:
  first is a king
  and
  second is a Jack

- you use different formulas:

one selection

$$P(A \text{ and } B) = \frac{\#\text{ in } A \text{ and } B}{\#\text{ in } S}$$

2 selections (or more)

$$P(A^{1st} \text{ and } B^{2nd}) \rightarrow P(A^{1st}) * P(B^{2nd})$$ B independent from A

$$\searrow P(A^{1st}) * P(B^{2nd}|A)$$ B dependent from A

notice these formulas were on formula sheet but not directly

Problem 1: 16 pts

Answer each question for the following table. For probability, state your answer as **un-simplified** fractions AND as decimals (following our rounding rule).

	Low Obesity	Average Obesity	High Obesity	Total
Hypertension	24	33	46	103
No Hypertension	109	101	87	297
Total	133	134	133	400

Let **T** = Hypertension, **N** = No Hypertension, **L** = Low Obesity, **A** = Average Obesity, **H** = High Obesity ← careful, don't confuse w/ "Hypertension"

(2 pt) (a) Find $P(T)$:

$P(T) = \frac{103}{400} = 0.258$

(2 pt) (b) Find $P(A \text{ and } N)$:

$P(A \text{ and } N) = \frac{101}{400} = 0.253$

(2 pt) (c) Find $P(H \text{ or } N)$:

$P(H \text{ or } N) = P(H) + P(N) - P(H \text{ and } N)$

$= \frac{133}{400} + \frac{297}{400} - \frac{87}{400} = \frac{343}{400} = 0.858$

(2 pt) (d) Find the probability that a person has hypertension given that the person is of Average Obesity.

$P(T \mid A) = \frac{P(T \text{ and } A)}{P(A)} = \frac{\frac{33}{400}}{\frac{134}{400}} = \frac{33}{134} = 0.246$

(2 pt) (e) Are the events "Hypertension" and "Low Obesity" independent? Explain why or why not.

The events T and L are not independent because they influence each other's probabilities. Moreover, notice that these two events share 24 people in common. Independent events must be disjoint which these sets are not (again, they share 24 people in common).

(2 pt) (f) Are the events "No Hypertension" and "Average Obesity" mutually exclusive? Explain why or why not.

The events N and A are not mutually exclusive since they are not disjoint. Both sets share 101 people in common.

(2 pt) (g) Two adults are randomly selected without replacement, what is the probability both are of Average Obesity?

$P(A \text{ and } A) = P(A) * P(A) = \frac{134}{400} * \frac{133}{399} = 0.112$

(2 pt) (h) Three adults are randomly selected with replacement, what is the probability at least one has Hypertension?

$P(\text{at least one } T) = 1 - P(\text{no } T)$

$= 1 - P(\bar{T} \text{ and } \bar{T} \text{ and } \bar{T})$

$= 1 - \left(\frac{297}{400}\right)^3$

$= 0.591$

↳ so put back so all $P(\bar{T})$ are same

$P(\bar{T}) = \frac{297}{400}$

*note $\bar{T}$ same as N

$P(\bar{T} \text{ and } \bar{T} \text{ and } \bar{T}) = P(\bar{T}) * P(\bar{T}) * P(\bar{T})$

$= \frac{297}{400} * \frac{297}{400} * \frac{297}{400} = \left(\frac{297}{400}\right)^3$

$= (0.7424)^3$

$= 0.409344\ldots$

Problem 2: 4 pts (1 pts each)

Fill in the blanks:

(a) 0 $\leq P(A) \leq$ 1

(b) $P(A) = 0$ means event A is an impossible event, cannot occur

(c) $P(A) = 1$ means event A is a certain event, is surely to occur

Multiple-choice. Select the correct answer:

(d) The shaded area in the Venn Diagram represents

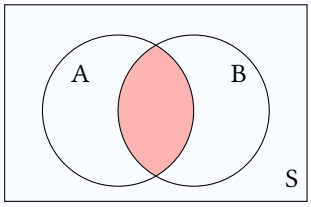
(A) $A|B$

(B) A and B (B)

(C) A or B

(D) $B|A$

(E) the entire sample space S

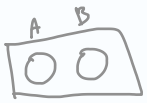


Problem 3: 6 pts (1 pts each)

TRUE or **FALSE** (please spell out/write the entire word for credit).

(Hint: you might find it helpful to use Venn Diagrams to help you arrive at an answer)

(a) True If A and B are mutually exclusive then $P(A \text{ and } B) = 0$.



(b) False If A and B are mutually exclusive then $P(A \text{ or } B) = 1$. see ↗

(c) False If $P(A) > P(B)$ then event ~~B~~ A is more likely than event ~~A~~ B.

(d) True If our population is 40,000 students, then a sample of 35 can be treated as being independent even if we sample without replacement. 5% rule for cumbersome calculations: $\frac{35}{40{,}000} = 0.00875$ way less 5%

(e) True If $P(A) = 0.00081$ then A is an unusual event.

(f) False We always have $P(A \text{ or } B) \leq P(A \text{ and } B)$.

"and" is smaller than "or"

Problem 4: 6 pts

(2 pt) (a) Consider the following table of values: What does p have to be to make the following a **probability distribution**? Show your mathematical reasoning.

x	$P(x)$
-12	0.222
20	0.235
23	0.324
35	p

$\sum P(x) = 1$

$0.222 + 0.235 + 0.324 + p = 1$

$0.781 + p = 1$

$-0.781 \quad -0.781$

$p = 0.219$

p must be 0.219 to make a probability distribution

↳ must satisfy:

① $\sum P(x) = 1$

② $0 \leq P(x) \leq 1 \rightsquigarrow 0 \leq p \leq 1$

Note · $\sum P(x) = 1$ ✓

· $0 \leq p \leq 1$ ✓

(4 pt) (b) Find the **mean** (μ) and the **standard deviation** (σ) of the probability distribution. Show your mathematical reasoning.

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(x)$
0	0.33	$0 \cdot 0.33 = 0$	$0^2 \cdot 0.33 = 0$
1	0.30	$1 \cdot 0.3 = 0.3$	$1^2 \cdot 0.3 = 0.3$
2	0.25	0.5	$= 1$
3	0.12	0.36	$= 1.08$
		$\sum x \cdot P(x) = 1.16$ (mean)	$\sum x^2 \cdot P(x) = 2.38$ (needed for σ)

mean $\mu = \sum x \cdot P(x)$

$\mu = 1.2$

Note no units given in this problem.

standard deviation $\sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2}$

$\sigma = \sqrt{2.38 - (1.16)^2}$

$\sigma = 1.0170545\ldots$

$\sigma = 1.0$

Problem 5: 26 pts

Let X be a random variable that counts the number of Heads can may occur out of three coin flips (assume that the coin is fair).

(2 pt) (a) Using set notation, write the **sample space** of the experiment of flipping 3 coins. *(Hint: use the H or T notation. So one outcome is HHT, for example)*

Sample space is set of all possible outcomes

$S = \{TTT, HTT, THT, TTH, HHT, HTH, THH, HHH\}$

0 Heads | 1 Heads | 2 Heads | 3 Heads

Note #S = 8 outcomes. (out of 3 coin flips.)

(2 pt) (b) What does x represent? What are all the values can it be?

x = # of Heads out of 3 coin flips

$x = 0, 1, 2, 3$

also: x are the values that the random variable X can take.

(4 pt) (c) Is X a **binomial probability distribution**? Explain why or why not.

Yes, X is a binomial prob. dist. because it satisfies the 4 conditions:

1. $n=3$ fixed # trials ✓
2. each trial is independent ✓
3. only two outcomes: either Heads or not ("S" / "F") ✓
4. probability of remains constant in each trial ($p = 1/2$) ✓

(4 pt) (c) Give the **probability distribution** associated to X.

Use sample space from part (a) to find probabilities

x	$P(x)$	$x \cdot P(x)$	$x^2 \cdot P(X)$
0	$1/8 = 0.125$	0	0
1	$3/8 = 0.375$	0.375	0.375
2	$3/8 = 0.375$	0.75	1.5
3	$1/8 = 0.125$	0.375	1.125
	$\Sigma = 1$ ✓	$\Sigma = 1.5$	$\Sigma = 3$

Note you only need the first two columns of the table to give the probability distribution. However, the other columns are there to help you compute μ & σ in parts (f) and (g).

(2 pt) (d) What is the probability of getting **at most** two Heads?

x 0 1 2 3

$P(X = 0,1,2) = P(X \le 2) = 0.125 + 0.375 + 0.375 = 0.875$

another way: $P(X \le 2) = \text{binomcdf}(n=3, p=0.5, x=2) = 0.875$ ← same ☺

(2 pt) (e) What is the probability of getting **at least** two Heads?

x 0 1 2 3

$P(X = 2,3) = P(X \ge 2) = 0.375 + 0.125 = 0.5$

another way: $P(X \ge 2) = 1 - P(X \le 1) = 1 - \text{binomcdf}(3, 0.5, 1) = 0.5$ ← same ☺

(2 pt) (f) What is the **mean**?

$\mu = \Sigma x \cdot P(x)$

$= 0 + 0.375 + 0.75 + 0.375$

$\mu = 1.5$ Heads out of 3 ← Note units are same units as x or X

μ is "average" or "expectation"

(2 pt) (g) What is the **standard deviation**?

$\sigma = \sqrt{\Sigma [x^2 \cdot P(x)] - \mu^2}$

$= \sqrt{3 - (1.5)^2}$

$\sigma = 0.8660\ldots$

$\sigma = 0.9$ Heads out of 3

Problem 6: 6 pts

Fidelity life insurance will sell a \$100,000 one-year term life insurance policy to 30 year-olds males for a premium of \$161. Based on U.S. Department of Health and Human Services, it has been determined that there is a 99.86% chance that a randomly selected 30 year old male survives the year.

↳ lives ↳ 0.9986 probability

Find the **expected value** to the company (that is, from the company's point of view) of this policy for a 30 year old male. Include in your answer the corresponding probability distribution. Also, please write your final answer in a complete sentence ($M \to E$).

X = the amount gained or lost for the insurance company for one policy for one year for a 30yo. male client

	x	$P(x)$
(lives)	\$161	0.9986
(dies)	−\$99,839	0.0014
		$\Sigma = 1$ ✓

$E(X) = \Sigma x \cdot P(x)$

$= (161)(0.9986) + (-99839)(0.0014)$

$= 160.7746 - 139.7746$

$E(X) = \$21.00$

($M \to E$) "The insurance company expects to gain \$21.00 over many policies of the same type (one year, 30 y.o. males)."

Problem 7: 16 pts

Assume we have a standard deck of 52 cards (see below). Use proper probability notation and rules when writing your answer.

(2 pt) (a) When one card is drawn, what is $P(\text{Black and } 4)$?

$P(\text{Black and } 4) = \frac{2}{52} = 0.038$

Note don't specify how to give answer so you can give it as unsimplified fraction or as a decimal (rounded to 3 sig figs). But I'll grade what you box!

(2 pt) (b) When one card is drawn, what is $P(\text{Spade or Heart})$?

$P(\text{Spade or Heart}) = P(\text{Spade}) + P(\text{Heart}) - P(\text{Spade and Heart}) = \frac{13}{52} + \frac{13}{52} - \frac{0}{52} = \frac{26}{52} = 0.5$

none, disjoint

(2 pt) (c) When one card is drawn, what is $P(\overline{\text{Face}})$?

$P(\overline{\text{Face}}) = P(\text{not Face}) = 1 - P(\text{Face}) = 1 - \frac{12}{52} = \frac{52}{52} - \frac{12}{52} = \frac{52-12}{52} = \frac{40}{52} = 0.769$

(2 pt) (d) When two cards are drawn with replacement, find $P(\text{Two Queens})$?

$P(Q \text{ and } Q) = P(Q) * P(Q) = \frac{4}{52} * \frac{4}{52} = \frac{16}{2704} = 0.00592$

same!

note 3 sig fig

(2 pt) (e) When two cards are drawn with replacement, find $P(1^{st}$ is a Black card and 2^{nd} is a Face card)?

$P(\text{Black and Face}) = P(\text{Black}) * P(\text{Face}) = \frac{26}{52} * \frac{12}{52} = 0.115$

(2 pt) (f) When two cards are drawn **without** replacement, find $P(\text{Two Red Face cards})$?

$P(\text{Red Face and Red Face}) = P(\text{Red Face}) * P(\text{Red Face}) = \frac{6}{52} * \frac{5}{51} = 0.0113$

Red and Face

bc 1st Red Face not put back into deck.

(2 pt) (g) When two cards are drawn **without** replacement, find $P(2^{nd}$ card is Heart $\mid 1^{st}$ card is Black)?

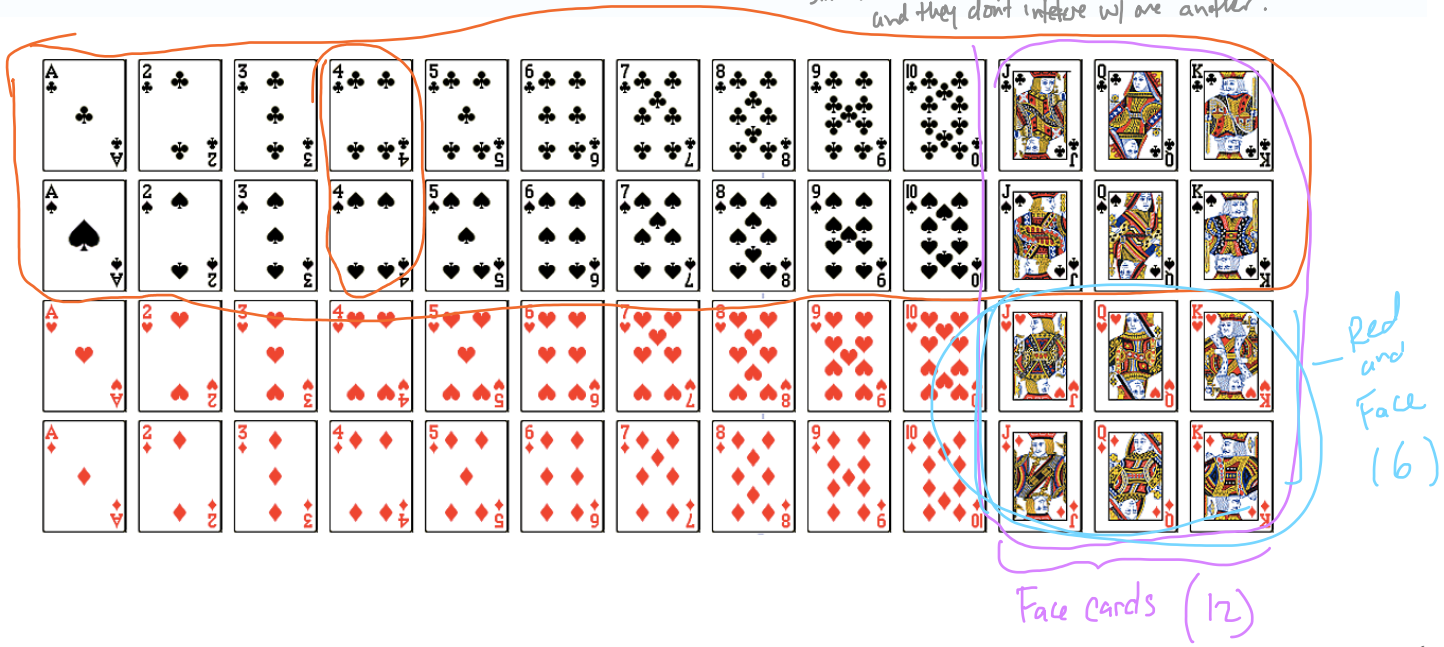
$P(\text{Heart} \mid \text{Black}) = P(\text{Black and Heart}) = P(\text{Black}) * P(\text{Heart}) = \frac{26}{52} * \frac{13}{51} = 0.127$

note since Hearts & Black cards are disjoint, this stays at 13 But the sample size does change & drops to 51 b/c one Black card is drawn.

(2 pt) (h) When two cards are drawn **without** replacement, find $P(\text{Two different colors})$?

$P(\text{One color and A different color}) = P(\text{One color}) * P(\text{a different color}) = \frac{26}{52} * \frac{26}{51} = 0.255$

Since there's only two colors Red or Black and they don't interfere w/ one another.



Problem 8: 12 pts

#S = 12

A fair, **12-sided die** is rolled. Let $\mathbf{A} = \{$ **odd** numbers $\}$, $\mathbf{B} = \{$ **even** numbers $\}$, and $\mathbf{C} = \{$ numbers **greater than** 7 $\}$.

(3 pt) (a) Using set notation, express the **sample space**.

$S = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ (C = greater than 7)

(3 pt) (b) Using set notation, express the event A and calculate $P(A)$.

event: $A = \{1,3,5,7,9,11\}$ $\quad P(A) = \frac{\#\text{ outcomes } A}{\#\text{ outcomes in } S} = \frac{6}{12} = 0.5$

$\#A = 6$ $\quad \#S = 12$

(3 pt) (c) Using set notation, express the event $\overline{C}$ and calculate $P(\overline{C})$.

$C = \{8,9,10,11,12\}$

$\overline{C} = \text{not } C = \{1,2,3,4,5,6,7\}$ (less than or equal to 7)

$P(\overline{C}) = \frac{\#\text{ in } \overline{C}}{\#\text{ in } S} = \frac{7}{12} = 0.583$

(3 pt) (d) Using set notation, express the event $A|\overline{C}$ and calculate $P(A|\overline{C})$.

event: $A|\overline{C} = A \text{ and } \overline{C}$

$A|\overline{C} = \{1,3,5,7\}$

$P(A|\overline{C}) = \frac{P(A \text{ and } \overline{C})}{P(\overline{C})} = \frac{\frac{4}{12}}{\frac{7}{12}} = \frac{4}{7} = 0.571$

Problem 9: 18 pts

Individual plays on a slot machine are independent. The probability of winning on any play is 0.18. (p)

X = # of wins in slot machine out of n trials

This is a binomial distribution

1. fixed trials $n = 30$ or 15
2. independent ✓
3. bi → win / lose ✓ $p = 0.18$ $P(\text{win})$, $q = 0.82$ $P(\text{lose})$
4. probability win constant ✓ $p = 0.18$

(3 pt) (a) What is the probability of winning three times in a row?

$n = 3$ binomial

$p = 0.18$

$x = 3$ (3 wins out of 3)

$P(X = 3) = \text{binompdf}(3, 0.18, 3) = 0.00583$

Note this is a very low probability the chance of winning 3 times is 0.583%, less than 1%!

For parts (b)-(e), suppose the slot machine is played 15 times in a row. ($n = 15$)

(3 pt) (b) What is the probability of winning **exactly** 10 times?

$x = 10$

$P(X = 10) = \text{binompdf}(n = 15, p = 0.18, x = 10) = 3.98 \times 10^{-5} = 0.0000398$

(3 pt (c) What is the probability of winning **at least** 10 times?

x 0 1 2 ⋯ 9 | 10 11 ⋯ 15

$P(X \geq 10) = 1 - P(X \leq 9) = 1 - \text{binomcdf}(n = 15, p = 0.18, x = 9) = 4.40 \times 10^{-5} = 0.0000440$

(3 pt (d) What is the probability of winning **less than** 8 times?

x 0 1 2 ⋯ 7 | 8 9 ⋯ 15

$P(X < 8) = P(X \leq 7) = \text{binomcdf}(n = 15, p = 0.18, x = 7) = 0.998$

(3 pt (e) Calculate the **mean** and **standard deviation**.

Since X is a binomial prob dist. we can use the simpler formulas: $\mu = np$ and $\sigma = \sqrt{npq}$

$n = 15$, $p = 0.18$, $q = 0.82$

$\mu = 15 \cdot 0.18$ $\quad \mu = 2.7$ wins in 15 games

$\sigma = \sqrt{15(0.18)(0.82)}$ $\quad \sigma = 1.49795\ldots$ $\quad \sigma = 1.5$ wins in 15 games

(3 pt (f) Are 6 wins out of 15 **significantly high**? Justify your answer with statistical reasoning and give your answer in a complete sentence $(M \to E)$.

2 ways to answer this:

Method 1 Range Rule of Thumb.

$x = 6$

sig high: $\mu + 2\sigma = 2.7 + 2(1.49) = 5.68$

Because $6 > 5.68$, yes 6 wins is sig high.

$(M \to E)$ "6 wins is significantly high because it is more than two standard deviations above the mean."

Method 2 Probability < 0.05

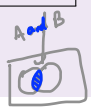
$P(X = 6) = \text{binompdf}(15, 0.18, 6) = 0.0285$

$P(X = 6) = 0.0285 = 2.85\%$

$(M \to E)$ "Since the probability of 6 wins out of 15 games is 2.85%, which is less than 5%, and 6 wins is greater than the mean = 2.7, we conclude that 6 wins is significantly high."

Formula Sheet

- $P(A \textbf{ or } B) = P(A) + P(B) - P(A \textbf{ and } B)$
- $P(A \textbf{ and } B) = P(A) * P(B)$
- $P(A \textbf{ and } B) = P(A) * P(B|A)$
- $P(B|A) = \dfrac{P(A \textbf{ and } B)}{P(A)}$
- $P(\bar{A}) = 1 - P(A)$



a lot of confusion!
short answer:
one selection vs 2 or more selections
use # in A and B / # in S
use *
vs

- $\mu = \sum[x \cdot P(x)]$
- $\sigma^2 = \sum(x - \mu)^2 \cdot P(x)]$
- $\sigma = \sqrt{\sum[x^2 \cdot P(x)] - \mu^2}$
- $E(X) = \sum[x \cdot P(x)]$

Post Exam Survey

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

1. When taking the exam I felt
   (a) Rushed. I wanted more time.
   (b) Relaxed. I had enough time.
   (c) Amazed. I had tons of extra time.
2. The week before the test I did all my homework on time: YES NO
3. The week before the test, in addition to the homework I followed a study plan. YES NO
   (a) I think this helped: YES NO
4. The day before the test I spend _____ hours studying and reviewing.
   (a) I think that was enough time: YES NO
5. The night before the test:
   (a) I stayed up very late cramming for the test
   (b) I stayed up very late, but I wasn't doing math
   (c) I didn't need to cram because I was prepared
   (d) I got a good night's sleep so my brain would function well.
6. I think I got the following grade on this test: _____
7. Strategies that worked well for me were (please elaborate):

8. Next time I will do an even better job preparing for the test by: