Section 10.1 and 10.2 Review

Objectives
- Review Systems of Equations
- Review the methods of solving Systems of Equations
- Review the number of solutions to Linear Systems of Equations

• Systems of Equations in 2 Variables

Defn 1

A **system of equations (SOE)** is a set of equations that involve the same variables. write as set of "points"
A **solution set** to a SOE is an assignment of values to each variable that makes each equation true; that is, a solution must solve every equation simultaneously.
A **linear SOE** means that the exponent of each variable is at most one.

Ex 1 SOEs Consider:

(a) $\begin{cases} 2x - y &= 5 \\ x + 5y &= 7 \end{cases}$ → $2x - 5 = y$ find y; substitute

$x + 5(2x - 5) = 7$ (one variable)

$x + 10x - 25 = 7$

$11x = 32 \Rightarrow x = \frac{32}{11}$ backsubstitute

(b) $\begin{cases} 2x - y &= 5 \\ x + y &= 4 \end{cases}$ Eliminate

$+$

$3x = 9$

$x = 3$ back sub

How to Solve We have three different ways to solve a SOE:
1. Substitution Method
2. Elimination Method
3. Graphing Method

Ex 2 SOEs Solve the previous examples using one of the methods.

Number of Solutions When we study **linear** SOEs in two variables, there are 3 possibilities for the solution set.
1. One
2. Infinitely Many
3. No solutions

• Systems of Equations in Several Variables

Defn 2

We usually write **several variables** using x as the base with indicies, such as x_1, x_2, x_3. So x_1 is the first variables and x_2 is the second variable, etc.
An equation in n variables requires us to also write coefficients. We use the letter a with indices to denote many coefficients. Therefore, we can write a **linear equation in n variables** as follows:

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = c$$

so $a_1, a_2, \ldots, a_n$ are the **coefficients** and the $x_1, x_2, \ldots, x_n$ are the **variables**.

Ex 3 SOEs If we have only 3 variables, we use x, y, z instead of x_1, x_2, x_3. If we have only 4 variables, we use x, y, z, w instead of x_1, x_2, x_3, x_4. linear

(a) $\begin{cases} x - 2y + 3z &= 1 \\ x + 2y - z &= 13 \\ 3x + 2y - 5z &= 3 \end{cases}$

(b) $\begin{cases} x + y + z + w &= 0 \\ x - y + w &= 5 \\ y - z - 2w &= 1 \\ 3x + y + 5z - w &= 4 \end{cases}$

Notice that some variables may be missing from some of the equations.

How to Solve It is much harder to solve a SOE in several variables than in simply two variables. However, we can use the following methods:

1. Back-substitution
2. Gaussian Elimination (studied in next chapter more fully)

Ex 4 SOEs

Solve the SOE using back-substituion: $\begin{cases} x - 2y - z &= 1 \\ y + 2z &= 5. \\ z &= 3 \end{cases}$

$x - 2(-1) - (3) = 1 \longrightarrow x = 2$

$y + 2(3) = 5 \rightarrow y = -1$

We say a SOE is in **triangular form** in this case.

$z = 3$ " back sub "

" one solution " $(x, y, z) = (2, -1, 3)$

Solution set

$\boxed{\{(2,-1,3)\}}$

What do you do if it is not in triangular form?

Rules for Solving SOEs When trying to solve a SOE you can use any of the following rules to arrive at an answer:

1. Combine equations: add a nonzero multiple of one equation to another equation to create a third equation.
2. Multiply an equation by a nonzero constant.
3. Interchange the position or order of any two equations.

Once you've found values of one variable, use back-substituion to solve for the remaining variables.

Ex 5 SOEs Solve the SOE:

$\begin{cases} ① & x + y + z &= 0 \\ ② & x - y + z &= 4 \\ ③ & 3x - z &= 6 \end{cases}$

①+②: $x + \cancel{y} + z = 0$

$+ \quad x - \cancel{y} + z = 4$

④ $2x + \cancel{2z} = 4$

③·2 : $6x - \cancel{2z} = 12$

$8x = 16$

$x = 16/8 = 2$

$\boxed{x = 2}$

back substitute

$2(2) + 2z = 4$

$4 + 2z = 4 \longrightarrow 2z = 0 \longrightarrow \boxed{z = 0}$

back substitute with also $x = 2$

$x + y + z = 0$

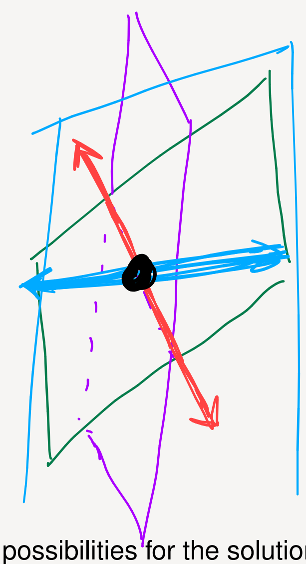
$2 + y + 0 = 0$

$\boxed{y = -2}$

Finally: $x = 2$
$y = -2$
$z = 0$

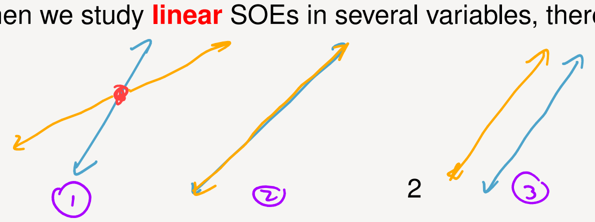
Write solution set:

$\boxed{\{(2, -2, 0)\}}$



Number of Solutions When we study **linear** SOEs in several variables, there are still only 3 possibilities for the solution set!

1. One
2. Infinitely Many
3. No solutions



3 variables linear eq are "planes"