Chapter 6: Discrete Probability Distributions

Section 6.2: Binomial Probability Distributions

BINOMIAL PROBABILITY DISTRIBUTION

Def A **binomial probability distribution** results from a procedure that meets the given requirements.

1. The procedure has a *fixed number of trials*.
2. The trials must be *independent*.
   (i.e. the outcome of an individual trial does not affect the probabilities in other trials.)
3. Each trial must have all outcomes classified into *two categories*. "Bi" two outcomes → "success", → "failure"
   (often referred to as success and failure)
4. The probability of a success remains *constant* in all trials.

Ex You roll a dice 10 times and want to determine if you rolled a 5 or not.
1) trials = 10
2) indep ✓
3) "bi" → S: roll a 5 → F: roll 1,2,3,4,6
4) P(5) = 1/6 constant.

NOTATION

Symbol	Represents
n	fixed number of trials
p	probability of success
q	probability of failure
x	specific number of success in n trials $x = 0, 1, 2, 3, \ldots, n$
$P(x)$	probability of getting exactly x successes among the n trials

$p + q = 1 \rightarrow \boxed{q = 1 - p}$

$\boxed{P(x) = \text{binompdf}(n, p, x)}$ exactly x successes

Ex: According to Wikipedia, 19% of Mexican residents are vegetarians. If we randomly survey 20 Mexican residents, what's the probability 3 of the Mexicans are vegetarians? Fill in the values given the information presented.

Two Possible Outcomes: vegetarian or not a vegetarian.

$n =$ 20 $p =$ 0.19 $q =$ 0.81 $x =$ 3

TWO METHODS TO FIND PROBABILITY OF A SPECIFIC VALUE FOR BINOMIAL DISTRIBUTIONS

1. FORMULA: $\boxed{P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}}$
2. Use Graphing Calculator (TI-83 or 84)

   Instructions: (a) 2nd ⇒ VARS ⇒ DISTR

   USE THIS → (b) $\boxed{\text{binompdf}(n, p, x)}$

Graphing Calculator for ${}_nC_x$
math ⇒ PROB ⇒ 3: ${}_nC_x$

C(n, x) = C(20, 3)

$\boxed{20}\ \boxed{nCr}\ \boxed{3}$

Ex: Use the problem from above (regarding vegetarianism in Mexico) to set up and evaluate using both methods.

$P(3) = {}_{20}C_3 (0.19)^{3} (0.81)^{20-3}$

$= (1140) \cdot (0.19)^3 \cdot (0.81)^{17}$

$= \boxed{0.217}$

$P(3 \text{ or } 3 \text{ or } \ldots \text{ or } 3) = 0.19 \cdot 0.19 \cdot 0.19 \cdot 0.81 \cdots 0.81$ + ⋯ +

0.19 0.19 0.19 0.81 0.81 ⋯ 0.81
V V V N N N ⋯ N V V or
N V V V N ⋯ N V N or
tons of possibilities or
N N ⋯ N V ⋯ N ⋯ V V ⋯ N V

${}_{20}C_3$ = all the ways 3 people selected to vegetarian out of 20 people.

Ex: We survey 5 GCC students and ask "Is this your first year here?" Assume that 20% of all GCC students are in their first year.

$n=5$ BINOMIAL Q $p = 0.20$ 20% → 0.20

(a) What is the probability that all 5 students are new?

$x = 5$

$P(x=5) = \text{binompdf}(5, 0.2, 5)$ (n, p, x)

$= 3.2E{-4} = 3.2 \times 10^{-4} = \boxed{0.00032}$

(b) What is the probability that 2 or 3 will be new?

$P(x=2 \text{ or } x=3) = P(x=2) + P(x=3)$

$= \text{binompdf}(5, 0.2, 2) + \text{binompdf}(5, 0.2, 3)$

$= \boxed{0.256}$

(c) What is the probability that at least one will be new?

$x = 1, 2, 3, 4, 5$ 0 1 2 3 4 5

$P(1 \le x \le 5) = P(\text{at least one}) = 1 - P(\text{none})$

$= 1 - P(x=0)$

$= 1 - \text{binompdf}(5, 0.2, 0)$

$= \boxed{0.672}$

(d) Create a probability distribution table for this exercise.

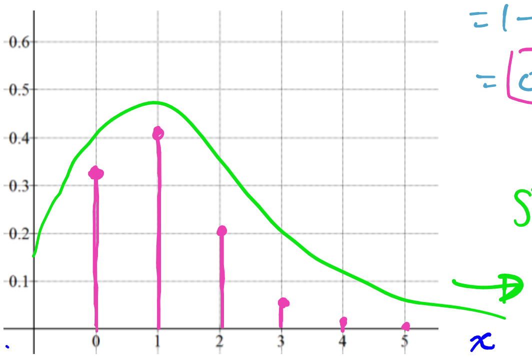
x	$P(x)$
0	0.328
1	0.410
2	0.205
3	0.0512
4	0.0064
5	0.00032

binompdf(5, 0.2, 0)

(n=5)

x = 0, 1, 2, 3, 4, 5

(e)

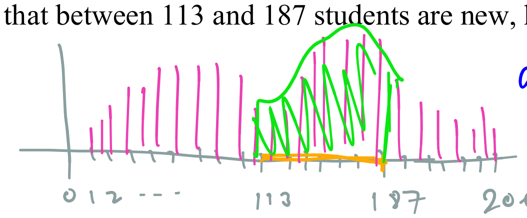


BAR GRAPH

*If we surveyed 200 students, and I ask the probability that between 113 and 187 students are new, how would you find your answer?

$n = 200$

$x = 113, 114, \ldots, 187$



add up each probability for x = 113, 114, etc until 187

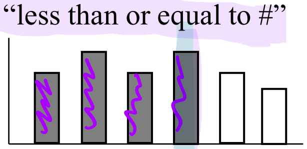
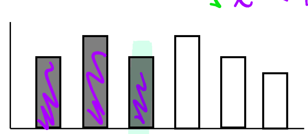
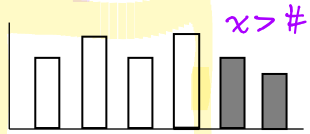
Finding a Cumulative Probability

Use Graphing Calculator (TI-83 or 84)

Instructions: (a) 2nd ⇒ VARS ⇒ DISTR   KEY

(b) $P(x \le \#) = binomcdf(n, p, \#)$ c for cumulative

Shortcut!

"no more than" or "at most #" or "less than or equal to #"	"fewer than #" or "less than #"	*Calculator
$x \le \#$  0 1 … #	$x < \#$  0 1 … #	$P(x \le \#) = \text{binomcdf}(n, p, \#)$ $P(x < \#) = \text{binomcdf}(n, p, \# - 1)$
"at least" or "no less than" or "greater than or equal to"	"more than" or "greater than"	*Calculator
$x \ge \#$  1 − tick # … x	$x > \#$  # - - -	$P(x \ge \#) = 1 - \text{binomcdf}(n, p, \# - 1)$ $P(x > \#) = 1 - \text{binomcdf}(n, p, \#)$

binomcdf(n, p, #−1)

Bi? Yes, b/c make FT / miss FT

$\nearrow p = 0.75$ $\nearrow n = 10$

Ex: A basketball player makes 75% of the free throws he tries. If the player attempts 10 free throws in a game, find the probability that:

(a) the player will make at most six free throws.

$\boxed{0\ 1\ 2\ 3\ 4\ 5\ 6}\ 7\ 8\ 9\ 10$

$P(x \le 6) = \text{binomcdf}(\overset{n}{10}, \overset{p}{0.75}, \overset{x}{6})$

$= \boxed{0.224}$

(b) the player will make at least eight free throws.

$\boxed{0\ 1\ 2\ 3\ 4\ 5\ 6\ 7}\boxed{8\ 9\ 10}$ 1 MINUS TRICK

$P(x \ge 8) = 1 - P(x \le 7)$

$= 1 - \text{binomcdf}(10, 0.75, 7) = \boxed{0.526}$

Ex: According to 2017 Washington Post article, approximately 53% of all U.S. households are wireless-only households (no landline). In a random sample of 20 households, what is the probability that… $p = 0.53$ $n = 20$ Bi? S / F

(a) fewer than 6 are wireless only?

$\boxed{0\ 1\ 2\ 3\ 4\ 5}\ 6\ 7 \cdots 20$

$P(x<6) = \text{binomcdf}(\overset{n}{20}, \overset{p}{0.53}, \overset{x}{5})$

$= 3.81\ E-6$ Notation

$= \boxed{0.00000381}$ "0+"

(c) more than 13 are wireless only?

$\boxed{0\ 1\ 2 \cdots 12\ 13}\boxed{14 \cdots 20}$ 1 MINUS TRICK

$P(x > 13) = 1 - \text{binomcdf}(20, 0.53, 13)$

$= \boxed{0.0958}$

Using Mean and Standard Deviation for Critical Thinking

FOR BINOMIAL

Mean Value

Formula: $\boxed{\mu = np}$

Variance

Formula: $\boxed{\sigma^2 = npq}$

Standard Deviation

Formula: $\boxed{\sigma = \sqrt{npq}}$

Rounding Rule: Use Stats Law of Rounding based on the values x

Ex: According to the U.S. Office of Adolescent Health, nearly 90% of adult smokers in America started smoking before turning 18 years old. $\nearrow p = 0.9$

(a) If 300 adult smokers are randomly selected, how many would we expect to have started smoking before turning 18 years old?

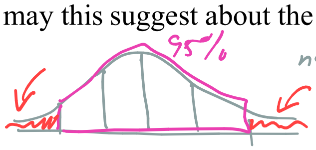
n x ↳ Expectation $E(x)$ = mean

$\mu = np = (300)(0.9) = 270$

$(\mu \to E)$ "We would expect 270 adult smokers out of 300 to have started smoking before 18 y.o."

(b) Would it be unusual (significantly high/low) to observe 240 smokers who started smoking before turning 18 years old in a random sample of 300 adult smokers? What may this suggest about the population that was observed?

unusual → 5% of data → Prob ≤ 0.05



normal curve

Empirical

$\mu-2\sigma$ $\mu-\sigma$ μ $\mu+\sigma$ $\mu+2\sigma$

Range Rule of Thumb

$x \le \mu - 2\sigma$

or $x \ge \mu + 2\sigma$

• $\sigma = \sqrt{np\overset{1-p}{q}}$

$= \sqrt{300(0.9)(1-0.9)} = 5.2$

• $\mu - 2\sigma = 270 - 2(5.2) = 259.6$

• compare: $240 < 259.6$ ✓

"yes, it is unusual to observe 240 adults started smoking under 18."