

Exam 3

Study Guide

§7.5–7.8, 11.1–11.5



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General Exam Info

Exams are a way for you to show me what you have learned (and please show all your steps so I can see this!) and to give you a sense of accomplishment! They are meant to be challenging and not just homework problems with the numbers changed. I really want to prepare you for university level math classes—so some exams may be longer or more challenging than others. Remember that I do grade fairly and my goal is to push you to succeed and excel in this class.

- Attendance required for all exams—I do NOT drop the lowest exam score.
- **“Make-up Exams”** are given only in extreme cases and at instructor’s discretion; a student is allowed at most one make-up exam. (Documentation must be provided for the instructor to even consider a make up exam. This means you would need a doctor’s note, etc.) A “Make-Up Exam” means you will be allowed to replace the missing score with the percentage you earn on the final exam. Please contact your instructor as soon as possible should there be a problem.
- **Your student ID is required for all exams.**
- During the exams—you will be required to leave your backpack and all non-test items at the front of the room, including cell phones and smart watches. Only your pencil/eraser and calculator will be allowed during the exam, and there will be a calculator check. Should you need to leave during the exam please ask for permission first before leaving and leave your cell phone with me. Not doing these things could result in a 0 on your exam.
- Once the exam is graded and returned, any problem you would like me to revisit must be brought to my attention by the next class session.
- Always keep your exams!

Exam 1 Date & Time

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|----------|-----------------|-------------------|
| • Exam 3 | Thursday, May 9 | 4:00 pm – 5:20 pm |
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ALSO, after the exam:

- As the test is only 80 minutes long, we will have a 10 minutes break, then continue with new material.
- I take attendance at the end of class on test days.

Exam 3 Specific Info

- Material covered: Anything from Exam 1 and 2 PLUS §7.5*, 7.7, 7.8, 11.1, 11.2, 11.3, 11.4, 11.5
- Section 7.5 will not be directly tested, but you need to know the material in this section.
- You will need a calculator (only scientific allowed)—you won’t be able to use your phone.
- Almost all questions have multiple parts
- Be prepared for the following types of questions: True-False-Sometimes, Multiple-Choice, and Free-response
- Be prepared to prove certain results (see below for more details)
- No cheat sheet is allowed on Exam 3, but I will provide a formula sheet (see below).
- Be prepared for questions with multiple parts that build on each other

Section 7.5

- Know the tools in the **integration toolbox**.
- Study the strategies for integration given in the section.
- This section will not be directly tested but I consider the material fundamental to integration. Basically practice, practice, practice!

Section 7.7

- Review the notation for definite integrals and **Riemann sums**.
- Be able to compute the **left-endpoint approximation (LEA)** given a graph.
 - When f increasing, the LEA is an underestimate for the integral.
 - When f decreasing, the LEA is an overestimate for the integral.
- Be able to compute the **right-endpoint approximation (REA)** given a graph.
 - When f increasing, the REA is an overestimate for the integral.
 - When f decreasing, the REA is an underestimate for the integral.
- Be able to compute the **midpoint-endpoint approximation (MPA)** given a graph.
- Be able to compute the **trapezoid approximation (TrapA)** given a graph.
- Know what **Simpson's Rule** is and the formula used to compute it.
- Memorize the formulas for LEA, REA, MPA, TrapA, and SimpA (good T/F and multiple choice questions)
- Know what **error** is: Error = Exact - Approximation
- Know the **error bounds** for MPA, TrapA, and SimpA. Be able to determine the number of n of approximations needed to guarantee an error to be within a given value.

Section 7.8

- Know what **improper integrals (type I)** are and how to compute them:
 - $\int_a^\infty f(x) dx$, $\int_{-\infty}^b f(x) dx$, or $\int_{-\infty}^\infty f(x) dx$
- Know what it means for an improper integral to **converge** or **diverge**.
- Memorize the **p -test** for Type I improper integrals. When the p -test applies, you can cite it and use it!
- You might be asked to prove a part of the p -test Type I.
- Know what **improper integrals (type II)** are and how to compute them:
 - $\int_a^c f(x) dx$, $\int_c^b f(x) dx$, or $\int_a^b f(x) dx$ with f VA at $x = c$
- Memorize the **p -test** for Type II improper integrals. When the p -test applies, you can cite it and use it!
- You might be asked to prove a part of the p -test Type II.
- Know the **comparison test** to determine if an improper integral C or D. The p -test is super helpful here. Also, know the basic inequalities I mentioned in class.

Chapter 7 Practice Problems

Page 564: # 1, 2, 19, 21, 29a,b

Page 574-575: # 1, 2, 9, 10, 13, 19, 27, 49–54 all, 57–59 all;

Page 577: True-False Quiz: # 5,6, 8–11 all;

Page 578–579: Exercises: # 41, 43, 45, 46, 48, 71, 77;

Section 11.1

- Know what a **sequence** is and what the n^{th} **term**.
- Know how to find the pattern of a sequence and give the n th term.
- Know how to compute **limits** of sequences.
- Know what it means for a sequence to **converge** or **diverge**.
- Know the different ways a sequence can diverge: by escaping to $+\infty$, $-\infty$, or by “oscillation”
- Know Theorem 2: Powers of r . This is used in the proof of p -test. Helpful for computing limits too.
- Know what **recurrence relations** and how to compute limits of sequences defined by recurrence relations.
- Know the terms: **increasing, decreasing, and monotonic** sequences.
- Be able to prove sequences are increasing or decreasing using both inequalities and using the ID test.
- Know the statement of Theorem 3: convergence implies bounded.
- Know the statement of Theorem 4: Monotone Convergence Theorem: monotone + bounded implies converges. Mainly, recognize when this is used for series proofs.

Section 11.2

- Know the definition of a **series** and what **partial sums** are. Know the notation given in class, not the book.
- Know what it means for a series to **converge** or **diverge**.
- Be able to find the sum of series if a sequence of partial sums is given.
- Know **decimal notation** and its relationship to rational numbers.
- Be able to convert rational numbers from decimal notation to fractions (using the **tens trick**) or from fractions to decimals (using a calculator using **long division** without a calculator)
- Memorize the entire statement of the **geometric series** (Def 2 and Theorem 2). Know when they converge and diverge.
- Know the proof of the geometric series for the case $|r| < 1$ (i.e. converges).
- Be able to determine whether series converge or diverge based on geometric series using Theorem 3 (basic properties of series).
- Know the definition of a **harmonic series** and that it diverges (Theorem 2).
- Know both parts of the **n th term test** (Theorem 4). Especially the test for divergence (part b).
- Be able to apply the test for divergence to series.

Section 11.3

- Know the statement of the **integral test**. Be sure to pay attention to the conditions that are required to use the test.
- Be able to show that the conditions of the integral test are satisfied when solving a problem using this test. And also be able to compute improper integrals correctly.
- Know what **p -series** are and Theorem 2 that tells you when they converge/diverge. Basically, by using integral test and p -test.
- Be able to determine when p -series C or D.
- Know that a **remainder** is the same thing as an error.
- Know Theorem 3: Remainder Estimate using the Integral Test. Know the conditions needed to apply it and how to apply it to estimate the number of terms needed for a partial sum/estimate to be accurate to a certain tolerance.
- Theorem 4 can't be tested since it's too hard to do calculations by hand.

Section 11.4

- Know the statement of the **comparison test**. Be sure to pay attention to the conditions required to apply the test.
- Be able to solve problems that ask to determine whether a series C or D using the comparison test.
- Know the statement of the **limit test**. Be sure to pay attention to the conditions required to apply the test.
- This a great test! A student favorite. It's basically the “**squint test**” made rigorous and easy to apply.
- Practice many examples using the limit test.

Section 11.5

- Know what **alternating series** are.
- Know the statement of the **alternating series test**. Be sure to pay attention to the conditions required to apply the test.
- Be able to check/verify the conditions are met when using the alternating series test.
- Be able to solve problems that show an alternating series converges using the alt series test.
- Know Theorem 2: Estimating Error of Alt Series. This is a great theorem since it's easy to remember!
- Be able to estimate the error/remainder term of alternating series using Theorem 2.

Chapter 11 Practice Problems

Page 824-825: True-False Quiz: # 1, 2, 3, 9, 11, 16, 17, 19, 20, 21

Page 825: Review Exercises: 1, 2, 3, 4, 6, 7, 9, 11, 12, 13, 14, 15, 16, 17, 21, 22, 27, 28*, 32, 34, 35**, 36**, 38*

*Note: 38 is optional

**Note: special instructions for 35 and 36: Ignore the estimating the partial sums part and only find the n so that the partial sum is accurate to (what's given in the problem).

Formula Sheet

- To be added later...