σ standard deviation
σ² variance

Chapter 9: Estimating the Value of a Parameter

Section 9.3: Estimating a Population Standard Deviation & Variance

Intro

We want to estimate the true population variance, σ^2, or the standard deviation, σ.

Why do we care? Coffee cup example that fills cups correctly on average but sometimes under fills and sometimes overfills and spills coffee! We want to avoid overspills, of course. We seek *consistency* as well as a good estimate on the mean, which is one reason why we would want to estimate the variance or standard deviation.

The distribution of sample proportions, $\hat{p}$, is normally distributed (Section 9.1).
The distributions of the sample means, $\bar{x}$, are distributed according to the t-distribution (Section 9.2).

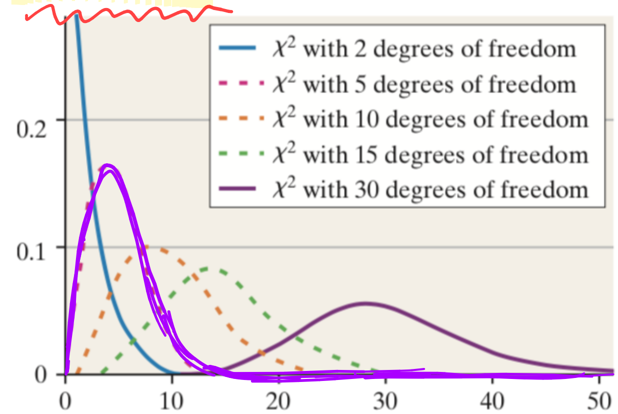
Question: How are the sample variance, s^2, or the sample standard deviation, s, distributed?

Chi Squared Distribution (Video: https://youtu.be/hcDb12fsbBU)

χ^2- Distribution Rules

χ "Greek"
"Chi" pronounced "Kai"

- The distribution is not symmetric, it is skewed RIGHT



- Z – RV normal
- Z^2 – square of Z

- Rounding Rule for χ^2: whatever the table gives
- Shape depends on the degree of freedom, *df*
- As sample size n increases, the shape becomes more nearly symmetric/normal.
- The values of χ^2 are non-negative.
- The mean is centered at the degrees of freedom, $\mu_{\chi^2} = df$.
- The variance is 2 times df, that is, $\left(\sigma_{\chi^2}\right)^2 = 2 \cdot df$. Hence, $\sigma_{\chi^2} = \sqrt{2 \cdot df}$.
- The simplest Chi-Squared Dist is: $\chi^2 = Z^2$ i.e the square of the normal distribution.
- Also used for important "goodness-of-fit test" which we'll learn in future chapters.

The Ti calculators do have "$\chi^2 cdf()$" programmed, but it doesn't have "$inv\chi^2()$". So you need to use a table to get the inverses for the critical values.

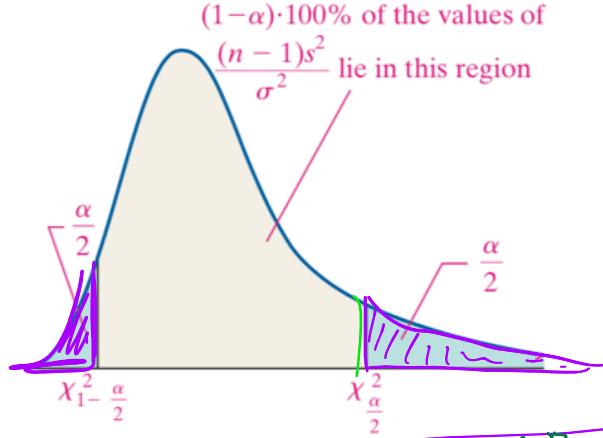
Worse, since the χ^2 –distributions is NOT symmetric, we need to find two separate critical values!

$Z_{\alpha/2} \to$ prop
$t_{\alpha/2} \to$ mean

NOTE: CI ~~not~~ of the form $\sigma^2 \pm E$ anymore! $\$$

$\chi^2_{\alpha/2} \to$ variance & st. dev

CONFIDENCE INTERVAL FOR THE POPULATION VARIANCE



Confidence Interval: $\boxed{\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}$

Point Estimate: $s^2 =$ Given or use 1-VAR STATS

Critical Values: Lower = $\chi^2_{1-\alpha/2}$ Upper = $\chi^2_{\alpha/2}$

Distribution used for critical values: χ^2-dist!

Requirements

1. The sample is a simple random sample (SRS)
2. The distribution of X is normal with mean \mu and standard deviation σ

$\chi^2_{\alpha/2}$ = RIGHT Critical Val UPPER
$\chi^2_{1-\alpha/2}$ = LEFT Crit Val LOWER

CONFIDENCE INTERVAL FOR THE POPULATION STANDARD DEVIATION

Confidence Interval: $\boxed{\sqrt{Lower} < \sigma < \sqrt{Upper}}$ (i.e. we take square roots of the CI for the variance)

Point Estimate: $s =$ Given or use 1-VAR STATS

Critical Values: Lower = $\chi^2_{1-\alpha/2}$ Upper = $\chi^2_{\alpha/2}$

Distribution used for critical values: χ^2-dist!

Requirements

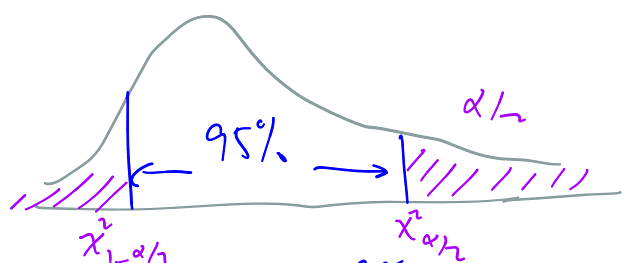
1. The sample is a simple random sample (SRS)
2. The distribution of X is normal with mean \mu and standard deviation σ

Ex1: Find the critical values that separate the middle 95% of a χ^2-distribution from each tail of percentage 2.5% with 18 degrees of freedom.

To do this, you need a table: https://people.richland.edu/james/lecture/m170/tbl-chi.html or use Table VIII in the back of our book.

df = 18

Upper RIGHT $\chi^2_{\alpha/2} = \chi^2_{0.025} = \boxed{31.526}$



CL = 0.95
$\alpha = 0.05$
$\alpha/2 = \boxed{0.025}$
$1-\frac{\alpha}{2} = 0.975$

Lower LEFT $\chi^2_{1-\frac{\alpha}{2}} = \chi^2_{0.975} = \boxed{8.231}$

Ex2: One way to measure the **risk** of a stock is through the **standard deviation** rate of return of the stock. The following data represent the weekly rate of return (in percent) of Microsoft for 15 randomly selected weeks. Compute the **90% confidence interval** for the risk of Microsoft stock[1].

σ

5.34	9.63	−2.38	3.54	−8.76	2.12	−1.95	0.27
0.15	5.84	−3.90	−3.80	2.85	−1.61	−3.31	

1 VAR STATS

$s = 4.697$

Check requirements

Identify point estimate

s = sample standard deviation

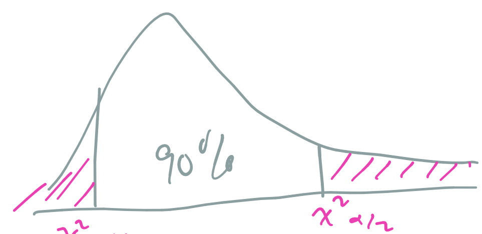
$\boxed{s = 4.697}$ weakly rate of return Microsoft per week

Determine critical values

UPPER crit Val RIGHT $\chi^2_{\alpha/2} = \chi^2_{0.05} = \boxed{23.685}$

$df = 15-1 = 14$

LOWER crit Val LEFT $\chi^2_{1-\alpha/2} = \chi^2_{0.95} = \boxed{6.571}$



$CL = 0.9$

$\alpha = 0.1$

$\alpha/2 = 0.05$

$1-\alpha/2 = 0.95$

Construct confidence interval

$$\sqrt{\frac{(n-1)s}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s}{\chi^2_{1-\alpha/2}}}$$

$n-1 = 14$

$s = 4.697$

$\chi^2_{\alpha/2} = 23.685$

$\chi^2_{1-\alpha/2} = 6.571$

$$\sqrt{\frac{(14)(4.697)}{23.685}} < \sigma < \sqrt{\frac{(14)(4.697)}{6.571}}$$

$$1.666 < \sigma < 3.163$$

$\boxed{CI: (1.666, 3.163)}$

Interpretation of CI

"We are 90% that the true standard deviation (risk) the weakly rate of return for Microsoft is between 1.666 and 3.163 (in percent)."

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[1] Source: Yahoo! Finance