Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor given any unauthorized assistance on this examination:

NAME (PRINT): __Solutions_____   SIGNATURE: _____

Directions

- YOU ARE ALLOWED TO USE ONLY A SCIENTIFIC CALCULATOR ON THIS EXAM.

- You have 85 minutes to complete this exam.

- The exam totals **100 points** with 5 points of extra-credit possible.

- There are 7 problems (plus one extra-credit problem at the end), many of them with multiple parts.

- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you your writing utensils.

- Cell phones must be turned off and put away in with your items in the front of the classroom.

- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.

- Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E). For True/False questions, you must spell out the entire word "true" or "false" in your answer.

- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*

- A problem which requires a **proof** means you must provide a general proof in complete sentences. Do not use logical short-hand in proofs.

- Leave answers in exact form (as simplified as possible), unless told otherwise.

- Put a | box around your final answer | where applicable.

- **PLEASE CHECK YOUR WORK!!!**
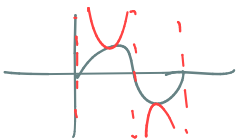
Score	Grade

This page is intentionally blank. It may be used for scratch paper. If you wish for me to grade your work on this page, please (i) label the problem you are working on, (ii) box your answer, (iii) indicate in the original problem's location that you will continue your work on this page.
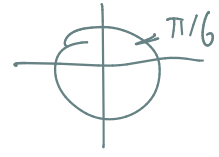
Problem 1: 20 pts (2 pts each)

Fill-in the blank: *(No work needed)*

(handwritten: ← need this to be defined)

(a) The **inverse property** $\sin\left(\sin^{-1}(x)\right) = x$ is true for $x \in$ ___$[-1, 1]$___

(handwritten: $\cot(x) = \frac{\cos(x)}{\sin(x)}$ need $\sin(x) \neq 0$)

(b) The **domain** of $\cot(x)$ is ___all $x \neq \pi n$ for $n \in \mathbb{Z}$___

(c) The **asymptotes** of $y = \csc(x)$ on the principal branch occur at ___$x = 0, x = \pi, x = 2\pi$___

(d) The **reciprocal identity** for the secant function is ___$\sec(x) = \frac{1}{\cos(x)}$___

(e) The **terminal point** corresponding to the $t \in \mathbb{R}$ when $t = -\frac{23}{6}\pi$ is ___$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$___

(handwritten: $-\frac{23\pi}{6} = \frac{-24\pi + \pi}{6} = -4\pi + \pi/6$)

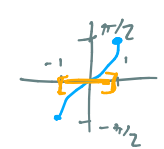*(handwritten: $\pi/6$ with circle diagram)*

TRUE or FALSE (please spell out/write the entire word for credit). *(No work needed)*

(a) ___FALSE___ The function $\sin(x)$ is **periodic** with period 2π means $\sin(x) + 2\pi \neq \sin(x)$.

(b) ___TRUE___ The function $\tan(x)$ is **odd** since $\tan(-x) = -\tan(x)$.

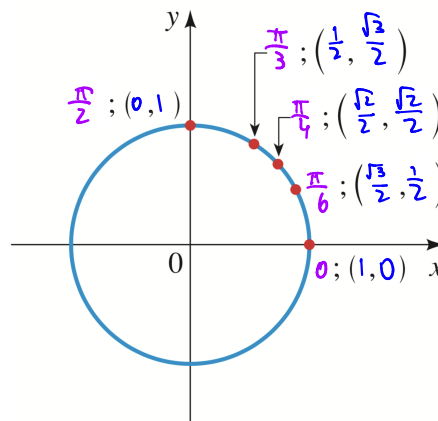*(handwritten: $\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$)*

(c) ___TRUE___ The identity $\tan^2(x) - \cot^2(x) = \sec^2(x) - \csc^2(x)$ is true.

(handwritten: $+1$ -1)

(d) ___TRUE___ The graph of $y = -5\sin\left(3x + \frac{3}{2}\pi\right)$ has **amplitude** 5, **period** $\frac{2}{3}\pi$, and is the graph of $y = \sin(x)$ **shifted horizontally** to the left by $\frac{\pi}{2}$.

(handwritten: $3\left(x + \frac{\pi}{2}\right)$ ← HS)

(e) ___FALSE___ The domain of the **arcsine** function, $\sin^{-1}(x)$, is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(handwritten: ← range $[-1,1]$ domain   with graph)

Problem 2: 15 points - 1 pt each blank

Fill out **Quadrant I** of the unit circle. Include the terminal points (x, y) corresponding to the real numbers $t \in [0, \pi/2]$ discussed in class. Please put the t to the left of the semicolon (;).

(handwritten unit circle diagram with points:)
- $\frac{\pi}{3} ; \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
- $\frac{\pi}{2} ; (0, 1)$
- $\frac{\pi}{4} ; \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
- $\frac{\pi}{6} ; \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
- $0 ; (1, 0)$

Do not need to provide the corresponding angle in degrees, only in radians (i.e. a real number).

Problem 3: 6 pts

Which of the statements below are true **identities**? *(Select all that apply; please box entire answer (letter and statement)) (Show some work for full credit)*

~~(A)~~ $\csc^2(x) + \sec^2(x) = 1.$ → undefined @ $x=0$ so not identity!

(B) $(\sin(a) + \cos(a))^2 = 1 + 2\sin(a)\cos(a).$ → $(\sin(a) + \cos(a))^2 = \sin^2(a) + 2 \cdot \sin(a)\cos(a) + \cos^2(a)$

$\underbrace{\sin^2(a) + \cos^2(a)}_{=1} = 1 + 2\sin(a)\cos(a)$ ✓

(C) $\dfrac{1}{1 - \sin^2(t)} = \tan^2(t) + 1.$ → LHS = $\dfrac{1}{1-\sin^2(t)} = \dfrac{1}{\cos^2(t)} = \sec^2(t) = 1 + \tan^2(t) = $ RHS ✓

(D) $\cos\left(\dfrac{\pi}{2} - x\right) = \sin(x).$ → cofunction formula ✓

~~(E)~~ $\dfrac{1 - \sin^2(u)}{\sin^2(u) - 1} \cdot \dfrac{1 - \csc^2(u)}{\cot^2(u)} = -1.$

↳ E) LHS = $\dfrac{\cos^2(u)}{(-)\cos^2(u)} \cdot \dfrac{(-)\cot^2(u)}{\cot^2(u)} = \boxed{+1}$ false!

- $\sin^2(u) + \cos^2(u) = 1$
- $\tan^2(u) + 1 = \sec^2(u)$
- $1 + \cot^2(u) = \csc^2(u)$

A) can also plug-in value: try $\pi/4$:

$\csc^2\left(\dfrac{\pi}{4}\right) + \sec^2(\pi/4)$

$= \dfrac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} + \dfrac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} =$

$= \dfrac{1}{\frac{2}{4}} + \dfrac{1}{\frac{2}{4}} = \dfrac{1}{\frac{2}{2}} + \dfrac{1}{\frac{2}{2}} = 2 + 2 = \boxed{4}$ $\neq 1$

Problem 4: 24 pts

Evaluate the following trigonometric functions. Given an exact value.

(a) $\cot\left(\dfrac{5\pi}{6}\right) = \dfrac{\cos\left(\frac{5\pi}{6}\right)}{\sin\left(\frac{5\pi}{6}\right)} = \dfrac{-\sqrt{3}/2}{1/2} = \boxed{-\sqrt{3}}$

$5\pi/6$, $\pi/6 \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(b) $\tan^{-1}(-1) = \boxed{-\dfrac{\pi}{4}}$

$y = \tan^{-1}(-1) \longleftrightarrow \tan(y) = -1$

$\dfrac{\sin(y)}{\cos(y)} = -1$

$\dfrac{-\sqrt{2}/2}{\sqrt{2}/2} = -1$ ✓

$3\pi/4$

$-\pi/4 \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

Recall domain tan⁻¹ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ so use $-\pi/4$ NOT $\frac{3\pi}{4}$.

(c) $\csc\left(\tan^{-1}\left(\dfrac{3}{4}\right)\right) = \csc(y) = \boxed{\dfrac{5}{3}}$

set $= y$

$y = \tan^{-1}\left(\frac{3}{4}\right) \longleftrightarrow \tan(y) = \dfrac{3}{4}$

$x, \, 3, \, y, \, 4$ → $3^2 + 4^2 = x^2$

$9 + 16 = x^2$

$25 = x^2 \rightarrow x = 5$

(d) $\sin\left(-\dfrac{11\pi}{3}\right) = \sin\left(\dfrac{\pi}{3}\right) = \boxed{\dfrac{\sqrt{3}}{2}}$

$-\dfrac{11\pi}{3} = \dfrac{-12\pi + \pi}{3} = -4\pi + \boxed{\dfrac{\pi}{3}}$ ref #

(e) $\sec(33\pi) = \dfrac{1}{\cos(33\pi)} = \dfrac{1}{\cos(\pi)} = \dfrac{1}{-1} = \boxed{-1}$

$33\pi = 32\pi + \pi = \boxed{\pi} + 16(2\pi)$

ref #

(f) $\sin^{-1}\left(\sin\left(\dfrac{13\pi}{6}\right)\right) = \sin^{-1}\left(\dfrac{1}{2}\right) = \boxed{\dfrac{\pi}{6}}$
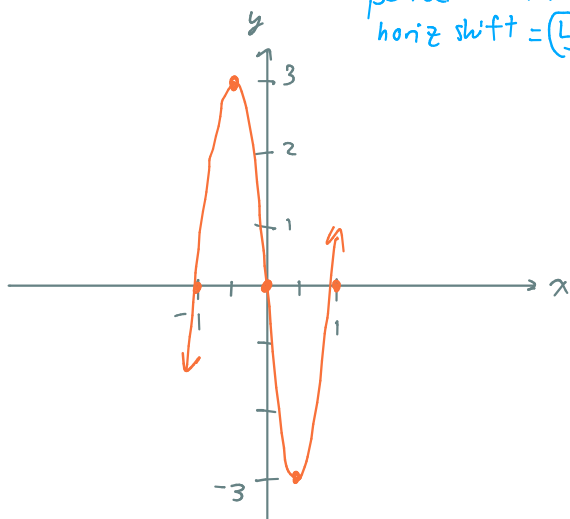
$\dfrac{13\pi}{6}, \pi/6$

$\dfrac{13\pi}{6} = \dfrac{12\pi + \pi}{6} = 2\pi + \pi/6$

Problem 5: 24 pts

Graph the following trigonometric functions. Include at least one full period. Be sure to label and scale your axes appropriately.
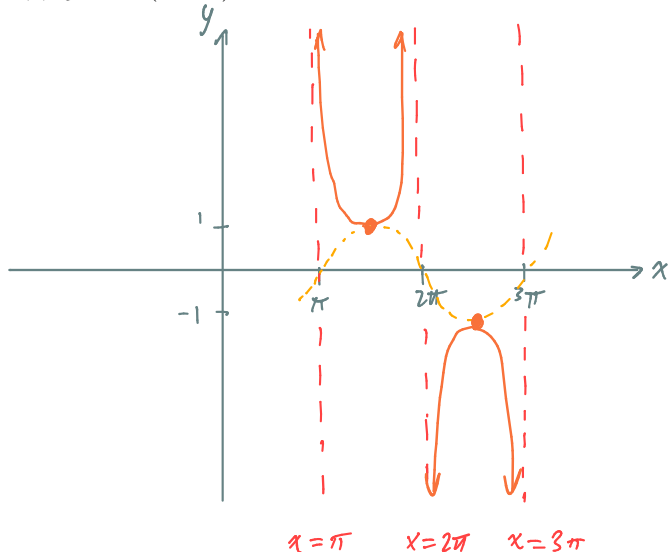
(a) $y = 3\sin(\pi(x+1))$
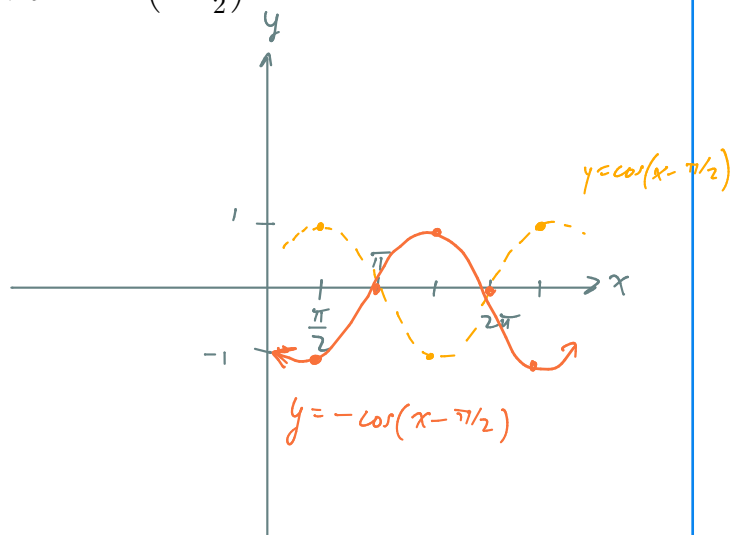
amp $= 3$
period $= 2\pi/\pi = 2$
horiz shift $= \boxed{L}\ 1$

(c) $y = -\cos\left(x - \dfrac{\pi}{2}\right)$

horiz shift $= \boxed{R}\ \pi/2$
vertical reflection

$y = \cos(x - \pi/2)$

$y = -\cos(x - \pi/2)$

(b) $y = \csc(x - \pi)$

horiz shift : $\boxed{R}\ \pi$

$x = \pi \qquad x = 2\pi \qquad x = 3\pi$

(d) $y = \tan\left(\dfrac{1}{2}x\right)$

period $= \dfrac{\pi}{\frac{1}{2}} = 2\pi$

$x = -\pi \qquad x = \pi$
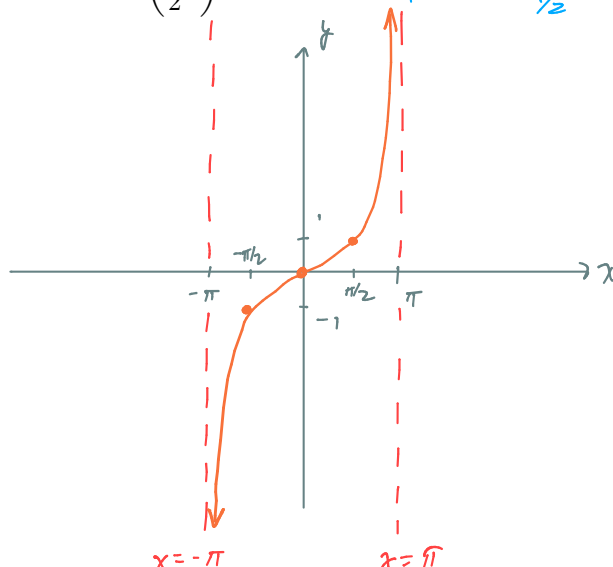
Problem 6: 6 pts

Prove the following identity: $\dfrac{\cos(\theta)}{1-\sin(\theta)} = \dfrac{\sin(\theta)-\csc(\theta)}{\cos(\theta)-\cot(\theta)}$.

Show all steps. Prove it as discussed in class. That is, start with one side and show, after a series of valid steps, that you get the other side.

Proof RHS $= \dfrac{\sin(\theta)-\csc(\theta)}{\cos(\theta)-\cot(\theta)}$

$= \dfrac{\left(\sin(\theta) - \dfrac{1}{\sin(\theta)}\right)}{\left(\cos(\theta) - \dfrac{\cos(\theta)}{\sin(\theta)}\right)} \cdot \dfrac{\sin(\theta)}{\sin(\theta)}$

$\sin^2\theta + \cos^2\theta = 1$
$\sin^2\theta - 1 = -\cos^2\theta$

$= \dfrac{\sin^2(\theta) - 1}{\sin(\theta)\cos(\theta) - \cos(\theta)}$

$= \dfrac{-\cos^2(\theta)}{\cos(\theta)\left(\sin(\theta) - 1\right)}$

$= \dfrac{-\cos(\theta)}{\sin(\theta) - 1}$

$= \dfrac{\cos(\theta)}{(1 - \sin(\theta))}$
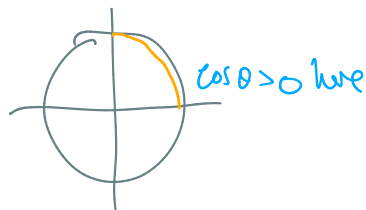
$= LHS. \ \square$

"end of proof box"

Problem 7: 5 pts

Make the **trigonometric substitution** $x = 3\sin(\theta)$ into the expression $\sqrt{9 - x^2}$ and simplify completely. Assume that $0 < \theta < \pi/2$.

$\sqrt{9 - x^2} = \sqrt{9 - \left(3\sin(\theta)\right)^2} = \sqrt{9 - 9\sin^2(\theta)} = \sqrt{9\left(1 - \sin^2\theta\right)} =$

$= \sqrt{9}\,\sqrt{1 - \sin^2\theta} = 3\sqrt{\cos^2\theta} = \boxed{3\cos\theta}$

$\cos\theta > 0$ here
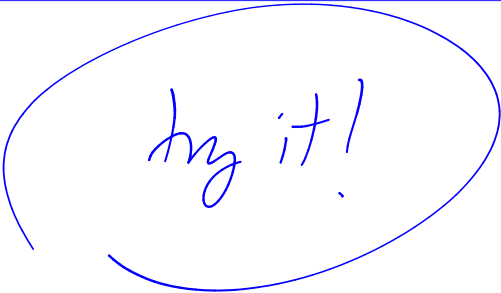
$\sin^2\theta + \cos^2\theta = 1$
$\cos^2\theta = 1 - \sin^2\theta$

Problem 8: Extra-credit (5 points)

You may attempt these only if every questions on the exam has an attempted solution. Otherwise they will not be graded.

(a) (2 pts) Sketch the function $y = x^2 \cdot \cos(x)$.

(b) (3 pts) Prove: $-x^2 \leq x^2 \cdot \cos(x) \leq x^2$ for all $x \in \mathbb{R}$.

try it!

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

1. When taking the exam I felt
    (a) Rushed. I wanted more time.
    (b) Relaxed. I had enough time.
    (c) Amazed. I had tons of extra time.

2. The week before the test I did all my homework on time: YES   NO

3. The week before the test, in addition to the homework I followed a study plan.   YES   NO
    (a) I think this helped: YES   NO

4. The day before the test I spend _____ hours studying and reviewing.
    (a) I think that was enough time: YES   NO

5. The night before the test:
    (a) I stayed up very late cramming for the test
    (b) I stayed up very late, but I wasn't doing math
    (c) I didn't need to cram because I was prepared
    (d) I got a good night's sleep so my brain would function well.

6. I think I got the following grade on this test: _____

7. Strategies that worked well for me were (please elaborate):

8. Next time I will do an even better job preparing for the test by: