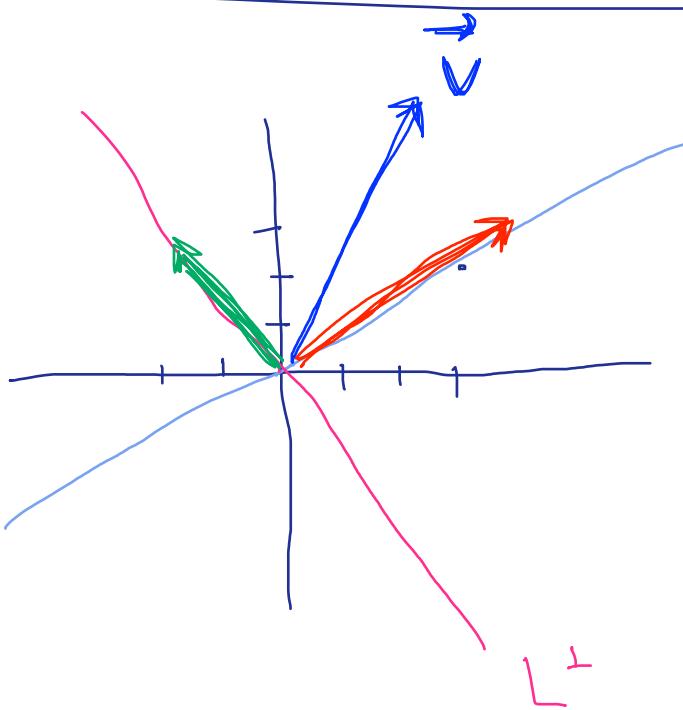


2.2

Example $L: y > \frac{2}{3}x$ $L^\perp: y = -\frac{3}{2}x$

Find $\text{proj}_L(\vec{v})$, $\text{proj}_{L^\perp}(\vec{v})$,

$\text{refl}_L(\vec{v})$, $\text{refl}_{L^\perp}(\vec{v})$



\star $\vec{v} = \text{proj}_L(\vec{v}) + \text{proj}_{L^\perp}(\vec{v})$

$\star \vec{v} \in L$ then $\text{proj}_L(\vec{v}) \parallel \langle 3, 2 \rangle$

(+) $\Leftrightarrow \text{proj}_L(\vec{v}) = a \langle 3, 2 \rangle$

$\star \vec{v} \in L^\perp$ then $\text{proj}_{L^\perp}(\vec{v}) \parallel \langle -2, 3 \rangle$

(++) $\Leftrightarrow \text{proj}_{L^\perp}(\vec{v}) = b \langle -2, 3 \rangle$

For some scalars $a, b \in \mathbb{R}$.

Use (4) & (+) (++) :

$$\vec{v} = \text{proj}_L(\vec{v}) + \text{proj}_{L^\perp}(\vec{v})$$

$$\langle x, y \rangle = a \langle 3, 2 \rangle + b \langle -2, 3 \rangle$$

$$\langle x, y \rangle = \langle 3a - 2b, 2a + 3b \rangle$$

SOE: $\begin{cases} 3a - 2b = x \\ 2a + 3b = y \end{cases}$ Given Find a, b .

$$\left[\begin{array}{cc|c} 3 & -2 & x \\ 2 & 3 & y \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & -5 & x-y \\ 0 & 13 & -2x+3y \end{array} \right]$$

$$R_1 - R_2 \rightarrow R_1$$

$$R_2 - 2R_1 \rightarrow R_2$$

$$\xrightarrow{\frac{1}{13}R_2 \rightarrow R_2} \left[\begin{array}{cc|c} 1 & -5 & x-y \\ 0 & 1 & \left(-\frac{2}{13}\right)x + \left(\frac{3}{13}\right)y \end{array} \right]$$

$$\xrightarrow{R_1 + 5R_2} \left[\begin{array}{cc|c} 1 & 0 & \left(\frac{3}{13}\right)x + \left(\frac{2}{13}\right)y \\ 0 & 1 & \left(-\frac{2}{13}\right)x + \left(\frac{8}{13}\right)y \end{array} \right]$$

$$a = \left(\frac{3}{13}\right)x + \left(\frac{2}{13}\right)y$$

$$b = \left(-\frac{2}{13}\right)x + \left(\frac{8}{13}\right)y$$

So: • $\text{proj}_L(\vec{v}) = a \langle 3, 2 \rangle = \left\langle \frac{9}{13}x + \frac{6}{13}y, \frac{6}{13}x + \frac{4}{13}y \right\rangle$

$$= \begin{bmatrix} \frac{9}{13} & \frac{6}{13} \\ \frac{6}{13} & \frac{4}{13} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \frac{1}{13} \begin{bmatrix} 9 & 6 \\ 6 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{\text{Proj}_L} = \frac{1}{13} \begin{bmatrix} 9 & 6 \\ 6 & 4 \end{bmatrix}$$

• $\text{proj}_{L^\perp}(\vec{v}) = b \langle -2, 3 \rangle = \left\langle \frac{4}{13}x + \frac{-6}{13}y, \frac{-6}{13}x + \frac{9}{13}y \right\rangle$

$$= \frac{1}{13} \begin{bmatrix} 4 & -6 \\ -6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{\text{Proj}_{L^\perp}} = \frac{1}{13} \begin{bmatrix} 4 & -6 \\ -6 & 9 \end{bmatrix}$$

- Using the vector eqs for reflections:

$$[\text{Ref}_L] = \frac{1}{13} \begin{bmatrix} 5 & 12 \\ 12 & -5 \end{bmatrix}$$

$$[\text{Ref}_{L^\perp}] = \frac{1}{13} \begin{bmatrix} -5 & -12 \\ -12 & 5 \end{bmatrix}$$