

Final Exam

Ch 6 – 13

Dec_12



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Honesty Pledge

On my honor, by printing and signing my name below, I vow to neither receive nor give any unauthorized assistance on this examination:

NAME (PRINT): _____ SIGNATURE: _____

Directions

- YOU ARE ALLOWED TO USE ONLY A SCIENTIFIC CALCULATOR ON THIS EXAM.
- You have 2 hours and 20 minutes to complete this exam.
- The exam totals **100 points**.
- There are 20 problems, some of which contain multiple parts. Each problem is worth 5 points. Points will be distributed fairly across multiple parts.
- Place all of your belongings in the front of the classroom and I will assign you a seat. Bring with you your writing utensils.
- Cell phones must be turned off and put away in with your items in the front of the classroom.
- Handwriting should be neat and legible. If I cannot read your writing, zero points will be given.
- Some questions contain multiple-parts which you must do individually and the parts are denoted by (a), (b), (c), etc. Some questions are multiple-choice and the choices are denoted with (A), (B), (C), (D), and (E). For True/False questions, you must spell out the entire word "true" or "false" in your answer.
- Make sure to ALWAYS SHOW YOUR WORK; you will not receive any partial credits unless work is clearly shown. *If in doubt, ask for clarification.*
- Leave answers in **exact form** (as simplified as possible), unless told otherwise.
- Put a

 box around your final answer where applicable.
- **PLEASE CHECK YOUR WORK!!!**

Score	Grade

Page 1 – Problems 1 through 14 – 5 points each problem

1. Evaluate the specified trigonometric functions without using your calculator. Give exact values.

(a) $\csc\left(\frac{19}{6}\pi\right)$ (b) $\cot\left(\frac{19}{6}\pi\right)$ (c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin\left(\cot^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

2. Graph the following trigonometric functions and state the period. Include at least one full period in your graph.

(a) $f(x) = 5 \cos(2x - \pi)$ (b) $g(x) = 2 \tan(2x - \pi)$

3. Prove the following trigonometric identities. You may use the Addition Formulas in your proofs.

(a) Prove: $\sin(2x) = 2 \sin(x) \cos(x)$

(b) Prove: $\cos(2x) = 1 - 2 \sin^2(x)$

4. Assume that $\theta \in (-\pi/2, \pi/2)$. If $x = 3 \sin(\theta)$, simplify the expression $\frac{x}{\sqrt{9 - x^2}}$.

5. Evaluate the following trigonometric functions. Give exact values.

(a) $\tan\left(\frac{\pi}{8}\right)$

(b) $\cos\left(\cos^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{1}{2}\right)\right)$

6. Prove the following trigonometric identity: $\frac{\sin(2x)}{1 + \cos(2x)} = \tan(x)$.

7. Solve the following trigonometric equation: $2 \sin^2(x) - 5 \sin(x) + 2 = 0$. Give all solutions.

8. At what POINTS do the functions $f(x) = \sin(x)$ and $g(x) = \cos(2x)$ intersect in the interval $[0, 2\pi)$?

9. (a) If $P = \left(4, -\frac{\pi}{4}\right)$ in polar coordinates, express P in rectangular/cartesian coordinates.

- (b) Plot the point P .

10. (a) Find the modulus, argument, and polar form of the complex number $2\sqrt{2} - 2\sqrt{2}i$.

- (b) Compute $(2\sqrt{2} - 2\sqrt{2}i)^8$. Express your answer in the form $a + bi$ where $a, b \in \mathbb{R}$.

11. Sketch the following parametric equations: $\begin{cases} x = 2 + 2 \cos(t) \\ y = 1 + \sin(t) \end{cases}$. Use arrows to indicate the direction of the curve as t increases.

12. Let $\vec{a} = 5\hat{i} - 3\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$ be two vectors in the plane.

- (a) Draw \vec{a} , \vec{b} , and $\vec{b} - \vec{a}$ on the same coordinate plane.

- (b) Is \vec{a} orthogonal to \vec{b} ? Justify your answer.

13. Let $\vec{u} = \langle -1, \sqrt{3} \rangle$ and $\vec{v} = \langle -\sqrt{3}, 1 \rangle$ be two vectors in the plane.

- (a) Find the dot product $\vec{u} \bullet \vec{v}$ and the angle between \vec{u} and \vec{v} (give exact angle in radians).

- (b) Find the projection $\text{proj}_{\vec{v}}(\vec{u})$.

14. (a) Write the form of the partial fraction decompositions: $\frac{2x + 1}{(x^2 + x)(x^2 - x - 2)}$ (NOTE: FORM ONLY)

- (b) Find the complete partial fraction decompositions: $\frac{27x}{x(x^2 - x - 2)}$

Page 2 – Problems 15 through 20 – 5 points each problem

15. (a) Solve the system of equations: $\begin{cases} x^2 + y^2 = 6 \\ x^2 - y^2 = 4 \end{cases}$. Write your answer as a solution set. How many solutions are there?
- (b) Sketch the solution set of $\begin{cases} 4y - 5x \leq 20 \\ 5x + y \geq 5 \\ 4y - x \geq 4 \end{cases}$ by graphing the boundary curves and shading the region. Be sure to indicate and label vertices, if any.

16. Solve the following system of equations using Gaussian Elimination and back-substitution: $\begin{cases} 2x & & + & 4z & = & 10 \\ 2x & + & y & + & z & = & 11 \\ 3x & + & 2y & & & = & 17 \end{cases}$

Write your answer as a solution set. How many solutions are there?

To receive full credit, you must indicate all elementary row operations used.

17. Sketch the graph of the following equations. Identify all important features appropriate to the object.

(a) $\frac{(x+2)^2}{9} + \frac{(y-3)^2}{25} = 1$ (b) $(x-3)^2 = -4(y-2)$

18. Write the equation of $4x^2 - y^2 - 16x + 8y - 16 = 0$ in standard form and sketch the graph. Identify all important features appropriate to the object.
19. Write a complete proof using Mathematical Induction of the following statement: for all natural numbers $n \in \mathbb{N}$:

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

20. (a) Use the Binomial Theorem to find $\left(x^3 - \frac{y}{3}\right)^6$
- (b) What is the 38th term of $\left(6\sqrt{x^3} + \frac{y}{2}\right)^{300}$? Give me the correct binomial coefficient and powers but you don't need to simplify completely!

Now that you have finished the exam, please take a few minutes to reflect on how you prepared for the exam and how you think you did. Then answer these questions.

- When taking the exam I felt
 - Rushed. I wanted more time.
 - Relaxed. I had enough time.
 - Amazed. I had tons of extra time.
- The week before the test I did all my homework on time: YES NO
- The week before the test, in addition to the homework I followed a study plan. YES NO
 - I think this helped: YES NO
- The day before the test I spend _____ hours studying and reviewing.
 - I think that was enough time: YES NO
- The night before the test:
 - I stayed up very late cramming for the test
 - I stayed up very late, but I wasn't doing math
 - I didn't need to cram because I was prepared
 - I got a good night's sleep so my brain would function well.
- I think I got the following grade on this test: _____
- Strategies that worked well for me were (please elaborate): _____
- Next time I will do an even better job preparing for the test by: