MATH 460: Probability & Statistics — September 2018

Chapter 3: Expectation & Chapter 6: Estimation Theory

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Sets

To begin our study of certainty, we first need to discuss the basic language it is written in–mathematical set theory.

Definition 1: Set Theory

A **set** is a collection of things, or **elements**. Elements of a set can be anything, such as numbers or people. We denote a set with curly brackets $\{$ and $\}$. So if we want to talk about the set of even numbers from 1 to 10, we can write it like this: $\{2, 10, 8, 6, 4\}$–note that we separate the elements by a comma and that the order doesn't matter.
Here are some important terms and notations:

- What is the empty set? Give two ways to denote it.
- S denotes the universe of all possible elements in consideration. We call it the **sample space**.
- What does $A \subset B$ mean? Given an example.
- What does A' (or A^c) mean? Given an example.
- What does $A \cup B$ mean? Given an example.
- What does $A \cap B$ mean? Given an example.
- What does $A - B$ mean? Given an example.
- What does it mean for two sets to be disjoint?

Activity 1: Set Theory

Do Venn Diagrams worksheet.

Consider the sets $A = \{4, 5, 6, 7, 9, 13, 16\}$ and $B = \{3, 6, 9, 12, 15\}$ where the sample space S consists of all positive integers less than or equal to 16. Find the following:

(a) $A \cup B$

(b) $A \cap B$

(c) A'

(d) $(A \cap B)'$

Random Experiments

An **experiment** is any activity or situation in which two or more outcomes will result. A **random experiment** is an experiement where there is uncertainty in which outcome will occur.

Sample Space

Definition 2: Sample Space

- What is a sample space? Give an example.
- What is an outcome? Give an example.

Events

Definition 3: Events and Sets

(a) What is an event?

(b) If A and B are events in S, then what does $A \cup B$ mean?

(c) If A and B are events in S, then what does $A \cap B$ mean?

(d) If A isn an event in S, then what does A' mean?

(e) If A and B are events in S, then what does $A - B$ mean?

(f) If A and B are events in S, then what does $A \cap B = \emptyset$ mean?

Activity 2: Events

Let S be the sample space of flipping a coin twice. Let A be the event "at least one tail occurs" and B be the event "the first toss results in a headl." Express A and B using the H and T notation and find:

(a) $A \cup B$

(b) $A \cap B$

(c) A'

(d) $A - B$

Permutations

Theorem 1: Permutations

When selecting n objects and order them r at a time, that is, permutation of n objects of length r, we have

$$_nP_r = \frac{n!}{(n-r)!} = n \cdot (n-1) \cdot (n-2) \cdots (n-(r-1)) \tag{1}$$

Activity 3: Permutations and Combinations

(a) In how many ways can 10 people be seated on a bench if only 4 seats are available?

(b) Castel and Joe are planning trips to three countries this year. There are 7 countries they would like to visit. One trip will be one week long, another two days, and the other two weeks. How many possibilities are there?

Combinations

Theorem 2: Combinations

When computing the n combinations of r objects, we have

$$ {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{{}_nP_r}{r!} \tag{2} $$

Activity 4: Permutations and Combinations

(a) In how many ways can 10 objects be split into two groups containing 4 and 6 objects, respectively?

(b) In how many ways can a team of 17 softball players choose three players to refill the water cooler?

Concept of Probability

Definition 4: Concept of Probability

In any random experiment there is always uncertainty as to whether a particular event will or will not occur. As a measure of the **chance**, or **probability**, with which we can expect the event to occur, it is convenient to assign a number between 0 and 1. If we are sure or certain that the event will occur, we say that its probability is 100%, or 1, but if we are sure that the event will not occur, we say that its probability is zero. If, for example, the probability is $\frac{1}{4}$, we would say that there is a 25% chance it will occur and a 75% chance that it will not occur. Equivalently, we can say that the odds against its occurrence are 75% to 25%, or 3 to 1.

There are two important procedures by means of which we can estimate the probability of an event.

- **CLASSICAL APPROACH:** If an event can occur in h different ways out of a total number of n possible ways, all of which are equally likely, then the probability of the event is: $\boxed{\text{Probability } = \frac{h}{n}}$.

Axioms of Probability

Definition 5: Concept of Probability

Suppose we have a sample space S.
To each event A, we associate a real number $P(A)$. Then P is called a **probability function**, and $P(A)$ the **probability of the event** A, if the following axioms are satisfied:

- **Axiom 1** For every event A,
$$ P(A) \geq 0 \tag{3} $$
- **Axiom 2** For the sample space S,
$$ P(S) = 1 \tag{4} $$
- **Axiom 3** If two events A and B are mutually exclusive,
$$ P(A \cup B) = P(A) + P(B) \tag{5} $$

Some observations:

- For any event A, the probability is between 0 and 1: $0 \le P(A) \le 1$. Of course, $P(A) = 0$ are for impossible events and $P(A) = 1$ are for events that are certain to happen.
- Because the sample space S consists of all possible outcomes for a chance experiment, in the long run an outcome that is in S must occur 100% of the time. This means that $P(S) = 1$.
- Axiom 3 can be applied to any number of mutually exclusive events. If $A_1, A_2, A_3, \ldots$ are all mutually exclusive events (that is $A_i \cap A_j = \emptyset$ for any two indices i and j) then

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots \tag{6}$$

  For example, if we roll a fair dice and A_1 is the event of rolling a 1, A_2 is the event of rolling a 2, etc then there are 6 mutually exclusive events and the sample space is $S = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_6$. Then

$$1 = P(S) = P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_6) = P(A_1) + P(A_2) + P(A_3) + \cdots + P(A_6). \tag{7}$$

Theorem 3: Theorems of Probability

For any event A,

(a) $P(A) + P(A') = 1$
This one makes sense since in words it says: any event either occurs or doesn't occur!

(b) $P(A') = 1 - P(A)$
This follows from Property (a) by subtracting $P(A)$ from both sides and noting that $A' = S - A$.

(c) If A and B are two events (not necessarily disjoint), then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{8}$$

Activity 5: Probability

Suppose A and B are two disjoint events in a sample space S and that $P(A) = .22$; $P(B) = .33$. Calculate the following probabilities.

(a) $P(A \cup B)$

(b) $P(A \cap B)$

(c) $P(A' \cup B)$

(d) $P(A' \cap B)$

(e) $P(A - B)$

Activity 6: Probability

A ball is drawn at random from a box containing 6 red balls, 4 white balls, and 5 blue balls. Determine the probability that it is (a) red, (b) white, (c) blue, (d) not red, (e) red or white.

Activity 7: Probability

Determine the probability for the following events.

(a) Roll a 7 or 11 from a pair of fair 6 sided dice.

(b) A non-defective television found next if out of 2,300 televisions already examined, 15 were defective.

(c) At least 1 tails appears in 5 tosses of a fair coin.

(d) The probability of drawing an ace or a club in a standard deck.

Conditional Probability

Definition 6: Conditional Probability

Let A and B be two events such that $P(A) > 0$. Denote by $P(B|A)$ the **probability of B given that A has occurred**. Since A is known to have occurred, it becomes the new sample space replacing the original S. From this we are led to the definition

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \tag{9}$$

or, equivalently,

$$P(B \cap A) = P(A) \cdot P(B|A) \tag{10}$$

In words, (11) says that the probability that both A and B occur is equal to the probability that A occurs times the probability that B occurs given that A has occurred.

Activity 8: Conditional Probability

(a) Five marbles are picked at random out of a jar containing 10 red marbles, 15 white marbles, 20 blue marbles, 25 orange marbles, and 30 purple marbles. What is the probability of picking one of each color, assuming you pick a marble one at a time?

(b) Drawing a king and a spades cards for first two draws from a well-shuffled 52 card deck.

Chapter 5: Sampling Theory

Organizing and Visualizing Data

Definition 7: Data

- **FREQUENCY DISTRIBUTION:**
  What is a **frequency distribution**?
- **BAR GRAPHS and HISTOGRAMS:** What is a histogram?

Measurements of Central Tendency

Definition 8: Mean-Median-Mode-Range

- **MEAN:** what is the mean?
- **MEDIAN:** what is median?
- **MODE:** what is mode? bimodal? trimodal?

Measurement of Dispersion

Definition 9: Standard-Deviation

A measure of how the values in a data set vary or deviate from the mean. Some notation to compute the standard deviation:

$$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{n}} \tag{11}$$

- x: a value from a data set
- $\bar{x}$: mean
- n: number of values in a data set
- Σ: "to sum or add" (Capital Greek letter "Sigma")
- σ: standard deviation (lower-case Greek letter "sigma")

Definition 10: Five-Number-Summary

- **RANGE:** what is the range?
- **FIVE NUMBER SUMMARY:** what is a five number summary?
  - **FIRST QUARTILE** Q_1: what is Q_1?
  - **THIRD QUARTILE** Q_3: what is Q_3?
- **Box-Whisker Plot:** what is a box-whisker plot?

Activity 9: Five-Number-Summary

(a) Find the five number summary, and draw a Box-Whisker plot for $S = \{42, 20, 31, 10, 5, 3, 2, 1, 67, 53, 44\}$.

(b) Find the standard deviation for the set from part (a).

Chapter 2: Random Variables

Discrete vs Continuous Variables

Definition 11: Discrete-vs-Continuous-Variable

- **Variable:** a function defined on the sample space. That is, given any event A from a sample space S, a random variable assigns a number to even A. Using function notation, we write this as $X(A)$.
- **Discrete variable:** a variable that can attain only specific values. Example: think the values of the roll of a dice.
- **Continous variable:** a variable can attain infinitely many values over a certain span or range. Example: the height of a person.
- **RANDOM variable:** a variable defined on a sample space that is comprised of random process or experiment–that is, experiment where you don't know what the outcome is until it is completed. Example: flipping a coin is a random experiment.

Binomial Distribution

Definition 12: Binomial-Distribution

- Let n be the total number of trails run in the experiment
- Let X be a random variable of a single "successful" trail
- Let p be the probability of the successful trail X
- Let q be the probability of trail X failing. (NOTE: $p + q = 1$, or $q = 1 - p$)
- Let x be the number of successful trials of X. So notice that x can take values from 0 up to n, i.e. $x = 0, 1, 2, 3, \ldots, n$.
- Let $P(X = x)$ denote the **probability of exactly x successful trails out n in a random experiment with independent trails**, then

$$\boxed{P(X = x) = \binom{n}{x} p^x q^{n-x}} \tag{12}$$

Recall that: $\binom{n}{x} =_n C_x = \frac{n!}{x!(n-x)!}$.

Activity 10: Binomial-Distribution-Probability

A die is tossed 3 times. What is the probability of

(a) No fives turning up?

(b) 1 five turning up?

(c) 3 fives turning up?

Definition 13: Binompdf-vs-Binomcdf

USING CALCULATOR TI83: binompdf(n,p,x)

$\boxed{DIST}$ key in yellow ($\boxed{2nd}$ > $\boxed{VARS}$) > Scroll to 10 "binompdf" or scroll to A "binomcdf"

- Binompdf is when we want exactly x trails to be successful so this is binomial distribution pdf. Thus this is 1-valued random variables.
- Binomcdf is when we want multiple values of x to be true. It is defined as

$$\text{binomcdf}(n, p, x) = P(X \leq x) \tag{13}$$

Notice the sneaky "$\leq$" less than or equal to sign in the binomcdf. This means:

$$\begin{aligned} \text{binomcdf}(n, p, x) &= P(X \leq x) = P(X = 0, 1, 2, \ldots, x) \\ &= P(X = 0) + P(X = 1) + P(X = 2) + \cdots + P(X = x) \end{aligned}$$

This can help us with "at most" and "at least" type of problems

Activity 11: Binomcdf-probability

What is the probability of at least four successful trials in a random experiment, with probability of success of a single trial being 25%?

Activity 12: Binomial-Distribution-Probability

Find the probability that in tossing a fair coin three times, there will appear

(a) three heads

(b) two tails and a head

(c) at least one head

(d) not more than one tail

Visualizing a Binomial Distribution

Activity 13: Visualizing-Binomial-Distribution

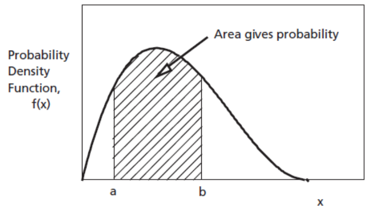
Let our experiment be shooting free-throws. Assume that the probability of making a freethrow is 70% and that these are independent events. Let X be the number of made in taking six shots.

(a) Use your calculator to find $P(X = x)$ for $x = 0, 1, 2, 3, 4, 5, 6$.

(b) Plot a histogram for the random variable X probability distribution.

(c) Describe any interesting features from your histogram.

Probability Density Functions



Definition 14: Probability-Density-Functions

Recall a random variable can be either discrete or continuous. If we want to know the probability of a random variable in certain range, then we

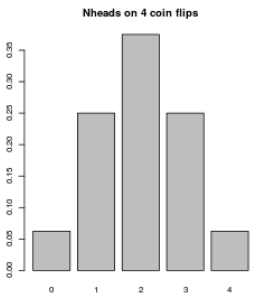
- **Probability Density Functions:** Let X be a continuous random variable. Then a **probability density (or probability distribution) function (pdf) of X** is a function $f(x)$ if:
  1. f is a continuous function on $\mathbb{R}$
  2. f is nonnegative, that is, $f(x) \geq 0$
  3. The probability that X takes on a value in the interval $[a, b]$ is the area above this interval and under the graph of the density function:

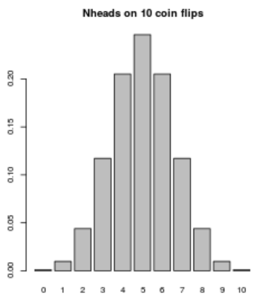
     $$P(a \leq X \leq b) = \text{Area under the curve between } x = a \text{ and } x = b \tag{14}$$

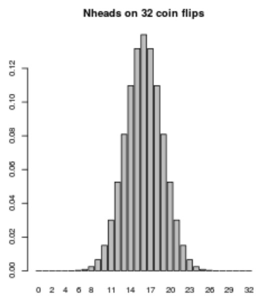
     The graph of $f(x)$ is often referred to as the density curve.
  4. The total area under the graph of $f(x)$ is 1. This corresponds to $P(S) = 1$.

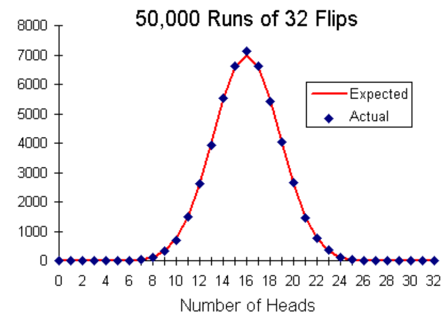
NOTE: for continuous pdf, $P(X = x) = 0$ why?

Normal Distribution







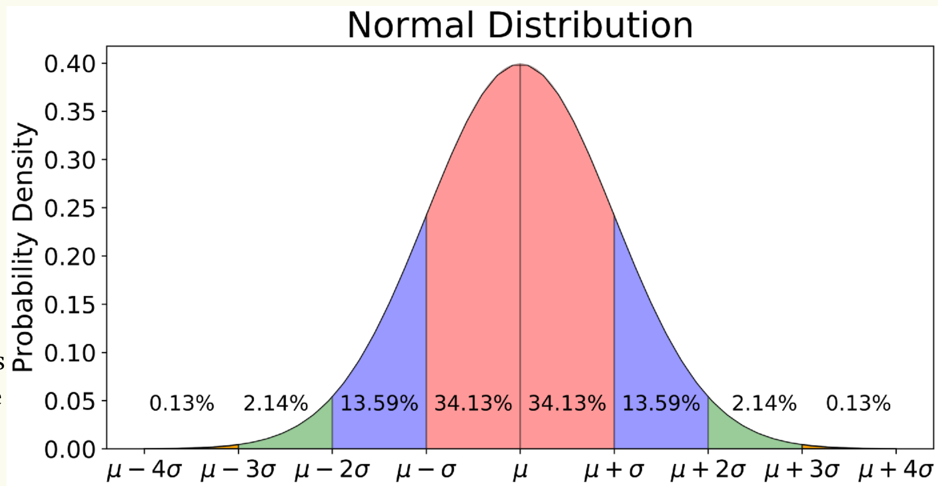


Definition 15: Normal-Distribution

- **Normal Distribution:** Is the PDF for a very special function, and it's graph is given by:

  Properties:

  - Symmetric, bell shaped
  - Continuous on all of $\mathbb{R}$
  - Mean: $\bar{x} = \mu$
  - Standard deviation: σ
  - The associated PDF $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$
  - Two parameters, μ and σ. Note that the normal distribution is actually a family of distributions since μ and σ determine the shape of the distribution.



- **Empirical Rule (68-95-99.7 Rule):** is a shorthand used to remember the percentage of values that lie within a band around the mean in a normal distribution with a width of two, four and six standard deviations, respectively. More accurately, 68.27%, 95.45% and 99.73% of the values lie within one, two and three standard deviations of the mean, respectively.

  In mathematical notation, these facts can be expressed as follows, where X is an observation from a normally distributed random variable, $\mu(=\bar{x})$ is the mean of the distribution, and σ is its standard deviation:

$$\text{Within 1 standard deviation of mean: } P(\mu - \sigma \leq X \leq \mu + \sigma) \approx 68\%$$
$$\text{Within 2 standard deviations of mean: } P(\mu - 2\sigma \leq X \leq \mu + 2\sigma) \approx 95\%$$
$$\text{Within 3 standard deviations of mean: } P(\mu - 3\sigma \leq X \leq \mu + 3\sigma) \approx 99.7\%$$

- Calculating probabilities outside of empirical rule: use table or calculator!

  USING CALCULATOR TI83: normalcdf(min, max, μ, σ)

  $\boxed{DIST}$ key in yellow ($\boxed{2nd}$ > $\boxed{VARS}$) > Scroll to 2. "normalcdf"

Why is the standard normal distribution useful?

- Many things actually are normally distributed, or very close to it. For example, height and intelligence are approximately normally distributed; measurement errors also often have a normal distribution
- The normal distribution is easy to work with mathematically. In many practical cases, the methods developed using normal theory work quite well even when the distribution is not normal.

- There is a very strong connection between the size of a sample N and the extent to which a sampling distribution approaches the normal form. Many sampling distributions based on large N can be approximated by the normal distribution even though the population distribution itself is definitely not normal.

Activity 14: Normal-Distribution

Weight (in grams) of bags of sugar from a factory are normally distributed, with a mean of 1000g, and standard deviation of 13g. Find the following.

(a) The probability that a randomly selected bad of sugar weighs in between 974g and 1000g.

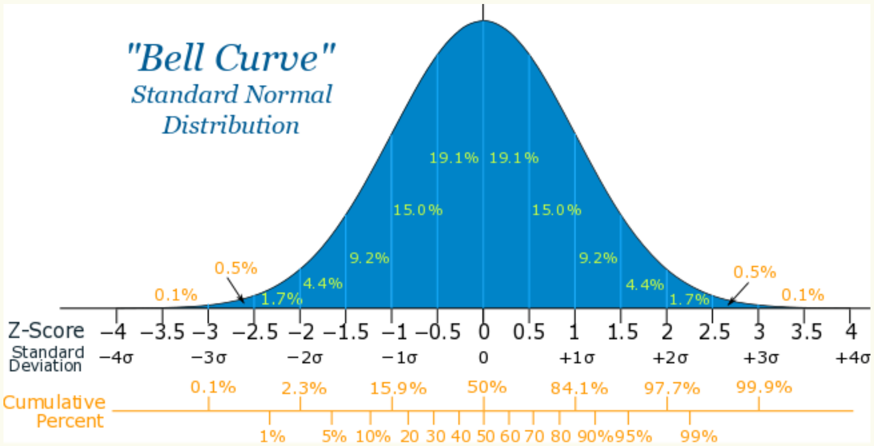
(b) The percentage of bags whose weight is above 1026g.

Standard Normal Distribution

Definition 16: Standard-Normal-Distribution

- **Standard Normal Distribution:** Is the PDF for a very special function, and it's graph is given by:

  Properties:
  - Symmetric, bell shaped
  - Continuous on all of $\mathbb{R}$
  - Mean: $\mu = 0$
  - Standard deviation: $\sigma = 1$
  - The associated PDF $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$



- Calculating probabilities outside of empirical rule: use table or calculator!

  USING CALCULATOR TI83: normalcdf(min, max) – NOTICE: when using Standard Normal we do not need to put μ and σ since the calculator already has is programed for $\mu = 0$ and $\sigma = 1$

  $\boxed{DIST}$ key in yellow ($\boxed{2nd}$ > $\boxed{VARS}$) > Scroll to 2. "normalcdf"

Definition 17: Using-Normal-Distribution-Z-Scores

- KEY: suffices to know the standard normal since we can go back and forth between normal and standard normal using the formulas:

$$\boxed{z = \frac{x-\mu}{\sigma}} \quad \text{and} \quad \boxed{x = z \cdot \sigma + \mu}$$

Activity 15: Convert-z-values

Convert each of the following between x and z values.

(a) $x = 35$ where $\mu = 40$, $\sigma = 2$

(b) $x = 130$ where $\mu = 100$, $\sigma = 12$

(c) $z = -0.57$ where $\mu = 14$, $\sigma = 1.5$

Activity 16: Standard-Normal-Distribution

Find the area under the standard normal curve between

(a) $z = 0$ and $z = 1.2$

(b) $z = -0.68$ and $z = 0$

(c) $z = -0.46$ and $z = 2.21$

(d) to the right of $z = -1.28$

Activity 17: Standard-Normal-Distribution

The mean weight of 500 male students at a certain college is 151 lb and the standard deviation is 15 lb. Assuming that the weights are normally distributed, find without using a calculator how many students weigh

(a) between 120 and 155 lb

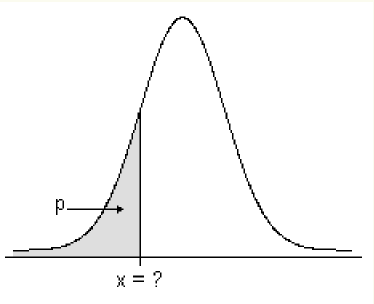
(b) more than 185 lb.

Inverse Normal Distribution

Definition 18: Inverse-Normal-Distribution

This is an informal name to the process of working backwards from a known (or given) probability to find an x-value.

Given probability p of the area under the curve of the PDF to the left of the value x_p, we want to find what the value x_p is. It really helps to draw a picture to help explain what we are doing:



Essentially, we solving for x in the equation: $P(X \leq x) = p$.

USING CALCULATOR TI83: invNorm(p,μ,σ)

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- Given probability p of the area under the curve of the PDF to the left of the value x_p, we can find what the value x_p is by using the “inverse normal distribution:”

$$\text{invNorm}(p, \mu, \sigma) = x_p \tag{15}$$

Activity 18: Inverse-Normal-Distribution

The time it takes employees to get to work from home (in minutes) is normally distributed with a mean of 30 minutes, and a standard deviation of 5 minutes. Find:

(a) the percentage of employees that take between 28 and 37 minutes to get to work (Hint: this is not an inverse problem)

(b) The number of minutes the longest it would take the bottom employee in the bottom 5% of the data to get to work. (Hint: this is an inverse problem)

Poisson Distribution

Definition 19: Poisson-Distribution

For a Poisson distribution, we make the following assumptions:

- X is a discrete random variable, i.e. it takes values 0,1,2, ...
  So X = the number of successes of a Poisson random variable.
- the number of successes in two disjoint time intervals is independent
- the probability of success during a small time interval is proportional to the entire length of the time interval

Then, if μ is the mean of the number of successes in the given time interval, the probability that X is successful x times is given by

$$\boxed{P(X = x) = \frac{e^{-\mu} \cdot \mu^x}{x!}} \tag{16}$$

Fact: A Poisson distribution has $\sigma = \sqrt{\mu}$.

USING CALCULATOR TI83: poissonpdf(μ,x)

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- Given mean μ and the number of successes x, the probability can be calculated by the "poisson distribution:"

$$\text{poissonpdf}(\mu, x) = P(X = x) \tag{17}$$

Activity 19: Poisson-Distribution

A Life Insurance (LI) salesman sells on average 3 LI policies per week. Assuming a Poisson Distribution, calculate the probability that in a given week she will sell:

(a) some policies

(b) 2 or more but less than 5 policies

(c) Assuming a five day workweek, what is the probability that in a given day, she will sell a policy?

Poisson Distribution

Definition 20: Poisson-Distribution

For a Poisson distribution, we make the following assumptions:

- X is a discrete random variable, i.e. it takes values 0,1,2, ... So X = the number of successes of a Poisson random variable.
- the number of successes in two disjoint time intervals is independent
- the probability of success during a small time interval is proportional to the entire length of the time interval

Then, if μ is the mean of the number of successes in the given time interval, the probability that X is successful x times is given by

$$\boxed{P(X=x)=\frac{e^{-\mu}\cdot\mu^{x}}{x!}} \tag{18}$$

FACTS:

(a) If n is an interval of time, and p is the probability of success, then $\mu = np$.

(b) A Poisson Distribution is often used to approximate the Binomial Distribution, when n is "large" and p is "small" (general rule is $n \geq 30$ and $p < 0.05$)

(c) A Poisson distribution has $\sigma = \sqrt{\mu}$.

USING CALCULATOR TI83: poissonpdf(μ,x)

$\boxed{DIST}$ key in yellow ($\boxed{2nd}$ > $\boxed{VARS}$) > Scroll to A. "poissonpdf("

- Given mean μ and the number of successes x, the probability can be calculated by the "poisson distribution:"

$$\text{poissonpdf}(\mu, x) = P(X = x) \tag{19}$$

Activity 20: Poisson-Distribution

A company makes electrical motors. The probability an electrical motor is defective is 0.01. What is the probability that a sample of 415 electrical motors will contain exactly five defective motors?

Activity 21: Visualizing-Poisson-Distribution

A 911 operator receives about six telephone calls between 8 a.m. and 10 a.m.

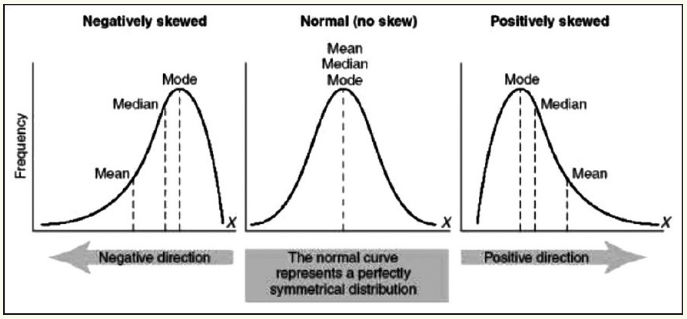
(a) What is the probability that she receives more than one call in the next 15 minutes?

(b) Plot the histogram for the probability $P(x) = P(X = x)$ for $x = 0, 1, 2, 3, \ldots$

x	$P(x)$
⋮	⋮

Skewness

Definition 21: Skewness



Chapter 3: Expectation

Definition 22: Expectation

- Expected value uses probability to tell us what outcomes to expect in the long run.
- For a discrete random variable X having the possible values $x_1, x_2, \ldots, x_n$, give the definition of the expectation of X:
- As a special case of (2), where the probabilities are all equal, the formula for expectation is?
- The expectation of X is very often called the mean of X and is denoted by μ_X, or simply μ, when the particular random variable is understood.
- The mean, or expectation, of X gives a single value that acts as a representative or average of the values of X, and for this reason it is often called a **measure of central tendency**.

Activity 22: Expectation

Suppose that a game is to be played with a single die assumed fair. In this game a player wins \$20 if a 2 turns up, \$40 if a 4 turns up; loses \$30 if a 6 turns up; while the player neither wins nor loses if any other face turns up.

(a) State what the random variable X is

(b) Find all the outcomes $x_1, \ldots, x_6$

(c) Find all the probabilities for each respective outcome

(d) Find the expected sum of money to be won (or lost).

(e) In a fair game, what do you think is a reasonable buy-in is in order to play the game?

Activity 23: Expectation

A game is played where a player rolls a six sided die and if the result is an even number, they win 4 times the number in dollars, but if the result is odd, they lose 6 times the number in dollars. Find the expected winnings (or losings).

(a) Find the expected winnings (or losings).

(b) Even if the game is free, should you play?

Chapter 6: Estimation Theory

Confidence Intervals

Definition 23: Inferential-Statistics

Explain these terms:

- **POPULATION vs SAMPLE:**
- **PARAMETER vs STATISTIC:**
- **INFERENTIAL STATISTICS:**
- **CONFIDENCE INTERVALS:** We realize that the point estimate is most likely not the exact value of the population parameter, but close to it. After calculating point estimates, we construct interval estimates, called **confidence intervals**.
- **ASSUMPTION:** For $n \geq 30$ the sample can be assumed to be nearly a normal distribution.

Definition 24: Confidence-Intervals

- **ASSUMPTION:** For $n \geq 30$ the sample can be assumed to be nearly a normal distribution.
- **POPULATION vs SAMPLE:** When you conduct a survey and calculate the sample mean, $\bar{x}$, and the **sample standard deviation**, S. You would use $\bar{x}$ to estimate the population mean and s to estimate the population standard deviation. The sample mean, $\bar{x}$, is the point estimate for the population mean, μ. The sample standard deviation, s, is the point estimate for the population standard deviation, σ.
- **STANDARD ERROR OF MEAN:** The **standard error of the mean** is given by $\frac{s}{\sqrt{n}}$.
- **CONFIDENCE:** Let C be a number from 0 to 1 and $C \cdot 100\%$ be a percentage between 0 and 100%.
- z**-SCORE:** Let z_C be the z-score such that the area between the interval $[-z_C, z_C]$ is C.
  To do this, you can use the inverse Normal distribution to find z_C but it is not simply C because we want the middle area. The formula is:

$$\boxed{z_C = \text{invNorm}\left(\frac{1+C}{2}\right)}$$

Table 6-1

Confidence Level	99.73%	99%	98%	96%	95.45%	95%	90%	80%	68.27%	50%
z_c	3.00	2.58	2.33	2.05	2.00	1.96	1.645	1.28	1.00	0.6745

- **CONFIDENCE INTERVAL:** the interval that contains the mean of the population μ with a confidence of $C \cdot 100\%$

$$\boxed{\bar{x} \pm z_c \cdot \frac{s}{\sqrt{n}}} \text{ or } \left[\bar{x} - z_c \cdot \frac{s}{\sqrt{n}}, \quad \bar{x} + z_c \cdot \frac{s}{\sqrt{n}}\right] \tag{20}$$

- In general:
  - The more confident (larger C) we are that we know where the population mean, μ is, then the bigger the interval will be!
  - The shorter the interval predicting where the population mean μ is, then we will be less confident (smaller C)!

Activity 24: Confidence-Interval

Find a $C \cdot 100\%$ confidence interval for μ for the given values:

(a) $\bar{x} = 75$, $s = 13.2$, and $n = 57$

(b) $\bar{x} = 315$, $s = 63$, and $n = 100$

Activity 25: Confidence-Interval

Below are the number of times per year 38 randomly selected employees for a large company feel overworked.

$$15, 23, 50, 31, 5, 27, 47, 43, 135, 164, 80, 123, 20, 34, 45, 56, 7, 12, 15,$$
$$16, 18, 64, 79, 84, 19, 32, 34, 56, 200, 0, 16, 61, 31, 52, 61, 70, 365, 105$$

(a) Find a 85% confidence interval for μ for the population mean of this data.

(b) Find a 90% confidence interval for the population mean of this data.