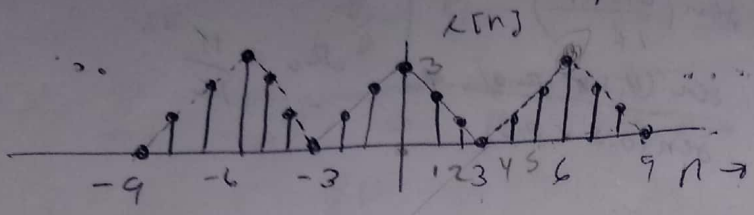
PR11. Montiel Cruz Jorge de Jesús

9.1.4 Encuentre la serie de Fourier discreta y el correspondiente espectro de amplitud y fase para $x[n]$



$3 - n = x[n]$

$x[0] = 3$

$x[1] = x[-1] = 2$

$x[2] = x[-2] = 1$

$x[3] = 0.$

$\Rightarrow 1, 2, 3, 2, 1, 0$

$$x[n] = \sum_{p=0}^{N_0-1} D_p e^{jp\Omega_0 n} \qquad \Omega_0 = \frac{2\pi}{N_0} \qquad N_0 = 6$$

$$\Omega = \frac{\pi}{3}$$

o bien

$$x[n] = \sum_{p=-2}^{3} D_r e^{jp\Omega n}$$

$$D_p = \frac{1}{6} \sum_{r=-2}^{3} x[r] e^{-r\frac{\pi}{3} jp}$$

$$Sx[n] = \sum_{p=-2}^{3} D_p e^{p\frac{\pi}{3} jn}$$

$$= D_{-2} e^{-\frac{2\pi}{3} jn} + D_{-1} e^{-\frac{\pi}{3} jn} + D_0 + D_1 e^{\frac{\pi}{3} jn} + D_2 e^{\frac{2\pi}{3} jn} + D_3 e^{\pi jn}$$

$$D_{-2} = \frac{1}{6} \sum_{r=-2}^{3} x[r] e^{\frac{2}{3}\pi jr} = \frac{1}{6}\left[e^{-\frac{4\pi}{3} j} + 2e^{-\frac{2}{3}\pi j} + 3 + 2e^{\frac{2}{3}\pi j} + e^{\frac{4\pi}{3} j} \right] = 0.$$

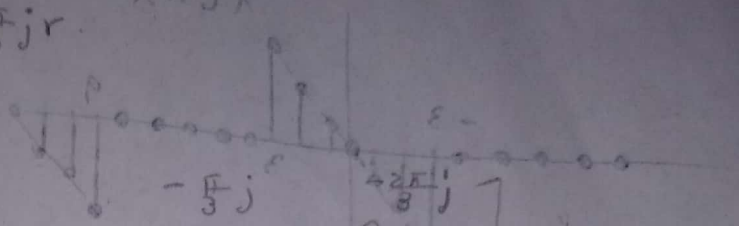
$$D_{-1} = \frac{1}{6} \sum_{r=-2}^{3} x[r] e^{\frac{\pi}{3} jr}$$

$$= \frac{1}{6}\left[e^{-\frac{2\pi}{3} j} + 2e^{-\frac{\pi}{3} j} + 3 + 2e^{\pi/3\, j} + e^{\frac{2\pi}{3} j} \right]$$

$$= 0.666$$

$$D_0 = \frac{1}{6}\sum_{r=-2}^{3} x[r] = \frac{9}{6} = \frac{3}{2}$$

$$D_1 = \frac{1}{6}\sum_{r=-2}^{3} x[r]\, e^{-\frac{\pi}{3}jr}$$



$$= \frac{1}{6}\left[e^{\frac{2\pi}{3}j} + 2e^{\frac{\pi}{3}j} + 3 + 2e^{-\frac{\pi}{3}j} + e^{-\frac{2\pi}{3}j} \right]$$

$$= 0.666$$

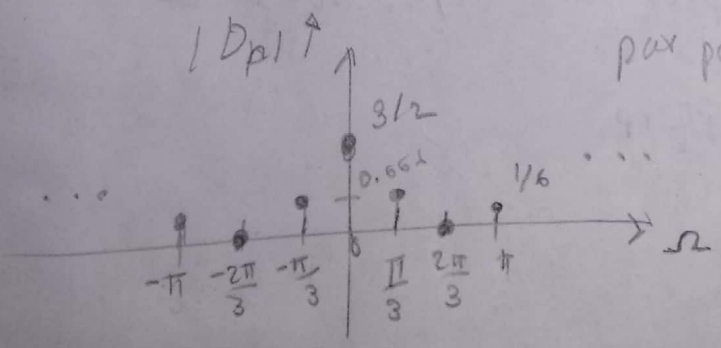
$$D_2 = D_{-2}^{*} = 0$$

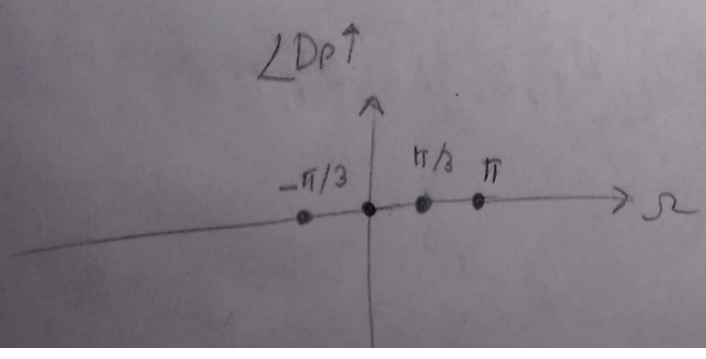
$$D_3 = \frac{1}{6}\sum_{r=-2}^{3} x[r]\, e^{-\pi jr} = \frac{1}{6}\left[e^{2\pi j} + 2e^{\pi j} + 3 + 2e^{-\pi j} + e^{-2\pi j} \right]$$

$$= \frac{1}{6}$$

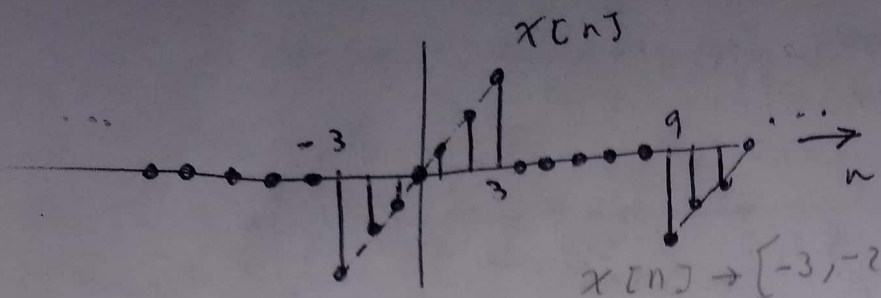
$$S_x[n] = 0.666\, e^{\frac{\pi}{3}jn} + \frac{3}{2} + 0.666\, e^{\pi/3 jn} + \frac{1}{6} e^{\pi jn}$$

par periodica





9.1.5 Repita para:



$x[n] \rightarrow [-3,-2,-1,\underset{\downarrow}{0},1,2,3,0,0,0,0,0]$

la serie de Fourier discreta está dada por:

$$S_x[n] = \sum_{p=-3}^{8} D_p e^{j s_0 p n} \quad \text{donde } s_0 = \frac{2\pi}{N_0} \quad N_0 = 12$$

$$\Rightarrow s_0 = \frac{\pi}{6}$$

así

$$S_x[n] = \sum_{p=-3}^{8} D_p e^{j\frac{\pi}{6}pn}$$

donde $D_p = \frac{1}{N} \sum_{r=-3}^{8} x[r] e^{-j\frac{\pi}{6}rp}$

de -3 a 8 solo en $[-3, 3]$ habrá valor para D_p pues de $[4, 8]$ $x[n] = 0$

así

$$D_p = \frac{1}{12} \sum_{r=-3}^{3} x[r] e^{-j\frac{\pi}{6}rp}$$

tenemos entonces

$D_{-3}, D_{-2}, D_{-1}, D_0, D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8$

Además:

$D_{-3} = D_3^*$ $\quad D_{-1} = D_1^*$

$D_{-2} = D_2^*$

$$D_{-3} = \frac{1}{12}\left(-3e^{\frac{\pi}{2}(-3)j} + 2e^{\frac{\pi}{2}(-2)j} - 1e^{\frac{\pi}{2}(-1)j} + e^{\frac{\pi}{2}j} + 2e^{\pi j} + 3e^{\frac{\pi}{2}(3)j}\right) = \frac{-1}{3}j$$

$$D_{-2} = \frac{1}{12}\left(-3e^{\frac{\pi}{3}(-3)j} - 2e^{\frac{\pi}{3}(-2)j} - 1e^{\frac{\pi}{3}(-1)j} + e^{\frac{\pi}{3}j} + 2e^{\frac{\pi}{3}(2)j} + 3e^{\frac{\pi}{3}(3j)}\right) = 0.4330j$$

$$D_{-1} = \frac{1}{12}\left(-3e^{\frac{\pi}{6}(-3)j} - 2e^{\frac{\pi}{6}(-2)j} - 1e^{\frac{\pi}{6}(-1)j} + e^{\frac{\pi}{6}j} + 2e^{\frac{\pi}{6}(2)j} + 3e^{\pi/6(3j)}\right) = 0.8720j$$

$D_0 = 0$

$D_1 = -0.8720j$

$D_2 = -0.4330j$

$D_3 = 1/3\,j$

$$D_4 = \frac{1}{12}\left(-3e^{-\frac{2\pi}{3}(-3)j} - 2e^{-\frac{2\pi}{3}(-2)j} - 1e^{-\frac{2\pi}{3}(-1)j} + e^{-\frac{2\pi}{3}j} + 2e^{-\frac{2\pi}{3}(2)j} + 3e^{-\frac{2\pi}{3}(3j)}\right)$$

$$= 0.144j$$

$$D_5 = \frac{1}{12}\left(-3e^{-\frac{5\pi}{6}j(-3)} - 2e^{-\frac{5\pi}{6}(-2)j} - 1e^{-\frac{5\pi}{6}(-1)j} + e^{-\frac{5\pi}{6}j} + 2e^{-\frac{5\pi}{6}(2)j} + 3e^{-\frac{5\pi}{6}(3)j}\right)$$

$$= -0.294j$$

$$D_6 = \frac{1}{12}\left(-3e^{-\pi(-3)j} - 2e^{-\pi(-2)j} - 1e^{-\pi(-1)j} + e^{-\pi j} + 2e^{-\pi(2)j} + 3e^{-\pi(3)j}\right) = 0$$

$$D_7 = \frac{1}{12}\left(-3e^{-\frac{7\pi}{6}(-3)j} - 2e^{-\frac{7\pi}{6}(-2)j} - e^{-\frac{7\pi}{6}(-1)j} + e^{-\frac{7\pi}{6}j} + 2e^{-7\pi/6(2j)} + 3e^{-\frac{7\pi}{6}(3)j}\right) = 0.294j$$

$$D_8 = \frac{1}{12}\left(-3e^{-\frac{4\pi}{3}(-3)j} - 2e^{-\frac{4\pi}{3}(-2)j} - e^{-\frac{4\pi}{3}(-1)j} + e^{-\frac{4\pi}{3}j} + 2e^{-\frac{4\pi}{3}(2j)} + 3e^{-\frac{4\pi}{3}(3j)}\right) = -0.144j$$

así

$$S_2[n] = \frac{1}{3} e^{-\pi/2} j\, e^{j\frac{\pi}{6}(-3)n} + 0.4330\, e^{\frac{\pi}{2}} j\, e^{j\frac{\pi}{6}(-2)n}$$

$$+ 0.8720\, e^{j\frac{\pi}{2}}\, e^{j\frac{\pi}{6}(-1)n} + 0.8720\, e^{-j\frac{\pi}{2}}\, e^{j\frac{\pi}{6}n}$$

$$+ 0.4330\, e^{-j\pi/2}\, e^{j\frac{\pi}{6}(2)n} + \frac{1}{3} e^{j\pi/2}\, e^{j\frac{\pi}{6}(3)n}$$

$$+ 0.144\, e^{j\frac{\pi}{2}}\, e^{j\frac{\pi}{6}(4)n} + 0.294\, e^{-j\frac{\pi}{2}}\, e^{j\frac{\pi}{6}(5)n}$$

$$+ 0.294\, e^{j\pi/2}\, e^{j\pi/6(7)n} + 0.144\, e^{-\pi/2} j\, e^{j\frac{\pi}{6}(8)n}$$