PR17 Montiel Cruz Jorge de Jesús

4.2.1 d)

encuentre la transformada de Laplace usando tablas y la propiedad de traslación en el tiempo de la transformada de Laplace unilateral.

$$x(t) = e^{-t} u(t-T)$$

la propiedad.

$$x(t-t_0)\, u(t-t_0) \rightarrow X(s)\, e^{-st_0} \quad t_0 > 0$$

y, de tabla

$$e^{\lambda t} u(t) \Longleftrightarrow \frac{1}{s-\lambda}$$

llevando $x(t)$ a la forma

$$x(t) = e^{-(t-T)} e^{-T} u(t-T) \quad \text{donde } e^{-T} = cte.$$

así pues.

$$\mathcal{L}\{x(t)\} = e^{-T}\mathcal{L}\{e^{-(t-T)} u(t-T)\} = e^{-T}\frac{1}{s+1} e^{-sT}$$

$$= \frac{e^{-T(1+s)}}{s+1}$$

Scanned by CamScanner

4.2.3 d) encuentre la inversa de Laplace

$$X(s) = \frac{e^{-s} + e^{-2s} + 1}{s^2 + 3s + 2}$$

$$X(s) = \frac{e^{-s} + e^{-2s} + 1}{(s+2)(s+1)} = (e^{-s} + e^{-2s} + 1)\left(\frac{A}{s+2} + \frac{B}{s+1}\right)$$

$$\Rightarrow A = \frac{1}{s+1}\Big|_{s=-2} = -1$$

$$B = \frac{1}{s+2}\Big|_{s=-1} = 1$$

$$A = -B = -1$$

$$X(s) = e^{-s}\left(\frac{-1}{s+2} + \frac{1}{s+1}\right) + e^{-2s}\left(\frac{-1}{s+2} + \frac{1}{s+1}\right)$$

$$+ \left(\frac{-1}{s+2} + \frac{1}{s+1}\right)$$

usando $x(t - t_0) \Leftrightarrow X(s)\, e^{-s t_0}$

$$\mathcal{L}^{-1}\{X(s)\} = \left(\left(-e^{-2(t-1)} + e^{-(t-1)}\right)u(t-1)\right) + \left(-e^{-2(t-2)} + e^{-(t-2)}\right)u(t-2)$$

$$- e^{-2t}u(t) + e^{-t}u(t)$$

Scanned by CamScanner

4.3.1 b) resuelva usando transformada de Laplace

$$(D^2+4D+4)y(t) = (D+1)x(t) \quad \ldots (1)$$

$$y(0^-) = 2 \qquad \dot{y}(0^-) = 1$$

$$x(t) = e^{-1}u(t)$$

$$y(t) \Leftrightarrow Y(s)$$

$$x(t) \Leftrightarrow X(s) = \frac{e^{-1}}{s}$$

$$\frac{d^2y(t)}{dt^2} \Leftrightarrow s^2Y(s) - sy(0^-) - \dot{y}(0^-)$$

$$= s^2Y(s) - 2s - 1$$

$$\frac{dy(t)}{dt} \Leftrightarrow sY(s) - y(0^-) = sY(s) - 2$$

$$X(s) = \frac{e^{-1}}{s}$$

$$\frac{dx}{dt} \Leftrightarrow sX(s) - x(0^-) = e^{-1} - 0 = e^{-1}$$

aplicando la transformada de Laplace a (1)

$$s^2Y(s) - 2s - 1 + 4sY(s) - 8 + 4y(0) = e^{-1} + \frac{e^{-1}}{s}$$

$$Y(s)(s^2+4s+4) = e^{-1} + \frac{e^{-1}}{s} + 2s + 9$$

$$\rightarrow Y(s) = \frac{(e^{-1}+9)}{(s+2)^2} + \frac{e^{-1}}{s(s+2)^2} + \frac{2s}{(s+2)^2}$$

$$* \quad \frac{e^{-1}}{s(s+2)^2} = e^{-1}\left(-\frac{1}{4(s+2)} - \frac{1}{2(s+2)^2} + \frac{1}{4s}\right)$$

$$* \quad \frac{2s}{(s+2)^2} = \frac{2}{s+2} - \frac{4}{(s+2)^2}$$

$$\longrightarrow Y(s) = \left(\frac{e^{-1}}{2}+5\right)\frac{1}{(s+2)^2} + \left(2 - \frac{e^{-1}}{4}\right)\frac{1}{s+2} + \frac{1}{4s}$$

Scanned by CamScanner

aplicando la inversa de Laplace

$$y(t) = \left(\left(\frac{e^{-1}}{2} + 5 \right) t e^{-2t} + \left(2 - \frac{e^{1}}{4} \right) e^{-2t} + \frac{1}{4} \right) u(t)$$

Scanned by CamScanner