PR05

Montiel Cruz Jorge de Jesús

a. Muestre que

$$r_{xy}(t) = r_{yx}(-t)$$

Por definición

$$r_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\,y(\tau)\,d\tau$$

sea $u = t+\tau \quad u-t=\tau \quad du = d\tau$

$$\Rightarrow \int_{-\infty}^{\infty} x(u)\,y(u-t)\,du = \int_{-\infty}^{\infty} y(-t+u)\,x(u)\,du = r_{yx}(-t)$$

$$r_{xy}(t) = r_{yx}(-t)$$

b. Muestre que.

$$r_{xx}(t) = r_{xx}(-t)$$

análogamente a a)

$$r_{xx}(t) = \int_{-\infty}^{\infty} x(t+\tau)\,x(\tau)\,d\tau \quad \text{si } u = t+\tau,\ u-t=\tau,\ d\tau = du \quad \Rightarrow \int_{-\infty}^{\infty} x(u)\,x(u-t)\,du$$

$$= \int_{-\infty}^{\infty} x(-t+u)\,x(u)\,du = r_{xx}(-t)$$

$$r_{xx}(t) = r_{xx}(-t)$$

c.

Sean las señales, $x(t)$ y $y = x(t+T)$ expresa $r_{xy}(t)$ y $r_{yy}(t)$ en términos de $r_{xx}(t)$

Sol.

$$r_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)y(\tau)\,d\tau = \int_{-\infty}^{\infty} x(t+\tau)x(\tau+T)\,d\tau \ldots (1)$$

$$r_{xy}(t) = \int_{-\infty}^{\infty} x(t+T+\tau)x(\tau+T)\,d\tau \ldots (2)$$

$$r_{xx}(t) = \int_{-\infty}^{\infty} x(t+\tau)x(\tau)\,d\tau \ldots (3)$$

de (3) si $t \to t+T$

$$\Rightarrow \quad r_{xx}(t+T) = \int_{-\infty}^{\infty} x(t+T+\tau)\,x(\tau+T)\,d\tau \quad \text{que es igual a (2)}$$

$$r_{yy}(t) = r_{xx}(t+T)$$

$$r_{xy}(t) = \int_{-\infty}^{\infty} x(t)\Big|_{t\to t+\tau}\, y(t)\Big|_{t\to\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} x(t+\tau)\,x(t+T)\Big|_{t\to\tau}\, d\tau$$

$$= \int_{-\infty}^{\infty} x(t+\tau)\,x(\tau+T)\,d\tau$$

$u = \tau + T \quad \tau = u - T$

$$\int_{-\infty}^{\infty} x(t+u-T)\,x(u)\,du = \int_{-\infty}^{\infty} x(p+u)\,x(u)\,du$$

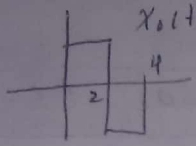
$p = t - T$

$$= r_{xx}(p) = r_{xx}(t-T)$$

d. Realiza la autocorrelación de

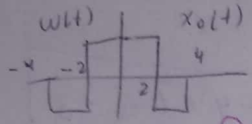
$$x_0(t) = u(t) - 2u(t-2) + u(t-4)$$

Sol.



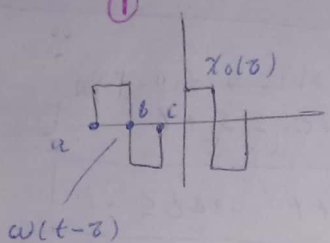
$$r_{x_0x_0}(t) = x_0(t) * x_0(-t)$$

$$x_0(-t) = w(t) \rightarrow r_{x_0x_0}(t) = x_0(t) * w(t)$$



$$x_0(t) * w(t) = \int_{-\infty}^{\infty} x_0(\tau)\, w(t-\tau)\, d\tau = r_{x_0x_0}(t)$$

①



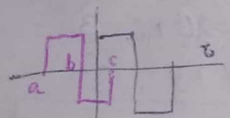
$a = t$

$b = 2+t$

$c = 4+t$

$$\boxed{r_{x_0x_0}(t) = 0 \qquad t < -4}$$

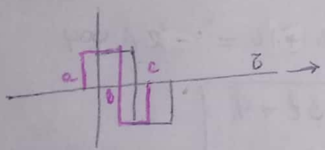
②



$$\rightarrow -\int_0^c d\tau = -c = -t-4 \qquad r_{x_0x_0}(t) = -t-4$$

$$\boxed{-4 \leq t < -2}$$

③



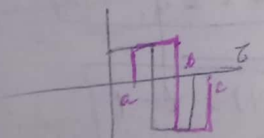
$$\rightarrow \int_0^b d\tau - \int_b^2 d\tau + \int_2^c d\tau = \int_0^{2+t} d\tau - \int_{2+t}^{2} d\tau + \int_2^{4+t} d\tau$$

$$= 2+t+t+2+t = 3t+4$$

$$r_{x_0x_0}(t) = 3t+4$$

$$\boxed{-2 \leq t < 0}$$

④



$$\rightarrow \int_a^2 d\tau - \int_2^b d\tau + \int_b^4 d\tau = 2-a+2-b+4-b$$

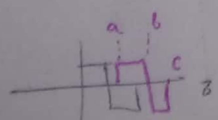
$$= 8-2b-a$$

$$= 8-(4+2t)-t = -3t+4$$

$$r_{x_0x_0}(t) = -3t+4$$

$$\boxed{0 \leq t < 2}$$

⑤

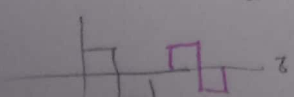


$$\rightarrow -\int_a^4 d\tau = a-4 = t-4 \qquad r_{x_0x_0}(t) = t-4$$

$$\boxed{2 \leq t \leq 4}$$

$$r_{x_0x_0}(t) = \begin{cases} -t-4 & -4 \leq t < -2 \\ 3t+4 & -2 \leq t < 0 \\ -3t+4 & 0 \leq t < 2 \\ t-4 & 2 \leq t \leq 4 \\ 0 & \text{otro caso} \end{cases}$$

⑥



$$\rightarrow \boxed{r_{x_0x_0}(t) = 0 \qquad 4 < t}$$

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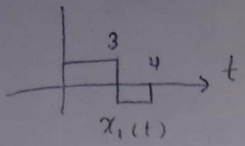
e. Realiza la autocorrelación de

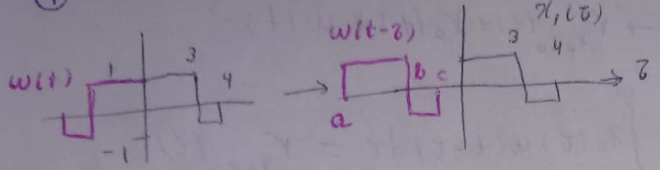
$x_1(t) = u(t) - 2u(t-3) + u(t-4)$



$r_{x_1x_1}(t) = x_1(t) * x_1(-t)$

$x_1(-t) = w(t)$

(1)



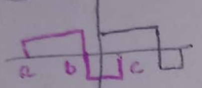
$a = t$

$b = 3+t$

$c = 4+t$

$$\boxed{r_{x_1x_1}(t) = 0 \qquad t \leq -4}$$

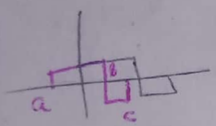
(2)



$$-\int_0^c d\tau = -c = -4-t$$

$$\boxed{r_{x_1x_1}(t) = -4-t \qquad -4 < t \leq -3}$$

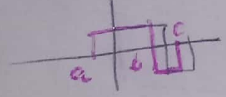
(3)



$$\int_0^b d\tau - \int_b^c d\tau = b + b - c = 2b - c = 2+t$$

$$\boxed{r_{x_1x_1}(t) = 2+t \qquad -3 < t \leq -1}$$

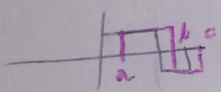
(4)



$$\int_0^b d\tau - \int_b^3 d\tau + \int_3^c d\tau = b + b - 3 + c - 3 = 2b + c - 6$$
$$= 3t + 4$$

$$\boxed{r_{x_1x_1}(t) = 3t+4 \qquad -1 < t \leq 0}$$

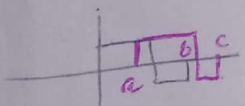
(5)



$$\int_a^3 d\tau - \int_3^b d\tau + \int_b^4 d\tau = 3 - a + 3 - b + 4 - b = -3t + 4$$

$$\boxed{\begin{matrix} r_{x_1x_1}(t) = -3t+4 \\ 0 < t \leq 1 \end{matrix}}$$

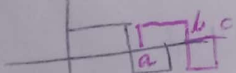
(6)



$$\Rightarrow \int_a^3 d\tau - \int_3^4 d\tau = 3 - a - 1 = 2 - t$$

$$\boxed{\begin{matrix} r_{x_1x_1}(t) = 2-t \\ 1 < t \leq 3 \end{matrix}}$$

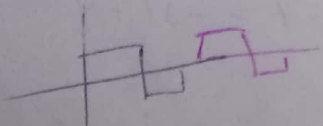
(7)



$$-\int_a^4 d\tau = a - 4 = t - 4$$

$$\boxed{\begin{matrix} r_{x_1x_1}(t) = t-4 \\ 3 < t \leq 4 \end{matrix}}$$

(8)



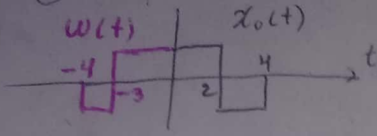
$$\boxed{\begin{matrix} r_{x_1x_1}(t) = 0 \\ t > 4 \end{matrix}}$$

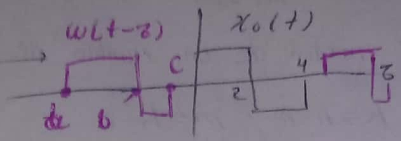
$$r_{x_1x_1}(t) = \begin{cases} -4-t & -4 < t \leq -3 \\ 2+t & -3 < t \leq -1 \\ 3t+4 & -1 < t \leq 0 \\ -3t+4 & 0 < t \leq 1 \\ 2-t & 1 < t \leq 3 \\ t-4 & 3 < t \leq 4 \\ 0 & \text{otro caso} \end{cases}$$

f.) Realiza la correlación de $\gamma_{x_0x_1}(t)$

$x_0(t) = u(t) - 2u(t-2) + u(t-4)$ $x_1(t) = u(t) - 2u(t-3) + u(t-4)$

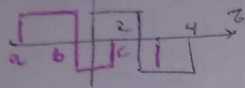
1. $\gamma_{x_0x_1}(t) = x_0(t) * x_1(-t)$ $x_1(-t) = w(t) \rightarrow \gamma_{x_0x_1}(t) = x_0(t) * w(t)$



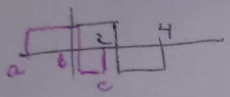


$\boxed{\gamma_{x_0x_1}(t) = 0 \text{ si } t \leq -4 \text{ ó si } t > 4}$

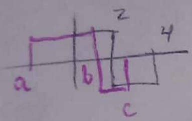
$a = t; \; b = 3+t; \; c = 4+t$

2. 

$-\int_0^c d\tau = -c = -4-t \qquad -4 < t \leq -3$

3. 

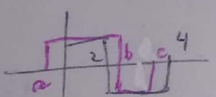
$\rightarrow \int_0^b d\tau - \int_b^c d\tau = 2b - c = 2+t \qquad -3 < t \leq -2$

4. 

$\rightarrow \int_0^b d\tau - \int_b^2 d\tau + \int_2^c d\tau = b + b - 2 - c - 2 = 2b + c - 4$

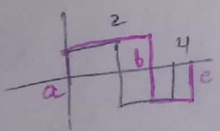
$= 3t + 6$

$-2 < t \leq -1$

5. 

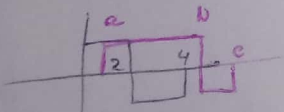
$\rightarrow \int_0^2 d\tau - \int_2^b d\tau + \int_b^c d\tau = 2 + 2 - b + c - b = 4 - 2b + c = 2 - t$

$-1 < t \leq 0$

6. 

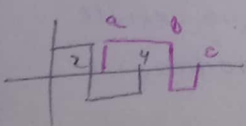
$\rightarrow \int_a^2 d\tau - \int_2^b d\tau + \int_b^4 d\tau = 2 - a + 2 - b + 4 - b = 8 - a - 2b = 2 - 3t$

$0 < t \leq 1$

7. 

$\rightarrow \int_a^2 d\tau - \int_2^4 d\tau = 2 - a - 2 = -a = -t$

$1 < t \leq 2$

8. 

$\rightarrow -\int_a^4 d\tau = a - 4 = t - 4$

$2 < t \leq 4$

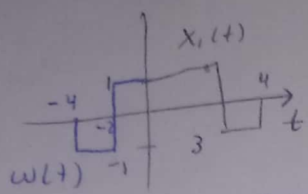
$$\gamma_{x_0x_1}(t) = \begin{cases} -4-t & -4 \leq t \leq -3 \\ 2+t & -3 < t \leq -2 \\ 3t+6 & -2 < t \leq -1 \\ 2-t & -1 < t \leq 0 \\ 2-3t & 0 < t \leq 1 \\ -t & 1 < t \leq 2 \\ t-4 & 2 < t \leq 4 \\ 0 & \text{otro caso} \end{cases}$$

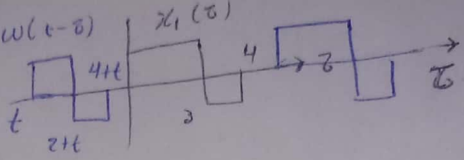
g) Realiza la correlación de $r_{x_1 x_0}(t)$

$x_1(t) = u(t) - 2u(t-3) + u(t-4)$ $x_0(t) = u(t) - 2u(t-2) + u(t-4)$

$r_{x_1 x_0}(t) = x_1(t) * x_0(-t) \to w(t) = x_0(-\tau) \to r_{x_1 x_0} = x_1(t) * w(t)$

①

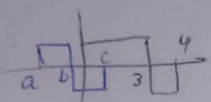




$r_{x_1 x_0}(t) = 0$ si $t \le -4$

ó si $t > 4$

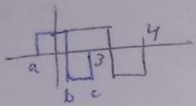
②



$a = t$
$b = 2+t$
$c = 4+t$

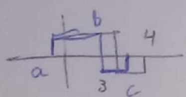
$\to -\int_0^c d\tau = -c = -4-t; \ -4 < t \le -2$

③



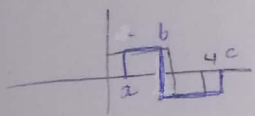
$\to \int_0^b d\tau - \int_b^c d\tau = b+b-c = 2b-c = t; \ -2 < t \le -1$

④



$\to \int_0^b d\tau - \int_b^3 d\tau + \int_3^c d\tau = 2b+c-6 = 3t+2; \ -1 < t \le 0$

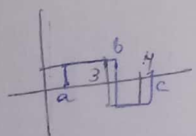
⑤



$\to \int_a^b d\tau - \int_b^3 d\tau + \int_3^4 d\tau = b-a+b-3+1 = 2b-a-2$

$= t+2; \ 0 < t \le 1$

⑥

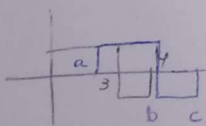


$\to \int_a^3 d\tau + \int_3^b (-1)\, d\tau + \int_b^4 d\tau = 3-a+3-b+4-b$

$= 10-2b-a = -3t+6$

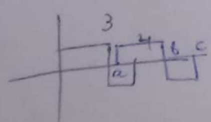
$1 < t \le 2$

⑦



$\to \int_a^3 d\tau + \int_3^4 (-1)\, d\tau = 3-a-1 = -a+2 = 2-t; \ 2 < t \le 3$

⑧



$\to -\int_a^4 d\tau = a-4 = t-4; \ 3 < t \le 4$

$$r_{x_1 x_0}(t) = \begin{cases} -4-t & -4 < t \le -2 \\ t & -2 < t \le -1 \\ 3t+2 & -1 < t \le 0 \\ t+2 & 0 < t \le 1 \\ -3t+6 & 1 < t \le 2 \\ 2-t & 2 < t \le 3 \\ t-4 & 3 < t \le 4 \\ 0 & \text{otro caso} \end{cases}$$