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1. Resolver el sistema de ecuaciones diferenciales

$$x' = 3x - 2y$$

$$y' = 3y - 2x$$

$$x(0) = y(0) = 1$$

Por Laplace

$$\left.\begin{array}{l} (1)\quad s\,X(s) - 1 = 3\,X(s) - 2\,Y(s) \\ (2)\quad s\,Y(s) - 1 = 3\,Y(s) - 2\,X(s) \end{array}\right\}$$

de (1)

$$-1 = 3\,X(s) - 2Y(s) - s\,X(s)$$

de (2)

$$-1 = 3Y(s) - 2\,X(s) - s\,Y(s)$$

Igualando

$$3\,X(s) - 2Y(s) - s\,X(s) = 3Y(s) - 2X(s) - s\,Y(s)$$

$$(5-s)\,X(s) = (5-s)\,Y(s)$$

$$X(s) = Y(s)$$

en (1)

$$s\,X(s) - 1 = 3\,X(s) - 2\,X(s)$$

$$-1 = (1-s)\,X(s) \Rightarrow X(s) = -\frac{1}{1-s} = \frac{1}{s-1}$$

cuya inversa de Laplace es e^{-t}

así que $X(t) = e^{t} = y(t)$

2. Resolver para $y(t)$

$$y'(t) - 2\int_0^t y(\tau)\,\text{sen}(t-\tau)\,d\tau = 1$$

$$y(0) = -1$$

Por Laplace

$$sY(s) + 1 - 2\mathcal{L}\{y(t) * \text{sen}(t)\} = \frac{1}{s}$$

$$= sY(s) + 1 - \frac{2Y(s)}{s^2+1} = \frac{1}{s}$$

$$Y(s) = \frac{(s^2+1)(1-s)}{s(s^3+s-2)} = \frac{-1-s^2}{s^3+s^2+2s} = \frac{-1-s^2}{s(s^2+s+2)}$$

Separando en fracciones parciales

$$= \frac{1-s}{2(s^2+s+2)} - \frac{1}{2s} = \frac{1}{2}\left(\frac{1}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}} - \frac{s}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}}\right) - \frac{1}{2s}$$

$$y(t) = \frac{-1}{2}e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{7}\,t}{2}\right) + \frac{1}{\sqrt{7}}e^{-t/2}\,\text{sen}\left(\frac{\sqrt{7}\,t}{2}\right) - \frac{1}{2}$$

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4. La función Gamma o $\Gamma(t)$ se define como

$$\Gamma(t) = \int_0^\infty e^{-u} u^{t-1} du, \quad t > 0$$

$$\Gamma(1) = 1$$

$$\Gamma(2) = 1$$

$$\Gamma(3) = 2$$

$$\Gamma(4) = 6$$

tenemos que $\Gamma(n+1) = n!$

Demuestre que.

$$\mathcal{L}\{t^r\} = \frac{\Gamma(r+1)}{s^{r+1}} \quad r > -1$$

Demostración

$$\mathcal{L}\{t^r\} = \int_0^\infty t^r e^{-st} dt \quad ; \text{ si } \quad u = st, \quad \frac{du}{s} = dt$$

$$= \int_0^\infty t^r e^{-u} \frac{du}{s} = \int_0^\infty \left(\frac{u}{s}\right)^r e^{-u} \frac{du}{s} = \frac{1}{(s^r)s} \int_0^\infty u^r e^{-u} du$$

$$= \frac{1}{s^{r+1}} \int_0^\infty u^r e^{-u} du = \frac{\Gamma(r+1)}{s^r+1}$$