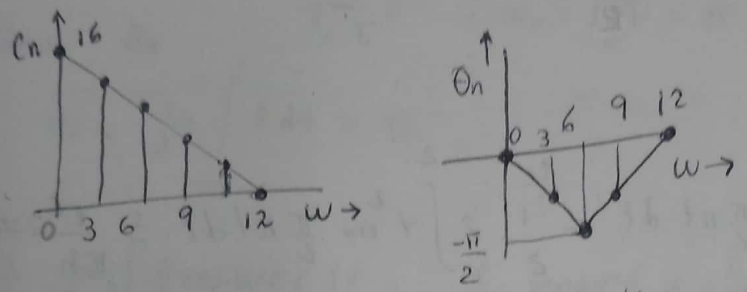
PRO 9 Montiel Cruz Jorge de Jesús

1. La figura 1 muestra el espectro trigonométrico de Fourier de una señal $f(t)$



a) escribe la serie de Fourier trigonométrica compacta de la señal $f(t)$

b) Escribe la serie de Fourier exponencial compleja de la señal $f(t)$ a partir de la serie de Fourier trigonométrica compacta.

c) Grafica el espectro exponencial de Fourier.

$$-mw + 16 = 0$$

$$\frac{16}{12} = m = \frac{4}{3} \rightarrow \frac{-4}{3}w + 16 \Rightarrow w = 3 \rightarrow 12$$

$$-mw = f.$$

$$\left(-\frac{1}{6}\right)\frac{-\pi}{2} = \frac{\pi}{12} \rightarrow \frac{-\pi}{12}w$$

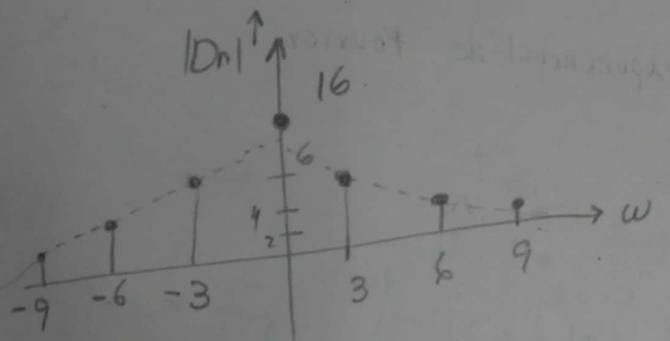
$w=0$	$C_n=16$
$w=3$	$C_n=12$
$w=6$	$C_n=8$
$w=9$	$C_n=4$
$w=12$	$C_n=0$

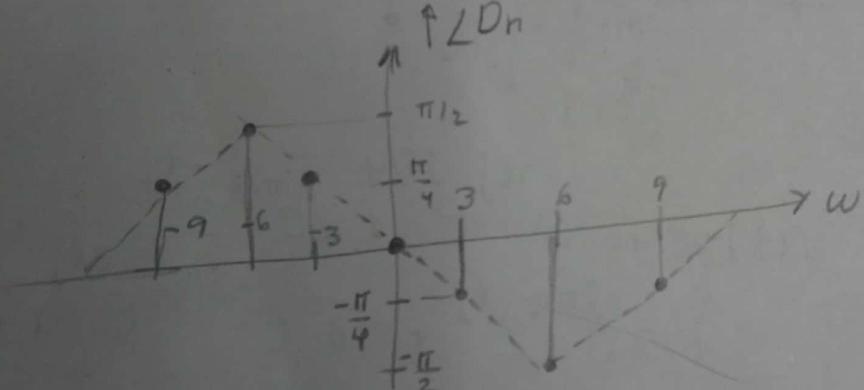
$w_0=0$	$\theta_n=0$
$w=3$	$\theta_n=-\frac{\pi}{4}$
$w=6$	$\theta_n=-\frac{\pi}{2}$
$w=9$	$\theta_n=-\frac{\pi}{4}$
$w=12$	$\theta_n=0$

a)
$$S_f(t) = 16 + 12\left(\cos\left(3t - \frac{\pi}{4}\right)\right) + 8\cos\left(6t - \frac{\pi}{2}\right) + 4\cos\left(9t - \frac{\pi}{4}\right)$$

para b)
$$S_f(t) = 16 + \frac{12}{2}\left[e^{3jt}e^{-j\pi/4} + e^{-3jt}e^{j\frac{\pi}{4}}\right] + \frac{8}{2}\left[e^{6jt}e^{-j\pi/2} + e^{-6jt}e^{j\pi/2}\right]$$
$$+ \frac{4}{2}\left[e^{9jt}e^{-j\frac{\pi}{4}} + e^{-9jt}e^{j\pi/4}\right]$$
$$= 16 + 6\left[e^{3jt}e^{-j\pi/4} + e^{-3jt}e^{j\pi/4}\right] + 4\left[e^{6jt}e^{-j\pi/4} + e^{-6jt}e^{j\pi/2}\right] + 2\left[e^{9jt}e^{-j\pi/4} + e^{-9jt}e^{j\pi/4}\right]$$

Para c).





espectro exponencial de Fourier

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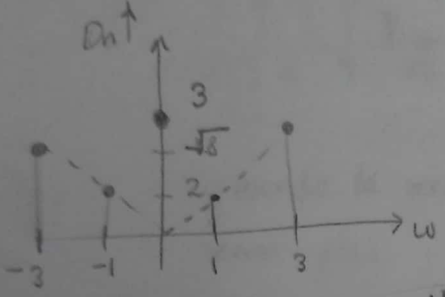
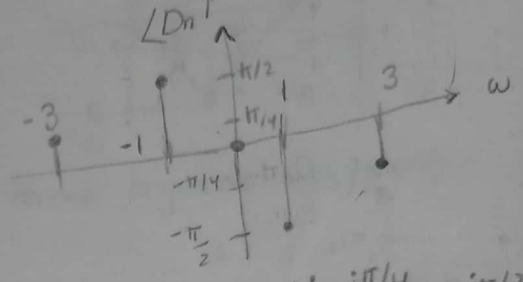
2. La serie de Fourier exponencial compleja de cierta función periódica es dada por

$$f(t) = (2+2j)e^{-3tj} + 2je^{-tj} + 3 - 2je^{tj} + (2-2j)e^{3tj}$$

$e^{j(at+\phi)}$

a) grafica el espectro exponencial de Fourier





$2+2j = \sqrt{8}\,e^{j\pi/4}$

$2-2j = -\sqrt{8}\,e^{-j\pi/4}$

$2j = 2e^{j\pi/2}$

$-2j = 2e^{-j\pi/2}$

$$f(t) = \sqrt{8}\,e^{-3tj}e^{j\pi/4} + 2e^{j\pi/2}e^{-tj} + 2e^{-j\pi/2}e^{tj} + \sqrt{8}\,e^{-j\pi/4}e^{3tj} + 3$$

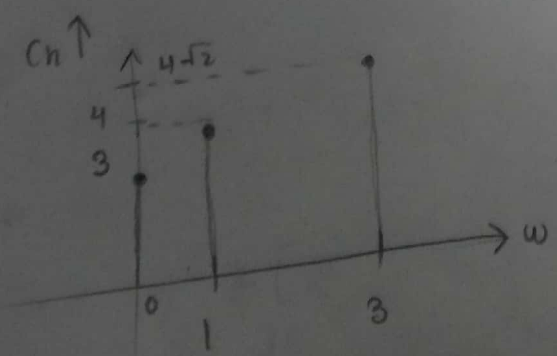
para la trigonométrica

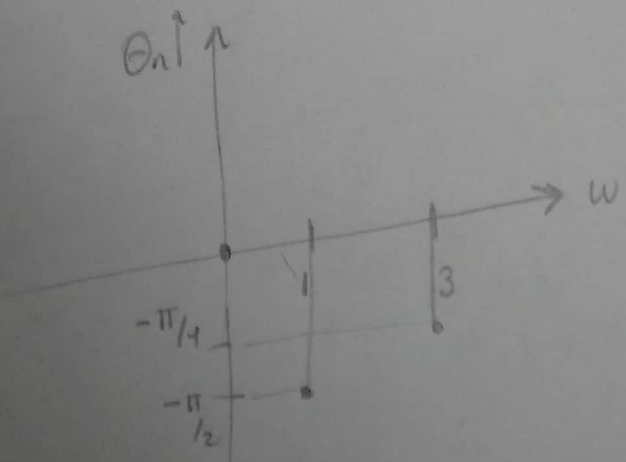
$C_n = 2\,\mathrm{Re}\{D_n\}$ $\qquad C_0 = D_0$

b) grafica el espectro trigonométrico de Fourier

para $f(t) = 2(\sqrt{8}\cos(3t - \pi/4)) + 2(2\cos(t - \pi/2)) + 3$

$= 4\sqrt{2}\cos(3t - \pi/4) + 4\cos(t - \pi/2) + 3$



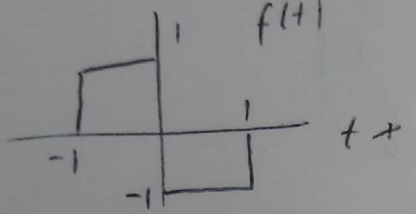


serie trigonométrica compacta.

c) $f(t) = 4\sqrt{2}\cos(3t - \frac{\pi}{4}) + 4\cos(t - \frac{\pi}{2}) + 3$

1. Represente $f(t)$ de la figura mediante su serie Legendre Fourier sobre el intervalo $(-1,1)$

Calcula solo los dos primeros coeficiente no cero de la serie.



usando

$$\int_{-1}^{1} P_m(t)\, P_n(t)\, dt = \begin{cases} \frac{2}{2m+1} & m=n \\ 0 & m\neq n \end{cases}$$

donde

$$P_n(t) = \frac{1}{n!\,2^n}\frac{d^n}{dt^n}(t^2-1)^n \qquad n=0,1,2,\dots$$

sabiendo que

$$f(t) = \sum_{n=0}^{\infty} a_n P_n(t)$$

si

$$P_m(t) f(t) = \sum_{n=0}^{\infty} a_n P_n(t) P_m(t)$$ integrando & suponiendo $m=n$.

$$\int_{-1}^{1} P_m(t) f(t)\,dt = \sum_{n=0}^{\infty} a_n \int_{-1}^{1} P_n(t) P_m(t)\, dt$$

para $m=0$

$$\int_{-1}^{0} dt + \int_{0}^{1}(-1)dt = 0 = \sum_{n=0}^{\infty} a_0 \frac{2}{1}$$

$$1 + (-1) = 0 \Rightarrow a_0 = 0.$$

$m=1$

$$\int_{-1}^{0} t\,dt - \int_{0}^{1} t\,dt = -\frac{1}{2} - \frac{1}{2} = -1 = a_1 \frac{2}{2+1}$$

$$a_1 = -\frac{3}{2}$$

a_2 si $m=2$

$$\int_{-1}^{0} \frac{1}{2}(3t^2-1)dt - \int_{0}^{1}\frac{1}{2}(3t^2-1)dt = \frac{1}{2}(t^3-t)\Big|_{-1}^{0} - \frac{1}{2}(t^3-t)\Big|_{0}^{1} = 0$$

$$a_2 = 0.$$

$a_3 \Rightarrow$ si $m=3$.

$$\int_{-1}^{0} \frac{1}{2}(5t^3-3t)\,dt - \int_{0}^{1} \frac{1}{2}(5t^3-3t)\,dt = a_3\,\frac{2}{7}$$

$$\frac{1}{2}\left(\left(\frac{5}{4}t^4-\frac{3}{2}t^2\right)\Big|_{-1}^{0} - \left(\frac{5}{4}t^4-\frac{3t^2}{2}\right)\Big|_{0}^{1}\right)$$

$$= \frac{1}{2}\cdot\left(\frac{5}{4}-\frac{3}{2}-\frac{5}{4}+\frac{3}{2}\right) = 0 \Rightarrow a_3 = 0$$

$$a_1 = -\frac{3}{2}$$