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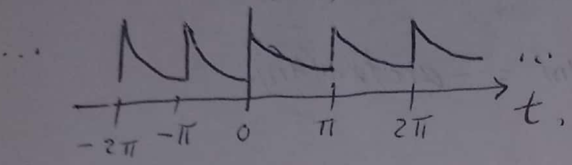
6.1 encuentre la serie de Fourier compacta de la señal $x(t)$ mostrada, grafique la amplitud y los espectros de fase para $x(t)$.

$$x(t) = e^{-t/2}$$



$T_0 = \pi$

$\omega_0 = \frac{2\pi}{T_0} = 2$ así $S_f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2nt) + b_n \text{sen}(2nt)$

donde

$$a_0 = \frac{1}{T_0}\int_{\langle T_0\rangle} x(t)\,dt \;;\; a_n = \frac{2}{T_0}\int_{\langle T_0\rangle} x(t)\cos(2nt)\,dt \;;\; b_n = \frac{2}{T_0}\int_{\langle T_0\rangle} x(t)\,\text{sen}(2nt)\,dt$$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} e^{-t/2}\,dt = -\frac{2}{\pi}\left(e^{-\pi/2} - 1\right) = 0.504$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\cos(2nt)\,dt = \frac{2}{\pi}\int_a^b e^{u}\cos(4nu)(-2du) = \frac{-4}{\pi}\int_0^{-\pi/2} e^{u}\cos(4nu)\,du$$

$u = -t/2$

$-2u = t \to \overset{t=0}{a = 0}$

$-2du = dt. \quad \overset{t=\pi}{b = -\pi/2}$

$$= \frac{4}{\pi}\int_{-\pi/2}^{0} e^{u}\cos(4nu)\,du.$$

$$a_n = \frac{4}{\pi}\left(\frac{e^{-\pi/2} - 1}{16n^2+1}\right) = 0.504\,\frac{2}{16n^2+1}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\,\text{sen}(2nt)\,dt$$ análogamente a a_n.

$u = -t/2 \quad du = \frac{-dt}{2}$

$$b_n = \frac{-4}{\pi}\int_0^{-\pi/2} e^{u}\,\text{sen}(4nu)\,du = \frac{4}{\pi}\int_{-\pi/2}^{0} e^{u}\,\text{sen}(4nu)\,du = \frac{4.032\,n}{16n^2+1} = \frac{0.504(8n)}{16n^2+1}$$

así

$$x(t) = 0.504\left[1 + \sum_{n=1}^{\infty} \frac{2}{1+16n^2}\left(\cos 2nt + 4n \operatorname{sen} 2nt\right)\right]$$

usando

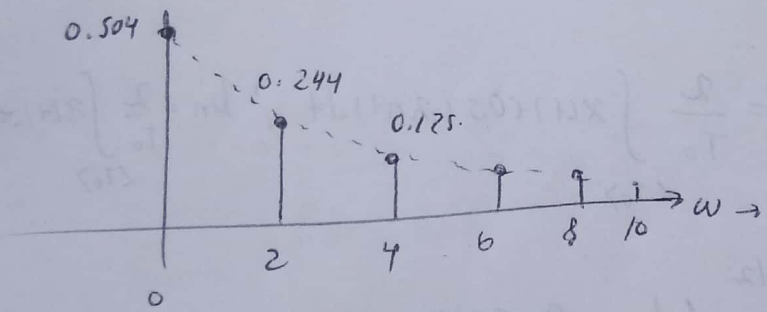
$$C_0 = a_0 = 0.504.$$

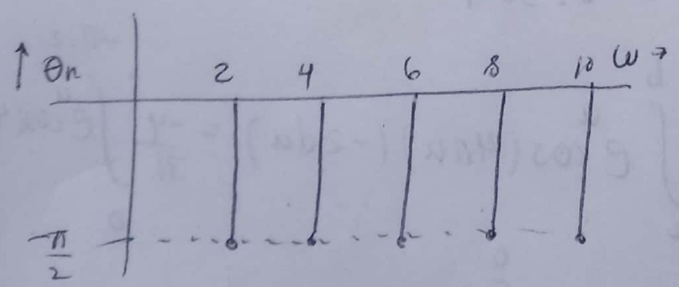
$$C_n = \sqrt{a_n^2 + b_n^2} = 0.504\left(\frac{2}{\sqrt{1+16n^2}}\right)$$

$$\theta_n = \arctan\left(\frac{-b_n}{a_n}\right) = \arctan(-4n) = -\arctan(4n)$$

así

$$x(t) = 0.504 + 0.504 \sum_{n=1}^{\infty} \frac{2}{\sqrt{1+16n^2}} \cos(2nt - \arctan(4n))$$





6.2. encuentre la serie de Fourier compacta de $x(t)$ y bosqueje el espectro de Fourier de fase y amplitud.

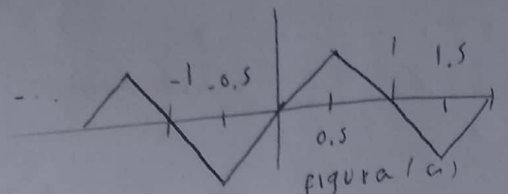


figura (a)

$T_0 = 2 \rightarrow \omega_0 = \frac{2\pi}{2} = 2$

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\pi t + b_n \operatorname{sen} \pi n t.$$

$$x(t) = \begin{cases} 2At & |t| < \frac{1}{2} \\ 2A(1-t) & \frac{1}{2} < t < \frac{3}{2} \end{cases}$$

sabemos que para una señal como la gráfica en

$$\frac{1}{T_0}\int_{\langle T_0\rangle} x(t)\,dt = 0 \quad \text{así} \quad a_0 = 0$$

Para a_n

$$a_n = \frac{2}{2}\int_{-1/2}^{3/2} x(t)\cos(\pi n t)\,dt = \int_{-1/2}^{1/2} 2At\cos\pi n t\,dt + \int_{1/2}^{3/2} 2A(1-t)\cos\pi n t\,dt$$

sabemos que función par * función impar = función impar.

$$y \quad \int_{-a}^{a} f(t)_{imp}\,dt = 0 \quad \text{para} \quad 2A\int_{1/2}^{3/2}\cos\pi n t = \frac{2A}{\pi n}\left(\operatorname{sen}\frac{3\pi n}{2} - \operatorname{sen}\left(\frac{\pi n}{2}\right)\right)$$

$= 0$ para $n > 0$ & $\in \mathbb{Z}$

así

$a_n = 0$

$$\text{Para } b_n = \int_{-1/2}^{1/2} 2At\operatorname{sen} n\pi t\,dt + \int_{1/2}^{3/2} 2A(1-t)\operatorname{sen} n\pi t\,dt$$

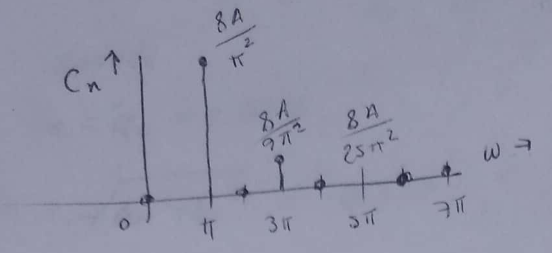
$$= \frac{8A}{n^2\pi^2}\operatorname{sen}\frac{n\pi}{2}$$

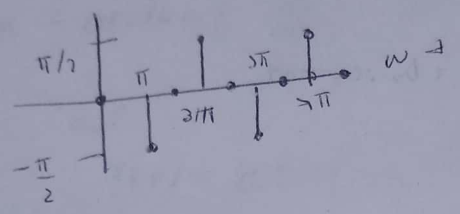
$$x(t) = \sum_{n=1}^{\infty}\frac{8A}{n^2\pi^2}\operatorname{sen}\left(\frac{\pi n}{2}\right)\operatorname{sen}\pi n t = \frac{8A}{\pi^2}\left[\operatorname{sen}\pi t - \frac{1}{9}\operatorname{sen}3\pi t + \frac{1}{25}\operatorname{sen}5\pi t - \ldots\right.$$

usando $\sin kt = \cos(kt - 90°)$

$-\operatorname{sen} kt = \cos(kt + 90°)$

$$x(t) = \frac{8A}{\pi^2}\left[\cos(\pi t - 90°) + \frac{1}{9}\cos(3\pi t + 90°) + \frac{1}{25}\cos(5\pi t - 90°) - \ldots\right]$$





6.3 una señal periódica es representada por la series de Fourier trigonométrica

$$x(t) = 2 + 3\cos 2t + 4\,\text{sen}\,2t + 2\,\text{sen}(3t + 30°) - \cos(7t + 150°)$$

exprese como la serie de Fourier trigonométrica compacta y grafique el espectro de amplitud y fase

usando

$$x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$$

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right)$$

$$3\cos 2t + 4\,\text{sen}\,2t = 5\cos(2t - 53.13°)$$

además

$$\text{sen}(3t + 30°) = \cos(3t - 60°)$$

y

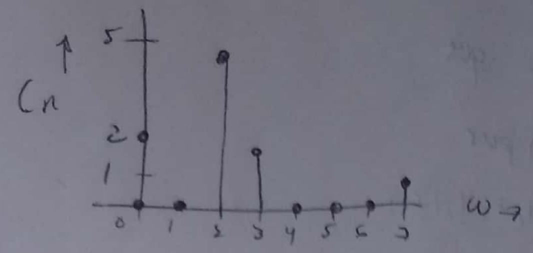
$$-\cos(7t + 150°) = \cos(7t - 30°)$$

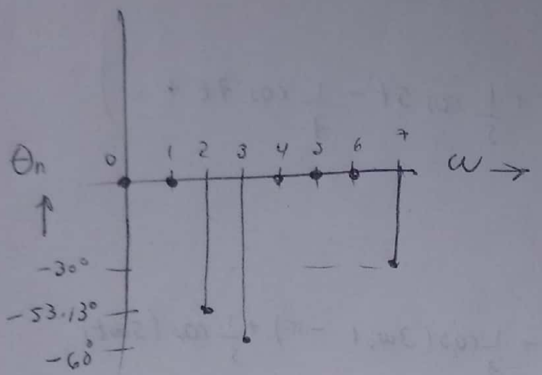
así $x(t) = 2 + 5\cos(2t - 53.13) + 2\cos(3t - 60°) + \cos(7t - 30°)$

Para este caso solo 4 componentes estan presentes (incluyendo dc)

para $\omega_3 = 2$ amplitud es 5, $\theta = -53.13$ · $\omega = 3$ $A = 2$ $\theta = -60$ · $\omega = 7$ $A = 1$ $\theta = -30$

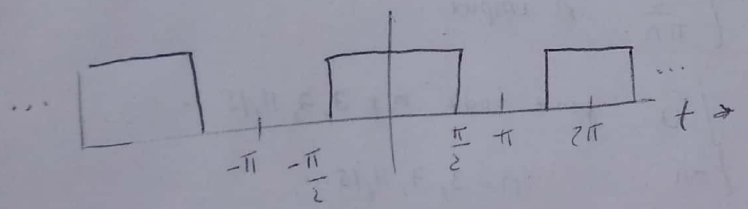
así





6.4.

encuentre la serie de Fourier trigonométrica compacta para el pulso cuadrado periódico mostrado, además grafique el espectro de fase y amplitud.



$T_0 = 2\pi \Rightarrow w_0 = 1.$

$$\rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \operatorname{sen} nt$$

donde

$$a_0 = \frac{1}{T_0}\int_{<T_0>} x(t)\,dt \rightarrow a_0 = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} dt = \frac{1}{2}$$ pues en otro intervalo es 0

$$a_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos nt\,dt = \frac{2}{n\pi}\operatorname{sen}\left(\frac{\pi n}{2}\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \operatorname{sen} nt\,dt = 0 \Longrightarrow$$ todos los términos con seno(wt) son 0.

$$x(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \operatorname{sen}\left(\frac{\pi n}{2}\right) \cos nt.$$

pero nos damos cuenta que

$$\operatorname{sen}\frac{\pi n}{2} = \begin{cases} 0 & \text{si } n \text{ par} \\ \frac{2}{n\pi} & n = 1, 5, 9, 13\ldots \\ -\frac{2}{n\pi} & n = 3, 7, 11, 15\ldots \end{cases}$$

así

$$x(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos t - \frac{1}{3}\cos 3t + \frac{1}{5}\cos 5t - \frac{1}{7}\cos 7t + \ldots\right)$$

usando

$$-\cos x = \cos(x - \pi)$$

$$\Rightarrow x(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_0 t + \frac{1}{3}\cos(3\omega_0 t - \pi) + \frac{1}{5}\cos(5\omega t)\right.$$

$$\left. + \frac{1}{7}\cos(7\omega_0 t - \pi) + \frac{1}{9}\cos(9\omega_0 t) + \ldots\right]$$

$$C_0 = \frac{1}{2}$$

$$C_n = \begin{cases} 0 & n \text{ par} \\ \frac{2}{\pi n} & n \text{ impar.} \end{cases}$$

$$\theta_n = \begin{cases} 0 & \text{para toda } n \neq 3, 7, 11, 15\ldots \\ -\pi & n = 3, 7, 11, 15\ldots \end{cases}$$

Pero si permitimos que C_n tenga valores negativos. es decir, que $\omega_n = 0$ para cualquier componente entonces sólo será necesario el espectro de amplitud.