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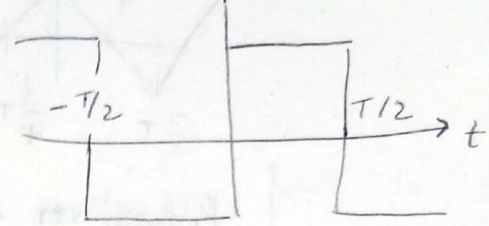
1.10 Encontrar la serie de Fourier para la función $f(t)$ definida por.

$$f(t) = \begin{cases} -1 & -\frac{T}{2} < t < 0 \\ 1 & 0 < t < \frac{T}{2} \end{cases}$$



y $f(t+T) = f(t)$

$T_0 = T$

$\omega_0 = \frac{2\pi}{T_0} \rightarrow \omega_0 = \frac{2\pi}{T} \rightarrow \omega_0 T = 2\pi$

Para a_n

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos \omega_0 n t \, dt = \frac{2}{T}\left[\int_{-T/2}^{0} -\cos \omega_0 n t \, dt + \int_{0}^{T/2} \cos \omega_0 n t \, dt\right]$$

$$= \frac{2}{T}\left[\frac{-1}{\omega_0 n} \operatorname{sen} \omega_0 n t \Big|_{-T/2}^{0} + \frac{1}{\omega_0 n} \operatorname{sen} \omega_0 n t \Big|_{0}^{T/2}\right]$$

$$= \frac{2}{T\omega_0 n}\left[-\operatorname{sen} \frac{2\pi n}{2} + \operatorname{sen} \pi n\right] = 0 \quad \text{para } n \neq 0$$

es fácil ver que el promedio de $f(t)$ es $0. \Rightarrow a_n = 0$

Para b_n $\quad b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t) \operatorname{sen} \omega_0 n t \, dt$

$$= \frac{2}{T}\left[-\int_{-T/2}^{0} \operatorname{sen} \omega_0 n t \, dt + \int_{0}^{T/2} \operatorname{sen} \omega_0 n t \, dt\right] = \frac{2}{T}\left[\frac{1}{\omega_0 n} \cos \omega_0 n t \Big|_{-T/2}^{0} - \frac{1}{\omega_0 n} \cos \omega_0 n t \Big|_{0}^{T/2}\right]$$

$$= \frac{2}{T\omega_0 n}\left[1 - \cos \pi n - \cos \pi n + 1\right] = \frac{2}{\pi n}\left[1 - \cos \pi n\right]$$

pero $\cos \pi n = (-1)^n$ para $n \in \mathbb{N} \rightarrow \frac{2}{\pi n}\left[1 - (-1)^n\right]$

así

$$f(t) = \frac{4}{\pi}\sum_{n = \text{impar}}^{\infty} \frac{1}{n} \operatorname{sen} n\omega_0 t$$

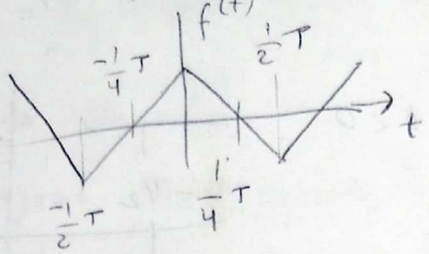
si n es impar.

$b_n = 0$

si par $b_n = \frac{4}{\pi n}$

ó $$f(t) = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}(1 - \cos(n\pi)) \operatorname{sen} \omega_0 n t$$

1.11 encontrar la serie de Fourier para la siguiente señal.



Podemos ver a f(t) analíticamente como.

$$f(t) = \begin{cases} 1 + \frac{4t}{T} ; & -\frac{T}{2} < t \leq 0 \\ 1 - \frac{4t}{T} ; & 0 \leq t < \frac{T}{2} \end{cases}$$

donde, de la figura es fácil ver que

$$a_0 = 0$$

así $$a_n = \frac{2}{T} \int_{-1/2 T}^{1/2 T} f(t) \cos(n\omega_0 t)\, dt$$

$$= \frac{2}{T} \left[\int_{-\frac{T}{2}}^{0} \left(1 + \frac{4t}{T}\right) \cos n\omega_0 t\, dt + \int_{0}^{T/2} \left(1 - \frac{4t}{T}\right) \cos n\omega_0 t\, dt \right]$$

$u = 1 + \frac{4t}{T}$ si $t = -\frac{T}{2}$ $u = -1$ si $t = 0$ $u = 1$ $t = \frac{(u-1)T}{4}$

$du = \frac{4}{T} dt$. * primera integral

$$= \frac{2}{T} \left[\frac{T}{4} \int_{-1}^{1} u \cos\left(n\omega_0 \frac{(u-1)T}{4}\right) du + \int_{0}^{T/2} \left(1 - \frac{4t}{T}\right) \cos n\omega_0 t\, dt \right]$$

$w = 1 - \frac{4t}{T}$ $dw = \frac{-4}{T} dt$

si $t = 0$ $w = 1$; $t = T/2$ $w = -1$

$$= \frac{2}{T} \left[\frac{T}{4} \frac{4}{n^2\pi^2} (1 - \cos n\pi) + \right.$$

$$+ \int_{1}^{-1} w \cos\left(n\omega_0 \frac{(w-1)T}{-4}\right)\left(\frac{T}{-4} dw\right)$$

$$= \frac{2}{T} \left[\frac{4T}{4n^2\pi^2} (1 - \cos n\pi) + \frac{T}{4} \int_{-1}^{1} w \cos\left(n\omega_0 \frac{(w-1)}{4}\right) dw \right]$$

$$= 2 \left[2 \left(\frac{1}{n^2\pi^2} (1 - \cos n\pi) \right) \right] = \frac{4}{n^2\pi^2} (1 - \cos n\pi)$$

Para b_n

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\,\text{sen}(n w_0 t)\,dt$$

$$= \frac{2}{T}\left[\int_{-T/2}^{0}\left(1 + \frac{4t}{T}\right)\text{sen}(n w_0 t)\,dt + \int_{0}^{T/2}\left(1 - \frac{4t}{T}\right)\text{sen}(n w_0 t)\,dt\right]$$

$$= \frac{2}{T}\left[\int_{-T/2}^{T/2}\text{sen}(n w_0 t)\,dt + \int_{-T/2}^{0}\frac{4t}{T}\,\text{sen}(n w_0 t)\,dt + \int_{0}^{T/2} -\frac{4t}{T}\,\text{sen}\, n w_0 t\,dt\right]$$

$$t = -\tau$$

$$= \frac{2}{T}\left[\cancel{\int_{-T/2}^{T/2}\text{sen}(n w_0 t)\,dt} + \int_{T/2}^{0}\frac{-4\tau}{T}\,\text{sen}(n w(-\tau))(-d\tau) + \int_{0}^{T/2}\frac{-4t}{T}\,\text{sen}\, n w_0 t\,dt\right]$$

$$0$$

$$= \frac{2}{T}\left[+\int_{0}^{T/2}\frac{4\tau}{T}\,\text{sen}(n w \tau)\,d\tau - \int_{0}^{T/2}\frac{4t}{T}\,\text{sen}\, n w_0 t\,dt\right] = 0.$$

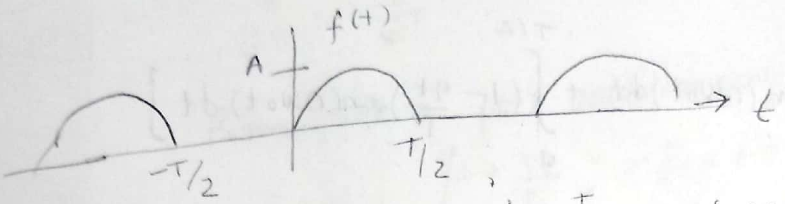
así

$$f(t) = \sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}(1-\cos n\pi)\cos(n w_0 t)$$

1.12. encuentre la serie de Fourier para

$$f(t) = \begin{cases} 0 & -\frac{T}{2} < t < 0 \\ A\,sen\,w_0 t & 0 < t \le \frac{T}{2} \end{cases}$$

y $f(t+T) = f(t)$, $w_0 = \frac{2\pi}{T}$



analizamos solo de $0 < t < \frac{T}{2}$ pues en otro intervalo es 0

$$a_0 = \frac{1}{T}\int_0^{T/2} A\,sen\,w_0 t\,dt = \frac{A}{Tw_0}\left(-\cos w_0 t\right)\Big|_0^{T/2} = \frac{A}{\pi}$$

$$a_n = \frac{2}{T}\int_0^{T/2} A\,sen(w_0 t)\cos(nw_0 t)\,dt$$

$u = w_0 t \quad dt = \frac{du}{w_0}$

$$= \frac{2A}{Tw_0}\int_0^{\pi} sen\,u\cos nu\,du = \frac{A}{\pi}\int_0^{\pi} sen\,u\cos nu\,du = \frac{A}{\pi}\int_0^{\pi}\left[\frac{sen((1+n)u) + sen((1-n)u)}{2}\right]du$$

Para $n=1 \rightarrow \frac{A}{2\pi}\int_0^{\pi} sen\,2u\,du = \frac{-A}{2\pi}(0) = 0$

si $a = 1+n$, $b = 1-n$

$$\frac{A}{2\pi}\int_0^{\pi}(sen\,au + sen\,bu)\,du = \frac{A}{2\pi}\left[-\frac{1}{a}\left[\cos a\pi - 1\right] - \frac{1}{b}\left(\cos b\pi - 1\right)\right]$$

$$a_n = \frac{A}{2\pi}\left[\frac{1-\cos((1+n)\pi)}{1+n} + \frac{1-\cos((1-n)\pi)}{1-n}\right]$$

para $n = 2, 3, 4, \ldots$

Para b_n

$$b_n = \frac{2}{T_0}\int_0^{T_0/2} A\,sen w_0 t\,sen\,nw_0 t\,dt = \frac{A}{T_0}\int_0^{T_0/2}\left[\cos((1-n)w_0 t) - \cos((1+n)w_0 t)\right]dt$$

$$= \frac{A}{T_0 w_0}\left[\frac{1}{1-n}\left(sen((1-n)\pi)\right) - \frac{1}{1+n}sen\left[(1+n)\pi\right]\right] = \frac{A}{(2\pi)}\left[\frac{sen((1-n)\pi)}{1-n} - \frac{sen((1+n)\pi)}{1+n}\right]$$

para cualquier n entera $n \geq 2$ $sen[(1-n)\pi] = 0$ y $sen[(1+n)\pi] = 0$

así $b_n = 0$, pero si $n = 1$.

$$\frac{2}{2}\int_0^{T/2} \operatorname{sen}^2 w_0 t \, dt = \frac{A}{2}$$

así pues

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\operatorname{sen}(w_0 t) + \sum_{n=2}^{\infty} \frac{A}{2\pi}\left[\frac{\operatorname{sen}((1-n)\pi)}{1-n} - \frac{\operatorname{sen}((1+n)\pi)}{1+n}\right]\cos(n w_0 t)$$

1.13. Desarrollar $f(t) = \operatorname{sen}^5 t$ en serie de Fourier

usamos

$$e^{\pm jn\theta} = \cos n\theta \pm j \operatorname{sen} n\theta$$

$$\cos n\theta = \frac{e^{jn\theta} + e^{-jn\theta}}{2}$$

$$\operatorname{sen} n\theta = \frac{e^{jn\theta} - e^{-jn\theta}}{2j}$$

así,

$$\operatorname{sen}^5 t = \left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right)^5 = \frac{1}{32j}\left(e^{j5t} - 5e^{j3t} + 10e^{jt} - 10e^{-jt} + 5e^{-j3t} - e^{-j5t}\right)$$

$$f(t) = \frac{5}{8}\operatorname{sen} t - \frac{5}{16}\operatorname{sen} 3t + \frac{1}{16}\operatorname{sen} 5t$$