

4.2.1 d)

encuentre la transformada de Laplace usando tablas y la propiedad de traslación en el tiempo de la transformada de Laplace unilateral.

$$x(t) = e^{-t} u(t-T)$$

la propiedad.

$$x(t-t_0) u(t-t_0) \rightarrow X(s) e^{-st_0} \quad t_0 \geq 0$$

y, de tabla

$$e^{\lambda t} u(t) \Leftrightarrow \frac{1}{s-\lambda}$$

llevando $x(t)$ a la forma

$$x(t) = e^{-(t-T)} e^{-T} u(t-T) \quad \text{donde } e^{-T} = \text{cte.}$$

así pues:

$$\begin{aligned} \mathcal{L}\{x(t)\} &= e^{-T} \mathcal{L}\{e^{-(t-T)} u(t-T)\} = e^{-T} \frac{1}{s+1} e^{-sT} \\ &= \frac{e^{-T(1+s)}}{s+1} \end{aligned}$$

✓

4.2.3 d) encuentre la inversa de Laplace

$$X(s) = \frac{e^{-s} + e^{-2s} + 1}{s^2 + 3s + 2}$$

$$X(s) = \frac{e^{-s} + e^{-2s} + 1}{(s+2)(s+1)} = (e^{-s} + e^{-2s} + 1) \left(\frac{A}{s+2} + \frac{B}{s+1} \right)$$

$$\Rightarrow A = \frac{1}{s+1} \Big|_{s=-2} = -1$$

$$B = \frac{1}{s+2} \Big|_{s=-1} = 1$$

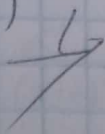
$$A = -B = -1$$

$$X(s) = e^{-s} \left(\frac{-1}{s+2} + \frac{1}{s+1} \right) + e^{-2s} \left(\frac{-1}{s+2} + \frac{1}{s+1} \right)$$

$$+ \left(\frac{-1}{s+2} + \frac{1}{s+1} \right)$$

usando $X(t-t_0) \Leftrightarrow X(s) e^{-s t_0}$

$$\mathcal{L}^{-1}\{X(s)\} = \left((-e^{-2(t-1)} + e^{-(t-1)}) u(t-1) \right) + \left((-e^{-2(t-2)} + e^{-(t-2)}) u(t-2) \right) - e^{-2t} u(t) + e^{-t} u(t)$$



4.3.1 b) resuelva usando transformada de Laplace

$$(D^2 + 4D + 4)y(t) = (D+1)x(t) \dots (1)$$

$$y(0^-) = 2 \quad \dot{y}(0^-) = 1$$

$$x(t) = e^{-t} u(t)$$

$$y(t) \Leftrightarrow y(s)$$

$$x(t) \Leftrightarrow x(s) = \frac{e^{-1}}{s}$$

$$\begin{aligned} \frac{d^2 y(t)}{dt^2} &\Leftrightarrow s^2 y(s) - s y(0^-) - \dot{y}(0^-) \\ &= s^2 y(s) - 2s - 1 \end{aligned}$$

$$\frac{d y(t)}{dt} \Leftrightarrow s y(s) - y(0^-) = s y(s) - 2$$

$$x(s) = \frac{e^{-1}}{s}$$

$$\frac{dx}{dt} \Leftrightarrow s x(s) - x(0^-) = e^{-1} - 0 = e^{-1}$$

aplicando la transformada de Laplace a (1)

$$s^2 y(s) - 2s - 1 + 4s y(s) - 8 + 4y(0) = e^{-1} + \frac{e^{-1}}{s}$$

$$y(s)(s^2 + 4s + 4) = e^{-1} + \frac{e^{-1}}{s} + 2s + 9$$

$$\rightarrow y(s) = \frac{(e^{-1} + 9)}{(s+2)^2} + \frac{e^{-1}}{s(s+2)^2} + \frac{2s}{(s+2)^2}$$

$$* \frac{e^{-1}}{s(s+2)^2} = e^{-1} \left(-\frac{1}{4(s+2)} - \frac{1}{2(s+2)^2} + \frac{1}{4s} \right)$$

$$* \frac{2s}{(s+2)^2} = \frac{2}{s+2} - \frac{4}{(s+2)^2}$$

$$\begin{aligned} y(s) &= \left(\frac{e^{-1}}{2} + 5 \right) \frac{1}{(s+2)^2} + \left(2 - \frac{e^{-1}}{4} \right) \frac{1}{s+2} \\ &\quad + \frac{1}{4s} \end{aligned}$$

aplicando la inversa de Laplace

$$y(t) = \left(\left(\frac{e^{-1}}{2} + 5 \right) t e^{-2t} + \left(2 - \frac{e^{-1}}{4} \right) e^{-2t} + \frac{1}{4} \right) u(t)$$