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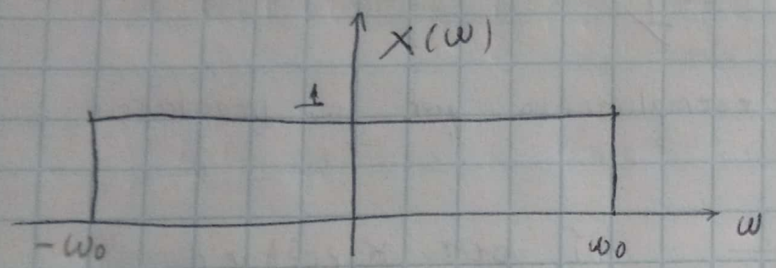
7.2. Muestre que la transformada inversa de Fourier de $X(\omega)$ mostrada en la figura 7.17 es $x(t) = \frac{\omega_0}{\pi} \operatorname{sinc}(\omega_0 t)$
Bosqueje $x(t)$



$$X(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(\omega)\, e^{j\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0} e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi j t}\, e^{j\omega t}\Big|_{-\omega_0}^{\omega_0} = \frac{1}{2\pi j t}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right)$$

$$= \frac{2j}{2\pi j t}\, \operatorname{sen} \omega_0 t = \frac{1}{\pi t}\, \operatorname{sen} \omega_0 t$$

$$= \frac{\omega_0}{\pi}\, \frac{\operatorname{sen} \omega_0 t}{\omega_0 t} = \frac{\omega_0}{\pi}\, \operatorname{sinc}(\omega_0 t)$$

7.3. Muestre que $\cos(w_0 t+\theta) \Leftrightarrow \pi(\delta(w+w_0)e^{-j\theta}+\delta(w-w_0)e^{j\theta})$

podemos ver al coseno como:

$$\cos(w_0 t+\theta) = \frac{1}{2}\left(e^{j(w_0 t+\theta)}+e^{-j(w_0 t+\theta)}\right)$$

así. la transformada de Fourier

$$\frac{1}{2}\int_{-\infty}^{\infty}\left(e^{j(w_0 t+\theta)}+e^{-j(w_0 t+\theta)}\right)e^{-jwt}\,dt = \frac{1}{2}\left(\int_{-\infty}^{\infty} e^{j(w_0 t+\theta)}e^{-jwt}\,dt+\int_{-\infty}^{\infty} e^{-j(w_0 t+\theta)}e^{-jwt}\,dt\right)$$

sabemos que.

$$e^{jw_0 t} \Leftrightarrow 2\pi\,\delta(w-w_0)$$

$$\int_{-\infty}^{\infty} e^{jw_0 t}e^{-jwt}\,dt = 2\pi\,\delta(w-w_0)$$

y que,

$$e^{-jw_0 t} \Leftrightarrow 2\pi(\delta(w+w_0))$$

$$\int_{-\infty}^{\infty} e^{-jw_0 t}e^{-jwt}\,dt = 2\pi\,\delta(w+w_0)$$

así pues

$$= \frac{1}{2}\left(2\pi\left(\delta(w-w_0)e^{j\theta}+\delta(w+w_0)e^{-j\theta}\right)\right) = \pi\left(\delta(w-w_0)e^{j\theta}+\delta(w+w_0)e^{-j\theta}\right)$$

$$\cos(w_0 t+\theta) \Longleftrightarrow \pi\,\delta(w-w_0)e^{j\theta}+\delta(w+w_0)e^{-j\theta}$$

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7.4 aplique la propiedad de dualidad a $e^{-at}u(t) \Leftrightarrow \frac{1}{a+j\omega}$ $a>0$

$e^{-a|t|} \Leftrightarrow \frac{2a}{a^2+\omega^2}$ $a>0$, $\cos\omega_0 t \Leftrightarrow \pi(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$

para mostrar que:

a. $\frac{1}{jt+a} \Leftrightarrow 2\pi e^{a\omega}u(-\omega)$

b. $\frac{2a}{t^2+a^2} \Leftrightarrow 2\pi e^{-a|\omega|}$

c. $\delta(t+t_0)+\delta(t-t_0) \Leftrightarrow 2\cos t_0\omega$

Propiedad de dualidad.

$X(t) \Leftrightarrow 2\pi x(-\omega)$.

considerar que

$X(\omega) \to X(t)\Big|_{t\to\omega}$ $x(-\omega) = x(t)\Big|_{t\to-\omega}$

a.

así $\frac{1}{a+jt} \Leftrightarrow e^{-a\omega}u(\omega)$

por propiedad de dualidad

$\frac{1}{a+jt} \Leftrightarrow 2\pi e^{a\omega}u(-\omega)$

b. $\frac{2a}{a^2+t^2} \Leftrightarrow e^{-a|\omega|}$ → aplicando la propiedad $\frac{2a}{a^2+t^2} \Leftrightarrow 2\pi e^{-a|-\omega|} = 2\pi e^{-a|\omega|}$

c. $\pi(\delta(t-t_0)+\delta(t+t_0)) \Leftrightarrow \cos t_0\omega$

aplicando la propiedad

$\delta(t-t_0)+\delta(t+t_0) = \frac{2\pi}{\pi}\cos(-t_0\omega) = 2\cos(t_0\omega)$