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2.26.

Verifique las siguientes ecuaciones:

a) $x[n] * h[n] = h[n] * x[n]$

b.) $\{x[n] * h_1[n]\} * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$

a. Por definición de convolución

$$x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

si hacemos

$$m = n-k. \rightarrow k = n-m$$

$$\Rightarrow x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[n-m]h[m] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$

$$= h[n] * x[n]$$

b. Sea $x[n] * h_1[n] = f_1[n]$ y $h_1[n] * h_2[n] = f_2[n]$

entonces $f_1[n] = \sum_{k=-\infty}^{\infty} x[k]h_1[n-k]$

$$\{x[n] * h_1[n]\} * h_2[n] = f_1[n] * h_2[n] =$$

$$= \sum_{m=-\infty}^{\infty} f_1[m]h_2[n-m] =$$

$$= \sum_{m=-\infty}^{\infty} \left[\sum_{k=-\infty}^{\infty} x[k]h_1[m-k]\right] h_2[n-m]$$

si hacemos $r = m-k \rightarrow m = r+k$

$$\Rightarrow \sum_{k=-\infty}^{\infty} \left[\sum_{r=-\infty}^{\infty} x[k]h_1[r]\right] h_2[n-r-k]$$

$$= \sum_{k=-\infty}^{\infty} x[k] \sum_{r=-\infty}^{\infty} h_1[r]h_2[n-r-k]$$

si $f_2[n] = \sum_{r=-\infty}^{\infty} h_1[r]h_2[n-r] \xrightarrow{n \rightarrow n-k} f_2[n-k] = \sum_{r=-\infty}^{\infty} h_1[r]h_2[n-r-k]$

entonces $\sum_{k=-\infty}^{\infty} x[k]f_2[n-k] = x[n] * f_2[n]$ pero $f_2[n] = h_1 * h_2$

así $= x[n] * \{h_1[n] * h_2[n]\}$

2.27. Demuestre que

a. $x[n] * \delta[n] = x[n]$

b. $x[n] * \delta[n-n_o] = x[n-n_o]$

c. $x[n] * u[n] = \sum_{K=-\infty}^{n} x[K]$

d. $x[n] * u[n-n_o] = \sum_{K=-\infty}^{n-n_o} x[K]$

a. $x[n] * \delta[n]$

por definición

$= \sum_{K=-\infty}^{\infty} x[K]\,\delta[n-K]$ y sabiendo que $\delta[n-K] = \begin{cases} 1 & \text{si } n=K \\ 0 & \text{si } n \neq K \end{cases}$

entonces $= x[n]$

$x[n] * \delta[n] = x[n]$

b. Por definición

$x[n] * \delta[n-n_o] = \sum_{K=-\infty}^{\infty} x[K]\,\delta[n-n_o-K]$ para el valor de $K = n-n_o$ la delta vale 1, para todos los demás es 0.

$\Rightarrow x[n] * \delta[n-n_o] = x[n-n_o]$

c. $x[n] * u[n]$

Por definición

$x[n] * u[n] = \sum_{K=-\infty}^{\infty} x[K]\,u[n-K]$

el escalón $u[n-K] = \begin{cases} 1 & n \geq K \\ 0 & n < K \end{cases}$

así, para que la suma sobreviva, debe cumplirse que $n \geq K$

$\Rightarrow = \sum_{K=-\infty}^{n} x[K]$

d. $x[n] * u[n-n_o] = \sum_{K=-\infty}^{\infty} x[K]\,u[n-n_o-K]$

$u[n-n_o-K] = \begin{cases} 1 & n-n_o \geq K \\ 0 & n-n_o < K \end{cases}$

$\Rightarrow = \sum_{K=-\infty}^{n-n_o} x[K]$

2.29. Calcule $y[n] = x[n] * h[n]$, donde.

a. $x[n] = \alpha^n u[n]$, $h[n] = \beta^n u[n]$

b. $x[n] = \alpha^n u[n]$, $h[n] = \alpha^{-n} u[-n]$ $0<\alpha<1$

a. Por definición

$$x[n] * h[n] = \sum_{K=-\infty}^{\infty} x[K]\, h[n-K] = \sum_{K=-\infty}^{\infty} \alpha^K u[K]\, \beta^{n-K} u[n-K]$$

$$= \sum_{K=-\infty}^{\infty} \alpha^K \beta^n \beta^{-K} u[K]u[n-K]$$ donde se debe cumplir que $n \geq K \geq 0$ para que el producto de esos escalones de 1. (0 otro caso)

$$\Rightarrow = \sum_{K=-\infty}^{\infty} \alpha^K \beta^{-K} \beta^n = \beta^n \sum_{K=-\infty}^{\infty} \left(\frac{\alpha}{\beta}\right)^K$$

$$\Rightarrow \beta^n \sum_{K=0}^{n} \left(\frac{\alpha}{\beta}\right)^K \qquad n \geq 0$$

sabiendo que $\displaystyle\sum_{K=0}^{N} a^N = \frac{1-a^{N+1}}{1-a}$ $a \neq 1$, $= N+1$ si $a=1$

entonces $\frac{\alpha}{\beta} \neq 1 \rightarrow \alpha \neq \beta$.

$$\beta^n \sum_{K=0}^{n} \left(\frac{\alpha}{\beta}\right)^K = \begin{cases} \beta^n \left(\dfrac{1-\left(\frac{\alpha}{\beta}\right)^{n+1}}{1-\frac{\alpha}{\beta}} \right) u[n] & \\ \beta^n (n+1)\, u[n] & \alpha = \beta \end{cases}$$

$$= \begin{cases} \dfrac{\beta^{n+1}\left(1-\alpha^{n+1}\beta^{-n-1}\right)}{\beta-\alpha} u[n] & \alpha \neq \beta \\ \beta^n (n+1)\, u[n] & \alpha = \beta \end{cases}$$

$$y[n] = \begin{cases} \dfrac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha} u[n] & \alpha \neq \beta \\ \beta^n (n+1)\, u[n] & \alpha = \beta \end{cases}$$

b. $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} \alpha^{k} u[k] \alpha^{-(n-k)} u[-(n-k)]$

$= \sum_{k=-\infty}^{\infty} \alpha^{-n} \alpha^{2k} u[k] u[k-n]$

Para $n \leq 0$ $\quad u[k]\,u[k-n] = \begin{cases} 1 & 0 \leq k \\ 0 & \text{otro caso} \end{cases}$

$\Rightarrow \quad y[n] = \alpha^{-n} \sum_{k=0}^{\infty} \alpha^{2k} = \alpha^{-n} \sum_{k=0}^{\infty} (\alpha^2)^k = \frac{\alpha^{-n}}{1-\alpha^2} \quad n \leq 0$

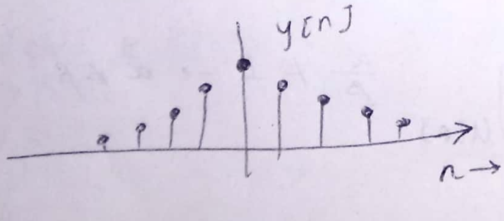
Para $n \geq 0$ se tiene que.

$u[k]\,u[k-n] = \begin{cases} 1 & n \leq k \\ 0 & \text{otro caso} \end{cases}$

$\Rightarrow \quad y[n] = \alpha^{-n} \sum_{k=n}^{\infty} (\alpha^2)^k = \alpha^{-n} \frac{\alpha^{2n}}{1-\alpha^2} = \frac{\alpha^{n}}{1-\alpha^2} \quad n \geq 0$

de esta manera, para cualquier n.

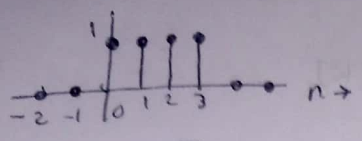
$y[n] = \frac{\alpha^{|n|}}{1-\alpha^2}$ para toda n.



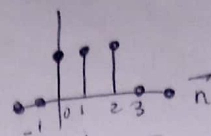
2.30 Evalúe $y[n] = x[n] * h[n]$

a) analíticamente

b) gráficamente



$x[n]$



$h[n]$

a.

tenemos que

$$y[n] = x[n] * h[n] = x[n] * \{\delta[n] + \delta[n-1] + \delta[n-2]\}$$

por distributividad de convolución

$$= x[n] * \delta[n] + x[n] * \delta[n-1] + x[n] * \delta[n-2]$$

aplicando

$$x[n] * \delta[n] = x[n] \quad \& \quad x[n] * \delta[n-n_0] = x[n-n_0]$$

$$y[n] = x[n] + x[n-1] + x[n-2]$$

Pero $x[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$

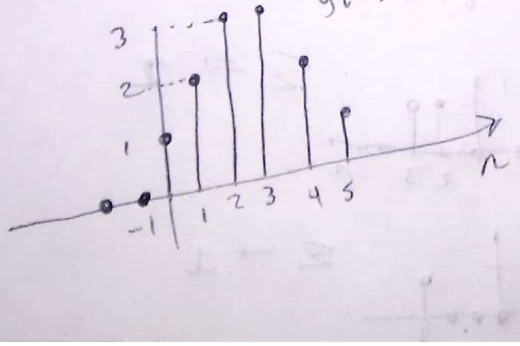
$$\Rightarrow \quad y[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] +$$
$$+ \delta[n-1] + \delta[n-2] + \delta[n-3] + \delta[n-4] +$$
$$+ \delta[n-2] + \delta[n-3] + \delta[n-4] + \delta[n-5]$$

simplificando

$$= \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 3\delta[n-3] + 2\delta[n-4] + \delta[n-5]$$

que es igual a

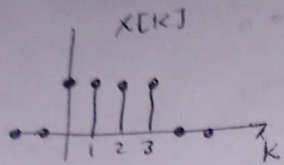
$$y[n] = \{\underset{\uparrow}{1}, 2, 3, 3, 2, 1\}$$

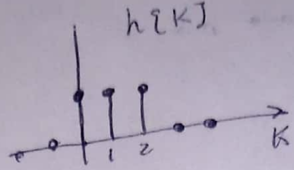


b. gráficamente

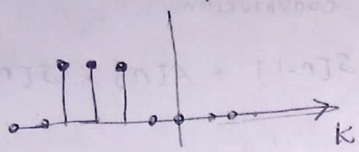
$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

así



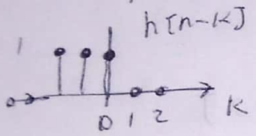


así $h[n-k]$ para $n<0$

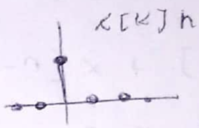


si $n<0$ el producto $x[k]h[n-k] = 0$.

para $n=0$



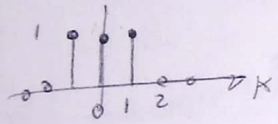
al hacer el producto con $x[k]$ →

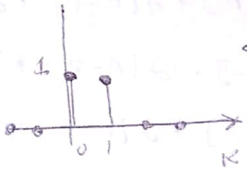


cuya suma es 1.

para $n=1$

$x[k]h[n-k]$ →

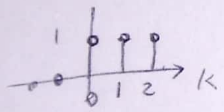




$\Sigma \Rightarrow 2$.

Para $n=2$.

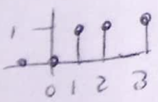
$x[k]h[n-k]$ →

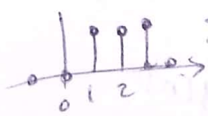


$\Sigma \Rightarrow 3$.

Para $n=3$

$x[k]h[n-k]$ →



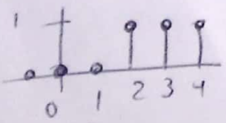


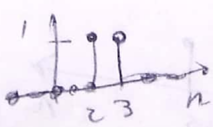
$\Sigma \Rightarrow 3$.

Para $n=4$.

$\Sigma \Rightarrow 2$

$x[k]h[n-k]$ →

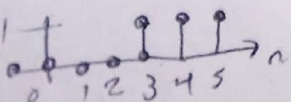


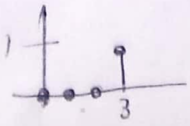


Para $n=5$

$\Sigma \to 1$

$x[k]h[n-k]$ →





Para $n>5$ ya no hay empalme y la suma es 0. como con $n<0$ así $y[n] = \{1, 2, 3, 3, 2, 1\}$ como era de esperarse