PR08

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encuentre la expresión de la serie de Fourier para

$f(t) = t$ en el intervalo $[-1, 1]$

Para a_0 $\quad T_0 = 2 \rightarrow \omega_0 = \frac{2\pi}{2} = \pi$

$$a_0 = \frac{1}{2}\int_{-1}^{1} t\,dt = 0.$$

$$a_n = \frac{2}{2}\int_{-1}^{1} t\cos\omega n t\,dt = \frac{t}{\omega n}\operatorname{sen}\omega n t + \frac{1}{\omega^2 n^2}\cos\omega n t \Big|_{-1}^{1}$$

$$= \frac{1}{\pi n}\cancel{\operatorname{sen}\pi n} + \frac{1}{\pi^2 n^2}\cos\pi n + \frac{(-1)}{\pi n}\cancel{\operatorname{sen}\pi n} - \frac{1}{\pi^2 n^2}\cos\pi n = 0$$

$\qquad 0 \qquad\qquad\qquad 0$

$a_n = 0$

$$b_n = \frac{2}{2}\int_{-1}^{1} t\operatorname{sen}\omega n t\,dt = \frac{-t}{\omega n}\cos\omega n t - \frac{1}{\omega^2 n^2}\operatorname{sen}\omega n t \Big|_{-1}^{1}$$

$$= \frac{-1}{\pi n}\cos\pi n - \cancel{\frac{1}{\pi^2 n^2}\operatorname{sen}\pi n} - \frac{1}{\pi n}\cos\pi n + \cancel{\frac{1}{\pi n^2}\operatorname{sen}\pi n}$$

$\qquad 0 \qquad\qquad\qquad 0$

$$= -\frac{2}{\pi n}(-1)^n$$

así

$$S_f(t) = \sum_{n=1}^{\infty} \frac{-2}{\pi n}(-1)^n \operatorname{sen}(\pi n t)$$

2. $S_f(t)$ para t^2 de $[-2,2]$

$$T_0 = 4 \rightarrow \omega_0 = \frac{\pi}{2}$$

para

$$a_0 = \frac{1}{4}\int_{-2}^{2} t^2 dt = \frac{1}{12}(8+8) = \frac{4}{3}$$

$$a_n = \frac{2}{4}\int_{-2}^{2} t^2 \cos \frac{\pi}{2}nt\, dt = \frac{1}{2}\, 2\int_{0}^{2} t^2 \cos \frac{\pi}{2}nt\, dt = \frac{t^2}{\frac{\pi}{2}n}\operatorname{sen}\frac{\pi}{2}nt +$$

$$+ \frac{2t}{\left(\frac{\pi}{2}\right)^2 n^2}\cos \frac{\pi}{2}nt - \frac{2}{\left(\frac{\pi}{2}\right)^3 n^3}\operatorname{sen}\omega_n t \Big|_0^2 = \frac{16}{\pi^2 n^2}(-1)^n$$

$b_n = 0$ pues $\int_{-2}^{2} t^2 \operatorname{sen}\left(\frac{\pi}{2}nt\right) dt = 0$ par $*$ impar $=$ impar.

así.

$$S_f(t) = \frac{4}{3} + \sum_{n=1}^{\infty} \frac{16}{\pi^2 n^2}(-1)^n \cos\left(\frac{\pi}{2}nt\right)$$