RCFB22. Montiel Cruz Jorge de Jesús

9.3 encuentre la DTFT de $x[n] = r^n u[n]$

$$X(\Omega) = \sum_{n=0}^{\infty} r^n e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} (r e^{-j\Omega})^n$$

es una serie geométrica infinita

$$X(\Omega) = \frac{1}{1 - r e^{-j\Omega}}$$

donde si:

$|r e^{-j\Omega}| < 1$ pero dado que $|e^{-j\Omega}| = 1$ entonces forzosamente $|r| < 1$ así

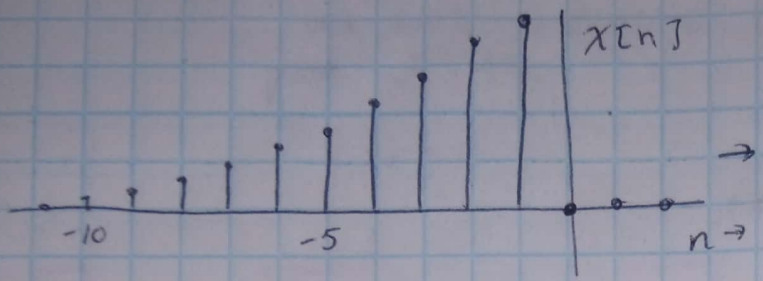
$$X(\Omega) = \frac{1}{1 - r e^{-j\Omega}} \quad |r| < 1 \quad \text{si } |r| > 1 \quad X(\Omega) \text{ no converge}$$

por Euler

$$X(\Omega) = \frac{1}{1 - r\cos(\Omega) + r j \operatorname{sen}\Omega} \rightarrow |X(\Omega)| = \frac{1}{\sqrt{(1 - r\cos(\Omega))^2 + (r \operatorname{sen}\Omega)^2}}$$

$$= \frac{1}{\sqrt{1 + r^2 - 2r\cos\Omega}}, \quad \angle X(\Omega) = \arctan\left(\frac{r \operatorname{sen}\Omega}{1 - r\cos\Omega}\right)$$

9.4. encontre DTFT de $\gamma^n u[-(n+1)]$ descrita a continuació



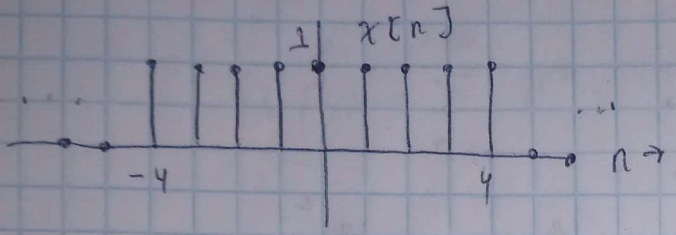
$$X(\Omega) = \sum_{m=1}^{\infty} \left(\frac{1}{\gamma} e^{j\Omega}\right)^m = \frac{1}{\gamma} e^{j\Omega} + \left(\frac{1}{\gamma} e^{j\Omega}\right)^2 + \dots$$

$$= \frac{1}{\gamma e^{-j\Omega} - 1} \qquad |\gamma| > 1$$

$$= \frac{1}{(\gamma\cos\Omega - 1) - j\gamma\operatorname{sen}\Omega} \rightarrow |X(\Omega)| = (1 + \gamma^2 - 2\gamma\cos\Omega)^{-1/2}$$

$$\angle X(\Omega) = \arctan\left(\frac{\gamma\operatorname{sen}\Omega}{\gamma\cos\Omega - 1}\right)$$

9.5 encuentre DTFT para el pulso discreto rectangular mostrado



$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} = \sum_{n=-4}^{4} \left(e^{(-j\Omega_0)}\right)^n$$

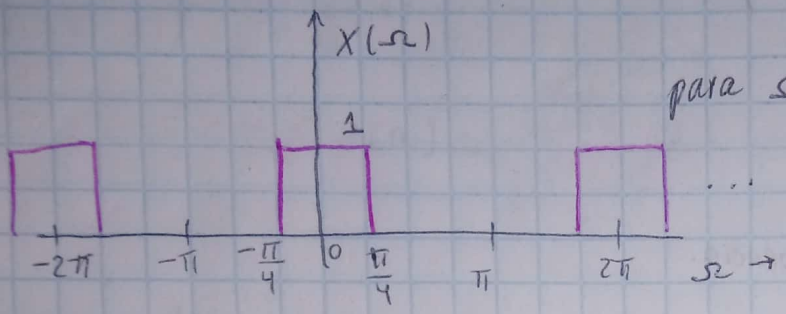
progresión geométrica con razón común $e^{-j\Omega}$

$$X(\Omega) = \frac{e^{-j5\Omega} - e^{j4\Omega}}{e^{-j\Omega} - 1} = \frac{e^{-j5\Omega} - e^{j4\Omega}}{e^{-j\frac{\Omega}{2}}\left(e^{-j\frac{\Omega}{2}} - e^{j\frac{\pi}{2}}\right)} = \frac{e^{-j5\Omega} - e^{j4\Omega}}{+2j\, e^{-j\frac{\Omega}{2}} \operatorname{sen}\frac{\Omega}{2}}$$

$$= \frac{e^{-j\frac{\Omega}{2}}\left(e^{-j\frac{9}{2}\Omega} - e^{j\frac{9}{2}\Omega}\right)}{2j\, e^{-j\frac{\Omega}{2}} \operatorname{sen}\frac{\Omega}{2}} = \frac{\operatorname{sen}\left(\frac{9}{2}\Omega\right)}{\operatorname{sen}\left(\frac{\Omega}{2}\right)}$$

✓

9.6 encuentre DTFT del pulso rectangular mostrado sobre una banda fundamental ($|\Omega| \leq \pi$) por $X(\Omega) = \text{rect}(\Omega / 2\Omega_c)$ para $\Omega_c \leq \pi$



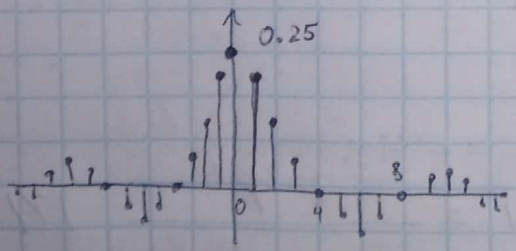
usando

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\pi\Omega} d\Omega$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega) e^{j\pi\Omega} d\Omega = \frac{1}{2\pi} \int_{-\Omega_c}^{\Omega_c} e^{j\pi\Omega} d\Omega$$

$$= \frac{1}{j2\pi n} e^{j\pi\Omega} \Big|_{-\Omega_c}^{\Omega_c} = \frac{\text{sen}(\Omega_c n)}{\pi n} = \frac{\Omega_c}{\pi} \text{sinc}(\Omega_c n)$$

$$x[n] = \frac{\Omega_c}{\pi} \text{sinc}(\Omega_c n)$$



espectro de Fourier