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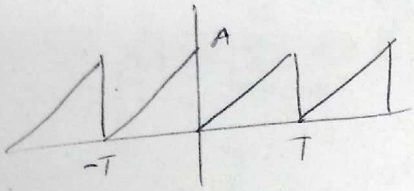
3.1 encontrar $\partial f(t)$ compleja para

$$f(t) = \frac{A}{T}t \quad 0 < t < T$$

$$f(t+T) = f(t)$$



sabemos que.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jnw_0 t} \quad \text{donde} \quad w_0 = \frac{2\pi}{T}$$

&

$$C_n = \frac{1}{T}\int_0^T f(t) e^{-jnw_0 t} dt$$

$$= \frac{A}{T^2}\int_0^T t e^{-jnw_0 t} dt = \frac{A}{T^2}\left(\left.\frac{t e^{-jnw_0 t}}{-jnw_0}\right|_0^T + \frac{1}{jnw_0}\int_0^T e^{-jnw_0 t} dt \right)$$

$$= \frac{A}{T^2}\left[\frac{T e^{-jn2\pi}}{-jnw_0} - \frac{1}{(jnw_0)^2}\left(e^{-jn2\pi} - 1\right)\right]$$

pero $e^{-jn2\pi} = 1 \Rightarrow C_n = \frac{jA}{nw_0 T} = \frac{jA}{2\pi n} = e^{j\pi/2}\frac{A}{2\pi n}$

Para $n=0$ se indetermina, así desde C_n.

$$C_0 = \frac{1}{T}\int_0^T f(t) dt = \frac{A}{T^2}\int_0^T t\, dt = \frac{1}{2}A$$

así

$$f(t) = \frac{A}{2} + j\frac{A}{2\pi}\sum_{n=-\infty}^{\infty}{}' \frac{1}{n} e^{jnw_0 t}$$

$$= \frac{A}{2} + \frac{A}{2\pi}\sum_{n=-\infty}^{\infty}{}' \frac{1}{n} e^{j(nw_0 t + \frac{\pi}{2})}$$

$\sum_{n=-\infty}^{\infty}{}'$ suma sólo incluye enteros diferentes de cero

3.2. Reducir el resultado de 3.1 a la forma trigonométrica

$$c_0 = \frac{1}{2} a_0 \quad , \quad c_n = \frac{1}{2}(a_n - j b_n)$$

$$C_{-n} = C_n^* = \frac{1}{2}(a_n + j b_n)$$

$$a_0 = 2C_0 = \frac{2A}{2} = A.$$

$$a_n = C_n + C_{-n} = C_n + C_n^* = 2\,\mathrm{Re}\,[C_n]$$

$$b_n = j(C_n - C_{-n}) = j(C_n - C_n^*) = -2\,\mathrm{Im}\,\{C_n\}$$

$$a_n = \frac{2A}{2\pi n} \cos\frac{\pi}{2} = 0.$$

$$b_n = -\frac{2A}{2\pi n} \,\mathrm{sen}\,\frac{\pi}{2} = -\frac{A}{\pi n}$$

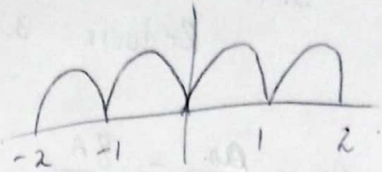
así.

$$f(t) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos n w_0 t + b_n \,\mathrm{sen}\, n w_0 t$$

$$= \frac{A}{2} - \frac{A}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \,\mathrm{sen}\, n w_0 t.$$

3.3 encontrar $Sf(t)$ compleja de

$f(t) = A\,\text{sen}\,\pi t \quad 0 < t < 1$



$f(t+T) = f(t) \Rightarrow T = 1$

$T = 1 \Rightarrow \omega_0 = 2\pi$

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n t}$$

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn2\pi t}\, dt$$

$$= \int_0^1 A\,\text{sen}\,\pi t\, e^{-j2\pi n t}\, dt$$

$$= A\int_0^1 \frac{1}{2j}\left(e^{-j\pi(2n-1)t} - e^{-j\pi(2n+1)t}\right) dt$$

$$= \frac{A}{2j}\left[\frac{e^{-j\pi(2n-1)t}}{-j\pi(2n-1)} - \frac{e^{-j\pi(2n+1)t}}{-j\pi(2n+1)}\right]\Bigg|_0^1$$

$e^{\pm j2\pi n} = 1$ y $e^{j\pi} = e^{-j\pi}$

$$C_n = \frac{-2A}{\pi(4n^2-1)}$$

cuando $n = 0$

$$C_0 = \frac{1}{T}\int_0^T f(t)\, dt = \frac{A}{T\pi}(-\cos\pi t)\Big|_0^T$$

$$= \frac{A}{\pi}-(\cos\pi - \cos 0)$$

$$= \frac{2A}{\pi}$$

así:

$$f(t) = -\frac{2A}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{4n^2-1}e^{j2\pi n t}$$

3.4

Reducir 3.3 a forma trigonométrica

$$C_0 = \frac{a_0}{2} = \frac{2A}{\pi}$$

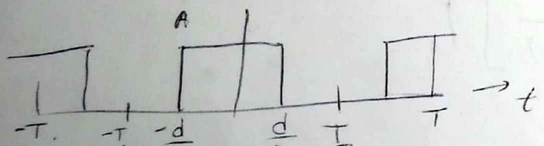
$$a_n = 2\,Re\{C_n\} = \frac{-4A}{\pi(n^2(4)-1)}$$

$$b_n = 2\,Im\{C_n\} = 0$$

así,

$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi}\sum_{n=1}^{\infty} \frac{1}{4n^2-1}\cos(n\,2\pi t)$$

3.7 encontrar los espectros de frecuencia para $f(t)$ de magnitud A y duración d.



La función la describimos como

$$f(t) = \begin{cases} A & \text{para } -\frac{d}{2} < t < \frac{d}{2} \\ 0 & \text{para } -\frac{T}{2} < t < -\frac{d}{2}, \quad \frac{d}{2} < t < \frac{T}{2} \end{cases}$$

$$\omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{A}{T}\int_{-d/2}^{d/2} e^{-jn\omega_0 t}\, dt = -\frac{A}{jn\omega_0 T}\, e^{-jn\omega_0 t}\Big|_{-d/2}^{d/2}$$

$$= \frac{-A}{jn2\pi}\left(e^{-jn\omega_0\frac{d}{2}} - e^{+jn\omega_0\frac{d}{2}}\right) = \frac{-A\,\cancel{2j}(-1)}{\cancel{2j}\,n\pi}\, sen\left(n\omega_0\frac{d}{2}\right)$$

$$C_n = \frac{A}{n\pi}\, sen\left(\frac{n\pi d}{T}\right)$$

C_n es real entonces el espectro de fase es 0

para el espectro de amplitud. $\frac{A}{n\pi}\, sen\left(n\omega_0\frac{d}{2}\right) \ldots (1)$

graficamos (1) contra $n\omega_0$

supongamos que $d = 1/20$ y $T = 1/4 \Rightarrow \omega_0 = 8\pi$

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Por tanto el espectro de amplitud

existe cuando

$\omega = 0, \pm 8\pi, \pm 16\pi, \ldots,$ etc.

vemos que

$\frac{d}{T} = \frac{1}{5}$ así, el espectro de amplitud

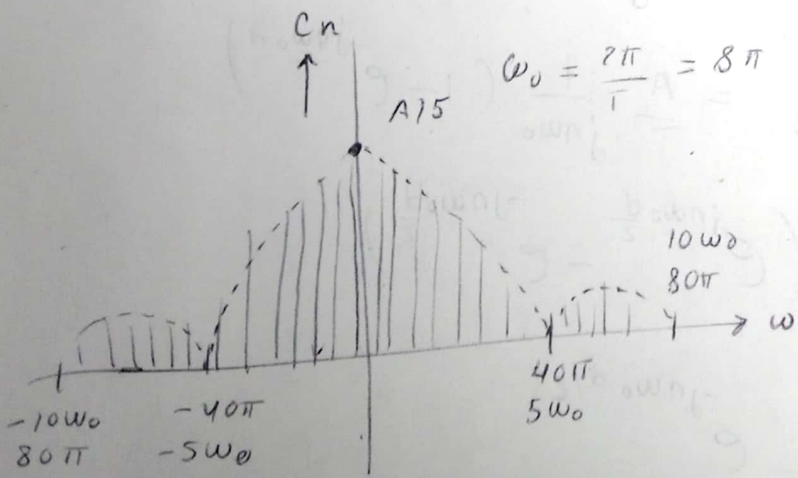
será cero en el valor de $n\omega_0$ para el cual

$$n\omega_0 \frac{d}{2} = m\pi \quad \text{ó} \quad n\pi\frac{d}{T} = \frac{n\pi}{5} = m\pi \quad (m = \pm 1, \pm 2, \ldots)$$

es decir cuando

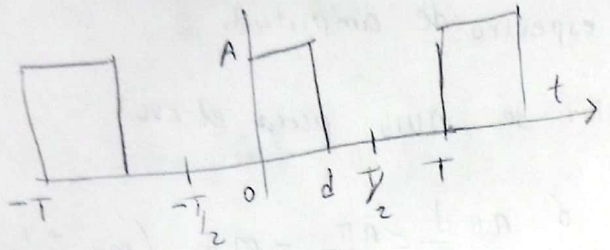
$$\omega = \pm 5\omega_0 = \pm 40\pi, \pm 10\omega_0 = \pm 80\pi \ldots$$

$$\omega_0 = \frac{2\pi}{T} = 8\pi$$



3.8

encontrar los espectros de frecuencia de la función periódica que se muestra



con $\omega_0 = 2\pi/T$

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt = \frac{A}{T}\int_0^d e^{-jn\omega_0 t}\, dt$$

$$\frac{A}{T}\,\frac{1}{-jn\omega_0}\, e^{-jn\omega_0 t}\Big|_0^d = \frac{A}{T}\,\frac{1}{jn\omega_0}\left(1 - e^{-jn\omega_0 d}\right)$$

$$\frac{A}{T}\,\frac{1}{jn\omega_0}\, e^{-jn\omega_0 \frac{d}{2}}\left(e^{jn\omega_0\frac{d}{2}} - e^{-jn\omega_0\frac{d}{2}}\right)$$

$$= \frac{Ad}{T}\,\frac{\operatorname{sen}\left(\frac{n\omega_0 d}{2}\right)}{\frac{n\omega_0 d}{2}}\, e^{-jn\omega_0 d/2}$$

$$= |C_n|\, e^{j\phi n}$$

$$|C_n| = \frac{Ad}{T}\,\frac{\operatorname{sen}\left(\frac{n\omega_0 d}{2}\right)}{\left(\frac{n\omega_0 d}{2}\right)}$$

si hacemos $d = \frac{1}{20}$ y $T = \frac{1}{4}$

$$\frac{d}{T} = \frac{1}{5}$$

$$\phi_n = \frac{-n\omega_0 d}{2} = -n\pi\frac{d}{T}$$