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2.1

Verifique

(a) $x(t) * h(t) = h(t) * x(t)$

(b) $\{x(t) * h_1(t)\} * h_2(t) = x(t) * \{h_1(t) * h_2(t)\}$

a. Por definición

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

si hacemos

$t - \tau = \lambda$, tenemos

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(t-\lambda) h(\lambda) d\lambda = \int_{-\infty}^{\infty} h(\lambda) x(t-\lambda) d\lambda$$

$$= h(t) * x(t)$$

b.

sea $x(t) * h_1(t) = f_1(t)$ y $h_1(t) * h_2(t) = f_2(t)$ entonces

$$f_1(t) = \int_{-\infty}^{\infty} x(\tau) h_1(t-\tau) d\tau$$

así.

$$\{x(t) * h_1(t)\} * h_2(t) = f_1(t) * h_2(t) = \int_{-\infty}^{\infty} f_1(\sigma) h_2(t-\sigma) d\sigma$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} x(\tau) h_1(\sigma-\tau) d\tau \right] h_2(t-\sigma) d\sigma$$

si hacemos $\lambda = \sigma - \tau$ entonces

$\Rightarrow \sigma = \lambda + \tau \quad ; \quad d\sigma = d\lambda$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x(\tau) h_1(\lambda) d\tau \; h_2(t-\lambda-\tau) d\lambda$$

$$= \int_{-\infty}^{\infty} x(\tau) \left[\int_{-\infty}^{\infty} h_1(\lambda) h_2(t-\lambda-\tau) d\lambda \right] d\tau$$

ya que. $\int_{-\infty}^{\infty} h_1(\lambda) h_2(t-\lambda) d\lambda = f_2(t)$ si sustituimos t por $t - \tau$. $\Rightarrow$ ...

$$\ldots \Rightarrow f_2(t-\tau) = \int_{-\infty}^{\infty} h_1(\lambda) h_2(t-\lambda-\tau) d\lambda \quad \Rightarrow$$

$$\int_{-\infty}^{\infty} x(\tau) f_2(t-\tau)\, d\tau = x(t) * f_2(t)$$

pero $f_2(t) = h_1(t) * h_2(t)$.

$\therefore$

$$= x(t) * \{h_1(t) * h_2(t)\}$$

$\Rightarrow$

$$\{x(t) * h_1(t)\} * h_2(t) = x(t) * \{h_1(t) * h_2(t)\}$$

2.2 Muestre que.

(a) $x(t) * \delta(t) = x(t)$

(b) $x(t) * \delta(t-t_0) = x(t-t_0)$

(c) $x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

(d) $x(t) * u(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$

a. por definición de convolución.

$$x(t) * \delta(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau =$$

por propiedad de Delta de Dirac.

$$= x(\tau)\big|_{\tau=t} = x(t)$$

b. $x(t) * \delta(t-t_0)$

Por definición.

$$x(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-t_0-\tau)\, d\tau$$

si $\delta(t-t_0) = y(t)$

$$\Rightarrow = x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau-t_0)\, d\tau$$

$$= x(\tau)\big|_{\tau = t-t_0} = x(t-t_0)$$

c. $x(t) * u(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$

$$x(t) * u(t) = \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\, d\tau =$$

pero el escalón

$u(t-\tau)$ que hace producto

con $x(\tau)$ es cero si $t<\tau$

así

$$\int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{t} x(\tau)\, d\tau$$

$$u(t-\tau) = \begin{cases} 1 & t>\tau \\ 0 & t<\tau \end{cases}$$

d. $x(t) * u(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$

$$x(t) * u(t-t_0) = \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau-t_0)\, d\tau = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau$$

el escalón

$u(t-\tau-t_0) = 0$

si $t-t_0-\tau<0$

$\Rightarrow t-t_0<\tau$

y 1 si $\tau<t-t_0$

$$u(t-t_0-\tau) = \begin{cases} 1 & \tau<t-t_0 \\ 0 & \tau>t-t_0 \end{cases}$$

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2.3

sean

$y(t) = x(t) * h(t)$ demuestre que:

$$x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$$

si $\quad x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \quad \ldots (1)$

entonces, si

$$x(t-t_1) * h(t-t_2) = \int_{-\infty}^{\infty} x(\tau-t_1)\, h(t-t_2-\tau)\, d\tau$$

sea $\tau - t_1 = \lambda \rightarrow \tau = \lambda + t_1$

$\Rightarrow$

$$x(t-t_1) * h(t-t_2) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-t_2-t_1-\lambda)\, d\lambda \quad (2)$$

si en (1)

cambiamos t por

$t - t_1 - t_2$ obtenemos (2)

así

$$x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$$

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2.9 Sea $h(t)$ el pulso triangular mostrado y $x(t)$ el tren de impulsos unitarios expresado como.

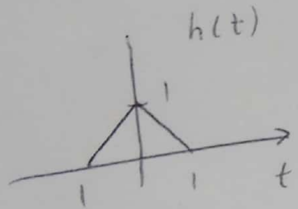
$$x(t) = \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

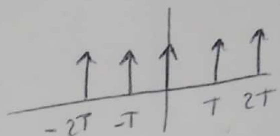
Determine y grafique. $y(t) = h(t) * x(t)$ para valores de T

a. $T=3$

b. $T=2$

c. $T=1.5$





$$y(t) = h(t) * \delta_T(t) = h(t) * \left[\sum_{n=-\infty}^{\infty} \delta(t-nT)\right]$$

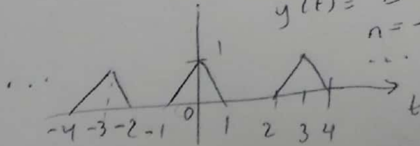
$$= \sum_{n=-\infty}^{\infty} h(t) * \delta(t-nT) = \sum_{n=-\infty}^{\infty} h(t-nT)$$

por propiedades de convolución con deltas.

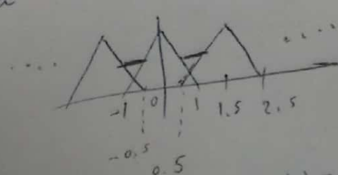
Para

a. $T=3$. $y(t) = \sum_{n=-\infty}^{\infty} h(t-3n)$ => $h(t)$ periodo $T=3$. en todo t

$$y(t) = \sum_{n=-\infty}^{\infty} h(t-3n)$$



Para $T=1.5$



$$y(t) = \sum_{n=-\infty}^{\infty} h(t-1.5n)$$

b. $T=2$.

$$y(t) = \sum_{n=-\infty}^{\infty} h(t-2n)$$