Temporary title: CMB power spectrum...

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ABSTRACT

An abstract for the paper. Describe the paper. What is the paper about, what are the main results, etc.

Key words. cosmic microwave background – large-scale structure of Universe

1. Introduction

Write an introduction here. Give context to the paper. Citations to relevant papers. You only need to do this in the end for the last milestone.

2. Milestone I

In this milestone we will look at the expansion history of a homogeneous and isotropic universe governed by the well known Friedmann equation 2. The universe we consider consists of baryonic matter (Ω_b), cold dark matter ($\Omega_{\rm CDM}$), radiation (Ω_γ), neutrinos (Ω_ν) and dark energy (Ω_Λ), where Ω is the mass/energy density divided by the critical density ($\rho_c = 3H^2/8\pi G$).

Since our universe is approximately homogeneous and isotropic on large scales, the solution we calculate will be

Since our goal in the end is to study the cosmic microwave background (CMB), the homogeneous solution of the universe is of great interest. This is because the CMB is close to being homogeneous with perturbations of order 10^{-5}.

2.1. Theory

The parameters we use for our universe are given below.

$$\boxed{\begin{aligned}
h &= 0.67,\\
T_{\rm CMB0} &= 2.7255\,K,\\
N_{\rm eff} &= 3.046,\\
\Omega_{\rm b0} &= 0.05,\\
\Omega_{\rm CDM0} &= 0.267,\\
\Omega_{k0} &= 0,\\
\Omega_{\nu0} &= N_{\rm eff}\cdot\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\Omega_{\gamma0},\\
\Omega_{\gamma0} &= 2\cdot\frac{\pi^2}{30}\frac{(k_bT_{\rm CMB0})^4}{\hbar^3c^5}\cdot\frac{8\pi G}{3H_0^2},\\
\Omega_{\Lambda0} &= 1-(\Omega_{k0}+\Omega_{b0}+\Omega_{\rm CDM0}+\Omega_{\gamma0}+\Omega_{\nu0}),
\end{aligned}} \tag{1}$$

where the subscript 0 denotes today's value. h is the dimensionless Hubble constant. More details can be found at Winther (2023).

The Friedmann equation is given by

$$H = H_0\sqrt{(\Omega_{b0}+\Omega_{\rm CDM0})a^{-3}+(\Omega_{\gamma0}+\Omega_{\nu0})a^{-4}+\Omega_{k0}a^{-2}+\Omega_{\Lambda0}}, \tag{2}$$

where a is the scale factor and $H = \frac{\dot{a}}{a}$. We will not use cosmic time (t) as our time variable. Instead, we use $x = \ln a$ as our dimensionless time variable. This implies that $a = e^x$ for conversion. Since $a(t = 0) = 0$ and $a(t = t_0) = 1$ we get $t = 0 \iff x = -\infty$ and $t = t_0 \iff x = 0$. The cosmic time as a function of x be found from the differential equation

$$\frac{dt}{dx} = \frac{1}{H}, \tag{3}$$

which can be solved numerically.

We also use a scaled Hubble parameter defined by $\mathcal{H} \equiv aH$ The evolution of the Ωs can be expressed as a function of a as showed below.

$$\begin{aligned}
\Omega_k(a) &= \frac{\Omega_{k0}}{a^2H(a)^2/H_0^2}\\
\Omega_{\rm CDM}(a) &= \frac{\Omega_{\rm CDM0}}{a^3H(a)^2/H_0^2}\\
\Omega_b(a) &= \frac{\Omega_{b0}}{a^3H(a)^2/H_0^2}\\
\Omega_\gamma(a) &= \frac{\Omega_{\gamma0}}{a^4H(a)^2/H_0^2}\\
\Omega_\nu(a) &= \frac{\Omega_{\nu0}}{a^4H(a)^2/H_0^2}\\
\Omega_\Lambda(a) &= \frac{\Omega_{\Lambda0}}{H(a)^2/H_0^2}.
\end{aligned} \tag{4}$$

2.2. Implementation details

Something about the numerical work.

2.3. Results

Show and discuss the results.

3. Milestone II

Some introduction about what it is all about.

3.1. Theory

The theory behind this milestone.

3.2. Implementation details

Something about the numerical work.

3.3. Results

Show and discuss the results.

4. Milestone III

In this milestone we calculate the evolution of the structures in the universe. In practice, this is done by solving the perturbed Einstein-Boltzmann equations numerically, which will give us the time evolution of different physical quantities of interest at different Fourier scales, k. We will go into more detail in the theory section. A detailed derivation of the relevant equations is given in Winther (2023).

Since the observed CMB is a measurement of the photon temperature field today, it is of great interest for us to calculate the photon temperature fluctuations in the universe at all times and Fourier scales. These fluctuations can be evaluated today to reconstruct the CMB power spectrum we observe.

4.1. Theory

Since the universe is not homogeneous, we will introduce perturbations to the distribution functions, f_i, for baryons, photons and CDM, i.e.

$$f_i(t, \boldsymbol{x}, \boldsymbol{p}) = \overline{f_i}(t, \boldsymbol{p}) + \delta f_i(t, \boldsymbol{x}, \boldsymbol{p}),$$

where $\bar{f_i}$ is the background distribution function and δf_i is the perturbation. These perturbations will perturb the energy momentum tensor, which in turn will perturb the metric. This will then change how particles move through space and time. We will therefore have a system of coupled differential equations. We can apply linear perturbation theory to the Einstein-Boltzmann equations in Fourier space with $x = \ln a$ as the time variable. We rewrite the equations in Fourier space because the differential equations become simpler. The photon temperature perturbations, $\Theta = \delta T/\bar{T}$ are expanded in Legendre multipoles, such that we are left with multipoles, Θ_ℓ of the photon distribution.

The photon

4.2. Implementation details

Something about the numerical work.

4.3. Results

4.3.1. Test results

The code produces the following results with the cosmological test parameters given in table 1. All the figures show plots at three different scales. In figures 1 and 2 we see the overdensity and the velocity, respectively, for both CDM and baryons. In figures 3 and 4 we see the photon temperature monopole, Θ_0, and the photon temperature dipole, Θ_1, respectively. The gravitational potential, Φ, is plotted in figure 5. All plots seem to agree with the plots produced by Winther (2023).

Table 1. Cosmological test parameters.

Parameter	Value
h	0.7
Ω_b	0.05
Ω_{CDM}	0.45
Ω_Λ	0.5
Ω_k	0
Ω_ν	0
T_{CMB}	2.7255 $[K]$

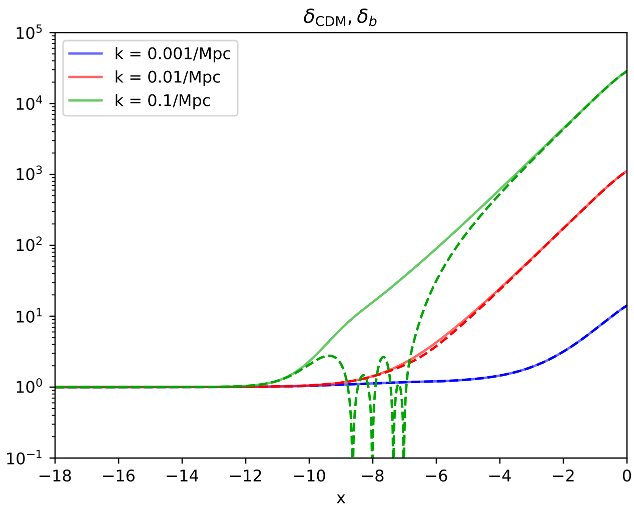

Fig. 1. The CDM overdensities, in solid lines, and the absolute value of the baryon overdensities, in dotted lines, are plotted at different scales, k.

4.3.2. Results

Gravity travels with the same velocity as light. Therefore, the conformal time, η, gives us the maximum reach of gravity, e.g. if η = 1 Mpc we know that scales up to 1 Mpc will cluster due to gravity. This is also reflected in our results, hopefully...

5. Milestone IV

Some introduction about what it is all about.

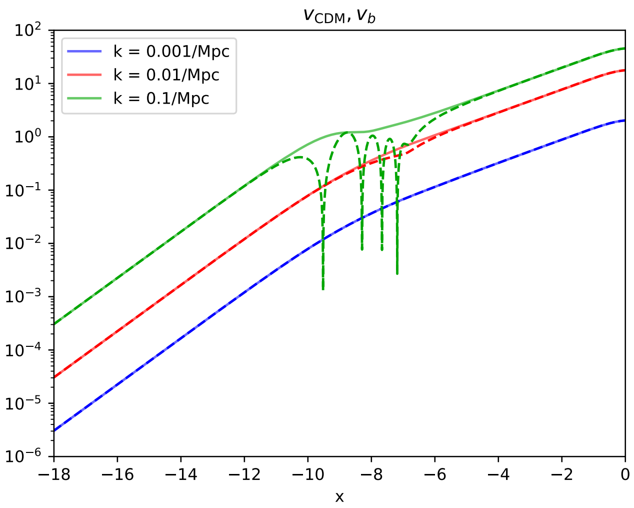

Fig. 2. The CDM velocities, in solid lines, and the absolute value of the baryon velocities, in dotted lines, are plotted at different scales, k.

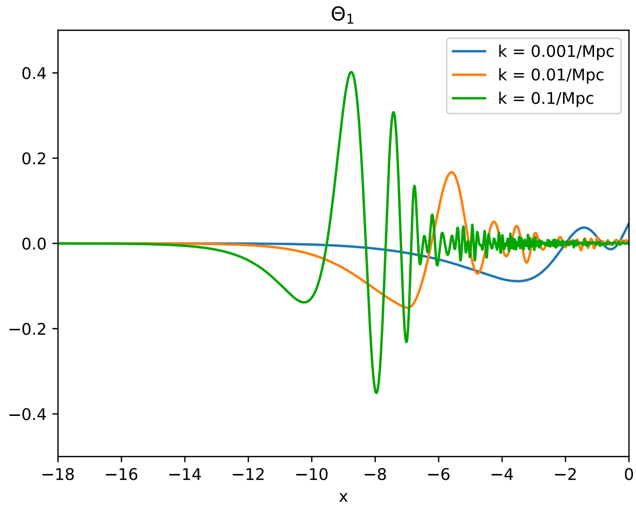

Fig. 4. The photon temperature dipole at different scales, k.

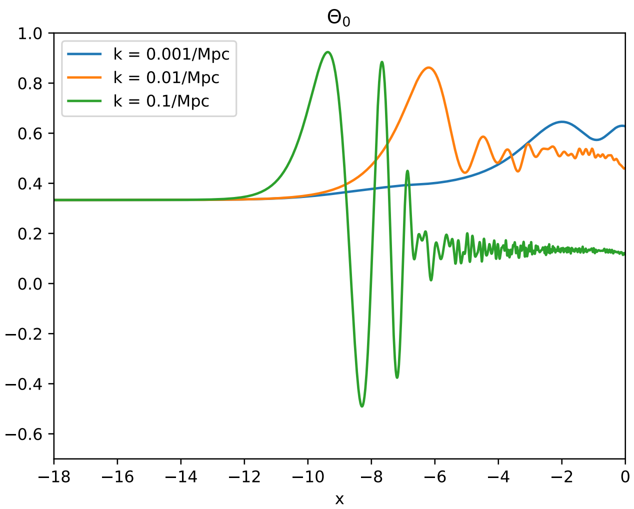

Fig. 3. The photon temperature monopole at different scales, k.

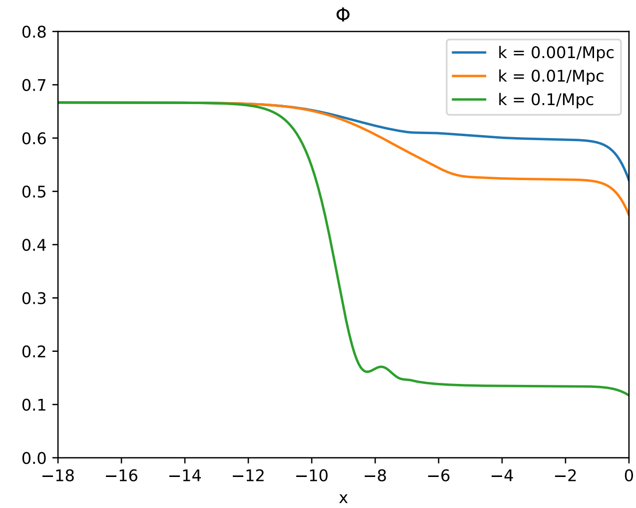

Fig. 5. The gravitational potential at different scales, k.

5.1. Theory

The theory behind this milestone.

5.2. Implementation details

Something about the numerical work.

5.3. Results

Show and discuss the results.

6. Conclusions

Write a short summary and conclusion in the end.

References

Winther, H. A. 2023, Numerical Project

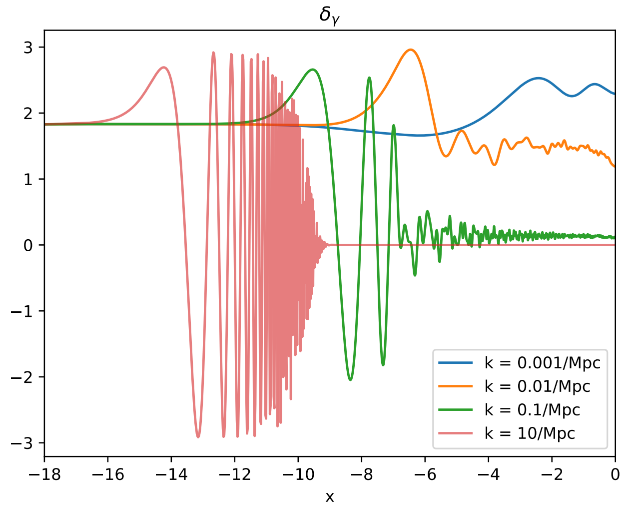

Fig. 6. The photon density perturbation, $\delta_\gamma = 4\Theta_0$, at four different scales, k.

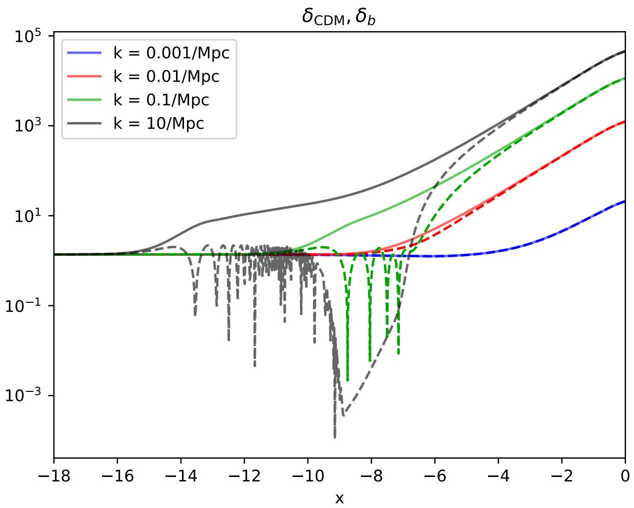

Fig. 7. The CDM overdensity and the absolute value of the baryon overdensity plotted at four different scales, k. The solid lines are for CDM and the dotted lines are for baryons.

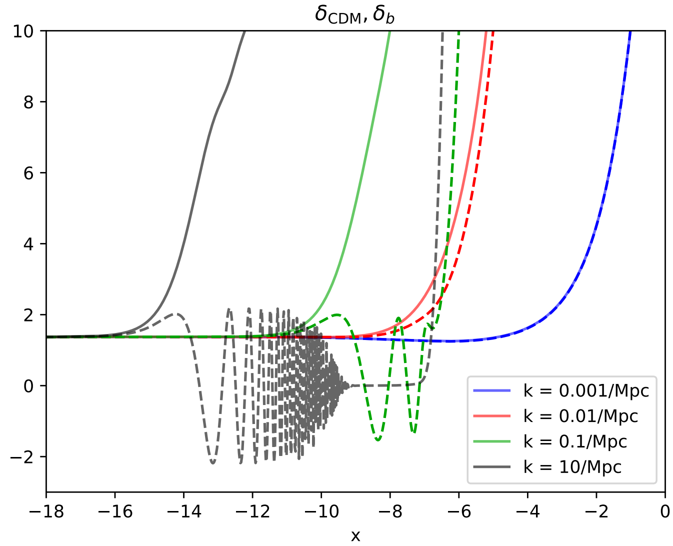

Fig. 8. A closer look at figure 7 with linear y-scale and with the correct sign for the baryon overdensity.

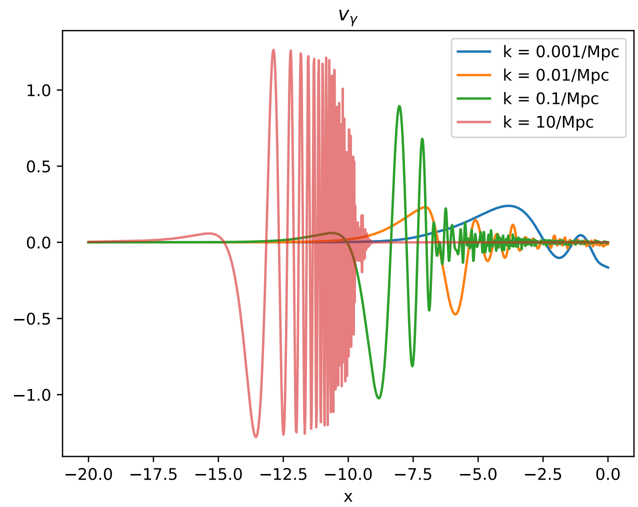

Fig. 9. The photon velocity perturbation, $v_\gamma = -3\Theta_1$, at four different scales, k.

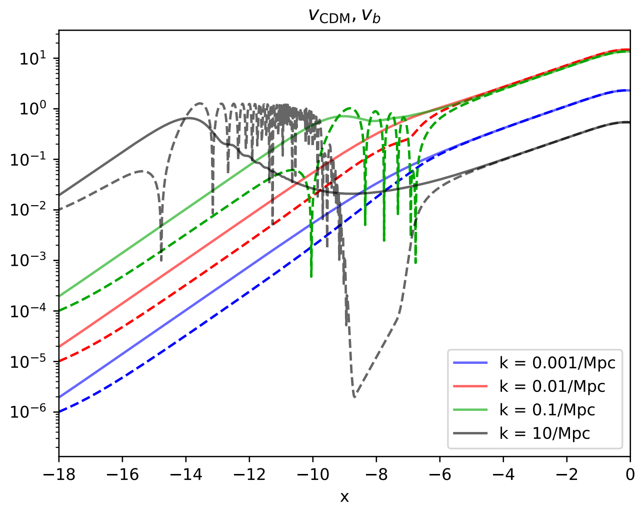

Fig. 10. The CDM velocities, in solid lines, and the absolute value of the baryon velocities, in dotted lines, are plotted at four different scales, k.

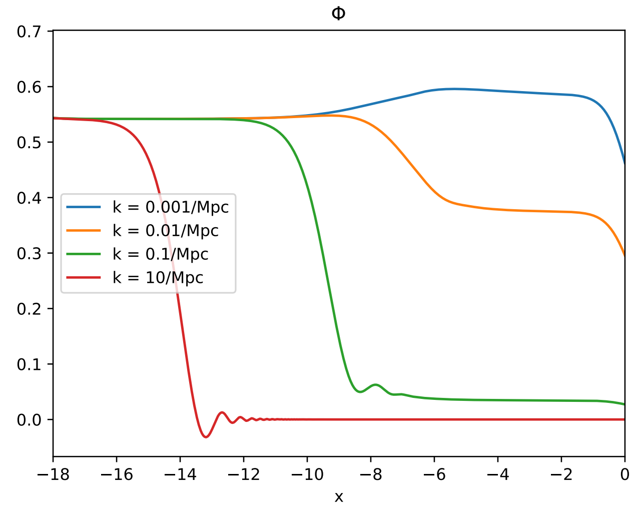

Fig. 12. The gravitational potential at four different scales, k.

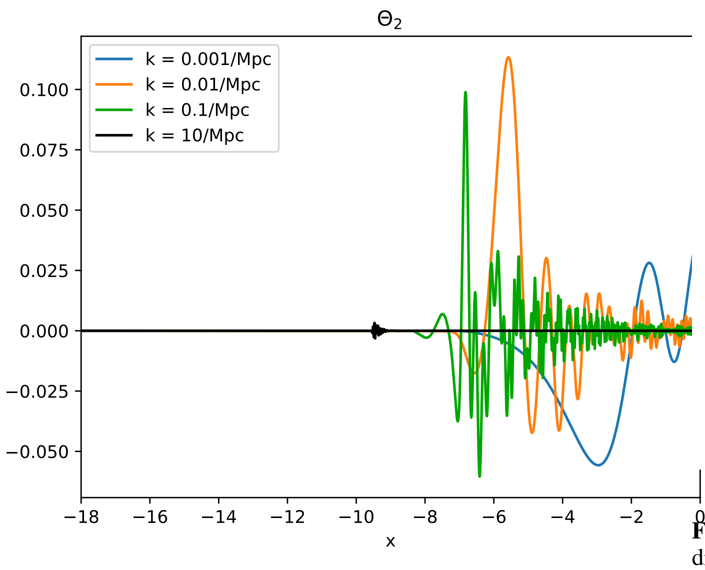

Fig. 11. The photon temperature quadrupole, Θ_2, at four different scales, k.

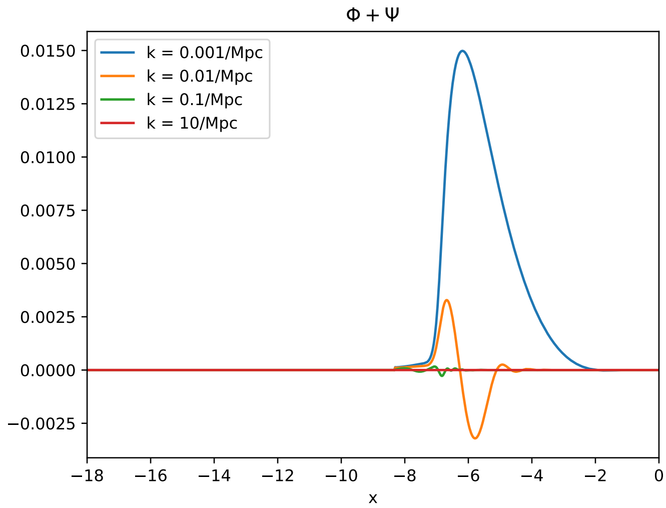

Fig. 13. The sum of the two gravitational potentials, Φ and Ψ, at four different scales, k.