Project 3: Car Demand and Home Market Bias

By Astrid Sophie Fugleholm, Jacob Strabo and Jørgen Høst

Characters: Approx 12000 (including spaces and math).

1 Introduction

Swedes only drive Volvos, Italians only drive Fiats and Germans only drive in VW. Or do they? In this paper, we investigate the potential home bias in the demand for cars by computing the own-price elasticity of demand. To do this, we use a conditional logit model on data of the top 40 most sold cars in 5 different markets for the past 30 years. We find a own-price elasticity of -0.11 for home-produced cars. Though statistically insignficant, the demand for home-produced cars is somewhat less price-sensitive than it is for foreign-produced cars. For the latter, we find an own-price elasticity of -0.25.

2 Data

We use panel data for $M = 5$ countries over $T = 30$ years (1970-1999) on the $J = 40$ highest-selling cars in those countries. We refer to a country-year pair as a market, indexed by $i = 1, ..., N$, where $N = M \cdot T = 150$. Our data has a total of $N \cdot J = 6,000$ observations. Our data contains 85 variables capturing various characteristics of the cars. These include price, technical attributes, type, manufacturer, country of manufacturer, etc. We use the share of individuals in market i that purchased car j as the market share, denoted by $\mathbf{y}_i := (y_{i1}, ..., y_{iJ})'$.

Aiming to predict market shares, we use 8 core car characteristics in our estimations, which are likely determinants of individuals' utility of cars. These include price (`logp`) (measured as log of price relative to per capita income), weight (`we`), cylinder volume (`cy`), horsepower (`hp`), an indicator for the car being produced domestically (`home`), and brand fixed effects. Including constants without car-specific variation only shifts levels and not relative ranking of utilities and thus the likelihood contribution, why we do not include these in our models (Schjerning (2021)). It may be that all cars have seatbelts, and that individuals value this highly, but preciously because all cars have them, this attribute alone does not change the order of choice.

3 Modelling Demand for Cars

To model the demand for cars, we define utility of household h from choosing car j in market i as

$$u_{ijh} = \mathbf{x}_{ij}\beta_0 + \varepsilon_{ijh}, \quad j = 1, ..., J, \tag{1}$$

where $\mathbf{x}_{ij}$ is a $K \times 1$ vector of observable market-car characteristics, and ε_{ijh} is an IID Extreme Value Error term observed by the household but not us, the econometrician. Equation (1) illustrates the idea that different car characteristics contributes differently to individuals' utility. If, for instance, the coefficient on horsepower is positive, a care with more horsepower will give individuals higher utility, ceterus paribus. Put differently, when a car's horsepower increases, the likelihood of individuals purchasing it increases.

The choice probability of j, i.e. the probability that car j gives individual i the highest utility is therefore:

$$\Pr(\text{household } h \text{ chooses car } j|\mathbf{X}_i) = \frac{\exp(\mathbf{x}_{ij}\beta_0)}{\sum_{k=1}^{J}\exp(\mathbf{x}_{ik}\beta_0)} \equiv s_j(\mathbf{X}_i, \beta_0), \tag{2}$$

where $\mathbf{X}_i = [\mathbf{x}_{i1}, \mathbf{x}_{i2}, ..., \mathbf{x}_{iJ}]'$ is a $J \times K$ matrix. The average choice probability of households purchasing a specific cars makes out the market share for that specific car. This means, that if the market share of a Fiat 500 is 10 pct. then the average choise probability of a household purchasing a Fiat 500 is 10 pct. Since the exponential function cannot handle large numbers, we max re-scale by subtracting from $\mathbf{x}_{ij}\beta$ the maximum utility from any alternative:

$$\Pr(\text{household } h \text{ chooses car } j|\mathbf{X}_i) = \frac{\exp(\mathbf{x}_{ij}\beta_0 - K_i)}{\sum_{k=1}^{J}\exp(\mathbf{x}_{ik}\beta_0 - K_i)}, \quad K_i = \max_{j\in\{1,2,...,J\}} \mathbf{x}_{ij}\beta_0. \tag{3}$$

Max re-scaling in this manner ensures numerical stability but does not change the choice probabilities as it amounts to subtracting constants from $\mathbf{x}_{ij}\beta_0$ in the exponential function.

4 Econometric Model

4.1 The Random Utility Model (RUM)

The Random Utility Model (RUM) provides a framework for modeling N individuals' utility-maximizing choice over a discrete set of J alternatives. Individual i's utility from choosing alternative j is given as

$$u_{ij} = v(x_{ij}, \theta) + \varepsilon_{ij} \tag{4}$$

$$y_i = \underset{j\in\{1,2,...,J\}}{\arg\max}\ u_{ij} \tag{5}$$

where $v(x_{ij}, \theta)$ is the deterministic utility for individual i from choosing alternative j given the characteristics of the individual and the alternative, and $\varepsilon_{ij} \sim$ IID Extreme Value Type I is the stochastic utility. Individual i chooses the alternative y_{ij} associated with the highest utility after observing both characteristics and taste shocks (the deterministic and stochastic part of utility from j).

4.2 The conditional logit model

The conditional logit model lies within the set of random utility models. Individuals' choices of alternatives depend on the characteristics of the alternatives, and all individuals are assumed to have the same preferences and thus respond similarly to changes in characteristics (Munk-Nielsen (2022)). A key aspect of the conditional logit model is that we cannot hope identify a constant in the regression.

We estimate our conditional logit model using the conditional maximum likelihood estimation (CMLE) nested in the class of M-estimators. The loglikelihood contribution for market i is

$$l(\mathbf{y}_i, \mathbf{X}_i; \beta) = \sum_{j=1}^{J} y_{ij} \log s_j(\mathbf{X}_i, \beta). \tag{6}$$

Minmizing the criterion function, i.e. the average negative log likelihood, gives us our estimator

$$\widehat{\beta} = \arg\min_{\beta} -\frac{1}{N}\sum_{i=1}^{N} l(\mathbf{y}_i, \mathbf{X}_i; \beta). \tag{7}$$

As the number of households H is assumed to be large enough for $s_j(\mathbf{X}_i, \beta_0) = y_{ij}$, we let the number of markets grow infinitely ($N \to \infty$) when considering asymptotic results. We have no reason to believe our this function to be non-convex, so we use the standard gradient-based Quasi-Newton (`BFGS`) algorithm in mimizing our log-likelihood function with respect to β.

4.2.1 Asymptotic Properties of M-Estimators

Theorem 12.2 in Wooldridge (2010a) states that the MLE $\widehat{\beta}$ is consistent for β_0 under 1) identification, and 2) a Uniform Law of Large Numbers (ULLN). Identification is assumed and implies that β_0 is identified, i.e. β_0 is the unique solution to the (population) minimization problem. Identification fails if we include variables that do not vary across alternatives but only countries and/or years, e.g. income per capita (Schjerning (2021)). The ULLN indicate that for any parameter β in the parameter space, the sample minimand converges to the population minimizer uniformly in probability. The sufficient conditions for the ULLN to hold are that 1) the parameter space is compact, 2) the criterion function in (7) is continuous in *beta*, and 3) some additional technical conditions (Sørensen (2022)).

Theorem 12.3 in Wooldridge (2010a) states that under the assumptions that four main and additional technical assumptions, the MLE is asymptotically normal:

$$\sqrt{N}(\widehat{\beta} - \beta_0) \to^d N(0, \mathbf{A}_o^{-1}\mathbf{B}_o\mathbf{A}_o^{-1}), \quad \mathbf{A}_o := \mathbb{E}[\mathbf{H}(\mathbf{X}, \beta_0)], \quad \mathbf{B}_o := \mathbb{E}[\mathbf{s}(\mathbf{X}, \beta_0)\mathbf{s}(\mathbf{X}, \beta_0)'] = \mathrm{Var}[\mathbf{s}(\mathbf{X}, \beta_0)]. \tag{8}$$

The four main assumptions are that 1) β_0 is interior to the compact parameter space; 2) the criterion function in (7) is twice continuous and differentiable on the interval of the parameter space; 3) the average score equals zero; and 4) the Hessian is positive definite (Sørensen (2022)).

4.2.2 Variance Computation

Given consistency and asymptotic normality, we compute the variance of our MLE using the sandwich formula:

$$Avar(\widehat{\beta}) = \mathbf{A}_o^{-1}\mathbf{B}_o\mathbf{A}_o^{-1}/N, \tag{9}$$

where $\mathbf{A}_o$ and $\mathbf{B}_o$ are defined as in (8) (Wooldridge (2010a)).

4.3 Home market bias

To assess how home bias affects the own- and cross-price elasticities of demand, we calculate these two measures using the output from our conditional logit model. The elasticity of demand for car j with respect to the price of car j (own-price elasticity) or another car k (cross-price elasticity) are given as

$$\mathcal{E}_{jj}(\mathbf{X}_i) := \frac{\partial s_j(\mathbf{X}_i, \beta_0)}{\partial p_{ij}} \frac{p_{ij}}{s_j(\mathbf{X}_i, \beta_0)} = \frac{\partial \nu_j}{\partial p_{ij}} s_j(s - s_j) \frac{p_{ij}}{s_j} = \frac{\partial \nu_j}{\partial p_{ij}} (1 - s_j) p_{ij} = \beta_p (1 - s_j) p_{ij} \tag{10}$$

$$\mathcal{E}_{jk}(\mathbf{X}_i) := \frac{\partial s_j(\mathbf{X}_i, \beta_0)}{\partial p_{ik}} \frac{p_{ik}}{s_j(\mathbf{X}_i, \beta_0)} = -\frac{\partial \nu_k}{\partial p_{ik}} p_{ik} s_k = -\beta_p p_{ik} s_k \tag{11}$$

where $p_{ij} \in \mathbf{X}_i$ is the price of car j in market i (Munk-Nielsen (2022), Train (2009)). The own- and cross-price elasticities measure the percentage change in demand for car j due to a one percentage increase in the price of car j and another car k, respectively. A lower own-price elasticity of demand for domestic relative to foreign car manufacturers indicates home bias in the market.[1]

4.3.1 The Independence of Irrelevant Alternatives (IIA) Assumption

The cross-price elasticity measures the *proportional* effect of changing an attribute of car j on the demand for all other cars. That is, we assume that the cross-elasticity is constant across markets i (Train (2009)). This proportionate shifting pattern of substitution follows from the Independence of Irrelevant Alternatives (IIA) assumption, which states that relative probabilities of choosing any two cars only depend on attributes of those cars and not on attributes of other cars (Wooldridge (2010b)). The IIA is a reasonable assumption if we believe there are not fundamental differences between alternative cars. However, the IIA is not a sensible assumption if substitution between some cars is more likely than between others (e.g., between gasoline and electric cars), and in that case, the logit model will produce unrealistic predictions of choices (Train (2009)).

4.3.2 Variance of Elasticities

We compute standard errors on elasticities using the Delta method. It is a tool for obtaining asymptotic properties on functions of asymptotically normal estimators $h(\widehat{\beta})$ based on those of $\widehat{\beta}$. Specifically, if $\sqrt{N}(\widehat{\beta} - \beta_0) \to^d N(0, \sigma_0^2)$ for some $\sigma^2 \in \mathbf{R}_{++}$, h is continuously differentiable at β_0 with nonzero derivative $h'(\beta_0)$, and $\widehat{\sigma}^2$ is a sequence of variance estimators consistent for σ_0^2, then

$$\sqrt{N}\{h(\widehat{\beta}) - h(\beta_0)\} \to^d N\left(0, [h'(\beta_0)]^2 \sigma_0^2\right), \quad \sigma_0^2 = \mathbf{A}_o^{-1} \mathbf{B}_o \mathbf{A}_o^{-1} \tag{12}$$

meaning that the random interval

$$\left[h(\widehat{\beta}) - 1.96 \cdot \frac{\hat{\upsilon}}{\sqrt{N}}, h(\widehat{\beta} + 1.96 \cdot \frac{\hat{\upsilon}}{\sqrt{N}}\right], \quad \hat{\upsilon}^2 := [h'(\widehat{\beta}]^2 \hat{\sigma}^2 \tag{13}$$

is an asymptotically valid 95 pct. confidence interval for $h(\beta_0)$, in this case our elasticities (Munk-Nielsen and Sørensen (2021)).

[1] If we could receive feedback on this, that would be immensely helpful. We are a little unsure how notation from Train (2009) translates into coded solution.

5 Results

Estimates of our conditional logit model and an OLS-estimation in Table 1 provide suggestive evidence of home bias. Given that prices are in logs, we can directly interpret OLS-estimates as elasticties. As expected higher prices yields lower market shares, and this effect is lower in for home-produced cars, though statistically insignificant at conventional significance levels. The logit coefficients alone do not have the same interpretation, but they do have the same sign.

Table 1: *Car characteristics and market share*

	Logit	OLS
cons_	.	10.33
	.	(9.27)
logp	-0.25**	-0.78
	(0.128)	(1.26)
home	1.42***	0.56
	(0.044)	(0.881)
logp_x_home	0.14**	0.18
	(0.061)	(1.675)
cy	-0.12	-0.77
	(0.074)	(1.978)
hp	-1.52***	10.24**
	(0.187)	(4.433)
we	0.65***	-3.5
	(0.188)	(3.798)
li	-0.04**	0.27
	(0.017)	(0.251)
he	-0.01***	-0.07
	(0.003)	(0.066)
Brand dummies	Yes	Yes

Note: p<0.1*, p<0.05**, p<0.01***. Standard errors computed using the "sandwich" formula in parentheses.

5.1 Evaluating Home Bias

We turn to Table 2 to show how a change in car prices influences market shares. Specifically, we are interested in the own-price elasticity of home-produced cars. Though insignificant, we find an estimate -0.11, somewhat higher than the own-price elasticity of foreign-produced cars at -0.25. We find no effect for the cross-price elasticity, the change in demand in car j for a price-change to car k.

It is important to note that the results indicate the effect of price changes on market share and not quantities sold. In the entire paper, where we normalize the market size, why we potentially could be in a situation where market share decreases for a company, but quantities sold increase.

Table 2: *Home bias*

	Elasticity
Own-price elasticity	-0.2
	(0.126)
Own-price elasticity - home produced	-0.11
	(0.134)
Own-price elasticity - foreign produced	-0.25**
	(0.126)
Cross-price elasticity	0.0
	(0.003)

Note: p<0.1*, p<0.05**, p<0.01***. Standard errors computed using the Delta method in parentheses.

6 Discussion

In section 4.3.1, we presented the assumption of Independence of Irrelevant Alternatives (IIA) which is assumed to hold in the (conditional) logit model, but may not be the case in this context. It has been suggested that the oil crisis of the 1970's has changed the long-run demand for fuel-efficient cars (Bonilla and Foxon (2009)). This implies that the relative substitution pattern which violates the IIA has also changed over time.

Further, the model potentially suffers from omitted variable issues, as the model does not consider characteristics of individuals nor countries (markets) that potentially can be the explanatory variables that drives car purchases. Furthermore the model only considers market shares, and not quantities sold. Had we instead had data on quantities sold of each car, then we would further be able to interpret on the total attribution of each variable on the aggregated and individual number of sold cars.

7 Conclusion

In this paper, we investigate the phenomenon of home-market bias in the demand for cars. To do this, we use a conditional logit model in which we feed observed characteristics of the 40 most sold cars in 5 countries over 30 years. While we find tentative evidence of home market bias, our estimates are not statistically significant at conventional significance levels.

References

David Bonilla and Timothy Foxon. Demand for new car fuel economy in the uk, 1970–2005. *Journal of Transport Economics and Policy (JTEP)*, 43(1):55–83, 2009.

Anders Munk-Nielsen. Discrete response: The logit model - advanced microeconometrics. 2022.

Anders Munk-Nielsen and Jesper Riis-Vestergaard Sørensen. Advanced microeconometrics, autumn 2021: The delta method. 11 2021.

Bertel Schjerning. Multinomial response 2 identification, 2021. URL `https://www.youtube.com/watch?v=BtsjO4VwOjw`.

Jesper Riis-Vestergaard Sørensen. M-estimation, i: Introduction and asymptotic properties. 2022.

Kenneth Train. *Discrete Choice Methods with Simulations*, chapter 3: Logit. Cambridge University Press, 2009.

Jeffrey M. Wooldridge. *Econometric Analysis of Cross Section and Panel Data*, chapter 12: M-Estimation. The MIT Press, 2nd edition, 2010a.

Jeffrey M. Wooldridge. *Econometric Analysis of Cross Section and Panel Data*, chapter 15: Discrete Response Models. The MIT Press, 2nd edition, 2010b.