METHODS 3: MULTILEVEL STATISTICAL MODELLING AND MACHINE LEARNING

COURSE OVERVIEW (FIRST HALF)

W1: Introduction

Setting up R and Python and recollection of the general linear model

W2: Linear Mixed Effects Models

Modelling random effects – and how do they differ from fixed effects?

W3: Generalized Linear Mixed Effects Models

What to do when the response variable is not continuous?

W4: Explanation and prediction

Why are good explanations sometimes bad?

W5: Evaluating and comparing models

How do we assess how models compare to one another?

Fall break:

Machine Learning and Python programming follows

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TODAYS PLAN

- Catch-up
- Output – logistic regression
- QQ-plots
- *(Model selection criteria*
    - *R-squared (revisited)*
    - *AIC/BIC*
    - *Out-of-sample-error)*
- Assignment tips
- Assignment work

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CATCH-UP

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- How are you hanging in? Last assignment? Git/GitHub?
- Extra readings
- Where would you want new messages?
- Bigger visualizations this time around
- Disclaimer

TODAYS PLAN

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logistic_model <- glm(formula = am ~ wt,
                      data = mtcars,
                      family = "binomial")
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- wt = weight (continuous)

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Uses link function

->

To log-odds, to have linear scale

OUTPUT – LOGISTIC REGRESSION

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am ~ wt
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> summary(logistic_model)

Coefficients:
              Estimate Std.
(Intercept)    12.040
wt             -4.024
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- Output in log-odds
  to have linear relationship
  between wt and am

OUTPUT – LOGISTIC REGRESSION

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Coefficients:
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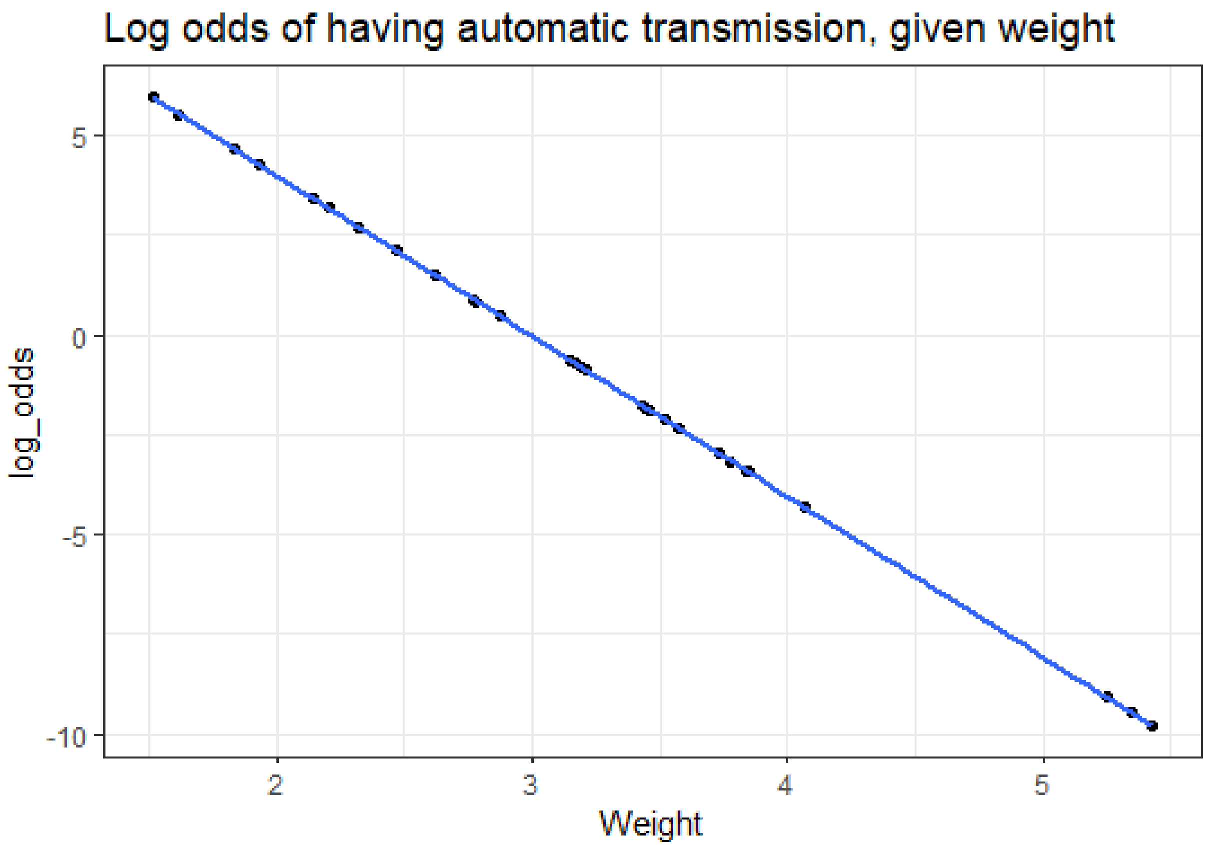


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OUTPUT – LOGISTIC REGRESSION

- Going through example of a Mazda

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OUTPUT – LOGISTIC REGRESSION

- Going through example of a Mazda

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> head(mazda)
          mpg cyl disp  hp drat   wt  qsec vs am gear carb
Mazda RX4  21   6  160 110  3.9 2.62 16.46  0  1    4    4
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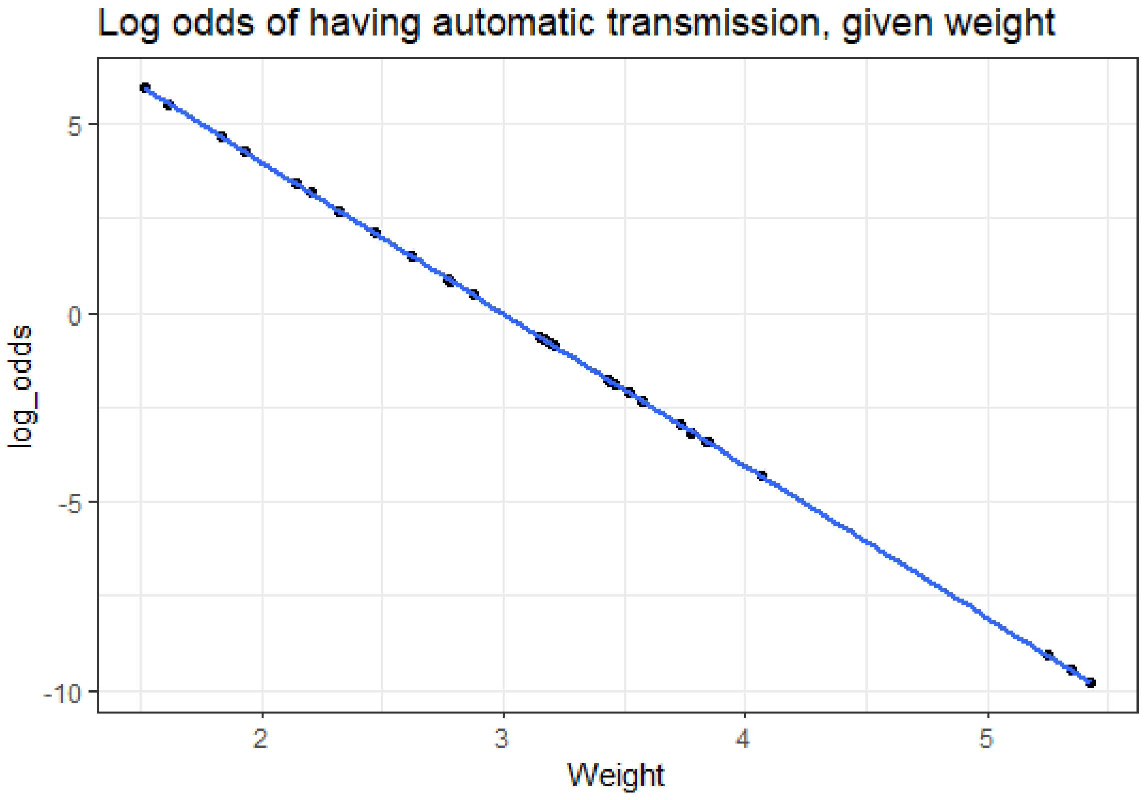

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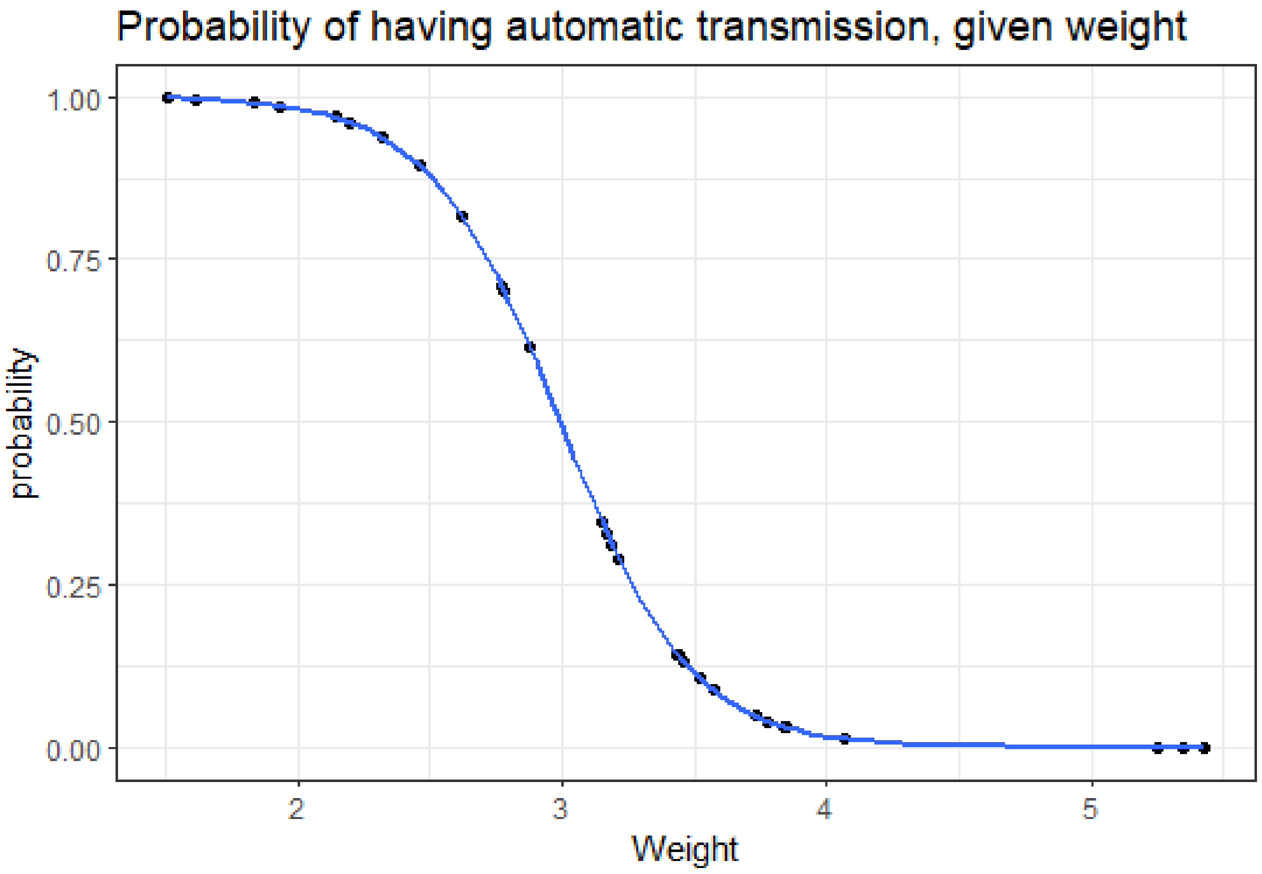

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"Inverse-link function"

```
inv.logit <- function(x) exp(x) / (1 + exp(x))
```

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inv.logit <- function(x) exp(x) / (1 + exp(x))

> inv.logit(1.49712)
[1] 0.8171445
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- logistic_model$fitted.values takes the inv-logit automatically

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- Going through example of a Mazda

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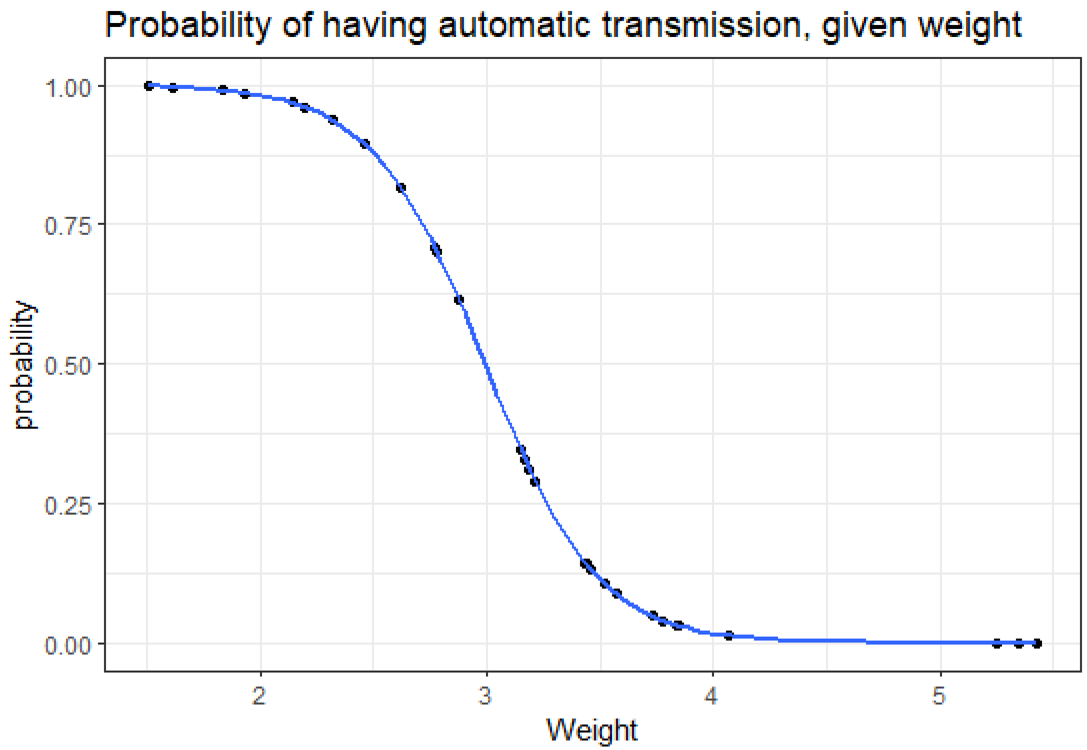
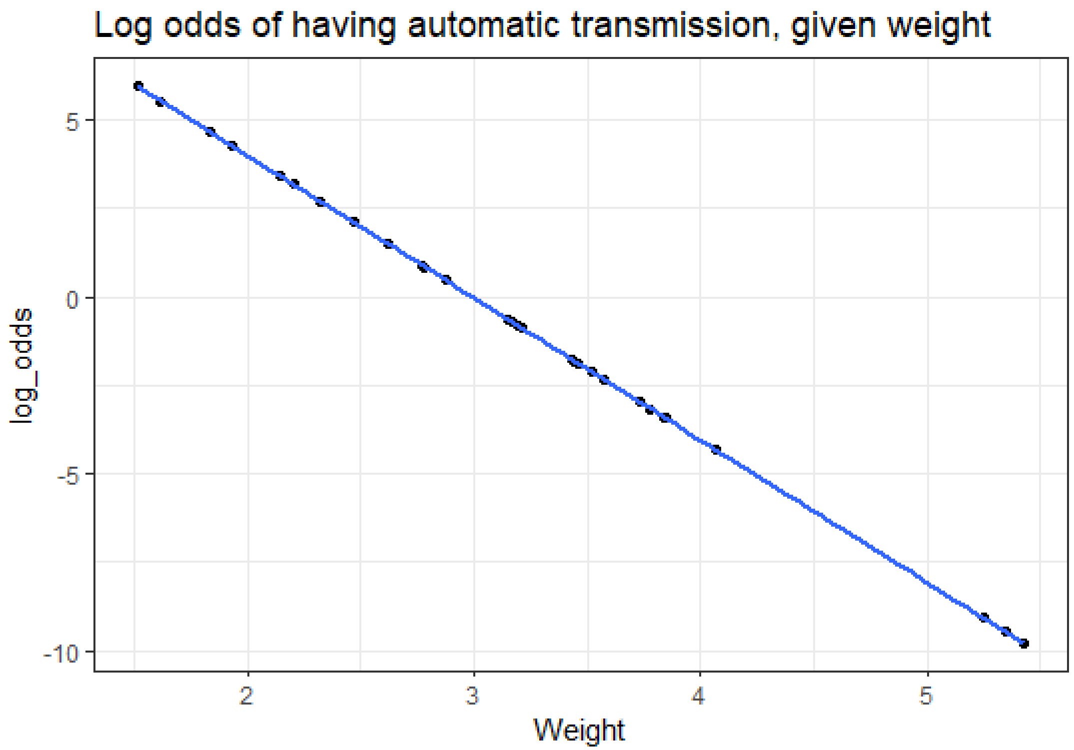
```
> logistic_model$fitted.values
       Mazda RX4    Mazda RX4 Wag
    8.172115e-01     6.157283e-01
```

OUTPUT – LOGISTIC REGRESSION





TODAYS PLAN

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- Output – logistic regression
- **QQ-plots**
- *(Model selection criteria*
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QQ-PLOTS

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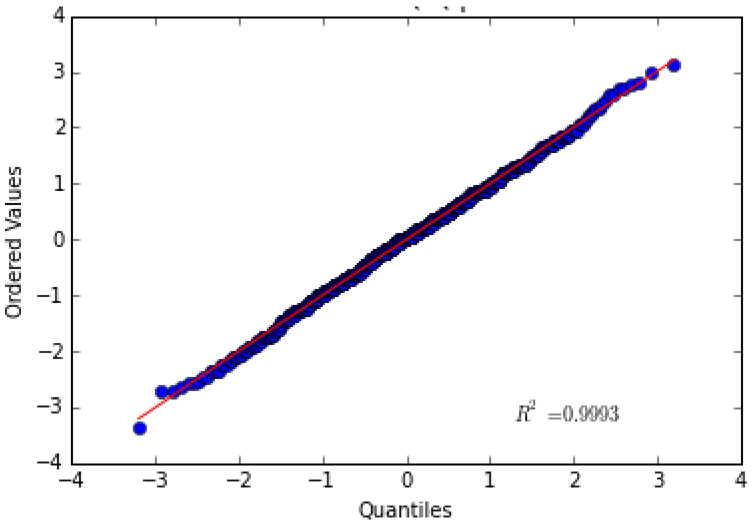

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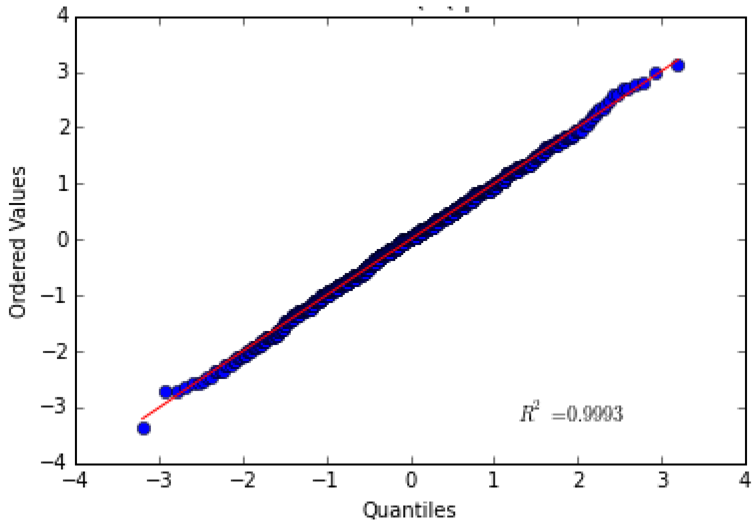
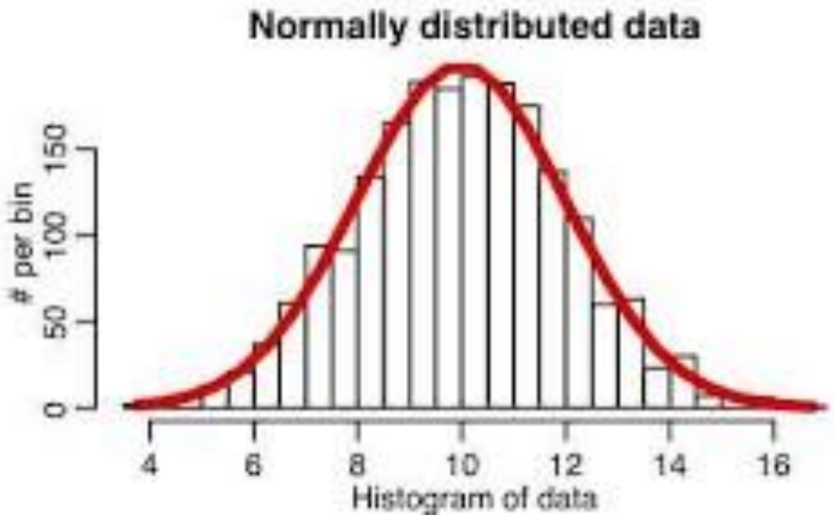
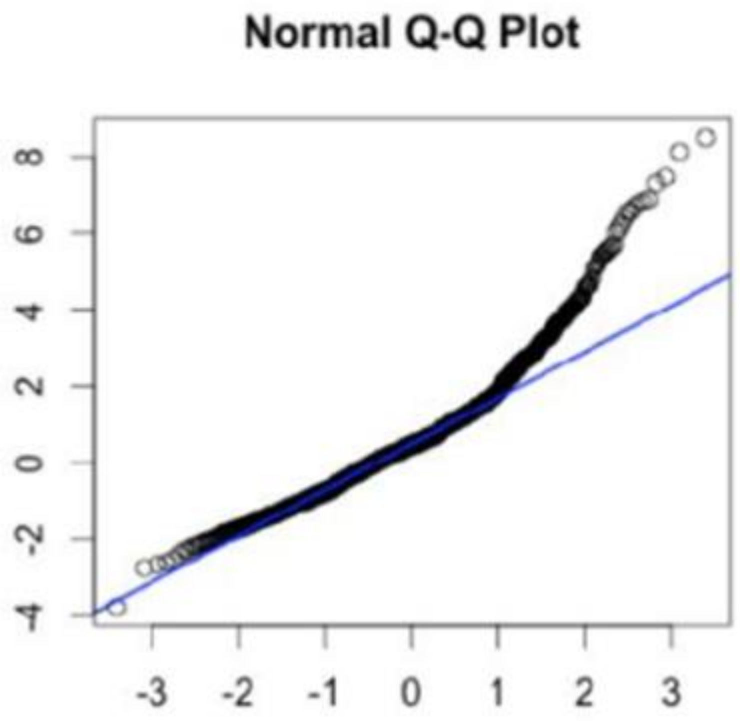


Ordered Values
Quantiles
R^2 =0.9993
4
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2
3
4



Normally distributed data
per bin
0
50
100
150
4
6
8
10
12
14
16
Histogram of data

QQ-PLOTS



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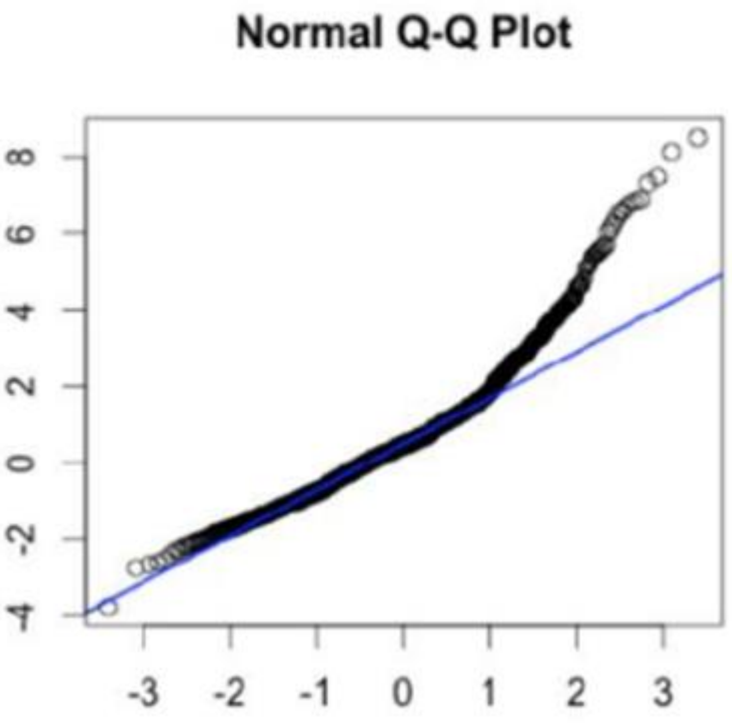

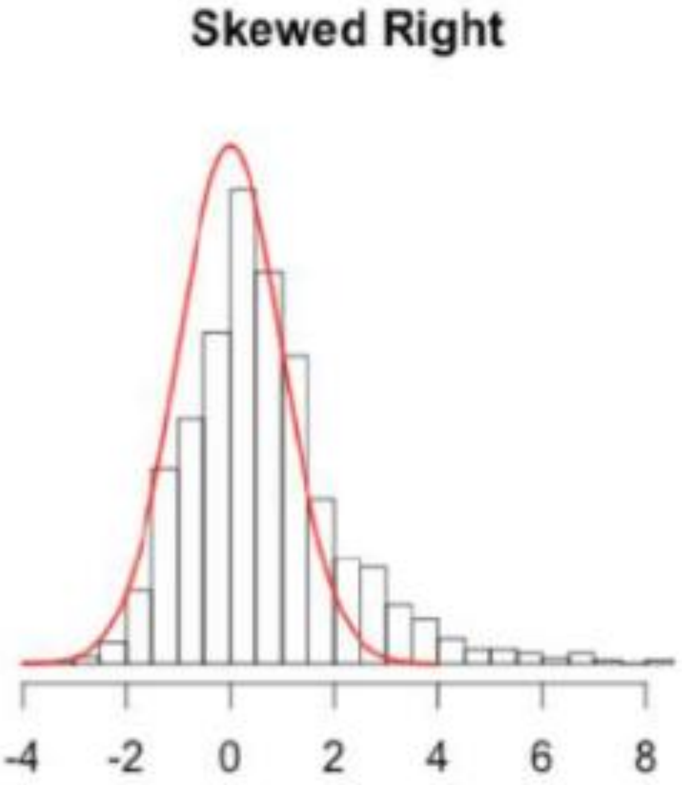

QQ-PLOTS

- Quantiles-quantile plots

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- What is a quantile?

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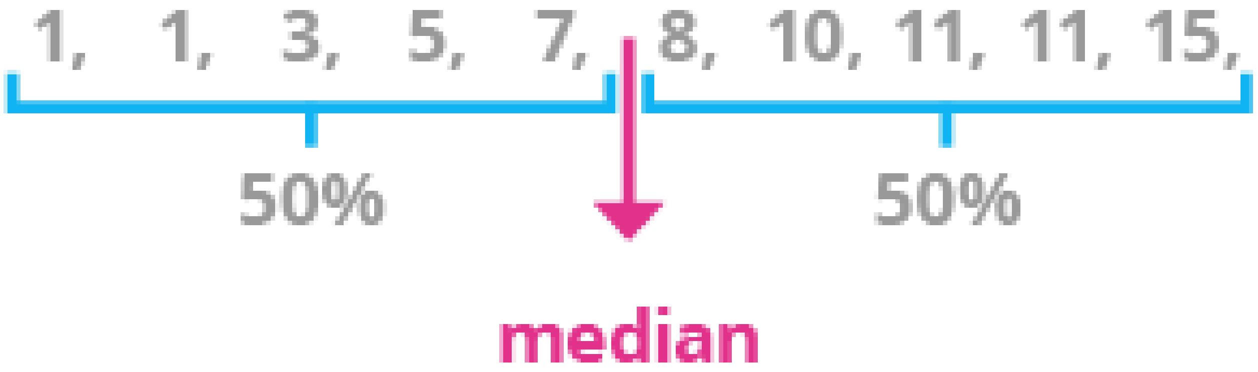

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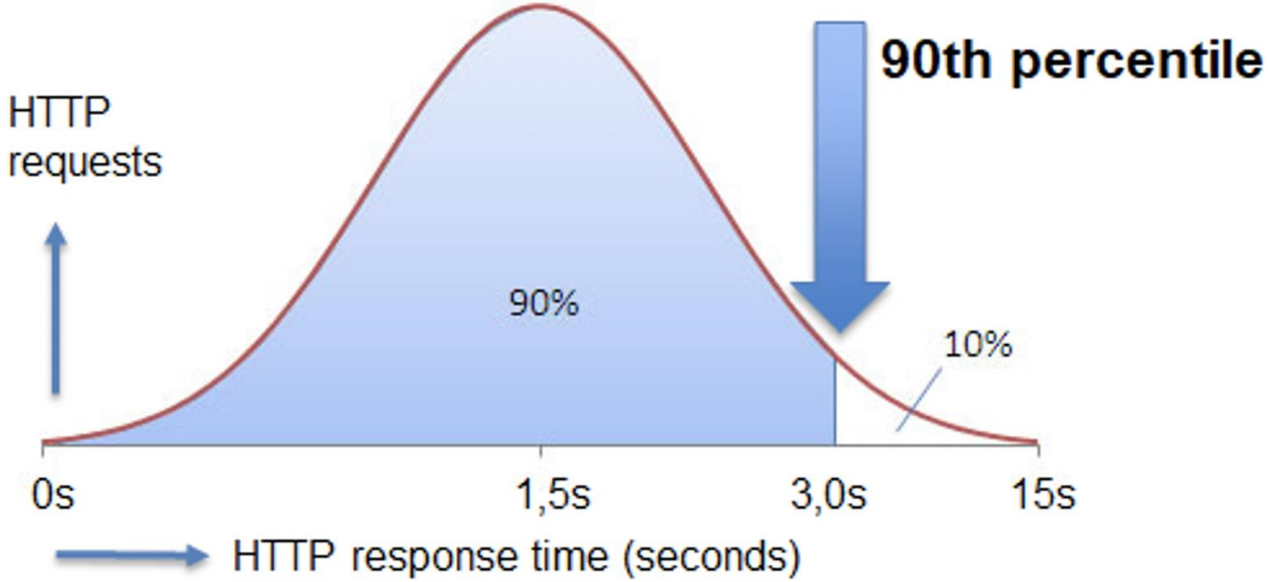
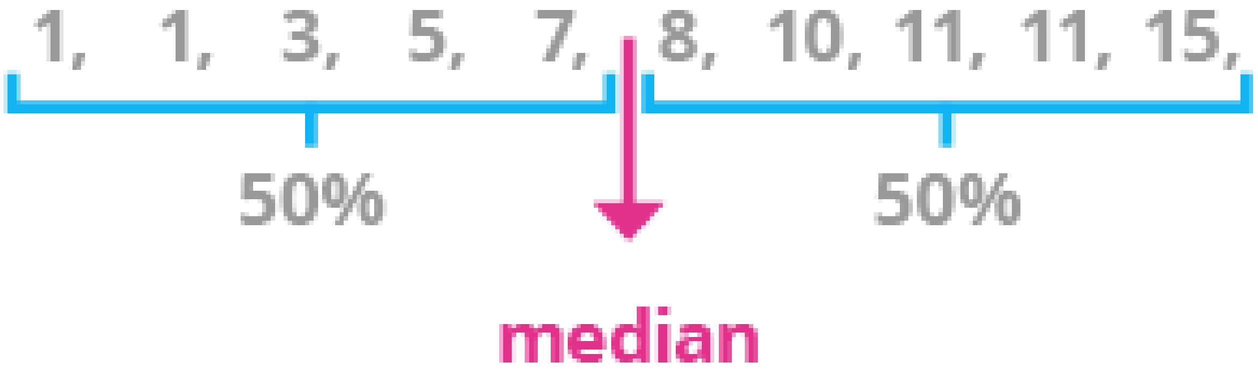




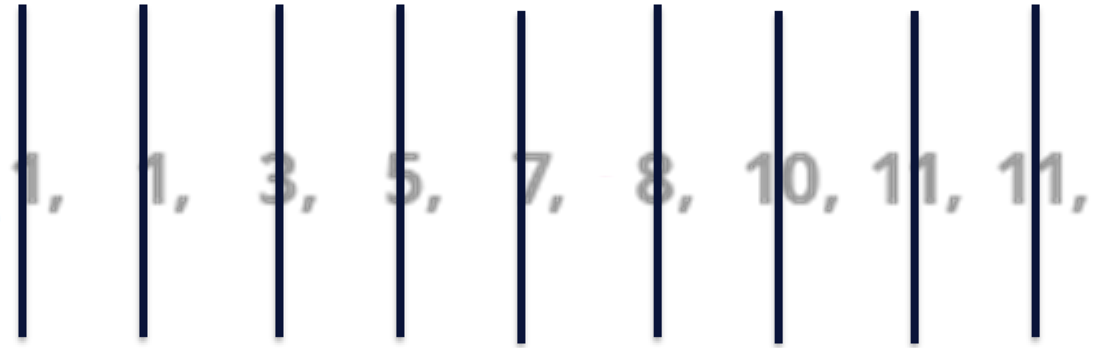
QQ-PLOTS

- Quantiles

1, 1, 3, 5, 7, 8, 10, 11, 11,

QQ-PLOTS

- Quantiles



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ASSIGNMENT TIPS

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- Understanding assignment data
    - Experiment paradigm

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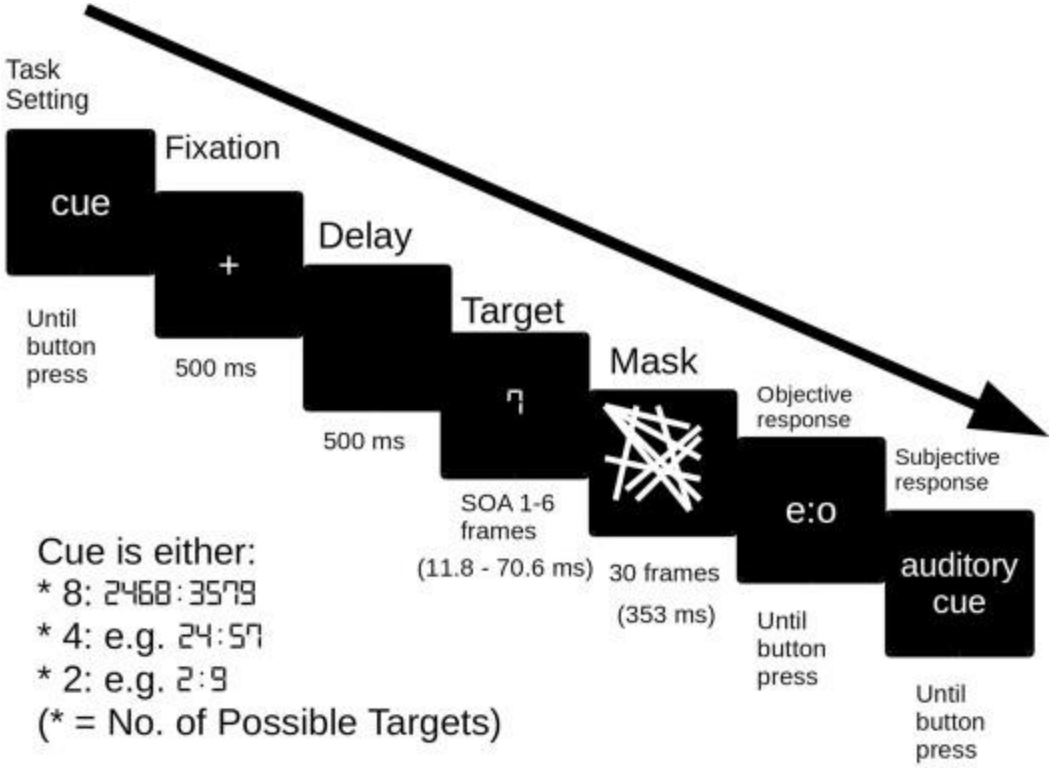
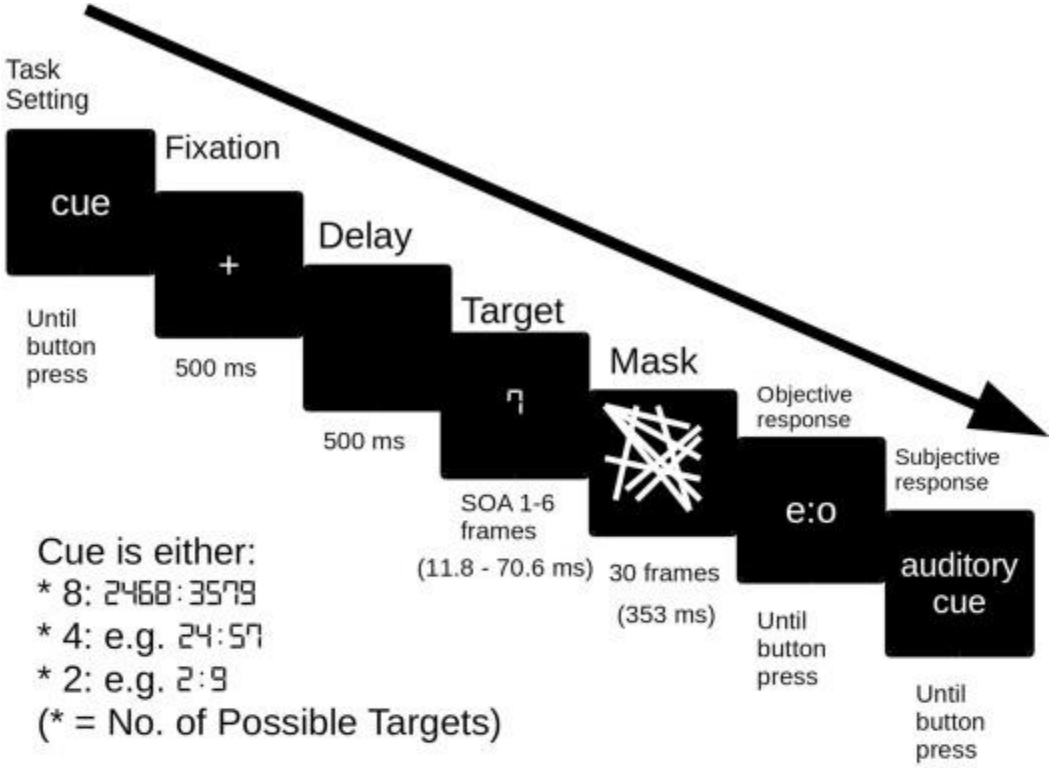
- Understanding the paradigm



ASSIGNMENT TIPS

- Understanding the paradigm
  - Task (8, quadrant, pairs)
  - PAS (how clearly was target seen?
  - rt.obj (Reaction time -> Even/Odd)
  - rt.sub (Reaction time -> PAS)



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- ggplot(mtcars, aes(mpg, disp)) +
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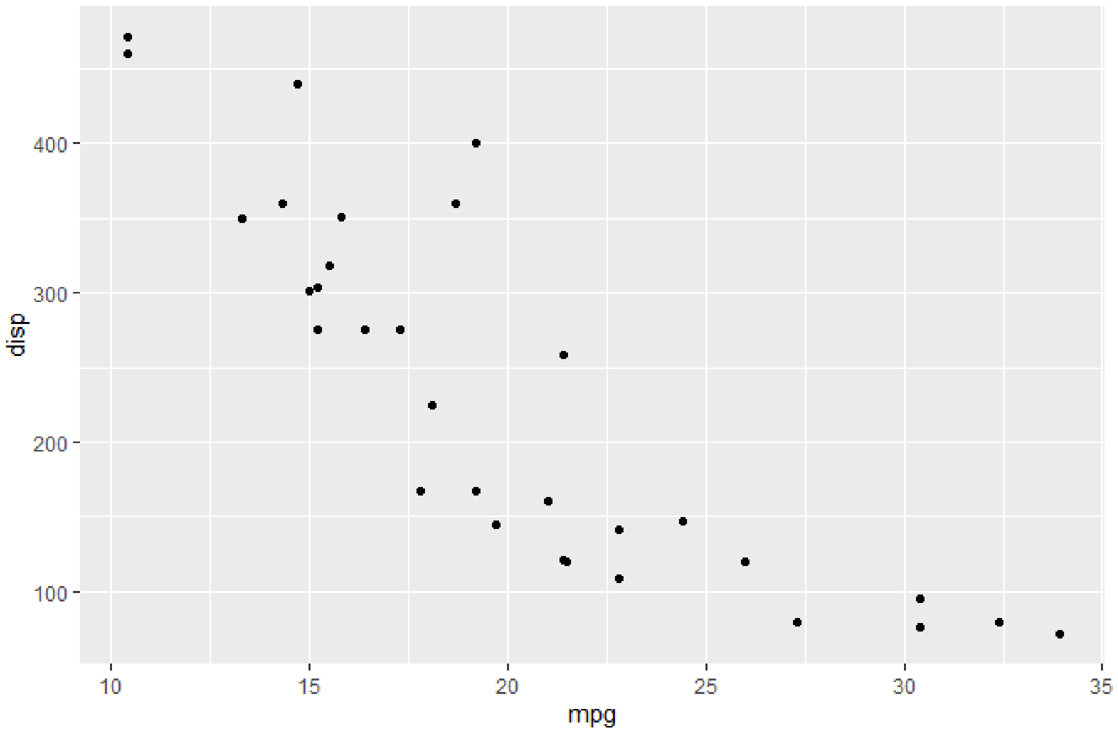

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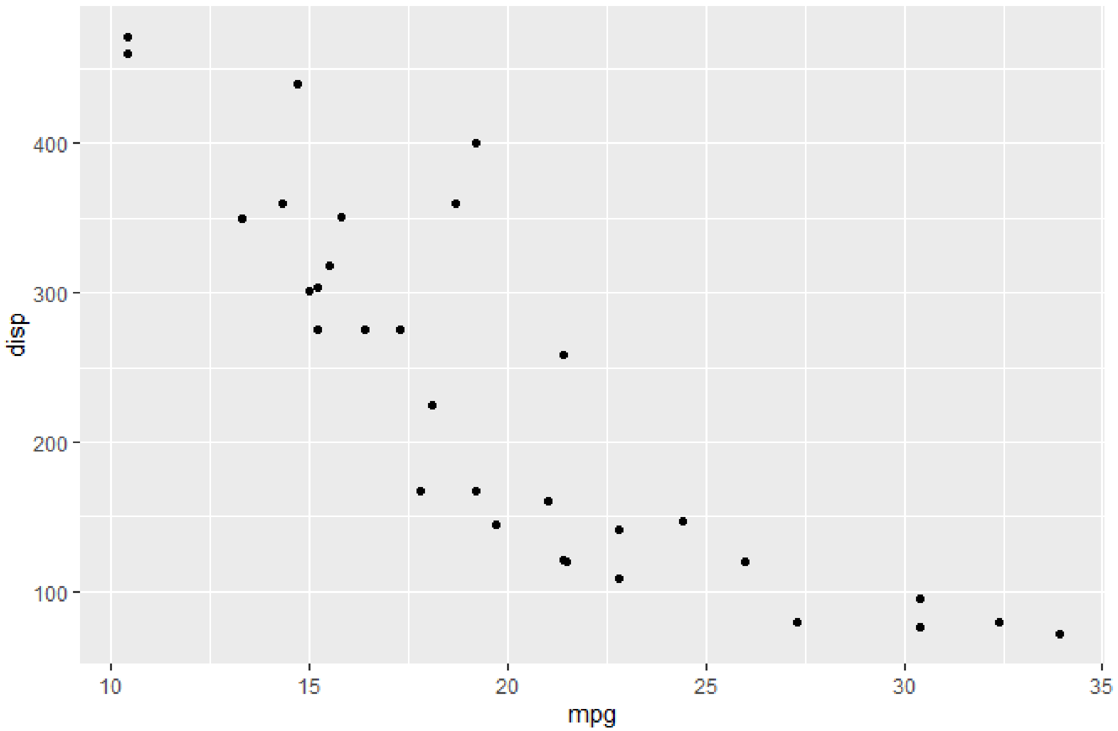

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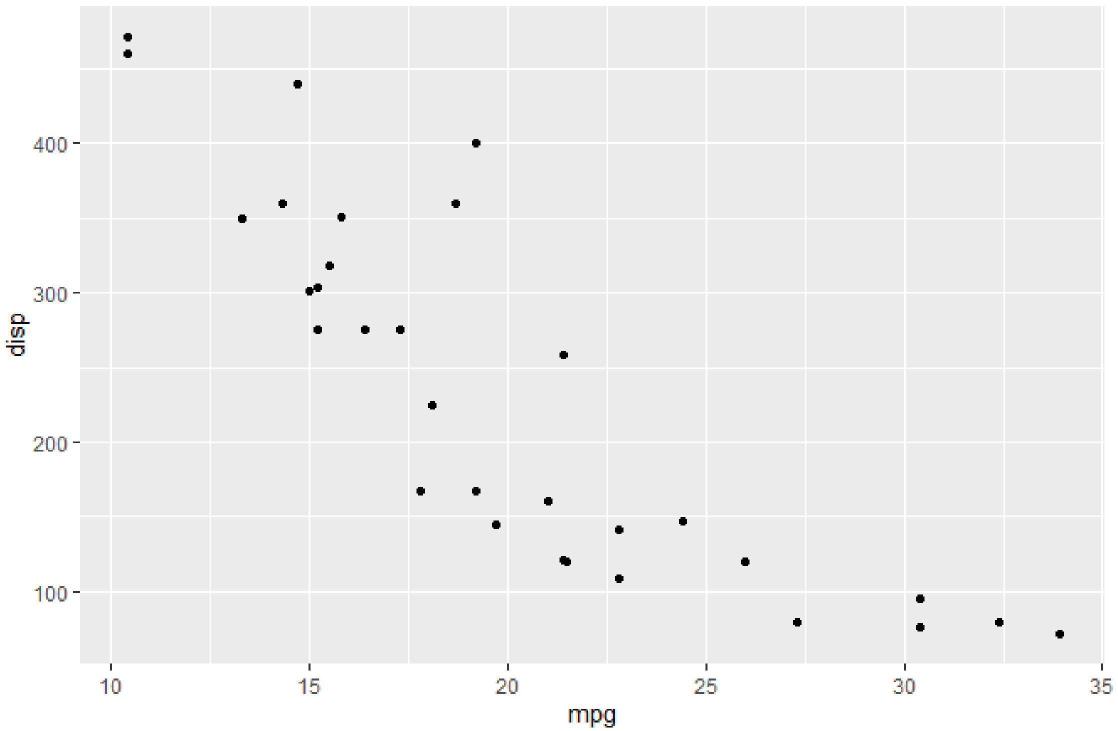

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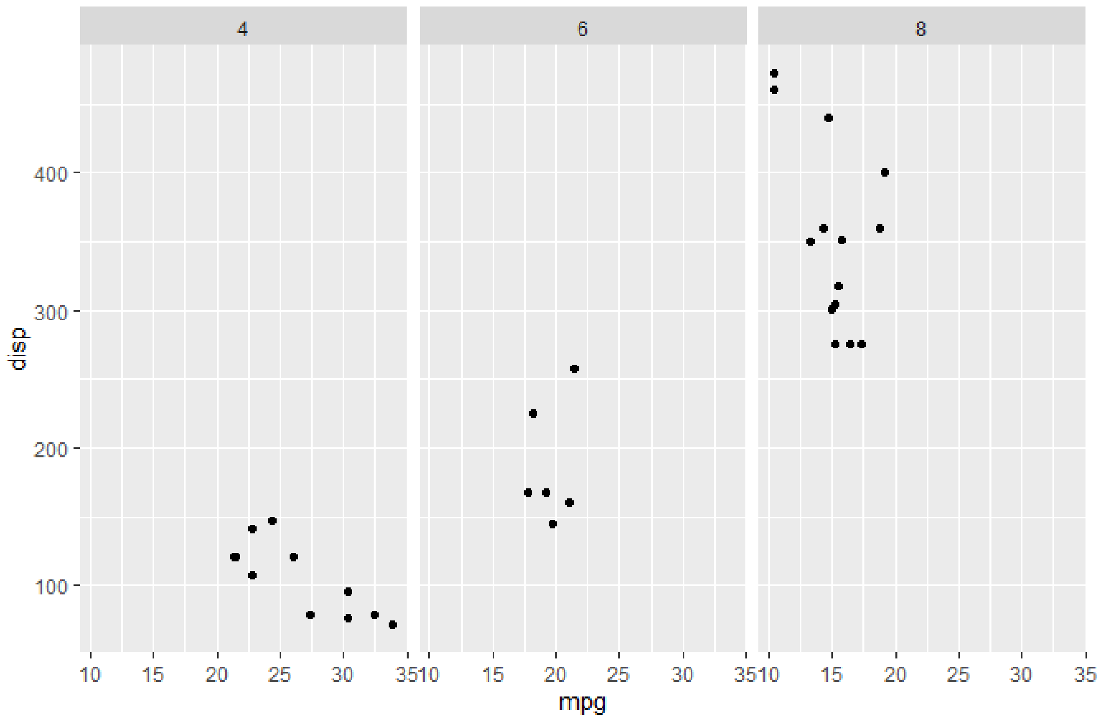

ASSIGNMENT TIPS

- Loading in the files
  - pwd in R = getwd()
  - ls in R = list.files()

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```
> getwd()
[1] "C:/Users/Lenovo/Desktop/Diverse dokumenter/C
nce/github_methods_3/week_03/non_student"
```

```
list.files("/experiment_2/")
[1] "001.csv" "002.csv" "003.csv" "004.csv" "005.csv"
[6] "006.csv" "007.csv" "008.csv" "009.csv" "010.csv"
11] "011.csv" "012.csv" "013.csv" "014.csv" "015.csv"
16] "016.csv" "017.csv" "018.csv" "019.csv" "020.csv"
21] "021.csv" "022.csv" "023.csv" "024.csv" "025.csv"
26] "026.csv" "027.csv" "028.csv" "029.csv"
```

BREAK TIME


THIS IS GIT. IT TRACKS COLLABORATIVE WORK ON PROJECTS THROUGH A BEAUTIFUL DISTRIBUTED GRAPH THEORY TREE MODEL.
COOL. HOW DO WE USE IT?
NO IDEA. JUST MEMORIZE THESE SHELL COMMANDS AND TYPE THEM TO SYNC UP. IF YOU GET ERRORS, SAVE YOUR WORK ELSEWHERE, DELETE THE PROJECT, AND DOWNLOAD A FRESH COPY.

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ASSIGNMENT

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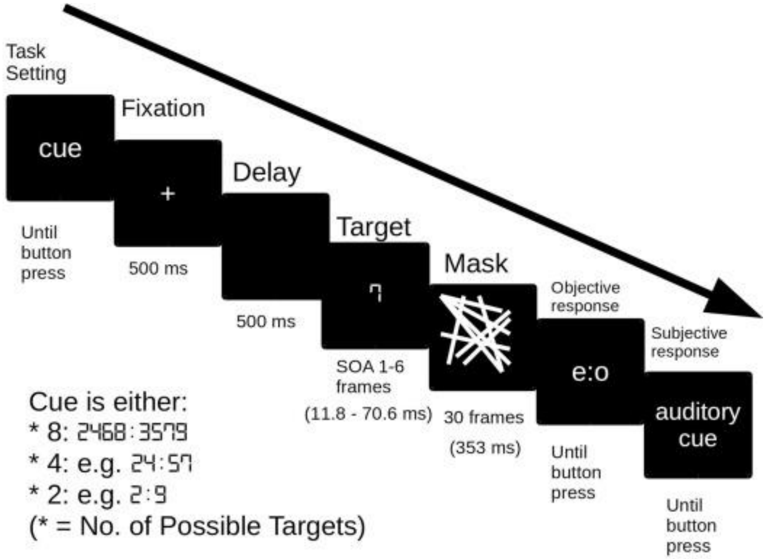
1. *Pull new assignment from upstream*
2. **Download files from BrightSpace, put into new folder "github_methods_3/week_03/"**
3. Create copy of "practical_exercise_3.Rmd", give unique filename ending
4. Work on new copied version of assignment



- list.files()
- getwd()
- + facet_wrap(~ subject)

AARHUS
UNIVERSITY

MODEL SELECTION CRITERIA

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- Why are we modeling?

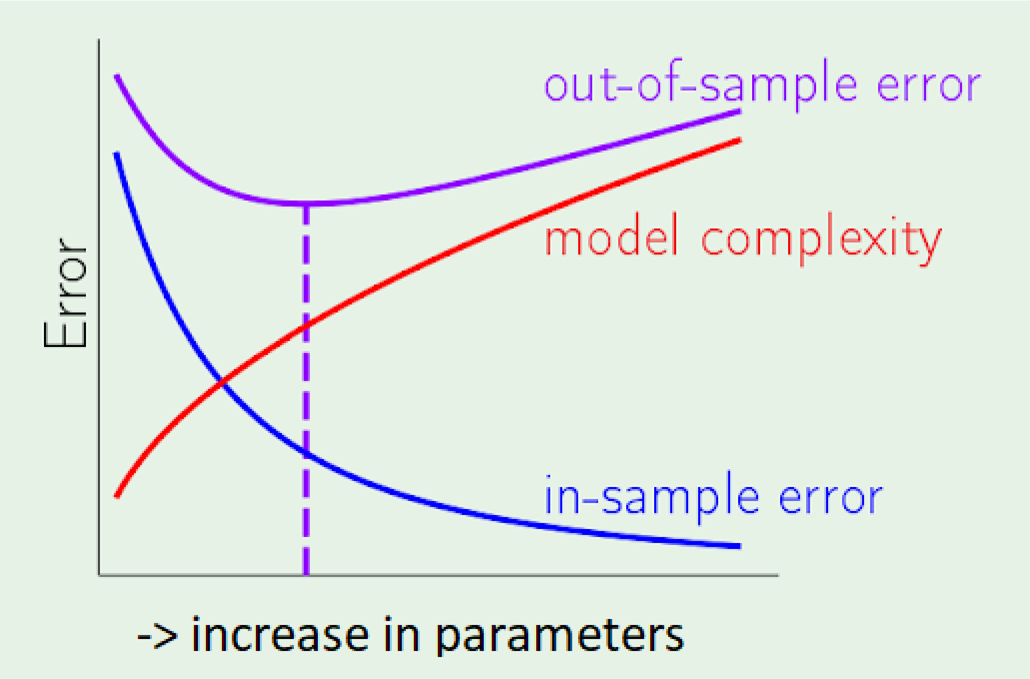
- What constitutes a good model?

MODEL SELECTION CRITERIA

- Why are we modeling?
  - *To be able to understand the world*
  - *Why understand? To be able to predict/manipulate*

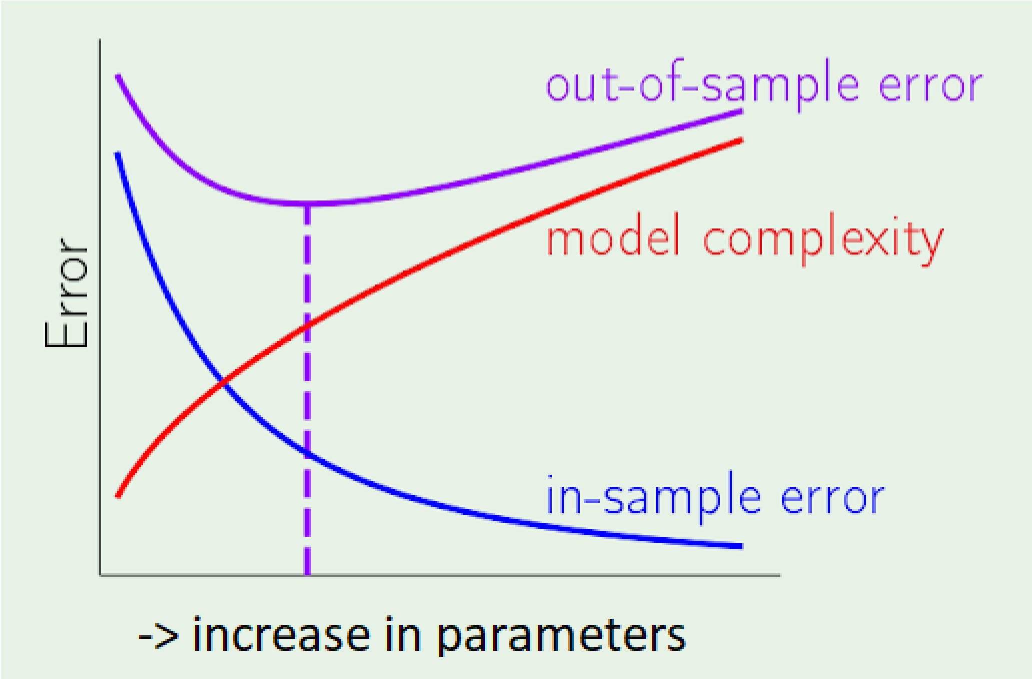
- What constitutes a good model?
  - *A model is good if it accurately models the world*
  - *… is good if it generalizes to new data*
  - *… is good if we accurately have estimated the underlying parameters of the population distribution*

MODEL SELECTION CRITERIA



MODEL SELECTION CRITERIA

- R-squared
- AIC, BIC
- Out-of-sample performance



R SQUARED

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \mu)^2}$$

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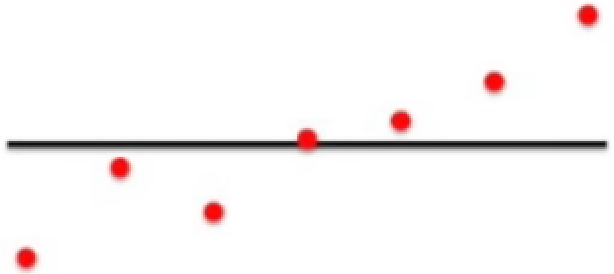

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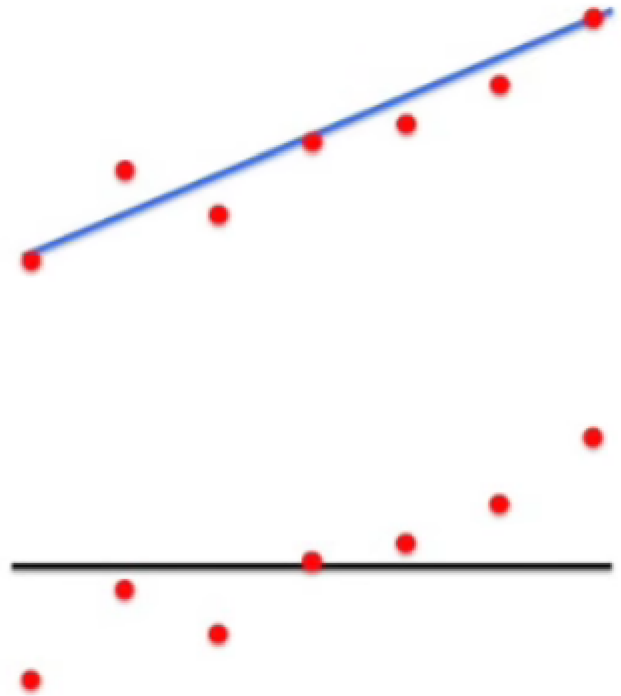

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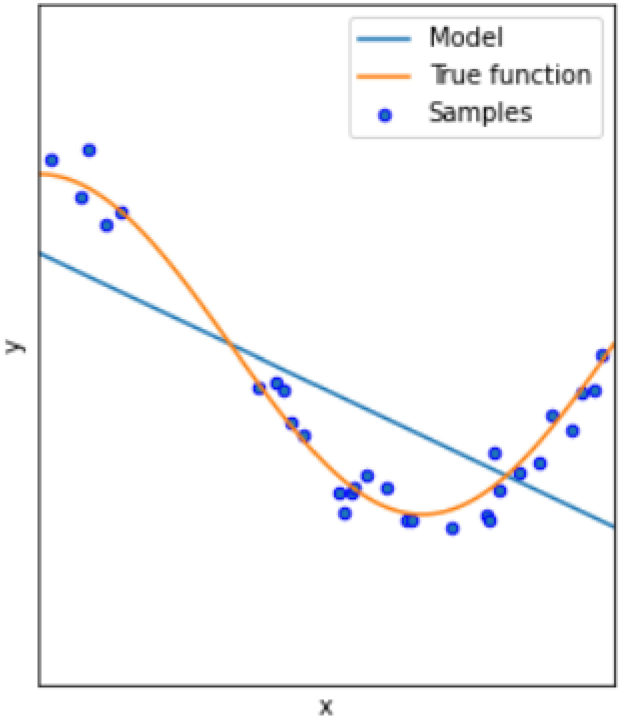

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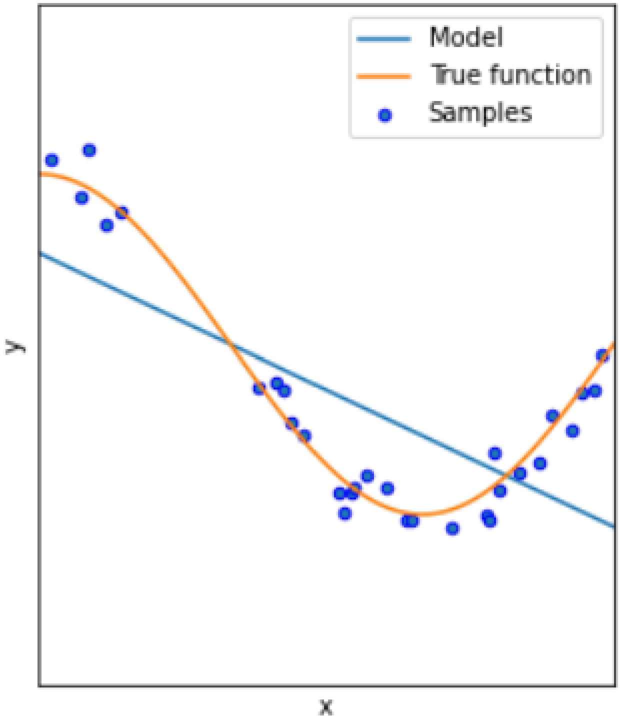

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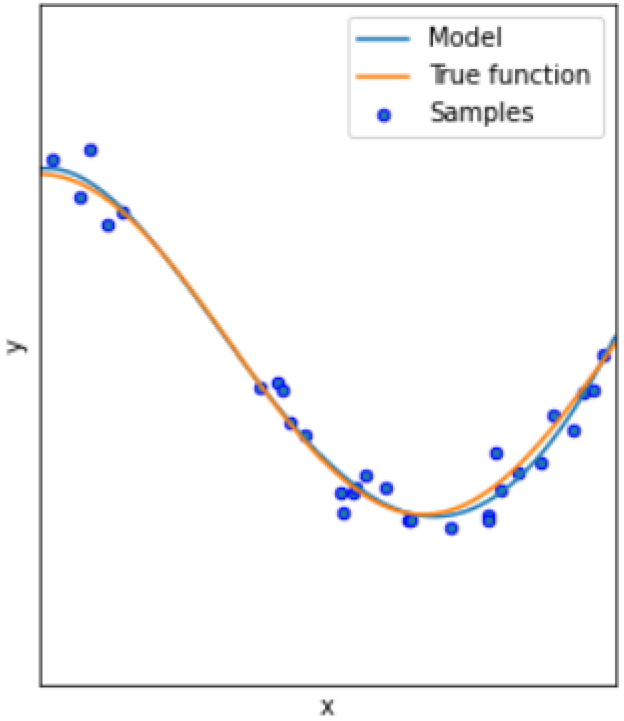

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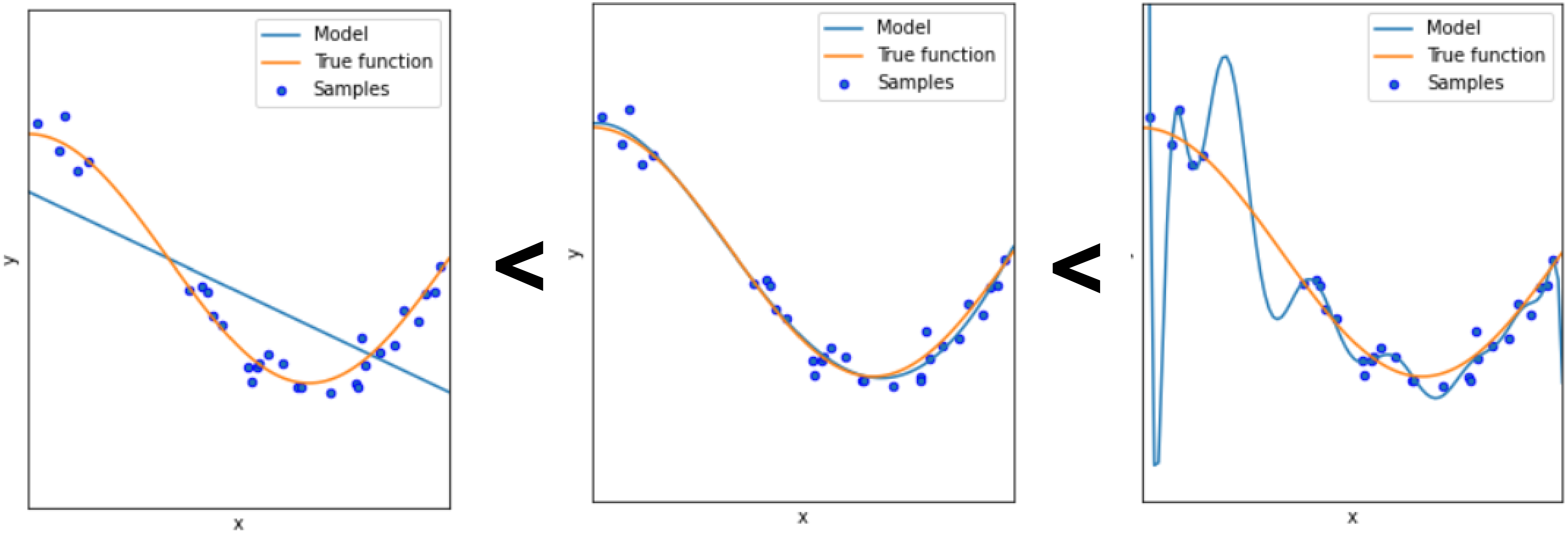
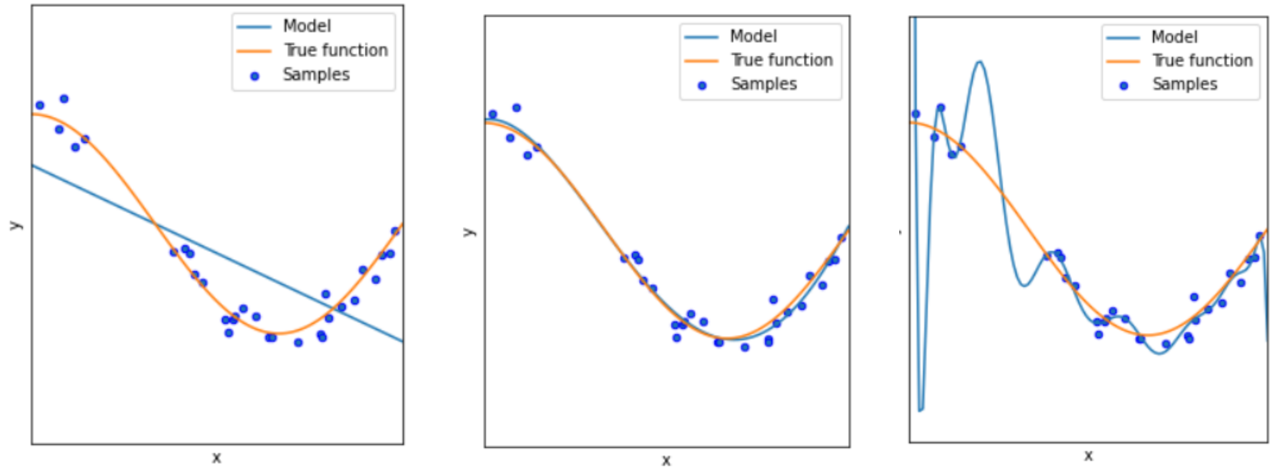
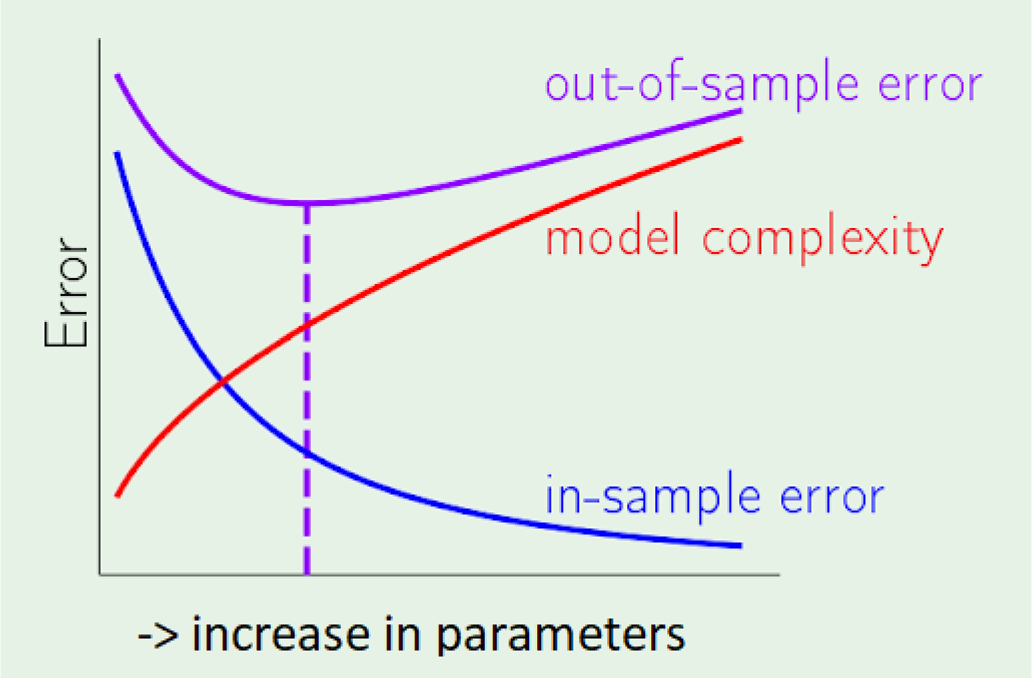


R SQUARED

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \mu)^2}$$





R SQUARED

- Can we use a criteria that accounts for overfitting?

AIC

AIC

$$AIC = -2\ln(L) + 2k$$

AIC

$$AIC = -2\ln(L) + 2k$$

$$=$$

$$\mathrm{AIC} \;=\; 2k - 2\ln(\hat{L})$$

AIC

$$AIC = -2\ln(L) + 2k$$

AIC - BIC

$$AIC = -2\ln(L) + 2k$$

$$\mathrm{BIC} = k\ln(n) - 2\ln(\widehat{L})$$

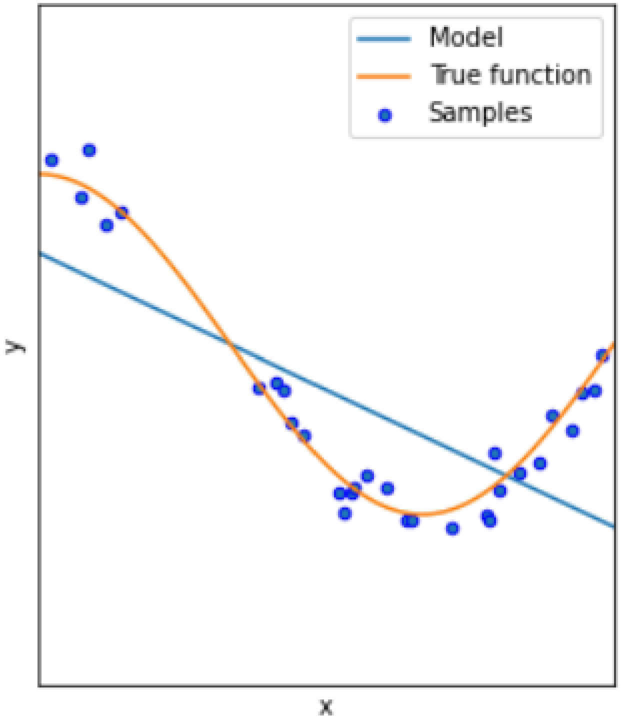
AIC - BIC

$$AIC = -2\ln(L) + 2k$$

$$\mathrm{BIC} = k\ln(n) - 2\ln(\widehat{L})$$

AIC - BIC

R-squared model rating:



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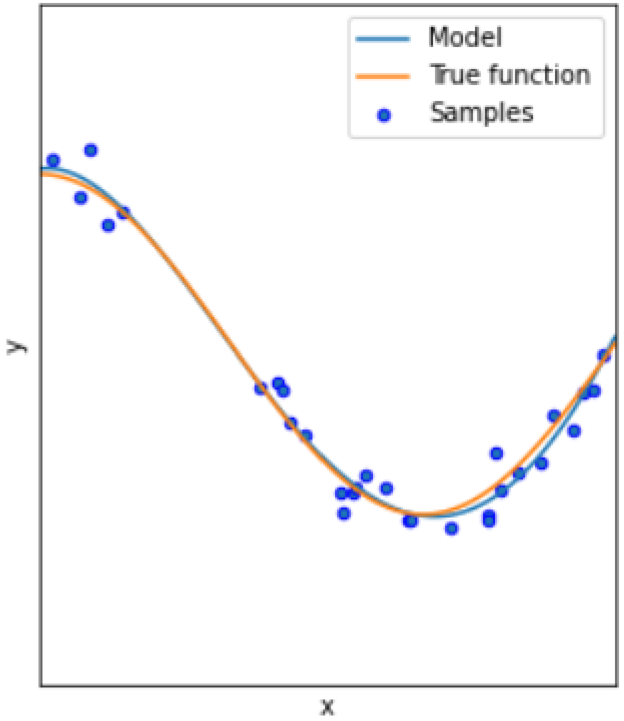

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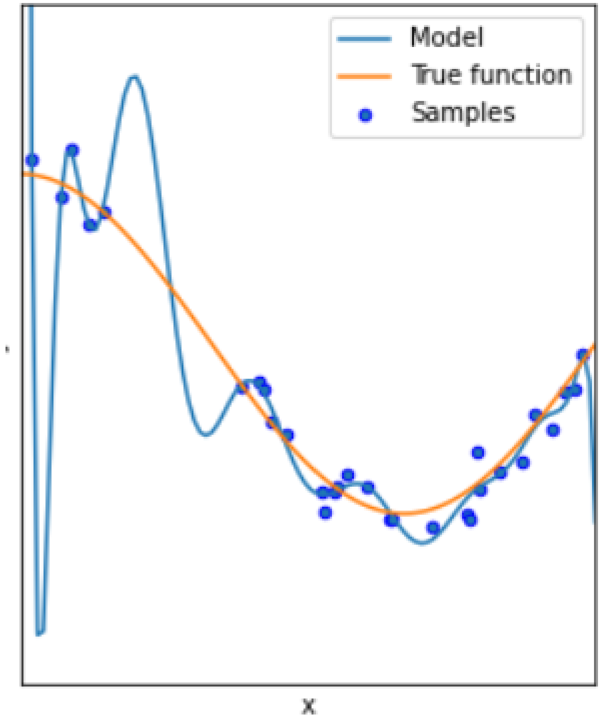
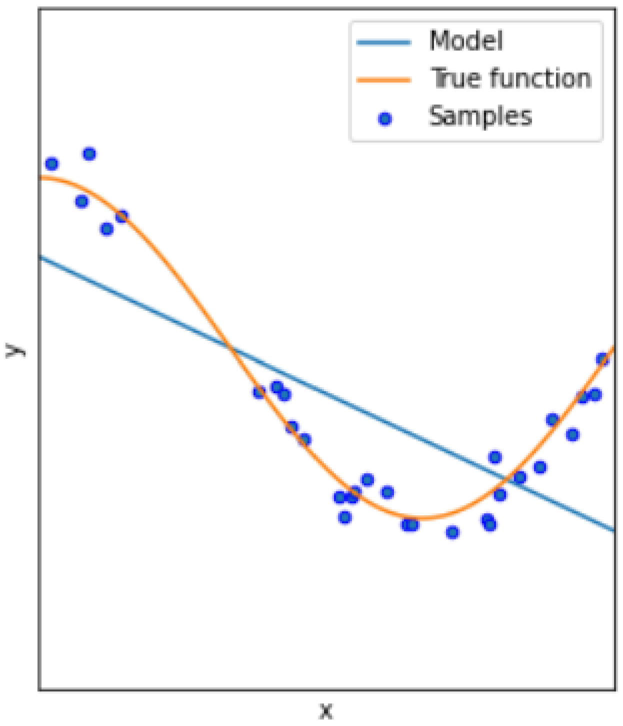


AIC - BIC

R-squared model rating:



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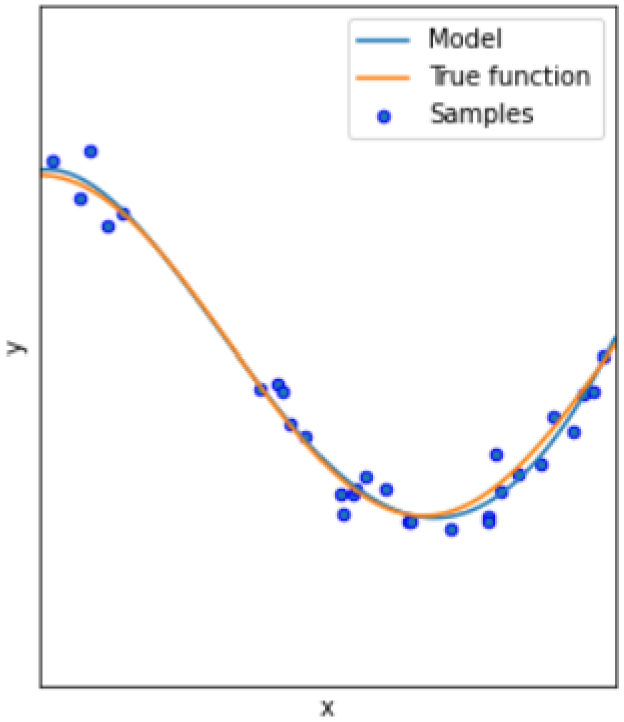

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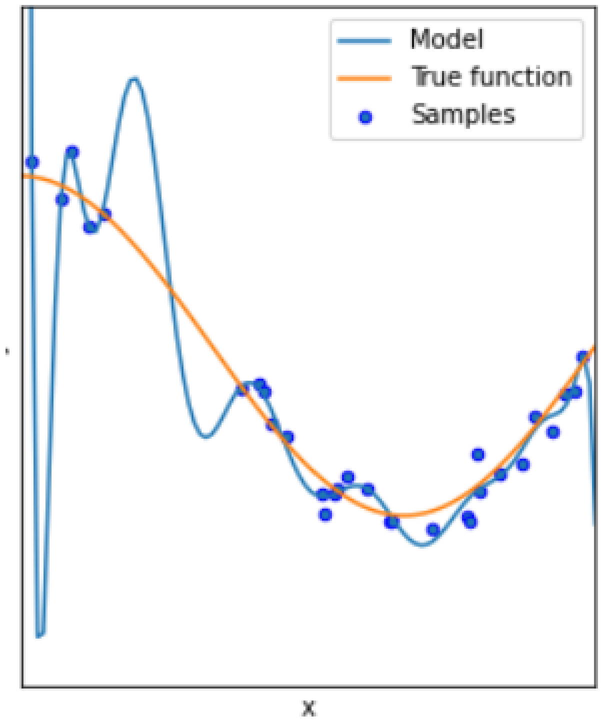
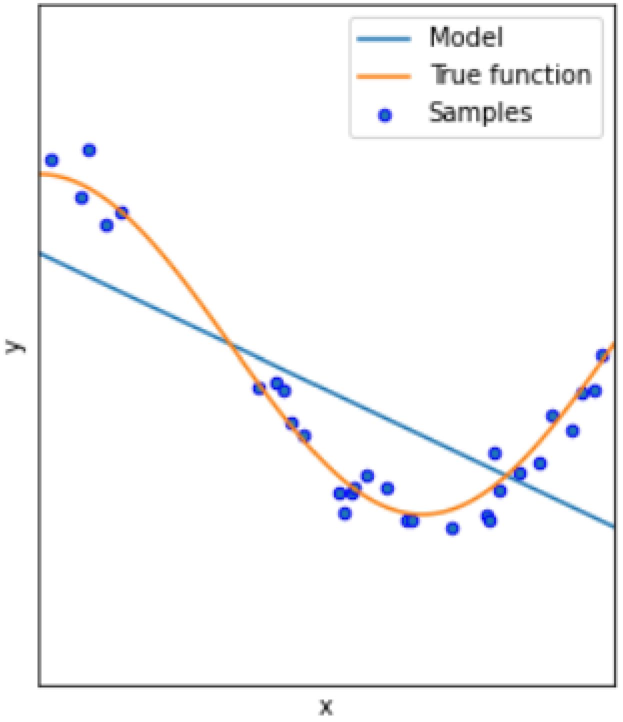


AIC - BIC

AIC / BIC model rating:



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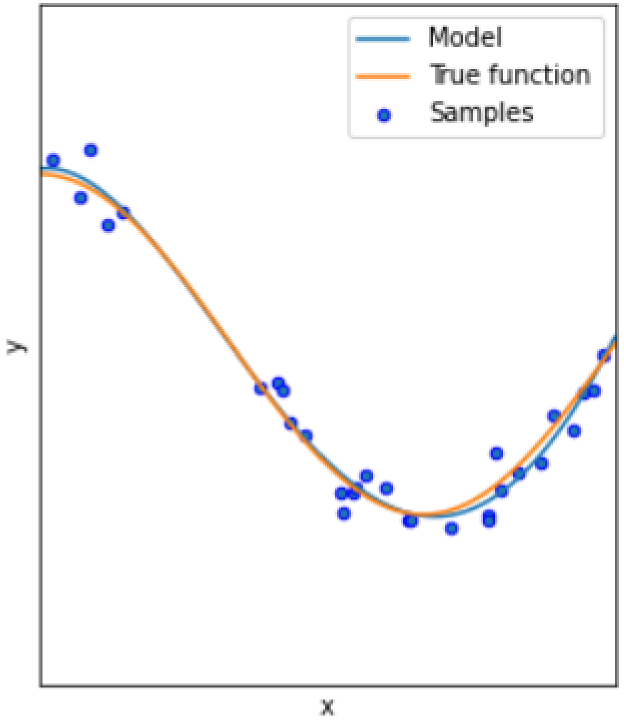

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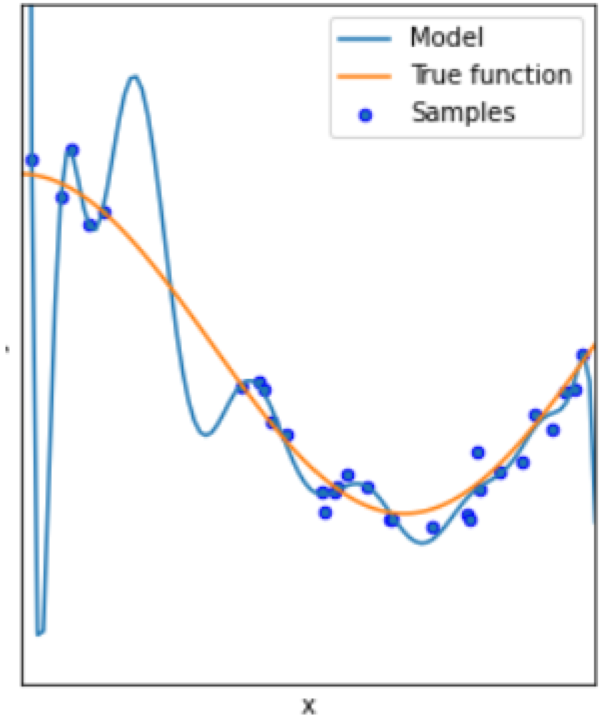
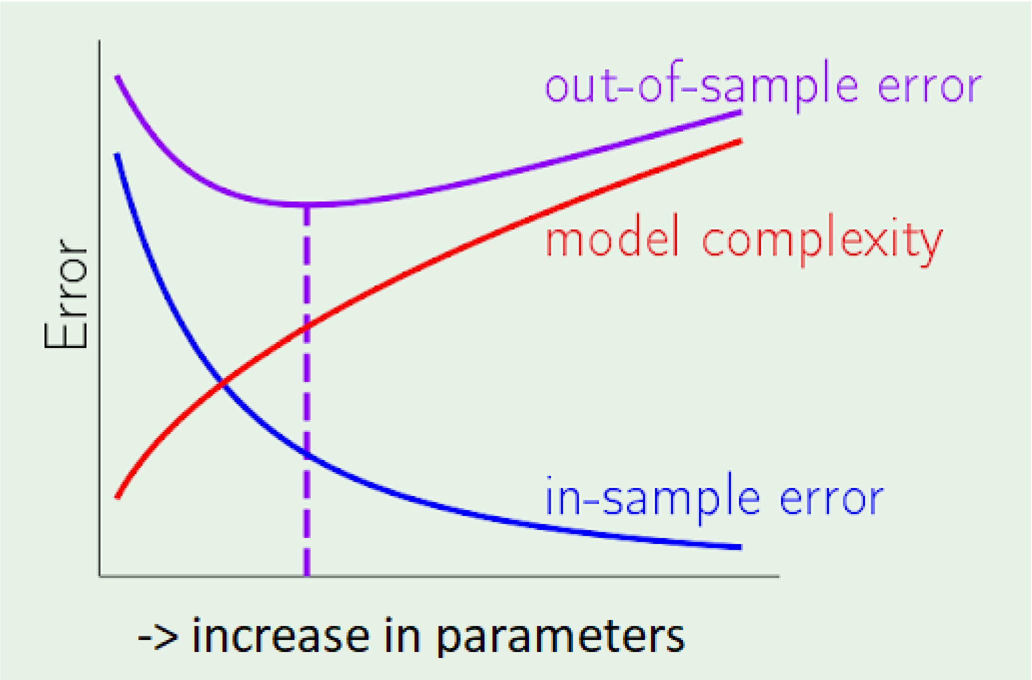
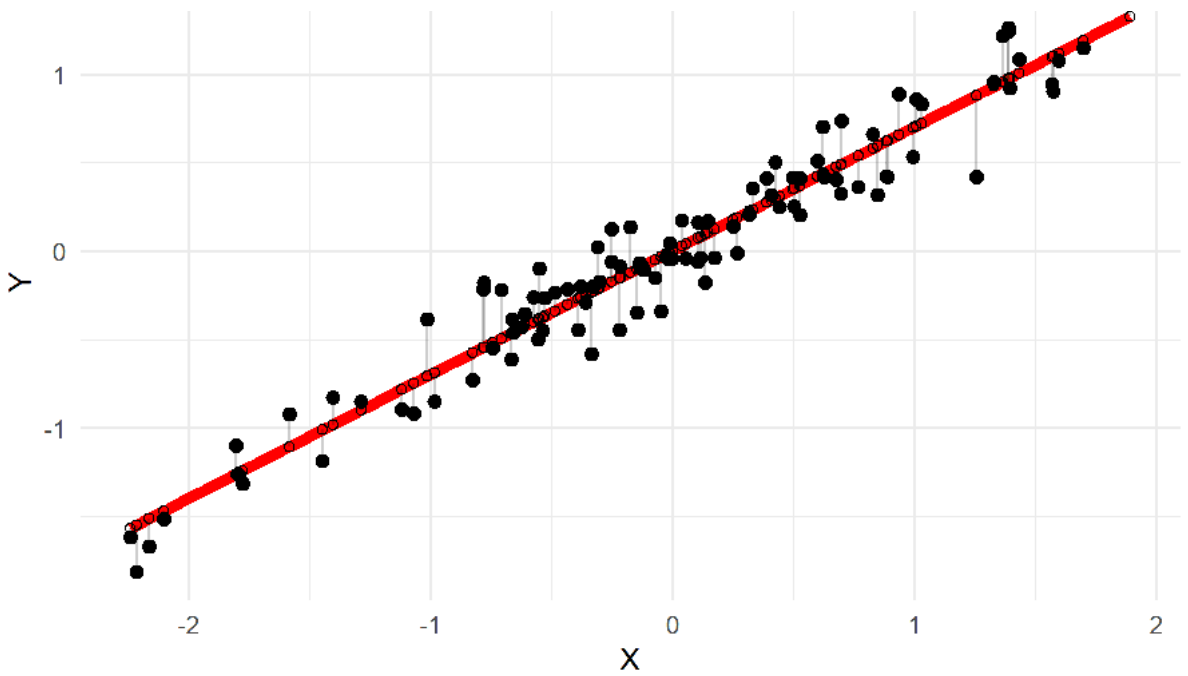


AIC - BIC

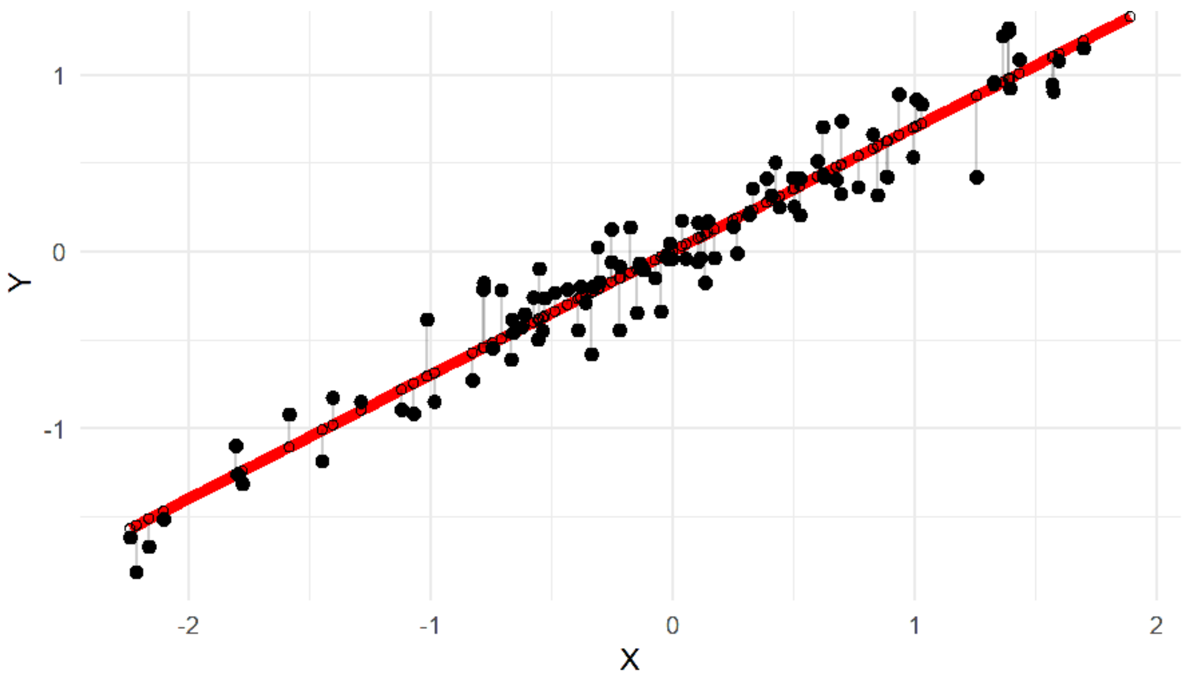

out-of-sample error
model complexity
Error
in-sample error
-> increase in parameters

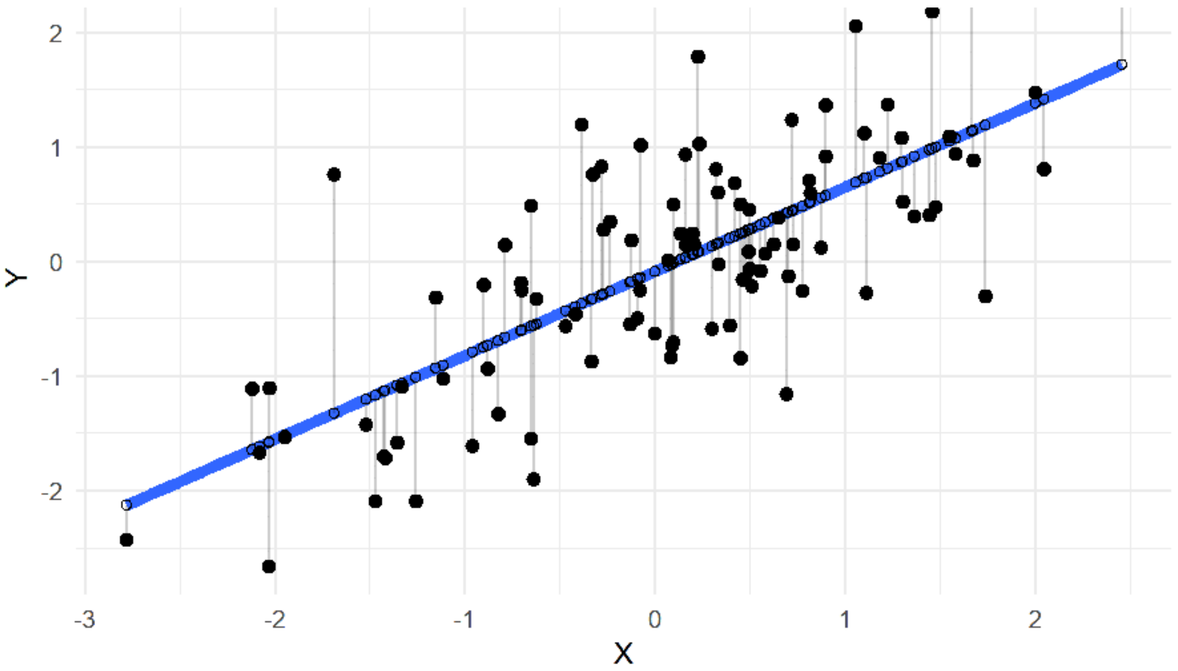
OUT-OF-SAMPLE ERROR

OUT-OF-SAMPLE ERROR

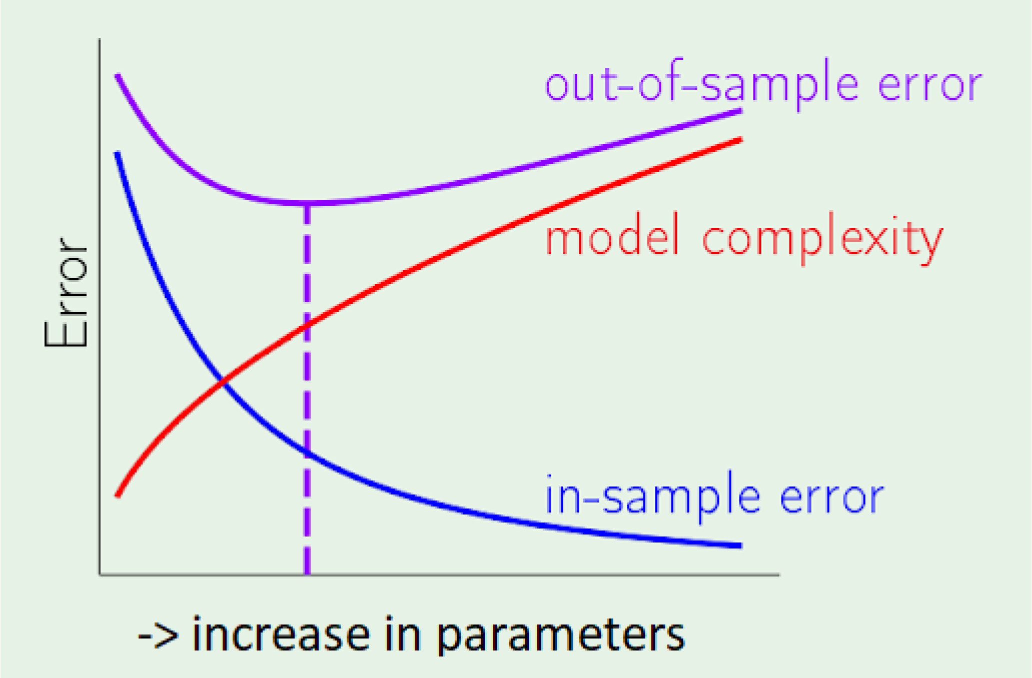


OUT-OF-SAMPLE ERROR





OUT-OF-SAMPLE ERROR



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