The poisson equation for each compartment:

$$-\Delta p_j = \frac{\mu_j}{\kappa_j} s_j = \sum_i t_{ij}(p_i - p_j),$$

with $t_{ij} = \frac{\mu_j}{\kappa_j} \gamma_{ij}$. Assume that the coefficients t_{ij} are sufficiently small, s.th.

$$\sum_i t_{ij}(p_i - p_j) \ll 1.$$

Rewrite the pressure as

$$p_j = \bar{p}_j + \tilde{p}_j$$

where $\bar{p}_j$ is assumed to be close to a Dirichlet boundary value

$$-\Delta \tilde{p}_j = \sum t_{ij}(\bar{p}_i - \bar{p}_j) + t_{ij}(\tilde{p}_i - \tilde{p}_j)$$

Seeing as the curvature is quite small, it seems natural to assume that

$$\frac{p_i(x) - p_j(x)}{\bar{p}_i - \bar{p}_j} \ll 1 \quad \forall x \in \Omega.$$

We therefore approximate the model by

$$-\Delta \tilde{p}_j = \sum_i t_{ij}(\bar{p}_i - \bar{p}_j)$$

where the RHS is a constant value.
While a positive Laplacian does not necessarily mean that the velocity field is pointing inwards in all directions (saddle points, etc), it seems reasonable given the constant boundary- and source.

Hence the magnitude and the direction of velocities in a specific compartment is determined by

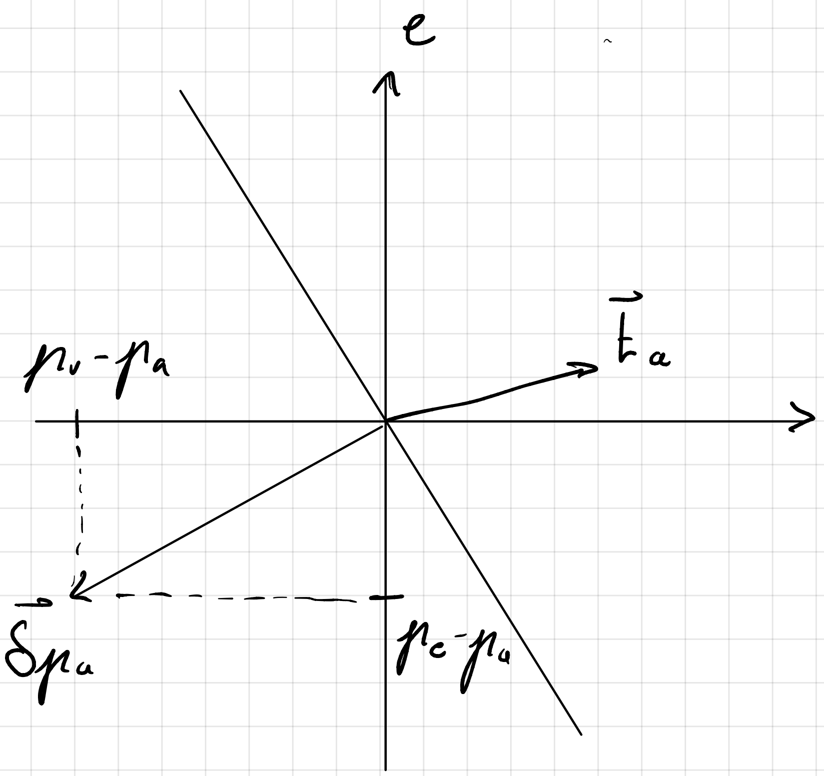
$$\sum_i t_{ij}(\bar{p}_i - \bar{p}_j) =: \vec{t}_j \cdot \overrightarrow{\delta p_j}$$

Hence the parameters with the largest effect on the fluid velocity in compartment j are the ones involved

$\max_{i \in J} |\bar{p}_i - \bar{p}_j|$ $\angle$

E.g. for a, e, v with

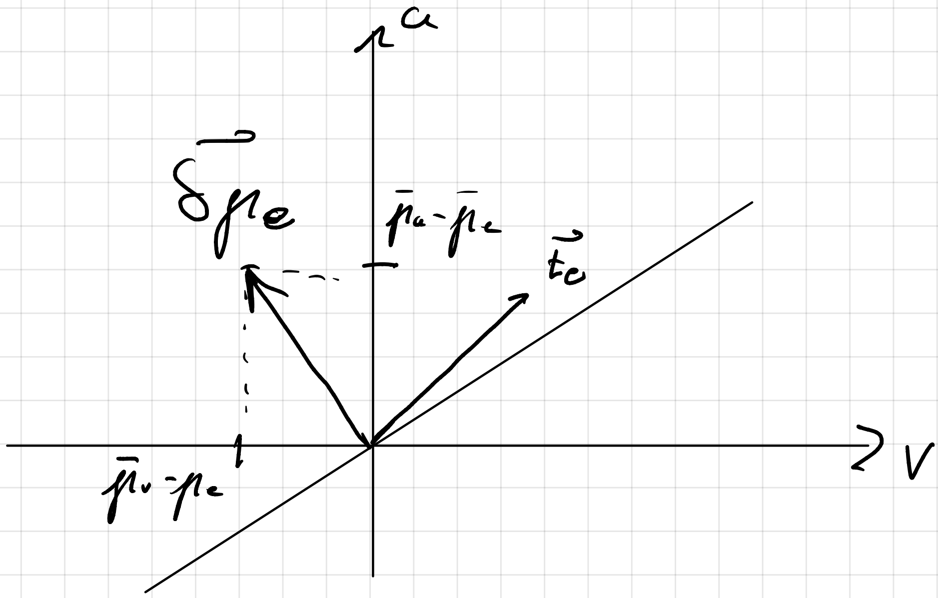
$\bar{p}_a > \bar{p}_e > \bar{p}_v$, then for the arterial comp.,



the RHS is necessarily negative, since all components t_{ia} are positive.

Then $\Delta p_a > 0$, and velocity field points inwards.

Similarly $\Delta p_v < 0$ and flow is outwards. However, for e,



In this case the hyperplane determined by the pressure-difference vector $\vec{\delta p_e}$ separates the positive quadrant, and the direction of the velocity-field in the ecs is determined by the transfer-coefficients t_{ve}, t_{ae}. In fact

$$\Delta \hat{p}_e > 0 \quad \text{if} \quad \frac{t_{ve}}{t_{ae}} > \frac{|\bar p_a - \bar p_e|}{|\bar p_v - \bar p_e|}$$

Main Conclusion (All compartments Dirichlet bdry).

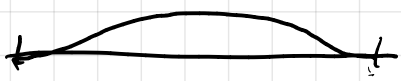
- Most important parameters for affecting pressure distribution in compartment

$$t_{ij} = \frac{\mu_j}{\kappa_j}\gamma_{ij}, \text{ where } i = \underset{k\in J}{\arg\max} |\bar p_k - \bar p_j|.$$

Complication

Assume that instead of a, v, e, we consider a, c, e, where c (capillaries) have a homogeneous Neumann boundary, indicating that the compartment is not in communication with the region, i.e.

$$\nabla p_c \cdot n = \nabla \tilde{p}_c \cdot n = 0.$$



This condition is incompatible with the assumption of constant forcing term, indicating that the assumption of

$$\sum_i t_{ic}(\bar{p}_i - \bar{p}_c) \not\approx \sum t_{ic}(\tilde{p}_i - \tilde{p}_c).$$

This is of course related to $\bar{p}_c$ not being determined by the boundary, and it will be determined by the pressures in the other compartments, and will end up with

$$\sum_i t_{ic} (\bar{p}_i - \bar{p}_c) \simeq 0$$

$$\Rightarrow \quad \bar{p}_c \simeq \left(\frac{\sum_i t_{ic} \bar{p}_i}{\sum_i t_{ic}} \qquad \text{assuming } t_{cc} = 0 \right)$$

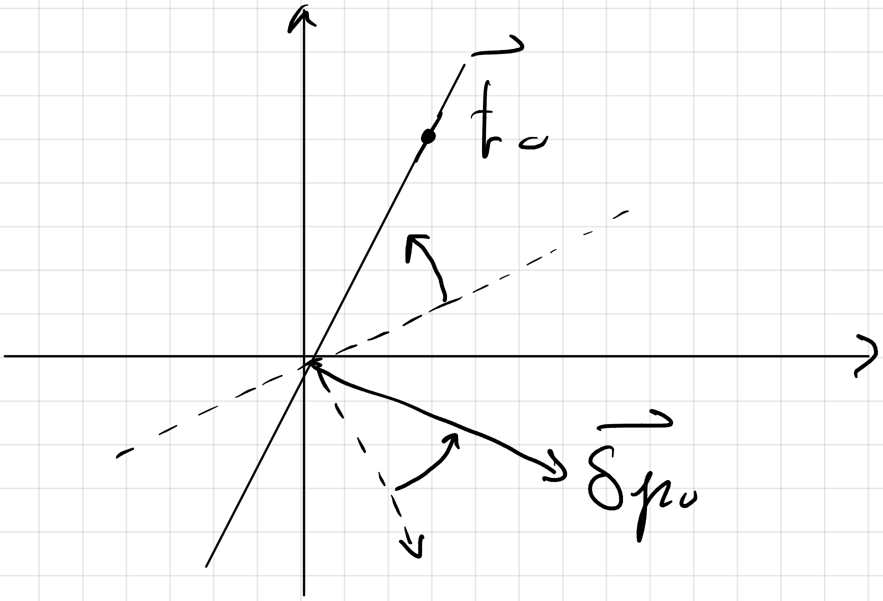
Such that the flow-direction in this compartment will be determined by the small-pressure variations;

$$\sum_i t_{ic} (\tilde{p}_i - \tilde{p}_c).$$

On the other hand, this also means that the RHS is very small and we might even assume $p_c \simeq \bar{p}_c$ everywhere,

Visually the formula for $\bar{p}_c$

may be interpreted visually by the pressure difference-vector orienting itself s.th. $\vec{t}_o$ lies on the hyperplane.



Now, since $\bar{p}_o$ is determined by the other compartments, then it does in turn affect the other ones:

$$-\Delta \tilde{p}_a = \sum_i t_{ia} (\bar{p}_i - \bar{p}_a)$$

$$= t_{va} (\bar{p}_v - \bar{p}_a) + t_{oa} \left(\frac{t_{ao} \bar{p}_a + t_{vo} \bar{p}_v}{t_{ao} + t_{vo}} - \bar{p}_o \right)$$

Hence all transfer coefficients connected to compartments with Neumann boundaries will have a significant effect on the remaining parameters.

Conclusions

1) Pay extra attention to transfer coefficients connected to compartments with homogeneous Neumann boundaries
2) For compartments with Dirichlet boundaries, transfer coefficients of compartments with the largest boundary-pressure differences