

The poisson equation for each component:

$$-\Delta p_j = f_j s_j = \sum_i t_{ij} (p_i - \bar{p}_j),$$

with $t_{ij} = \frac{\mu_i}{\kappa_j} \gamma_{ij}$. Assume that the coefficients t_{ij} are sufficiently small, s.t.

$$\sum_i t_{ij} (p_i - \bar{p}_j) \ll 1.$$

Moreover assume that p_i has some constant dimmed value \bar{p}_j , and write

$$p_j = \bar{p}_j + \tilde{p}_j$$

$$-\Delta \tilde{p}_j = \sum_i t_{ij} (\bar{p}_i - \bar{p}_j) + t_{ij} (\tilde{p}_i - \tilde{p}_j)$$

Seeing as the errore is quite small, it seems natural to assume that

$$\frac{p_i(x) - \bar{p}_j(x)}{\bar{p}_i - \bar{p}_j} \ll 1 \quad \forall x \in \Omega.$$

We therefore approximate the model by

$$-\Delta \tilde{p}_j = \sum_i t_{ij} (\bar{p}_i - \bar{p}_j)$$

where the RHS is a constant value.

While a positive Laplacian does not necessarily mean that the velocity field is pointing inwards in all directions (saddle points, etc), it seems reasonable given the constant boundary - and source.

Hence the magnitude and the direction of velocities in a specific convection is determined by

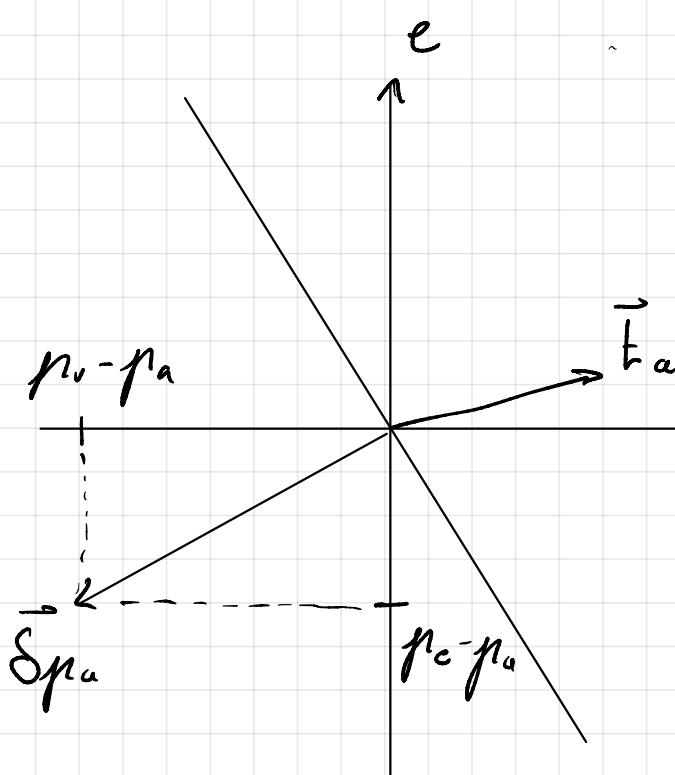
$$\sum_i t_{ij} (\bar{p}_i - \bar{p}_j) = \vec{t}_j \cdot \vec{s}_{pj}$$

Hence the parameters with the largest effect on the fluid velocity in convection j are the ones involved

$$\max_{i \in J} |\bar{y}_{\bar{i}} - \bar{y}_j|$$

E.g. for a, e, v with

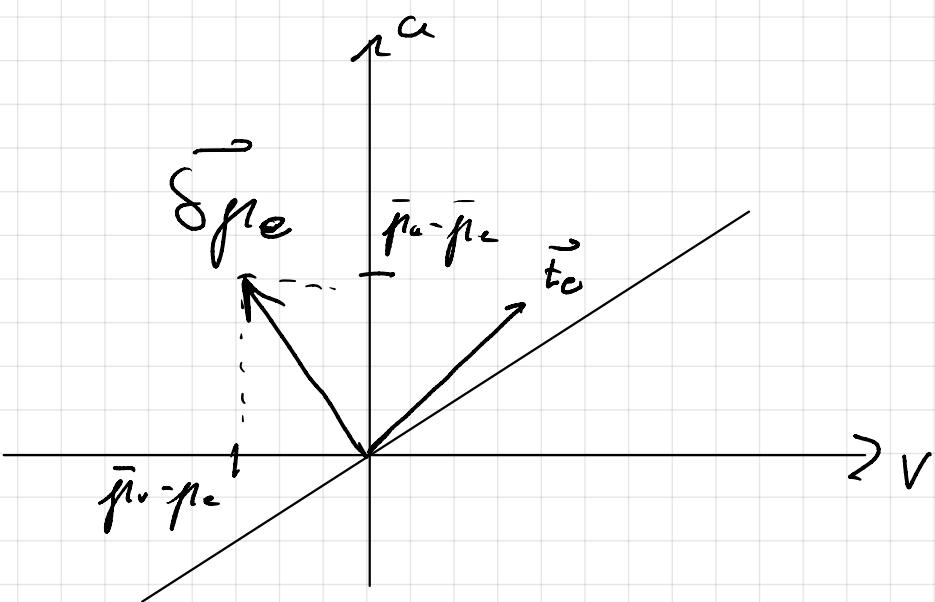
$\bar{\rho}_a > \bar{\rho}_e > \bar{\rho}_v$, then for the adiabat comp.,



the RHS is necessarily negative, since all components are positive.

Then $\Delta p_a > 0$, and velocity field points inwards.

Similarly $\Delta p_v < 0$ and flow is outwards. However, for e ,



In this case the hyperplane defined by the pressure-difference vector $\vec{\Delta p}_{re}$ separates the positive quadrant, and the direction of the velocity-field in the ees is defined by the transfer-coefficients t_{re} , t_{ae} . In fact

$$\Delta \bar{p}_{re} > 0 \quad \text{if} \quad \frac{t_{re}}{t_{ae}} > \frac{|\bar{p}_a - \bar{p}_{re}|}{|\bar{p}_v - \bar{p}_{re}|}$$

Main Conclusion (All compartments Dirichlet bdy).

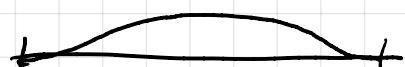
- Most important parameters for affecting pressure distribution in compartment

$$t_{ij} = \frac{\mu_i}{\kappa_j} \gamma_{ij}, \text{ where } i = \underset{k \in J}{\text{any max}} |\bar{p}_k - \bar{p}_j|.$$

Complication

Assume that instead of a, v, e , we consider a, c, e , where c (capillaries) have a homogeneous Neumann boundary, indicating that the compartment is not in communication with the region, i.e.

$$\nabla p_c \cdot n = \nabla \tilde{p}_c \cdot n = 0.$$



This condition is incompatible with the assumption of constant forcing term, indicating that the assumption of

$$\sum_i t_{ic} (\bar{p}_i - \bar{p}_c) \gg \sum_i t_{ic} (\tilde{p}_i - \tilde{p}_c).$$

This is of course related to \bar{p}_c not being determined by the boundary, and it will be determined by the

pressures in the other constraint,
and will end up with

$$\sum_i t_{ic} (\bar{p}_i - \bar{p}_c) \approx 0$$

$$\Rightarrow \bar{p}_c \approx \left(\frac{\sum_i t_{ic} \bar{p}_i}{\sum_i t_{ic}} \right) \quad \text{assuming } t_{cc} = 0$$

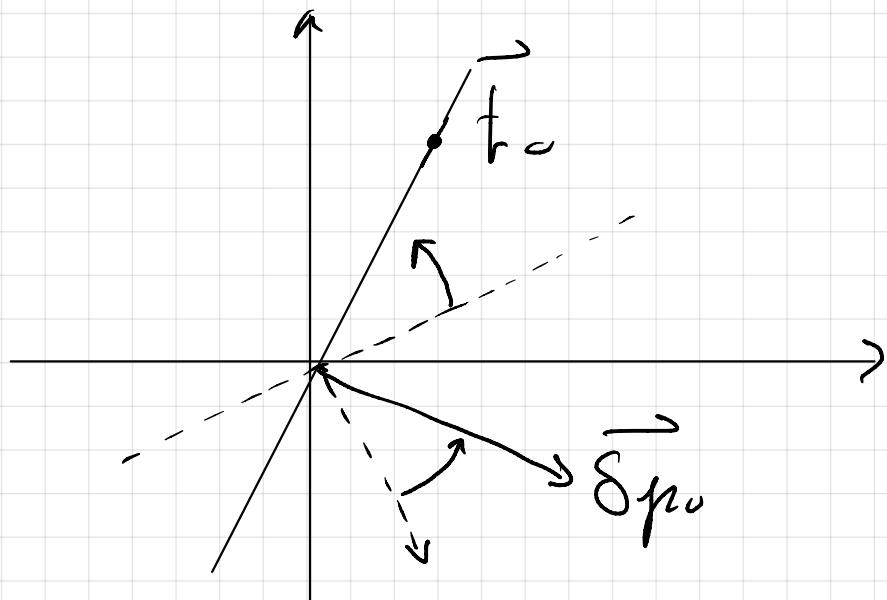
such that the flow-directions
in this constraint will be determined
by the small-pressure variations;

$$\sum_i t_{ic} (\tilde{p}_i - \tilde{p}_c)$$

On the other hand, this also means
that the RHS is very small and
we might even assume $p_c \approx \bar{p}_c$ everywhere.

Visually the formula for \bar{p}_c

may be interpreted visually by the pressure difference - vector orienting itself s.t. to lie on the hyperplane.



Now, since \bar{p}_{rc} is determined by the other compartments, then it does in turn affect the other ones:

$$-\Delta \tilde{p}_{\text{rc}} = \sum_i t_{ia} (\bar{p}_i - \bar{p}_{\text{rc}})$$

$$= t_m (\bar{p}_m - \bar{p}_{\text{rc}}) + t_{ca} \left(\frac{t_{ac} \bar{p}_a + t_{rc} \bar{p}_{\text{rc}}}{t_{ac} + t_{rc}} - \bar{p}_{\text{rc}} \right)$$

Hence all transfer coefficients connected to compartments with Neumann boundaries

will have a significant effect on the remaining parameters.

Conclusions

- 1) Pay extra attention to transfer coefficients connected to compartments with homogeneous Neumann boundary
- 2) For compartments with Dirichlet boundaries, transfer coefficients of compartments with the largest boundary-pressure differences

