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On post-processing day-ahead NWP forecasts using Kalman filtering

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ABSTRACT

Kalman filtering is an important concept in engineering and statistics. In the field of solar forecasting, it is well known as a numerical weather prediction (NWP) post-processing technique. However, it appears that this acknowledged post-processing technique needs some revisit. Since Kalman filtering is a sequential procedure, i.e., actual measurement from t is required to filter the forecast made for time $t + 1$, it changes the forecast horizon of NWP from day-ahead to hour-ahead. Hence, the previously claimed improvements over NWP forecasts are not interpretable. Two simple remedies are proposed, which address the forecast horizon problem, but the effectiveness of the remedies is thought to be minimal.

1. Background

Numerical weather prediction (NWP) is the dominant approach for day-ahead solar forecasting (Yang et al., 2018). In general, NWP is able to forecast 1-h or 3-h solar irradiance up to 72 h with a fair accuracy, which is otherwise difficult to achieve using satellite-based or ground-based methods. Nevertheless, NWP forecasts often carry significant model-led systematic bias. Hence, post-processing has become a standard practice since at least Lorenz et al. (2009), in which Lorenz et al. demonstrated a version of model output statistics (MOS) that improves the raw NWP forecast accuracy.

The main idea of MOS is to model the bias as a function of input variables such as zenith angle or forecast clear-sky index. Hence, when a new forecast arrives, MOS could estimate its bias using the fitted function. In most cases, MOS is used as a batch post-processing technique, since the fitted function can correct multiple forecasts without needing to update the function parameters.

Beside MOS, another popular post-processing technique is Kalman filtering, which is sequential in nature. Informally, one can consider it to be a technique that estimates the "true" state of a dynamical system from noisy measurement data. Kalman filtering is an important engineering concept that has been extended to almost all major scientific domains. For the interest of this short communication, Kalman filtering is only discussed with respect to post-processing the day-ahead NWP solar forecasts.

There are several representative papers in the solar forecasting literature that use Kalman filtering (e.g., Pelland et al., 2013; Diagne et al., 2014; Rincon et al., 2018). Some of these papers have received major attention from the renewable energy community. For instance, up until 2018–10–05, Pelland et al. (2013) has received 141 citations based on the number on Google Scholar. Similarly, Diagne et al. (2014) has received 46 citations, which is well above the median citation number of the solar forecasting papers published in 2014 (Yang et al., 2018). Despite the evident public acceptance of this technique, the Kalman filtering procedures described in some of these papers can be alarmingly misleading. Therefore, in this short communication, I explain some of the common misperceptions and useful applications of the Kalman filter as post-processing technique for day-ahead NWP solar forecasts.

2. Using Kalman filtering for post-processing

Kalman filtering is a well-developed technique with many variants. As a result, there is no agreed symbol usage. In this communication, the choice of symbols follows Box et al. (2015), which is a classic textbook on time series.

2.1. The general state-space model

The general state-space model consists of a *state equation*:

$$\boldsymbol{Y}_t = \boldsymbol{\Phi}_t \boldsymbol{Y}_{t-1} + \boldsymbol{a}_t, \tag{1}$$

and an *observation equation*, which also known as *measurement equation*:

$$Z_t = \boldsymbol{H}_t \boldsymbol{Y}_t + N_t, \tag{2}$$

where $\boldsymbol{a}_t$ and N_t are independent white-noise processes with variance $\boldsymbol{\Sigma}_a$ and σ_N^2, respectively. If the state vector $\boldsymbol{Y}_t \in \mathbb{R}^{r\times 1}$, then $\boldsymbol{\Phi}_t \in \mathbb{R}^{r\times r}$, $\boldsymbol{a}_t \in \mathbb{R}^{r\times 1}$, $\boldsymbol{H}_t \in \mathbb{R}^{1\times r}$, and both Z_t and N_t are scalars. Eq. (1) describes how the unobserved *state* of a dynamical system, $\boldsymbol{Y}$, evolves in time, whereas Eq. (2) describes the fact that an *observation*, Z, is

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modeled as a linear combination of state variables plus a noise term. $\boldsymbol{\Phi}_t$ is called the *transition matrix* or *system matrix*, which can vary with time.

Suppose observations $Z_1, \ldots, Z_t$ are made, the finite sample optimal estimate of $\boldsymbol{Y}_{t+h}$ is given by:

$$\widehat{\boldsymbol{Y}}_{t+h|t} = \mathbb{E}[Y_{t+h}|Z_t, \ldots, Z_1], \tag{3}$$

with variance:

$$\boldsymbol{V}_{t+h|t} = \mathbb{E}[(Y_{t+h} - \widehat{\boldsymbol{Y}}_{t+h|t})(Y_{t+h} - \widehat{\boldsymbol{Y}}_{t+h|t})^\top], \quad \in \mathbb{R}^{r\times r}. \tag{4}$$

Then, given the initial estimates of $\boldsymbol{Y}_0 \equiv \boldsymbol{Y}_{0|0}$ and $\boldsymbol{V}_0 \equiv \boldsymbol{V}_{0|0}$, the Kalman-filtered estimate of $\widehat{\boldsymbol{Y}}_{t|t}$ is given by:

$$\widehat{\boldsymbol{Y}}_{t|t} = \widehat{\boldsymbol{Y}}_{t|t-1} + \boldsymbol{K}_t(Z_t - \boldsymbol{H}_t\widehat{\boldsymbol{Y}}_{t|t-1}), \tag{5}$$

$$\boldsymbol{K}_t = \boldsymbol{V}_{t|t-1}\boldsymbol{H}_t^\top(\boldsymbol{H}_t\boldsymbol{V}_{t|t-1}\boldsymbol{H}_t^\top + \sigma_N^2)^{-1}, \tag{6}$$

$$\widehat{\boldsymbol{Y}}_{t|t-1} = \boldsymbol{\Phi}_t\widehat{\boldsymbol{Y}}_{t-1|t-1}, \tag{7}$$

$$\boldsymbol{V}_{t|t-1} = \boldsymbol{\Phi}_t\boldsymbol{V}_{t-1|t-1}\boldsymbol{\Phi}_t^\top + \boldsymbol{\Sigma}_a, \tag{8}$$

$$\boldsymbol{V}_{t|t} = (\boldsymbol{I} - \boldsymbol{K}_t\boldsymbol{H}_t)\boldsymbol{V}_{t|t-1}. \tag{9}$$

In the above equations, $\boldsymbol{K}_t \in \mathbb{R}^{r\times 1}$ is the *Kalman gain*, $\boldsymbol{I} \in \mathbb{R}^{r\times r}$ is an identity matrix, term $Z_t - \boldsymbol{H}_t\widehat{\boldsymbol{Y}}_{t|t-1}$ is called the *innovation*.

Although there are more general forms of the state-space model than the one described in Box et al. (2015), e.g., one may add a control matrix and control vector to Eq. (1), the above formulation appears to cover most, if not all, Kalman filter implementations in solar forecast post-processing applications.

2.2. An example implementation in the solar forecasting context

After presenting the general formulation, we relate it to the implementations done in the solar forecasting context. Diagne et al. (2014) is used as an example. The setup adopted by Diagne et al. (2014) is:

$$\boldsymbol{Y}_0 = (0, 0, 0)^\top, \tag{10}$$

$$\boldsymbol{H}_t = \left(1, \frac{\mathrm{GHI}_{\mathrm{NWP},t}}{1000}, \cos(\mathrm{SZA}_t)\right), \tag{11}$$

$$\boldsymbol{V}_0 = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}, \tag{12}$$

$$\boldsymbol{\Sigma}_a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{13}$$

$$\sigma_N^2 = 0.01, \tag{14}$$

$$\boldsymbol{\Phi}_t = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{15}$$

where $\mathrm{GHI}_{\mathrm{NWP},t}$ is the NWP forecast at time t, and SZA_t is the solar zenith angle at time t.

When $t = 1$, $\widehat{\boldsymbol{Y}}_{0|0}$ and $\widehat{\boldsymbol{V}}_{0|0}$ are known, $\widehat{\boldsymbol{Y}}_{1|0}$ and $\widehat{\boldsymbol{V}}_{1|0}$ can be obtained via Eqs. (7) and (8), respectively. At this stage, the bias at time $t = 1$ can already be calculated:

$$\widehat{\mathrm{bias}}_1 = 1000\widehat{Z}_1 = 1000\boldsymbol{H}_1\widehat{\boldsymbol{Y}}_{1|0}. \tag{16}$$

It is noted the calculation of $\widehat{\mathrm{bias}}_1$ requires $\boldsymbol{H}_1$, i.e., the NWP forecast made at time $t = 1$. Subsequently, the Kalman gain, $\boldsymbol{K}_1$, can be calculated through Eq. (6), and $\widehat{\boldsymbol{V}}_{1|1}$ is obtained from Eq. (9). When the actual bias, $1000Z_1$, becomes available, $\widehat{\boldsymbol{Y}}_{1|1}$ is the last item to be computed—via Eq. (5)—for the $t = 1$ iteration. During the next iteration, the filtering procedure repeats, i.e., starting from estimating $\widehat{\boldsymbol{Y}}_{2|1}$ and $\widehat{\boldsymbol{V}}_{2|1}$, and ending at estimating $\widehat{\boldsymbol{Y}}_{2|2}$.

The above procedure of Diagne et al. (2014) was largely based on Pelland et al. (2013), with differences in terms of initial value assignment and $\boldsymbol{H}_t$ definition—Pelland et al. (2013) used only 1 and $\mathrm{GHI}_{\mathrm{NWP},t}/1000$. There are other ways that one can define $\boldsymbol{H}_t$. For instance, Philippe Lauret suggested that using the clear-sky index,[1] the performance of the Kalman filtering procedure can be slightly improved (Lauret, 2018). As a result, the estimated bias, see Eq. (16), should be related to the estimated clear-sky GHI, instead of 1000.

2.3. The nature of post-processing

Having looked at the detailed filtering procedure, the nature of post-processing is discussed. The word “post-processing” in the general sense means any processing sequence after the main procedure is completed. Nevertheless, in solar forecasting, the meaning of “post-processing” should be stricter. More specifically, a post-processing technique should not change the forecast horizon. For example, if an NWP model issues hourly forecasts for the next 72 h at 18:00 today, all 72 post-processed forecasts must also be generated at 18:00 + today.[2]

2.4. The misleading papers

Whenever NWP is mentioned, most readers would immediately assume a day-ahead forecasting scenario. Usually, day-ahead forecasting covers a horizon of 0–72 h. However, given the hourly-update nature of Kalman filtering, such day-ahead forecast horizon becomes hour-ahead. Unfortunately, this important piece of information is often missing, unclear, or buried within the long text of a paper. For example, despite that Diagne et al. (2014) mentioned that their proposed method is “hourly forecasting,” owing to the ambiguity caused by the word “NWP,” many later papers have wrongfully taken Diagne et al. (2014) as a day-ahead forecasting paper (e.g., Ayet and Tandeo, 2018; Huang and Thatcher, 2017; Aybar-Ruiz et al., 2016; Lorenzo et al., 2015; Badosa et al., 2015).

The correct classification of the Kalman filtered solar forecasting approach in Diagne et al. (2014) would be an hour-ahead forecast. While benchmarking against day-ahead NWP as in Diagne et al. (2014) may be valuable from an academic perspective to test value-added by the Kalman filter, the proper benchmark for applications would be different hour-ahead forecasts, such as statistical or satellite forecasts. Applying the Kalman filter for day-ahead forecasting is questionable as discussed in the next section.

2.5. Two remedies

Since the main problem of Kalman filtering is that it changes the forecast horizon, to keep the original day-ahead horizon, multiple Kalman filters can be built. For instance, for 1–24-h horizons, 24 Kalman filters can be used, one for each horizon. This approach has been successfully demonstrated for post-processing of many meteorological parameters, such as ozone concentration (Delle Monache et al., 2006). Nevertheless, when day-to-day changes in prediction error are large, the performance of Kalman filter is limited (Delle Monache et al., 2011). On this point, the reader is referred to Yang et al. (2017), and the R code therein, for an implementation of multiple Kalman filters on GHI forecasts. Unfortunately, the performance of this kind of filtering was shown to be way worse than MOS.

Another possible solution is to consider a multi-step-ahead filtering. Since

[1] In this particular case, the clear-sky index can be calculated as $\mathrm{GHI}_{\mathrm{NWP}}$ divided by the clear-sky GHI estimated by the NWP model.

[2] The expression 18:00 + means slightly later than 18:00, due to the computation time required for post-processing. Nonetheless, this computation can usually be done in real-time, at least for MOS and Kalman filtering.

$$\widehat{Z}_{t+h|t} = \boldsymbol{H}_{t+h}\widehat{\boldsymbol{Y}}_{t+h|t}, \tag{17}$$

and

$$\widehat{\boldsymbol{Y}}_{t+h|t} = \boldsymbol{\Phi}_{t+h}\widehat{\boldsymbol{Y}}_{t+h-1|t}, \tag{18}$$

if $\boldsymbol{\Phi}_{t+h} \equiv \boldsymbol{\Phi}$, $\forall\, t$, h, then

$$\widehat{Z}_{t+h|t} = \boldsymbol{H}_{t+h}\boldsymbol{\Phi}^h\widehat{\boldsymbol{Y}}_{t|t}, \tag{19}$$

one can generate an estimate of the h-step-ahead bias. In other words, the Kalman gain is updated every h steps. Notwithstanding, the expected performance of this strategy is questionable due to the "outdated" $\widehat{\boldsymbol{Y}}_{t|t}$, especially when h gets large. It is thought further studies are necessary.

3. Conclusion

Kalman filtering as an NWP post-processing technique is herein investigated. Due to its sequential nature, the filter changes the original day-ahead forecast horizon of NWP to hour-ahead. To that end, many previously reported post-processing improvements by Kalman filtering against day-ahead forecasts are irrelevant. Further benchmarking against other hour-ahead forecasts is required. Whether Kalman filtering can eventually be considered as a competitive alternative to day-ahead MOS is questionable and requires further investigation. At this moment, Kalman filtering through an ordered set of analog forecasts (see Delle Monache et al., 2011) seems promising. Hence, some connections to the field of meteorology need to be made.

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