A critical review on the state-of-the-art of Lambert's problem solvers

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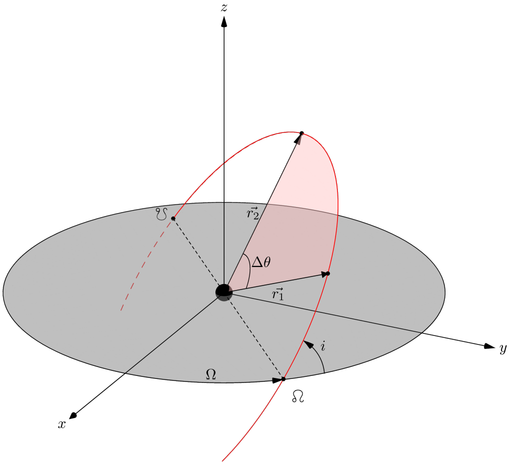
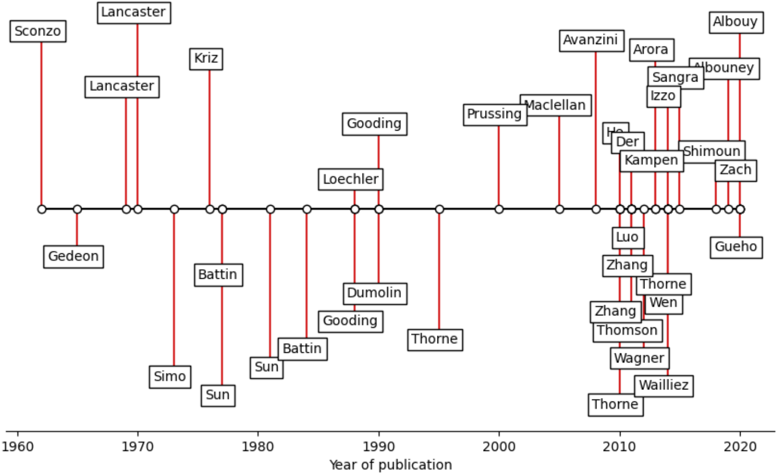
The Lambert's problem

- Initially posed by Johann Heinrich Lambert in a letter to Leonhard Euler.
- Many applications: orbit determination, trajectory design and intercepting maneuvers.



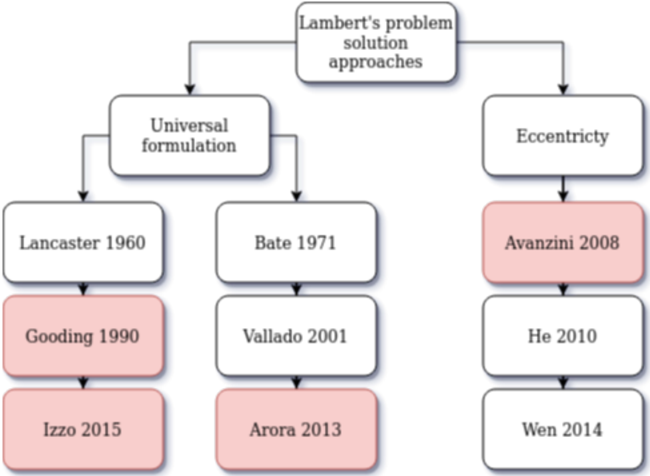
The revival of the problem

Since the 60s, many solutions have been proposed in the form of computer algorithms.



Classification according to free-parameter and inheritance

The following classification can be performed over selected algorithms according to the free-parameter employed and their inheritance on other solvers.



Elements comparison

Any Lambert's solver requires from the following four elements: a free-parameter, an initial guess sub-routine, a root finder and a velocity construction procedure.

The following table provides each one of these elements for the algorithms considered.

	Method			
	Gooding	Avanzini	Arora	Izzo
Free-parameter	x	e_T	k	ξ
Initial guess	Bi-linear	$x = 0$	Rational formulae	Linear approximation
Root finder	Halley	Regula-falsi	Halley	Householder
Velocity vectors	v_r and v_θ	COE to RV	f and g functions	v_r and v_θ

Performance comparison: the metrics

The following metrics were established to assess the performance of the solvers:

- Number of iterations as function of τ and $\Delta\theta$.
- Only direct transfer arcs ($M = 0$) are considered with $\rho = 2$.
- Time per iteration.
- Total computation time.
- Absolute tolerance 10^{-5} and relative one 10^{-7}.

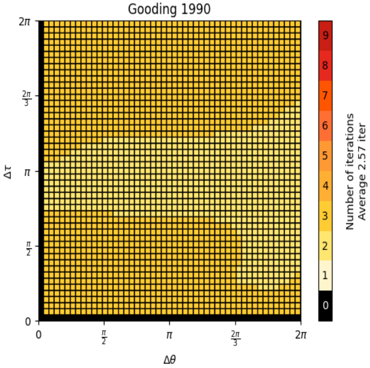
Time measurements might be affected by machine specifications and running processes.

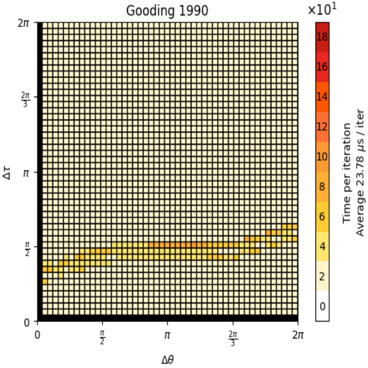
Source code files

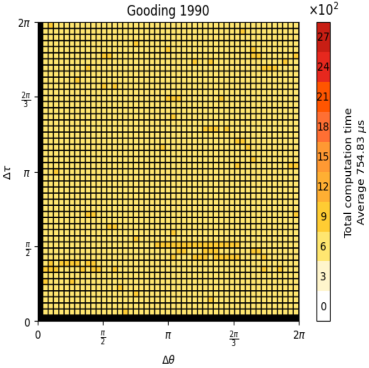
All solvers were implemented in Python and provided in the form of a library
https://github.com/jorgepiloto/lamberthub

Solver performance: Gooding 1990

Routines as in the original article: TLAMB, XLAMB and VLAMB. Maximum of three iterations. Lowest time per iteration. Accurate and robust.

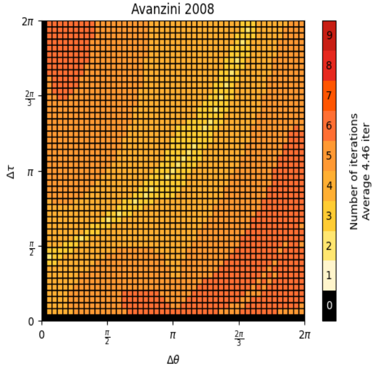


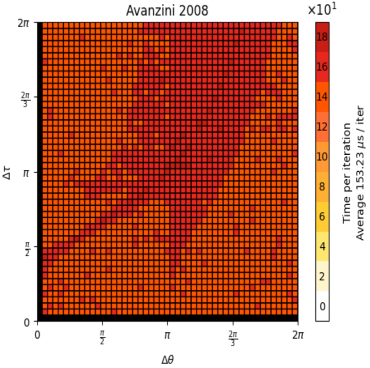


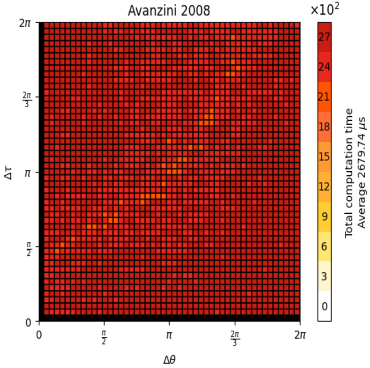


Solver performance: Avanzini 2008

First of its class. Provides a deep understanding of the problem's geometry. Simple to implement. Lack of initial guess. Low order convergence root solver.

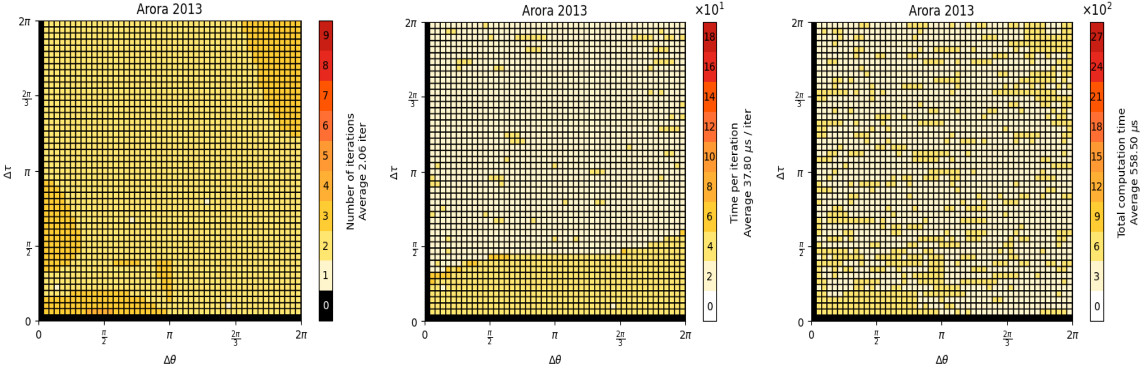






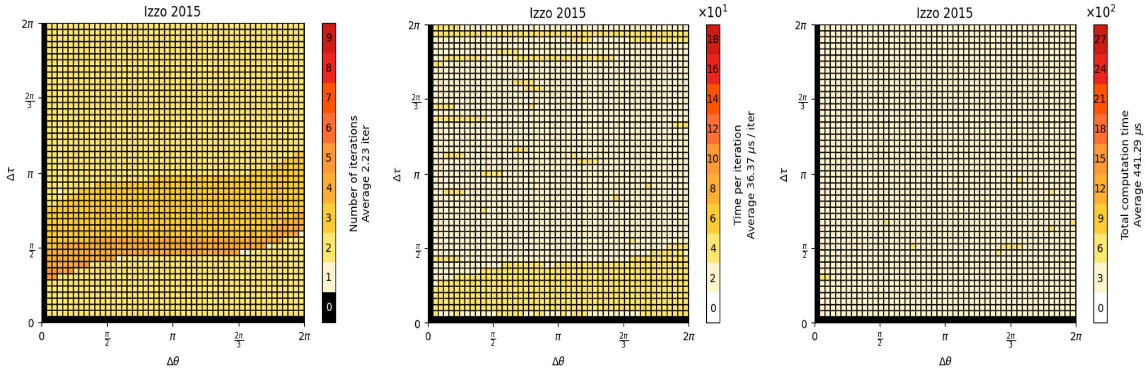
Solver performance: Arora 2013

Very robust initial guess procedure, although too much time devoted to it. Low number of iterations due and high order convergence method. Stable and accurate.



Solver performance: Izzo 2015

Very popular solver. Lowest total computation time. Accurate, robust and easy to implement. Shows the best overall performance.



Obtained results and conclusion

The following conclusions can be done regarding any Lambert's problem solver:

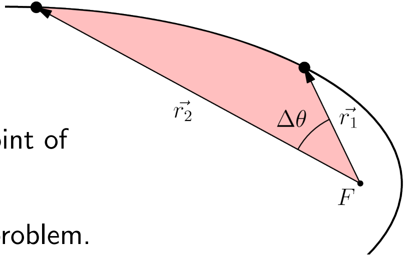
- A robust initial guess is crucial, although the procedure must remain as simple as possible.
- High-order convergence root finders are preferred.
- Performance is driven by high iteration workload with low total computation time.

Method	**Average iterations**	**Time per iter**	**Total time**	**Iter workload**
Gooding 1990	2.57	23.78 μs/iter	754.83 μs	8.10 %
Avanzini 2008	4.46	153.23 μs/iter	2679.74 μs	25.50 %
Arora 2013	2.06	37.80 μs/iter	558.50 μs	13.94 %
Izzo 2015	2.32	36.37 μs/iter	441.29 μs	19.12 %

Future work

All the results presented in here might be expanded by:

- Adding more solvers.
- Addressing the multi-revolution scenario.
- Explore a performance from GPU parallelization point of view.
- The initial value problem, also known as Kepler's problem.
- Memory cost and profiling.



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Questions?