AN EXPERIMENTAL COMPARISON OF BIASED AND UNBIASED RANDOMKEY GENETIC ALGORITHMS*

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Abstract

Random key genetic algorithms are heuristic methods for solving combinatorial optimization problems. They represent solutions as vectors of randomly generated real numbers, the so-called random keys. A deterministic algorithm, called a decoder, takes as input a vector of

random keys and associates with it a feasible solution of the combinatorial optimization problem for which an objective value or fitness can be computed. We compare three types of random-key genetic algorithms: the unbiased algorithm of Bean (1994); the biased algorithm of Gonçalves and Resende (2010); and a greedy version of Bean's algorithm on 12 instances from four types of covering problems: general-cost set covering, Steiner triple covering, general-cost set k -covering, and unit-cost covering by pairs. Experiments are run to

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construct runtime distributions for 36 heuristic/instance pairs. For all pairs of heuristics, we compute probabilities that one heuristic is faster than the other on all 12 instances. The experiments show that, in 11 of the 12 instances, the greedy version of Bean's algorithm is faster than Bean's original method and that the biased variant is faster than both variants of Bean's algorithm.

Keywords: genetic algorithm, biased random-key genetic algorithm, random keys, combinatorial optimization, heuristics, metaheuristics, experimental algorithms

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NOPQRSTUVWXYZabcdefg hijk l m n o p q r s t u v w x y z A B C D E FGHIJKLMNOPQRSTUVWXYZa bcdefghijklmnopqrstuvw $x y z \iota J A B \Gamma \Delta E Z H \Theta I K \Lambda M N \Xi O$ ΠΡΘΣΤΥΦΧΨΩ ∇αβγδεζηθι κλμνξοπρςστυφχψωδεθχ φρωΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠ Ρ Θ Σ Τ Υ Φ Χ Ψ Ω ∇ α β γ δ ε ζ η θ ι κ λ μνξοπρς στυφχψω δεθκφο π ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡΘ ΣΤΥΦΧΨΩ ∇αβγδεζηθικλμ νξοπρςστυφχψωδεθκφρω ΑΒΓΔΕΖΗΘΙΚΛΜΝΞΟΠΡΘΣ **ΤΥΦΧΨΩ**∇αβγδεζηθικλμνξ οπρςστυφχψω δεθκφο π ΑΒΓ ΔΕΖΗΘΙΚΛΜΝΞΟΠΡΘΣΤΥ **ΦΧΨΩ** ∇αβγδεζηθικλμνξοπ ρςστυφχψω δεθκφο π F F O 1 2 3 45678901234567890123456 789**0123456789**0123456789

Japonês

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APÊNDICE

Espécies de morcegos registradas em subbosque em 15 localidades amostradas na Amazônia e utilizadas nas análises de similaridade e auto-correlação espacial de morcegos da Amazônia: IWO = Iwokrama Forest, Guiana; PAR = Paracou, Guiana Francesa; ARA = Baixo Arataye, Guiana Francesa; FLAM = Floresta Nacional do Amapá, Amapá, Brasil; PNMT = Parque Nacional das Montanhs do Tumucumaque, Amapá, Brasil; BAR = Barcelos e Santa Isabel do Rio Negro; PNJ = Parque Nacional do Jaú, Amazônas, Brasil; MAN = Manaus, Amazônas, Brasil; XIN = Baixo Xingú, Pará, Brasil; PNA = Parque Nacional da Amazônia, Amazonas/Pará, Brasil; PNSD = Parque Nacional da Serra do Divisor, Acre, Brasil; BAL = Balta, Ucayali, Peru; MNU = Parque Nacional de Manu, Madre de Dios, Peru; CUZ = Reserva Cuzco Amazônico, Madre de Dios, Peru e MTP = Médio Teles Pires, Mato Grosso, Brasil.

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