From Frequentist Problems Towards Bayesian Solutions

Part 2: Introduction to Bayesian statistics (with JASP)

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Slides at: https://rebrand.ly/Nagoya2019-Part2
GitHub: https://github.com/jorgetendeiro/Nagoya-Workshop-10-Aug-2019

Today

- Suggested reading material
- Frequentist versus bayes
- Bayes rule
- Jasp
- References

Suggested reading material

Introduction to Bayes

Papers:

- Etz & Vandekerckhove (2018): The “Harry Potter” paper.
  Very accessible introduction, with examples.
- Etz et al. (2018): “How to become a Bayesian in eight easy steps: An annotated reading list”.
  Yes, Alexander Etz writes ‘readible’ papers *very* well. Strongly advised read, but it takes quite some time to process.
- Kruschke (2013): Besides providing an excellent introduction to core concepts, Kruschke offers a discussion over the “testing” versus “estimation” tension. I personally like Kruschke’s position on this matter.

Books:

- Kruschke (2015): The “puppies” book.
  Accessible book, with plenty of examples and code. Perfect as a first pick.
- McElreath (2016): From what I read thus far, this book is a jewel.
- Lambert (2018): I’m currently half way. Seems perfect for teaching (hence learning!).
- Gelman (2014): More advanced read (perhaps not the first pick), but truly a master piece.

Learn about JASP

Website
https://jasp-stats.org/how-to-use-jasp/.
It includes:

- Tutorial section
- YouTube channel to see (and hear) it in action.
- Forum to post lingering questions.
- Teaching with JASP includes much more material.

Video tutorials
Etz (who else?) is making videos as we speak.
https://alexanderetz.com/jasp-tutorials/.

Papers

- Marsman & Wagenmakers (2017, in *European Journal of Developmental Psychology*).
- Wagenmakers et al. (2018, in a special issue in *Psychonomic Bulletin & Review*).

Frequentist *versus* Bayes

Frequentist paradigm

Concept of probability:

- Long-run frequency of a procedure.

> The probability of a fair coin landing up heads is 50%.

- One cannot state anything about one single event in the long-run sequence.

> What is the probability that the next coin toss lands heads?

- Recall the definitions of a p-value and confidence interval: They are based on long-run frequencies.
  Conclusion: What can we really conclude from one p-value or one confidence interval?…

Bayesian paradigm

Concept of probability:

- Degree of belief.
- Expression of uncertainty about the true state of affairs.
- Subjective: Different people have different beliefs.
- Data are used to update one’s belief, by means of the *laws of probability*.
- Applies to both single and repetitive events.

But how do we update our belief in light of data?

Bayes' rule

Bayes' rule

Let $\mathcal{A}$ denote something we want to study.
This can be:

- A parameter, like the mean μ of a population.
- A hypothesis, like $\mu > 100$.

Bayes' rule:

$$p(\mathcal{A}|\text{data}) = \frac{p(\mathcal{A})p(\text{data}|\mathcal{A})}{p(\text{data})}$$

- $p(\mathcal{A})$: Prior probability.
- $p(\text{data}|\mathcal{A})$: Data likelihood.
- $p(\text{data})$: Marginal likelihood.
- $p(\mathcal{A}|\text{data})$: Posterior probability.

Bayes' rule and frequentism

Important:

- The Bayes' rule is a mathematical necessity, it follows from the axioms of probability.
- Frequentists do not dispute this formula!

Bayes' rule and model comparison

Say we have in total two competing hypotheses, $\mathcal{H}_0$ and $\mathcal{H}_1$.

We can apply the Bayes' rule to either hypothesis:

$$p(\mathcal{H}_0|\text{data}) = \frac{p(\mathcal{H}_0)p(\text{data}|\mathcal{H}_0)}{p(\text{data})} \quad , \quad p(\mathcal{H}_1|\text{data}) = \frac{p(\mathcal{H}_1)p(\text{data}|\mathcal{H}_1)}{p(\text{data})}.$$

Now divide both equations:

$$\underbrace{\frac{p(\mathcal{H}_0|\text{data})}{p(\mathcal{H}_1|\text{data})}}_{\text{Posterior odds}} = \underbrace{\frac{p(\mathcal{H}_0)}{p(\mathcal{H}_1)}}_{\text{Prior odds}} \times \underbrace{\frac{p(\text{data}|\mathcal{H}_0)}{p(\text{data}|\mathcal{H}_1)}}_{BF_{01}}$$

where BF_{01} is the Bayes factor.

The Bayes factor is a measure of the *relative evidence* in the data for either model. E.g., if $BF_{01} = 5$:

- The data are 5 times as likely under $\mathcal{H}_0$ than under $\mathcal{H}_1$.
- After looking at the data, we now support $\mathcal{H}_0$ five times as much.

Bayes factor

A whole lot can be said about Bayes factors.

They have fervent followers (e.g., Kass & Raftery, 1995; Dienes, 2014; Morey, Romeijn, & Rouder, 2016; E.-J. Wagenmakers et al., 2018).

But there are also critics, including myself (Tendeiro & Kiers, 2019).

JASP

Note of caution

JASP is ‘Bayes factor’-oriented.

I personally dislike it, as I think parameter estimation offers a far a clearer, all-inclusive, paradigm.

- To see why I think this, see our preprint: Kiers & Tendeiro (2019).

Worked-out example

Let's jump to JASP now!

The pet example that I will use is the first experiment of Bem (2011):

Precognitive detection of erotic stimuli.

- $n = 100$ (50 men, 50 women), 36 trials per subject.
- In each trial:
  - Two curtains shown side by side.
  - One curtain hides a picture, the other hides a blank wall.
  - Erotic and nonerotic pictures randomly intermixed.

Main research hypothesis:

> Subjects are able to "feel" where the erotic pictures are more often than chance (!!!).

Some results from Bem (2011)

> Across all 100 sessions, participants correctly identified the future position of the erotic pictures significantly more frequently than the 50% hit rate expected by chance: 53.1%, $t(99) = 2.51$, $p = .01$, $d = 0.25$. In contrast, their hit rate on the nonerotic pictures did not differ significantly from chance: 49.8%, $t(99) = -0.15$, $p = .56$.

> The difference between erotic and nonerotic trials was itself significant, $t_{\text{diff}}(99) = 1.85$, $p = .031$, $d = 0.19$.

> (…) the correlation between stimulus seeking and psi performance was .18 ($p = .035$).

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