Original equation. k_2 and k_{-2} depend on the N_t concentration. The k_3 reaction produces a bi-product P that is not present initiatlly. Therefore k_{-3} is initially zero.

$$\ldots [\mathrm{M_i^r}] \xrightleftharpoons[\mathrm{k_{-1i}}]{\mathrm{k_{1i}}} [\mathrm{M_i^o}] \xrightleftharpoons[\mathrm{k_{-2i}}]{\mathrm{k_{2i}}} [\mathrm{M_i^o}][\mathrm{N_t}] \xrightleftharpoons[\mathrm{k_{-i3}}]{\mathrm{k_{i3}}} [\mathrm{M_{i+1}^r}] \ldots \tag{1}$$

Simplified equation

$$\ldots [\mathrm{M_i^r}] \xrightleftharpoons[\mathrm{k_{-1i}}]{\mathrm{k_{1i}}} [\mathrm{M_i^o}] \xrightleftharpoons[\hat{k}_{-2i}]{\hat{k}_{2i}} [\mathrm{M_{i+1}^r}] \ldots \tag{2}$$

The parameters $k{-}1i$ and $k1i$ are known and are functions of i. The $\hat{k}$ parameters should depend on a term ΔG_i. If I make the last step irreversible

$$\ldots [\mathrm{M_i^r}] \xrightleftharpoons[\mathrm{k_{-1i}}]{\mathrm{k_{1i}}} [\mathrm{M_i^o}] \xrightarrow{\hat{k}_{2i}} [\mathrm{M_{i+1}^r}] \ldots \tag{3}$$

the ODE system looks like this:

$$\begin{aligned}
\frac{\mathrm{d}[M_i^r]}{\mathrm{d}t} &= k_{-1i}[M_i^o] + k_{2(i-1)}[M_{i-1}^o] - k_{1i}[M_i^r] \\
\frac{\mathrm{d}[M_i^o]}{\mathrm{d}t} &= k_{1i}[M_i^r] - (k_{-1i} + \hat{k}_{2i})[M_i^o] \\
\frac{\mathrm{d}[M_{i+1}^r]}{\mathrm{d}t} &= k_{2i}[M_i^o] - (\hat{k}_{-2i}[M_{i+1}^o] + k_{1(i+1)})[M_{i+1}^r]
\end{aligned}$$