FINDING THE OPTIMAL GEOMETRY OF THE HIPÓDROMO DE LA ZARZUELA

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ABSTRACT

This paper analysis the possibility for a re-design and optimization of the roof of the Hipódromo de la Zarzuela, designed by Eduardo Torroja and located in Madrid. This roof is a masterpiece, since it is aesthetically pleasing, efficient and a perfect example of a thin concrete shell. The hyperboloid shape of the roof allows it to carry heavy loads for a relatively thin structure. When Torroja made the design and calculations, computational methods did not exist. All the designing and calculations were done by hand and by using physical models. In this paper, the aim is to find the optimal geometry of the roof by using computational methods. This result will be compared with Torroja's design.

Keywords: *thin shell structure, Eduardo Torroja, Hipódromo de la Zarzuela, hyperboloid, geometry, FEA*

1. INTRODUCTION

A shell structure is the most efficient method to span a certain distance. A very thin shell can cover the same area as a considerably thicker beam. The Hipódromo de la Zarzuela requires a cantilevering roof. The best way to reduce the loads on this cantilevering roof is by minimizing its selfweight. By using a thin shell for the roof, the thickness can be minimized, and thus the selfweight is minimized. Using this knowledge, Torroja decided to construct a thin shell concrete roof for the Hipódromo de la Zarzuela.

Figure 1: Hipódromo de la Zarzuela [11]

Torroja used hand calculation and physical models for all the designing and optimizing. Because these methods are not exact, Torroja made a complete full-scale model of a part of the roof to test it. This full-scale model failed under a life load of 6,05 kN/m^2 in the test [2], [5].

The exact geometry of the roof is not known in literature. Certain parameters of the roof's geometry are known. The aim of this paper is to determine the magnitude of the unknown parameters to create the optimal geometry of the roof. This will be done in two ways. First, the structural system behind the roof structure will be explored. This will result in a basic understanding of the mechanics of the roof. This way, a first estimate can be made for the optimal magnitudes of the unknown parameters. Secondly, these estimates will be used to create four different geometries in the parametric software Grasshopper. These different geometries will be imported in the Finite Element Method analysis program DIANA FEA.

Using this analysis in DIANA FEA, the different geometries will be compared. This way the optimal geometry can be found. Lastly, this optimal geometry will be compared to the test done by Torroja on the full-scale model.

2. STATE OF THE ART

2.1 Structural Concept

As already discussed, the Hipódromo de la Zarzuela requires a cantilevering roof. Torroja choose to balance this cantilevering roof with the slab of the top promenade using a tie member. The choice for a hyperbolic paraboloid thin shell roof ensures the roof to be thin and thus have a low self-weight. This makes it possible to balance it with the slab of the top promenade using a tie member, see figure 2.

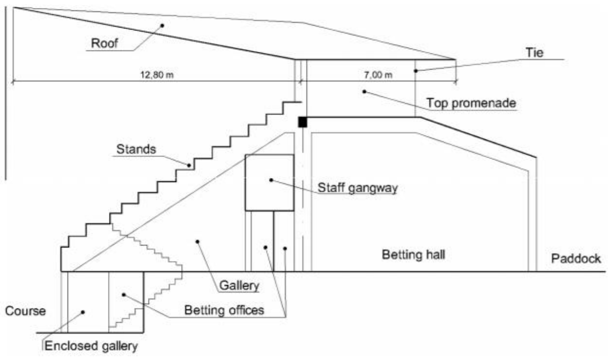

Figure 2: Section showing the functions of the spaces [2]

Figure 3 shows a simplified model of the forces in the structure. This simplified model shows the tie member between C and D, and the main column ABF supporting the roof. At point A, a hinge is created in one direction. The model clearly shows how minimizing the thickness of the roof between A and K will also result in less tension in the tie member between C and D. [2]

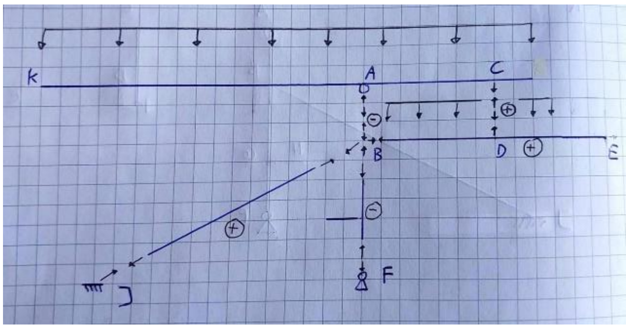

Figure 3: Simplified model of forces in the structure

To create a cantilever, it is beneficial to use a curved surface rather than a straight surface. A straight surface results in a V-shaped roof section. A curved surface results in smaller forces in the roof section, while the roof is still in compression.

Using graphic statics, it is easy to visualize this advantage of using a curved surface. For this, a longitudinal cross section is made. Since this cross section is symmetrical, only half of it will be modelled.

The V-shaped roof section is given a negligible curvature, to make the force polygon more explicit.

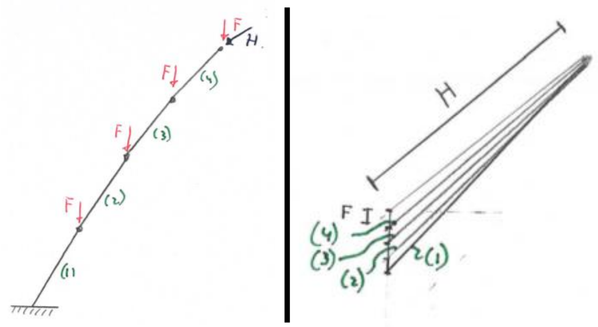

Figure 4: Force polygon of V-shaped roof

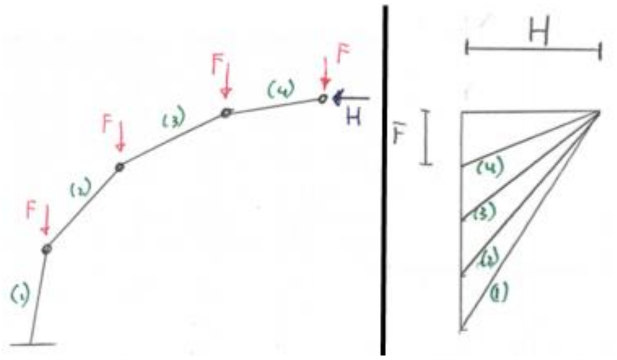

Figure 5: Force polygon of curved roof

Figure 4 shows the force polygon for the V-shaped roof section and figure 5 shows the force polygon for the curved roof section.

This graphic static calculation has a horizontal force H pushing at the top of the cantilevering roof. This Horizontal force is the hoop force created by the principle tension stresses in the roof.

The force polygons for the V-shaped roof section shows very big forces in the structure compared to the applied load F. The force polygon for the curved roof section has substantially smaller forces in the structure. Therefore, the curved shape is a brilliant shape to use for the roof sections.

To determine the exact placement of the rebars, Torroja divided the roof in three areas.

- o Part of the roof between A and K.

- o The part of the roof close to A.
- o The joint between tie CD and the roof. [2]

By considering the roof as a cantilever, Torroja obtained the internal forces in the roof. Using these forces and the Vening-Meinesz's integral, Torroja was able to find the stresses. Figure 6 shows these principle stresses. The circular stresses around the main support are the tension stresses. While the forces approaching the main support are the compression stresses. [2]

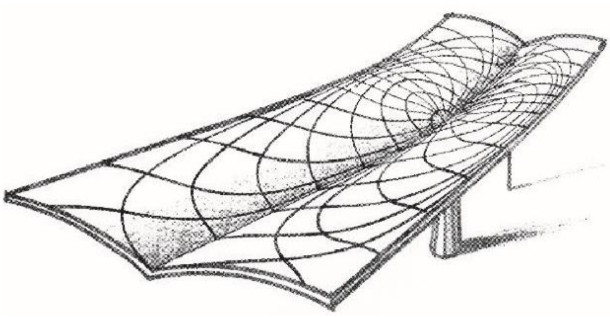

Figure 6: Distribution of principle stresses [2]

These found principle stresses match with the expectation. Because, in anticlastic shell roofs, the tension stresses follow the concave curvature, while the compression stresses follow the convex curvature. [6]

It is optimal to have tension stresses near the edges of the surface instead of compression stresses, since compression stresses can cause buckling in the edge of the surface.

A hyperbolic paraboloid surface is anticlastic. This mean that the Gaussian curvature is negative and thus it cannot be developed out of a flat shape. So, it would not be possible to create this roof out of a flat sheet of steel. Thus, a formwork and concrete are used to create the right shape of the surface. Since concrete is a highly plastic material it can take up any shape inside the formwork.

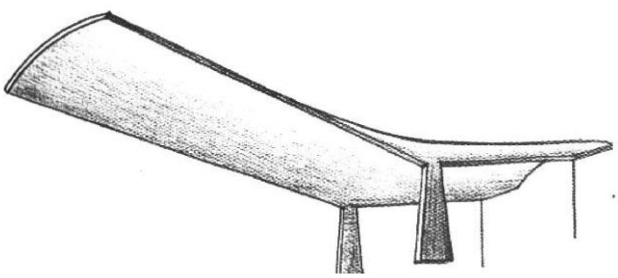

Figure 7: Roof section: Type 1 [4]

To save costs, it is beneficial to create roof section. Because then the formwork can be reused for each roof section.

To construct the full roof, multiple roof sections such as shown in figure 7 must create a continuous roof. The roof section shown in figure 7 will create a lot of problems when constructing the full roof, since the formwork of one section will need to rest on the supports created in the previous section. Torroja solved this problem by cutting the roof section shown in figure 7 in half and then connecting these two parts at the line of the support. This results in the roof section as shown in figure 8.

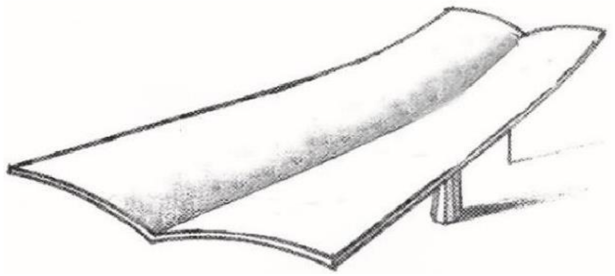

Figure 8: Roof section: Type 2 [4]

The roof section consist of a main column and tie member and two hyperbolic paraboloids, which Torroja called lobes. These roof sections are stable on itself and using these sections, the full roof can be built using one formwork multiple times. [1]

In this paper, these roof sections will be created using the parametric software Grasshopper and will be analyzed using the Finite Element Method analysis program DIANA FEA.

2.2 Efficiency

To check the efficiency of the design, it will be compared to another proposal design for the Hipodrómo de la Zarzuela. This proposal was made by F. Heredero, J. Golfin and C. Fernández-Casado. Their entry had a system of one-way floor slabs supported by transverse stiffening ribs and longitudinal beams. [1]

Fernández-Casado's design required a total volume of 3,04 m^3 concrete and a surface of formwork of 34,4 m^2 per linear meter of grandstand. While Torroja's final design required a total volume of 2,40

m^3 concrete and a surface of formwork of 21,2 m^2 per linear meter of grandstand. So, 27% less concrete and 62% less formwork was used in Torroja's design

Another advantage of Torroja's design was that by using less concrete and formwork, smaller foundations where needed. Furthermore, Torroja's design is considered as an aesthetically pleasing structure. The project won the National Architecture Price.

A disadvantage of Torroja's design is that the formwork has a complex form, which result in a more expensive formwork and more labor required to use. [2]

To conclude, the design made by Torroja was a very efficient design. Since labor costs where relatively cheap compared to material costs in these times, it was also an economic design.

2.3 Geometry

The complete geometry of the hyperbolic paraboloid is not specified in literature. However, the following dimensions are specified. The thickness of the elements varies over the length and is 6 cm at the end and 75 cm at the support. Each transverse roof section consists of two circumference arches, which allows a simple laying out. Near the support the middle part rises 1,40 m and has a radius of 2,75 m. While at the front of the roof section, the rise is 0,44 m and the radius is 6,70 m. For the backside of the lobe, the radius and rise is assumed to be equal to the middle part [1], [3], [4].

For the thickness at the back of the hyperbolic paraboloid a thickness of 38,5 cm is used. This thickness is found by using the ratio of thickness of the front of the roof section and the thickness of the roof near the support and using the difference in length between the main support and the front of the roof section and the main support and the back of the roof section, see equation (1)

$$0{,}75 - \left((0{,}75 - 0{,}06) * \frac{6{,}75}{12{,}75}\right) = 0{,}385 \text{ m.} \quad (1)$$

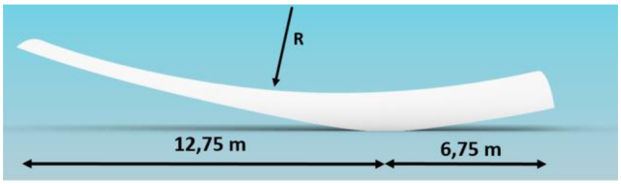

Figure 10: Side view of a roof section

The unspecified parameters are the radius of the curve at the back of the roof section, the height at the back of the roof section and the height at the front of the roof section. These parameters are named R1, H1 and H2 respectively. Figure 9 shows a grasshopper render specifying these parameters. The roof section in this figure is later transformed in a roof section of type 2, see figure 11.

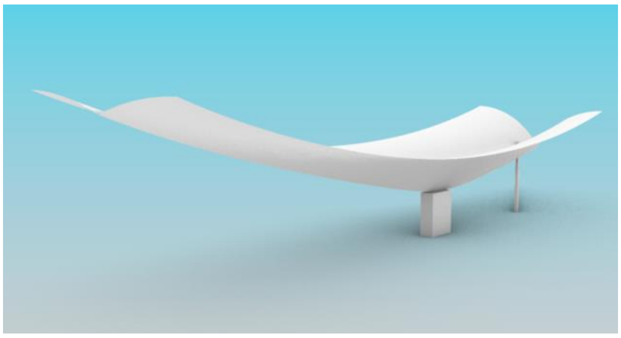

Figure 11: Roof section created in Grasshopper

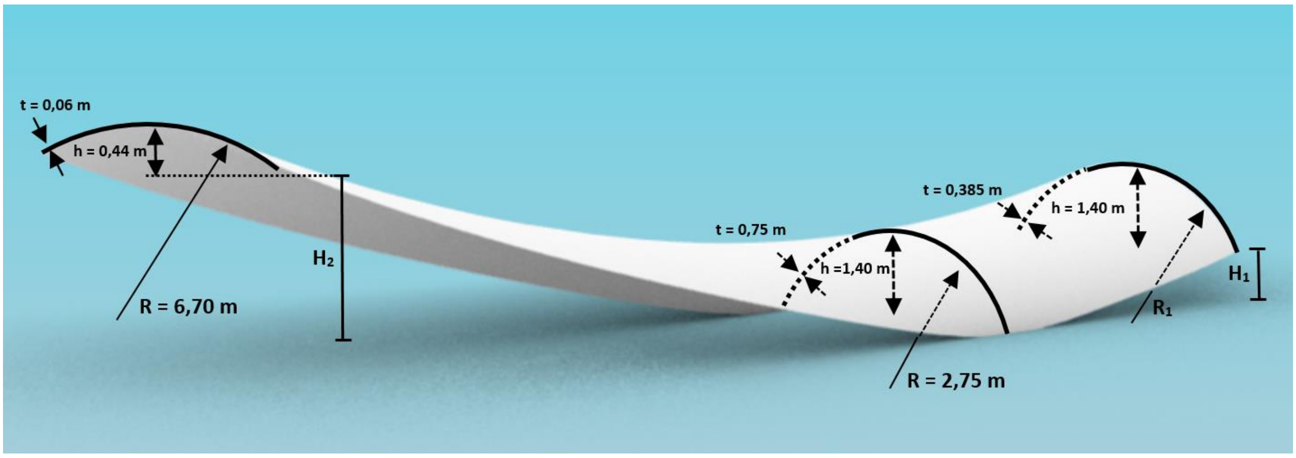

Figure 9: Geometry of a roof section

The parametric model works as follows: all the specified and unspecified parameters are put in at the start apart from the thicknesses. Then, parts of three circles are created, see figure 9. These partial circles are connected using a curve perpendicular to the top point of all three circles. This curve has a radius dependent on the chosen parameters, see figure 10.

To create a section of the roof, the hyperboloid shape shown in figure 9 is split and moved, to create the shape of a lobe. Then, the main support and the structural ties are added. This model is shown in figure 11.

2.4 Variants

The goal of this paper is to determine the unknown parameters R1, H1 and H2 that would create the optimal geometry. To do this, four variants are created, each with different magnitudes for the unknown parameters. First, variant 1 is created by measuring the unknown parameters and estimating the magnitudes.

Variant 2, 3 and 4 each have one of the parameters changed compared to variant 1. By comparing all these variants in a finite element analysis in DIANA, the optimal variant can be found.

	Variant 1	Variant 2	Variant 3	Variant 4
R1 [m]	2,75	4,00	2,75	2,75
H1 [m]	0,50	0,50	0,50	1,00
H2 [m]	2,50	2,50	4,00	2,50

Table 1: Variants and their corresponding parameters

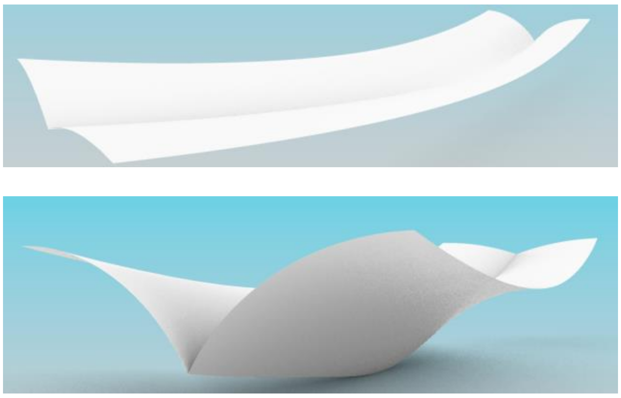

Figure 12: Variant 1

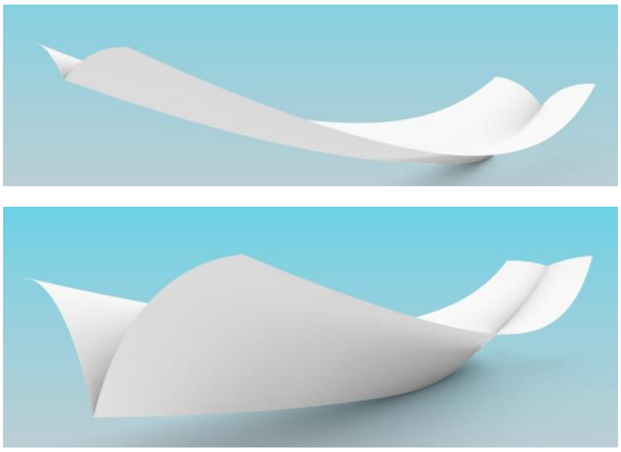

Figure 13: Variant 2

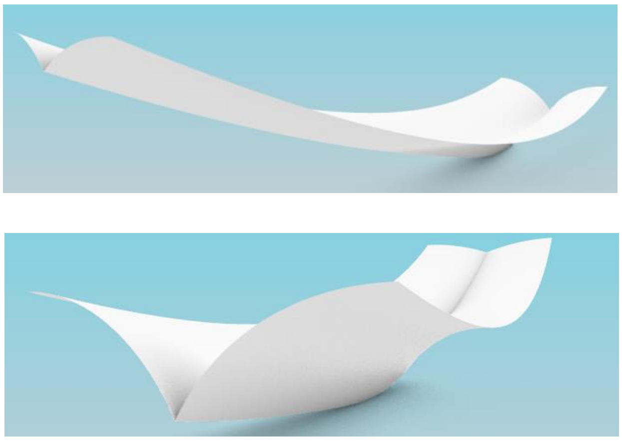

Figure 14: Variant 3

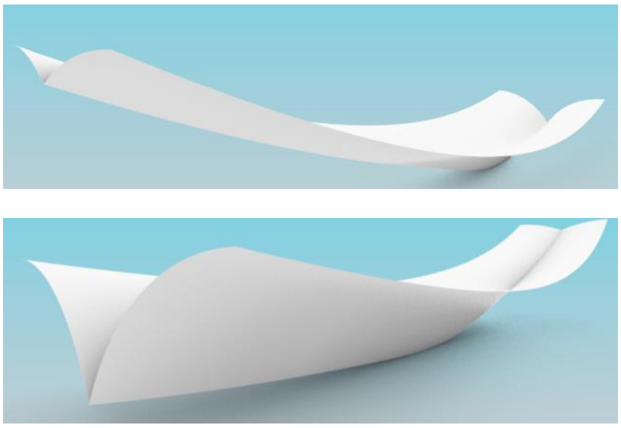

Figure 15: Variant 4

To be able to compare these variants to the existing roof, the lobe shape is repeated in Grasshopper. This way, the shape of the full roof is created. Figure 16 and 17 shows a comparison between the created roof in variant 1 and the existing roof.

Figure 16: Existing roof [12]

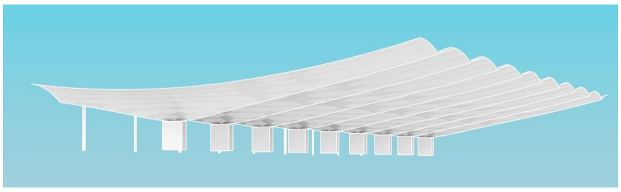

Figure 17: Full roof created in Grasshopper

3. PROJECT AND RESULTS

3.1 Full-scale test

Because Torroja's calculations were not exact, a complete full-scale model of the lobe was constructed. For this test a selfweight of 2,80 kN/m^2

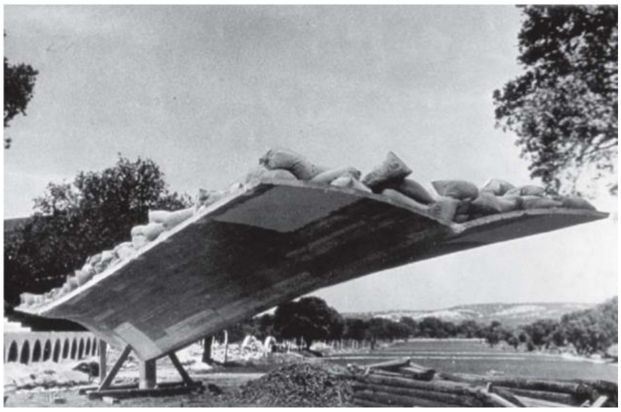

Figure 18: Full-scale test of the lobe [4]

was used. In this time the codes prescribed a snow & vertical wind of 0,70 kN/m^2.

The life load at which the lobe failed in the test was 6,05 kN/m^2, see figure 18.

This test also showed the following results:

- The maximum deflection was 150 mm at the highest point.
- The suspected failure was by compression of the concrete around the central column.
- No rebars have yielded.
- the mounting system formworks is acceptable. [4]

3.2 DIANA FEA input

For the analysis of the roof, the geometry as stated was modeled using Grasshopper software. The obtained geometry was copied to Diana. In Diana a Finite Element Analysis was done to obtain the forces stresses and moments in the geometry for all four variants.

The following parameters were used in Diana. For the E-modulus, it was found that buildings constructed around this time where working with concrete that is best resembled by C12/15 concrete [6].

So, modulus of Elasticity = 27100 N/mm^2, Poison ratio = 0.15, mass density of concrete = 2400 kg/m^3, vertical wind load is 0,70 kN/m^2.

The input in DIANA FEA is elaborated in table 2.

	DIANA FEA
Element type	regular curved shells
Element size [m]	0,2
Mesher type	Hexa/Quad
Concrete type	C12/15
Poison ratio	0,15
mass density [kg/m3]	2400
Loads	Self-weight, life load (6,05kN/m2)
Supports	Main support (constraints: R1, R3, T1, T2, T3) Structural tie (constraints: T3)
Analysis	Linear Elastic

Table 2: DIANA FEA input

3.3 Results

The lobes of the four different variants were all tested using the DIANA finite element analysis software. This analysis provided information regarding the magnitude of forces, stresses and moments in the lobes. The variants are compared to each other regarding these forces stresses and moments. The aim is to find the most optimal variant this way.

Figures 19, 20, 21, 22, 23, 24, 25 and 26 show the stresses, forces, displacements and moments for the top part of the geometry of all four variants. Variant 1, 2, 3 and 4 are shown respectively

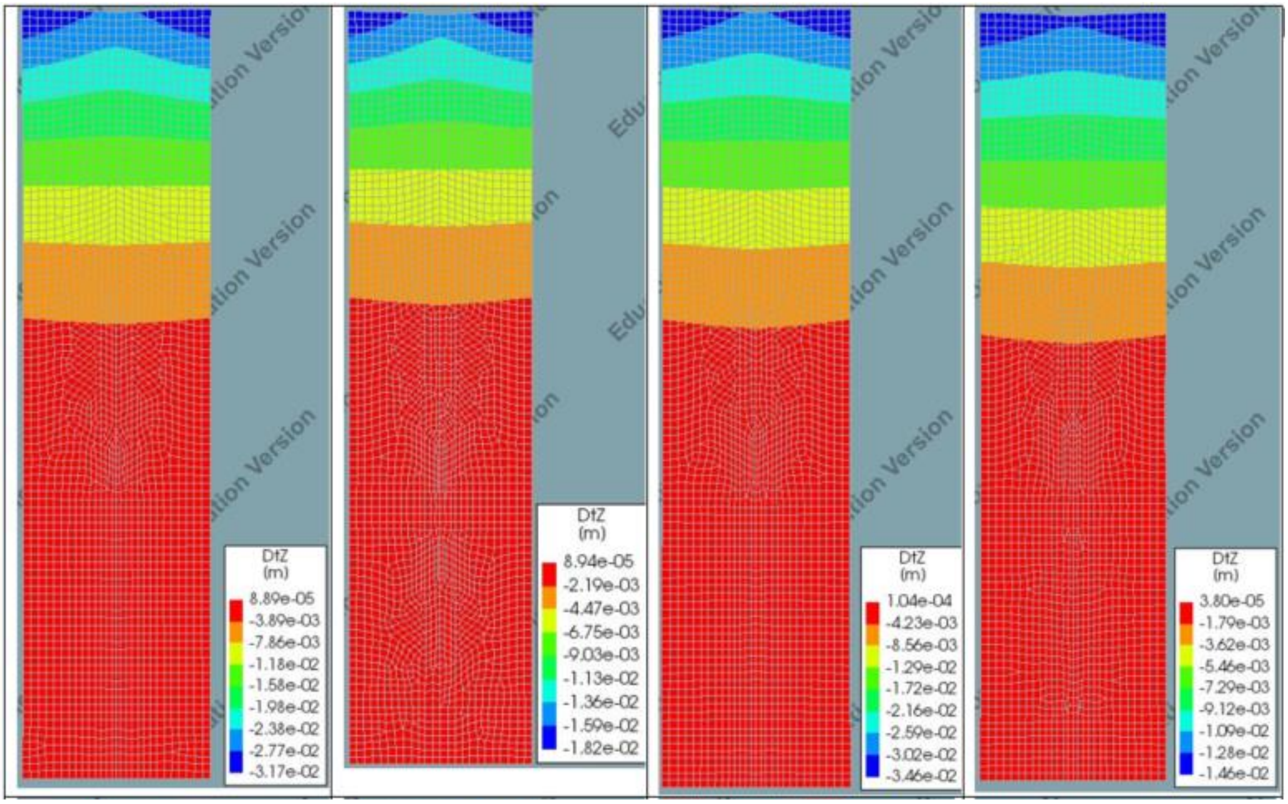

Figure 19: Deflection Dtz

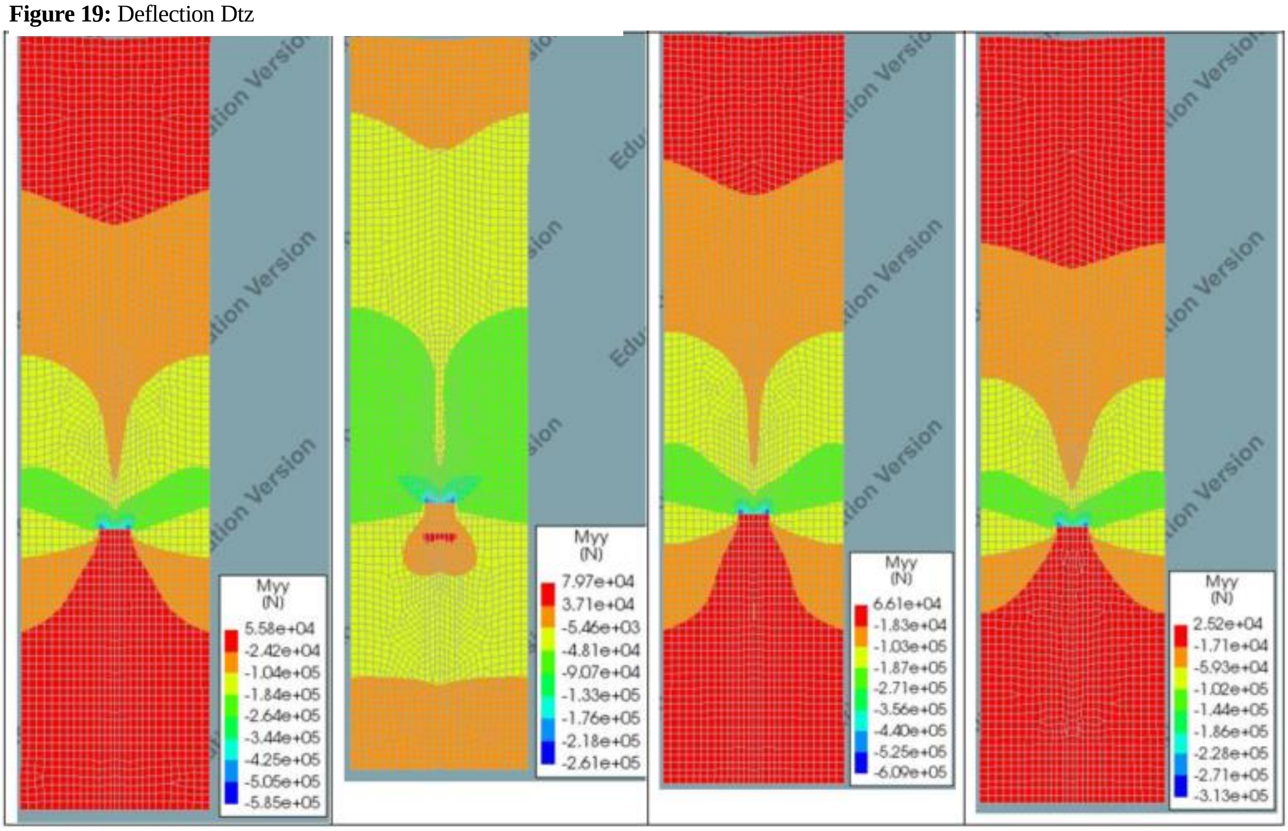

Figure 20: Moment Myy

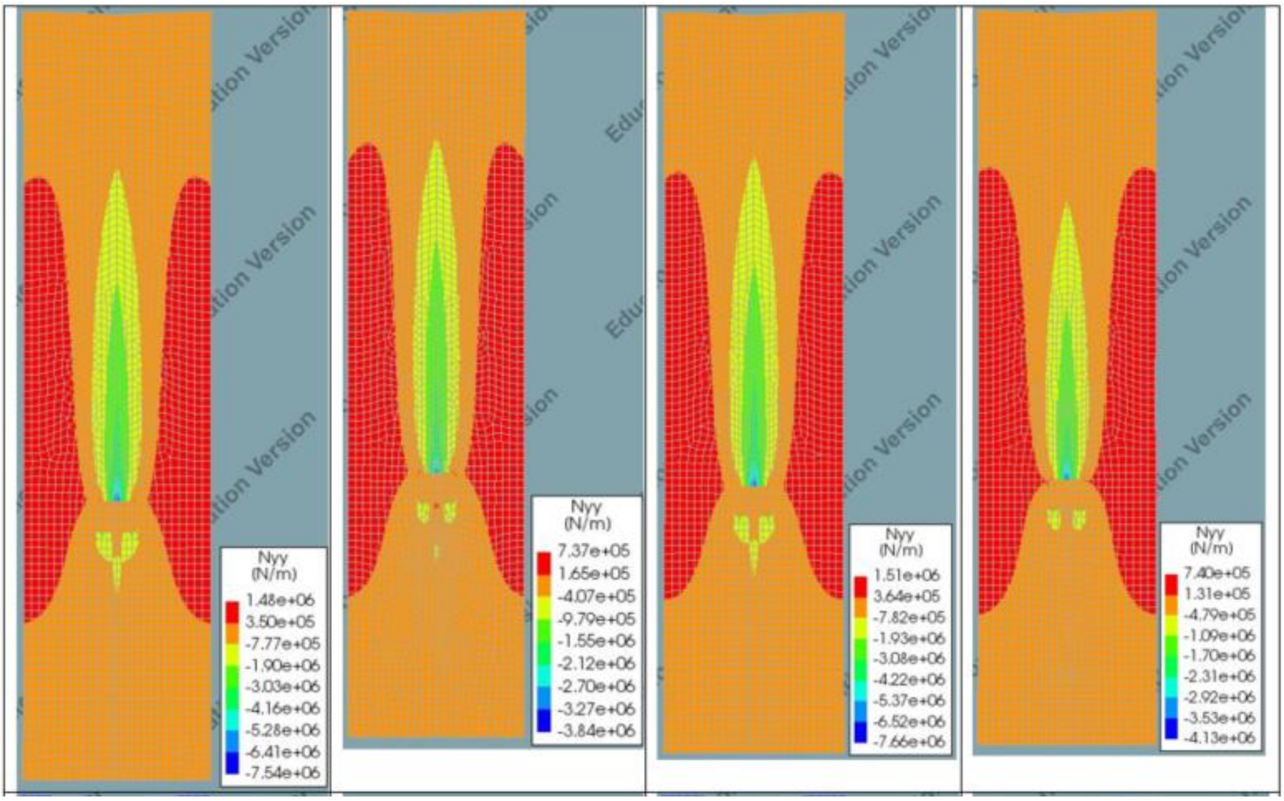

Figure 21: Normal Force Nyy

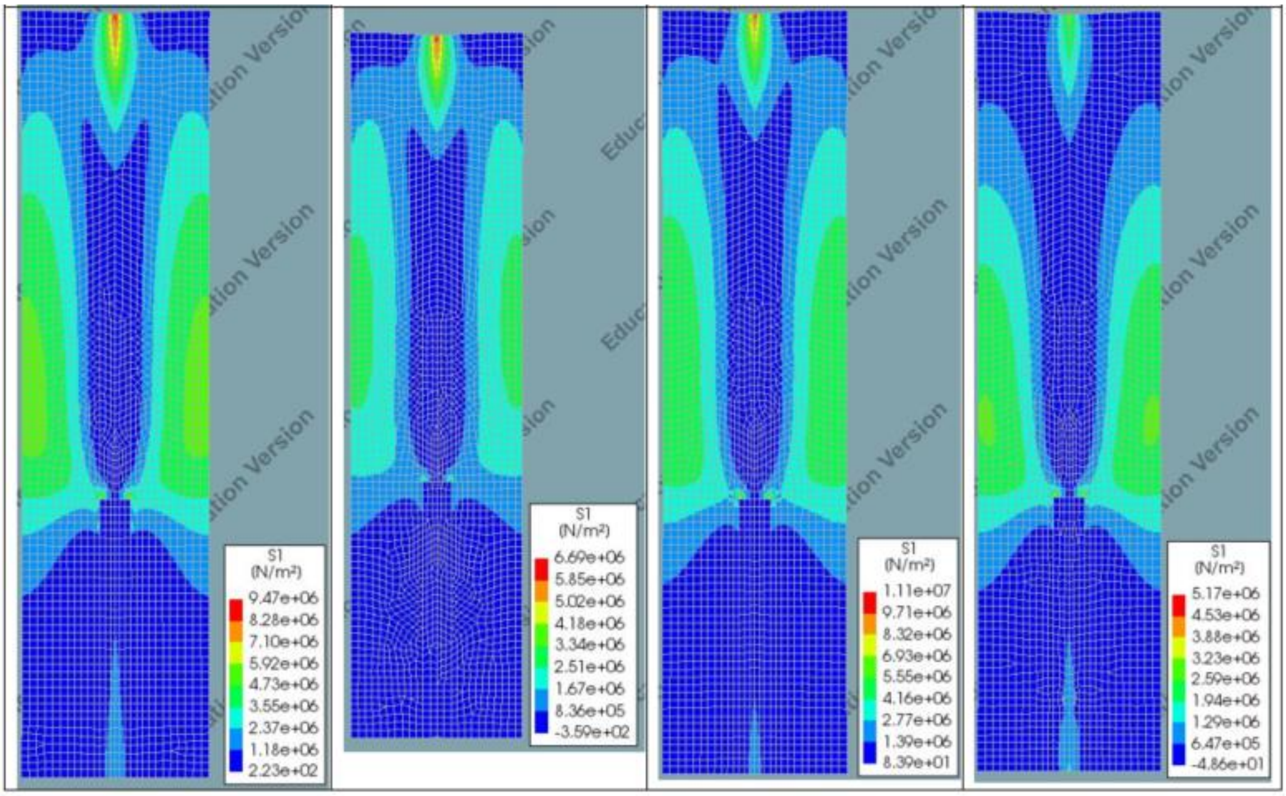

Figure 22: principle stress S1

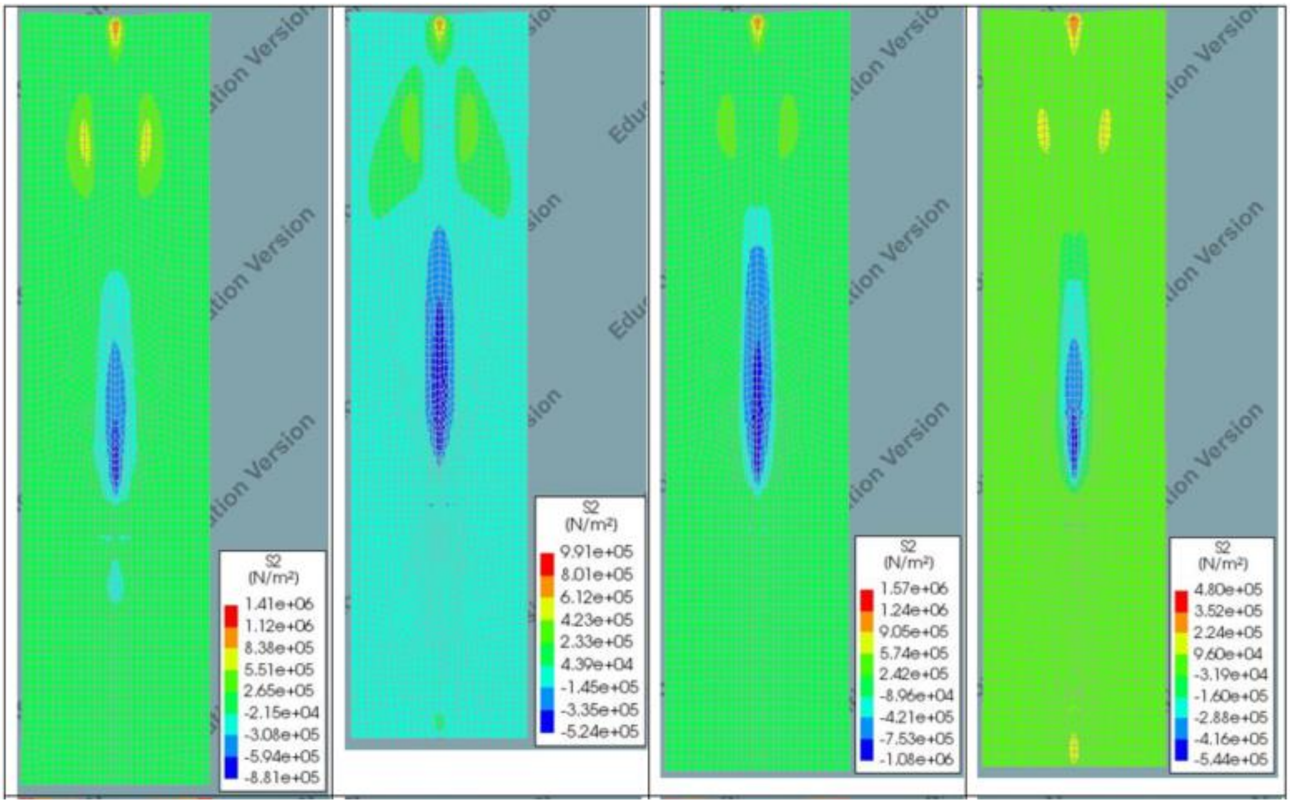

Figure 23: Principle stress S2

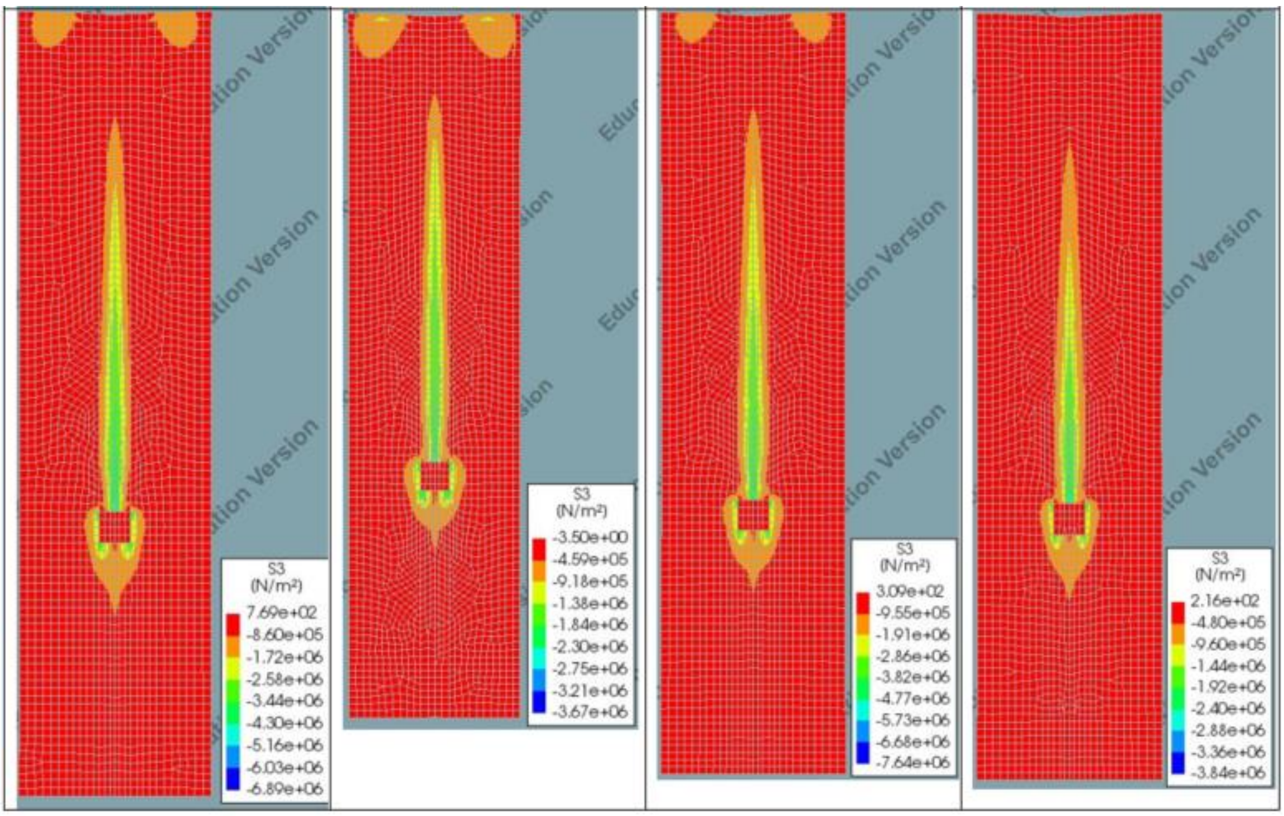

Figure 24: Principle stress S3

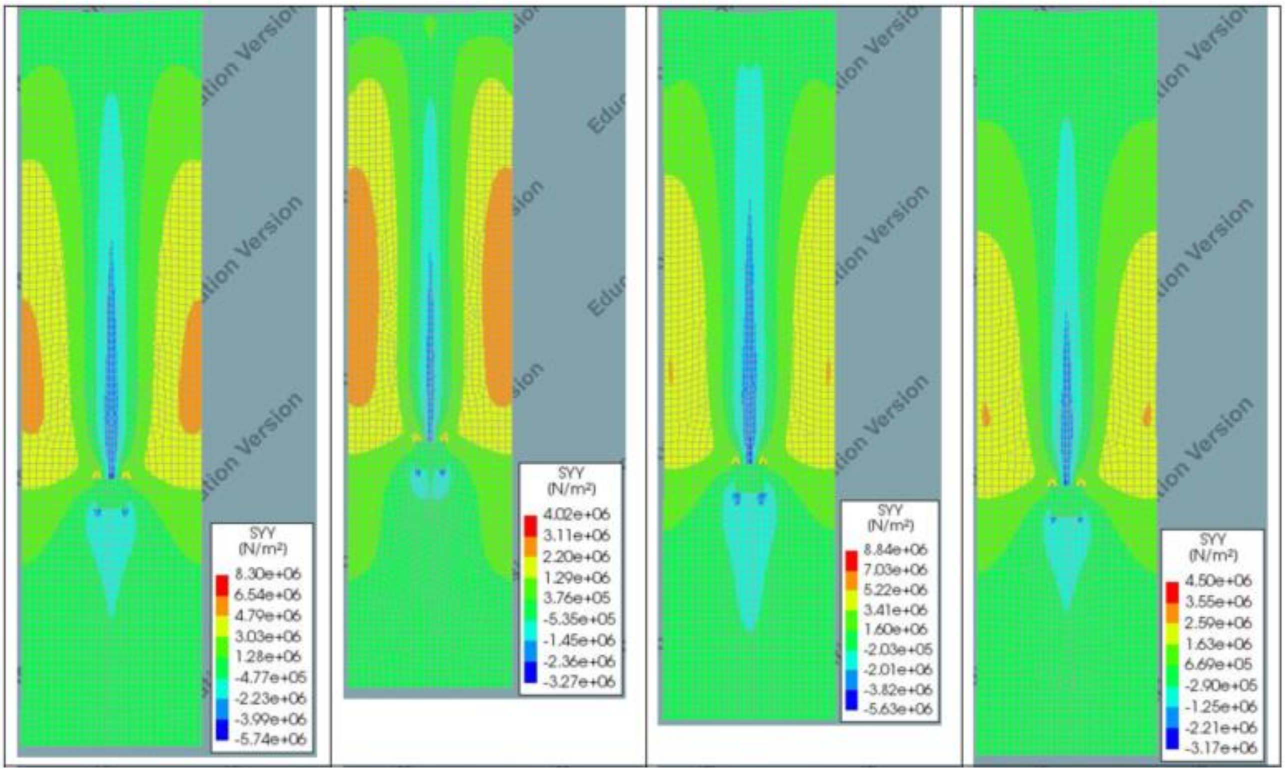

Figure 25: Stress Syy

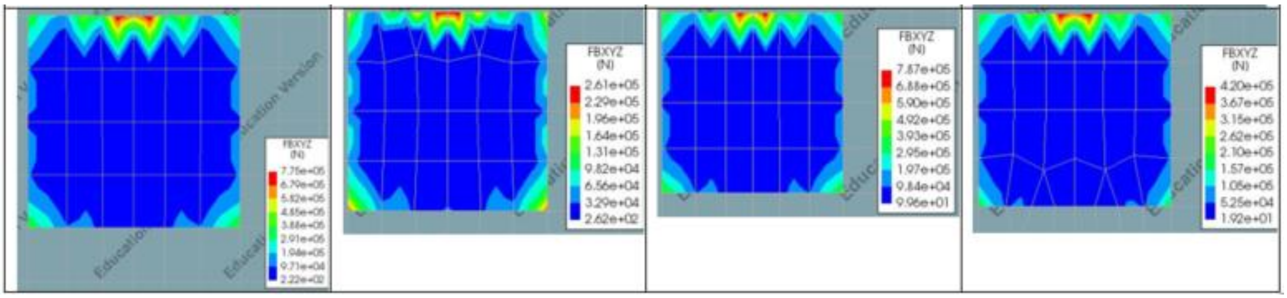

Figure 26: Compression force in the column

	Variant 1	Variant 2	Variant 3	Variant 4
R1 [m]	2,75	4,00	2,75	2,75
H1 [m]	0,50	0,50	0,50	1,00
H2 [m]	2,50	2,50	4,00	2,50
DtZ [mm]	32	18	35	15
Myy [kN] +	56	80	66	25
Myy [kN] -	585	261	609	313
Nyy [kN/m] +	1480	737	1510	740
Nyy [kN/m] -	7540	3840	7660	4130
S1 [kN/m2] +	9470	6690	11110	5170
S1 [kN/m2] -	X	360	X	49
S2 [kN/m2] +	1410	991	1570	480
S2 [kN/m2] -	881	524	1008	544
S3 [kN/m2] +	769	X	309	22
S3 [kN/m2] -	6890	3670	7640	3840
SYY [KN/m2] +	8300	4020	8840	4500
SYY [KN/m2] -	5740	3270	5630	3170
FBXYZ [kN] +	775	261	787	420

Table 3: Variants and their corresponding parameters

Table 3 shows the results of this analysis. For each force, stress and moment, the lowest positive and negative value is marked in green. In two cases the lowest value was absent, these cases will not be taken into account in this analysis.

The results of the finite element analysis clearly show that either variant 2 or variant 4 is the optimal geometry. The suspected failure of the full-scale test was caused by compression of the concrete around the central column. This compression in the central column is given by the FBXYZ value, see table 3. Since this value is the lowest for variant 2, and variant 2 scores the best on the most different forces, stresses and moments, variant 2 is the optimal geometry out of the four variants.

The full-scale test done by Torroja can also be compared to the found results of the Finite Element Analysis by looking at the maximum deflection. In the full-scale test this maximum deflection was 150 mm. For variant 2, under the same loading this maximum deflection was 18 mm. At first glance this looks like a big difference. But the main difference between the two models is that there are no inaccuracies in the geometry and surface of the Finite Element Analysis. For buckling a factor is used to compensate for the difference between a perfect surface and a surface with inaccuracies. This factor, known as the knockdown factor, is 1/6. So, if we assume the same to be true for deflections, we will end up with a deflection of 108 mm for variant 2. Considering the concrete type is assumed to be C12/15 and in the full-scale model more factors would influence the model, it looks like the Finite Element Analysis depicts the magnitude of the maximum deflection quite well.

4. DISCUSSION

In the Finite Element Analysis, only four variants are checked, so the geometry can be even more optimized. First, variant 1 is created by measuring the unknown parameters and estimating the magnitudes. Variant 2 and 4 both proved to be a more optimal geometry than variant 1. Variant 2 has the largest R1 value, while variant 4 has the largest H1 value. So, both a larger value for R1 and H1 compared to variant 1 will be expected to provide an optimal geometry. An increase of the H2 value compared to variant 1 does not result in a more optimal geometry.

Playing with the magnitudes of the known parameters can also result in a more optimal geometry. This is something which can be determined in future research.

In this research, the loading of the lobe is compared to the full-scale test. In future research, the geometries can also be compared when asymmetric loading is present.

Torroja was able to create a very smart and aesthetically pleasing solution by using an anticlastic surface for the roof of the racetrack.

5. CONCLUSION

Since Torroja used simplified models for the designing and optimizing of the roof, a complete full-scale model of the lobe had to be made to test it. This full-scale model failed under a life load of 6,05 kN/m^2 in the test.

The aim of this paper was to determine the magnitude of the unknown parameters to create the optimal geometry of the roof. Four different geometries were created in the parametric software Grasshopper. These different geometries were analyzed in the Finite Element Method analysis program DIANA FEA. With this analysis an optimal geometry was found.

By looking at the maximum deformation of the geometry it is concluded that the finite element analysis of the model can be assumed to be in accordance with the results of the full-scale test.

Out of the four tested variants, the geometry in variant 2 is found to be the optimal, most of the forces stresses and moments in this geometry are smaller than for the other variants. The geometry of variant 2 also results in the smallest compression force in the central column. Since this compression force was the suspected failure of the full-scale test, variant 2 is concluded to be the best geometry of the four.

Variant 4 is the best geometry on certain aspects. So, it is expected that a combination of variant 2 and 4 will result in an optimal geometry. Variant 2 has the largest R1 value, while variant 4 has the largest H1 value. So, both a larger value for R1 and H1 compared to variant 1 will be expected to provide an optimal geometry.

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