SC42155 Modeling of Dynamical Systems

Modelling and Simulation Mini Project

1 Physical Modelling (4pt=3+1)

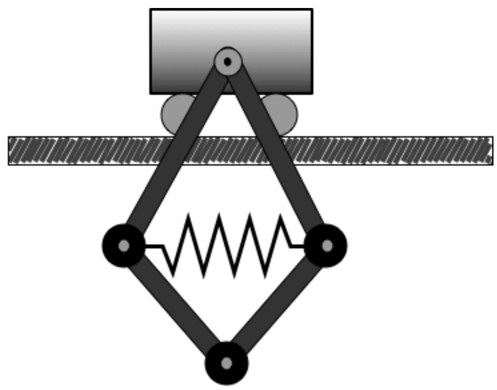

Figure 1: Complex pendulum on a cart system.

Consider the system in Figure 1, consisting of a cart, powered by a DC motor, of which a pendulum consisting of 4 links and three masses interconnected (black circles). Two of the masses of the pendulum are interconnected with a spring. Observe that at each of the joints connecting two links each of the links can rotate freely, that is, the angle that these links form with each other is not constant. Do not consider the links idealized, that is, assume the links have some mass uniformly distributed, so that you can easily compute their moments of inertia. You may assume the masses (black circles) at the end of the links as point masses.

1. Derive the physical equations connecting the voltages at the motor with the state of the mechanical device.
2. Incorporate mechanical losses at the joints and in the friction with the ground on the vehicle.
3. Select appropriate modelling approaches and justify your choices, describing the major steps of the modelling process. Detailed derivations should be placed in an Appendix.
4. You may incorporate also the presence of a hydraulic brake on the cart.

1.1 Simulations (1pt)

Next, you need to implement the derived model in some simulation software in order to validate the model.

1. Select reasonable dimensions and parameter values for your model.
2. Select a set of informative inputs to apply to the system model and evaluate whether the model produces reasonable predictions of the real system's behaviour.
3. Provide simulations that illustrate the mechanical and electrical losses of the system.
4. Discuss the simulation results.

2 Computing Modelling (3pt=2+1)

Consider now that angular velocity sensors are placed at the joint of each of the first two links connecting them to the cart (on the light grey circle). The sensors produce an output of 0 when the angular velocity of the link is within some safe bounds $|\omega| < W$, and a 1 when those bounds are exceeded. An alarm is signalled if the sensor measures at least 3 consecutive 1s. Assume that it is a low-quality sensor with an average error rate of less or equal to 1 bits every 8 bits, that is, in every window of 8 bits there is at most 1 bit flipped from its actual value.

1. Construct a regular expression and a DFA capturing the language of all sequences at which one can be certain of the safety speed having been exceeded.
2. Construct a Mealy or Moore machine filtering the signal from the sensor and producing a symbol T when it is certain that the sensor signalled an alarm, D when in doubt, and O when velocities are certainly within safe bounds.
3. Describe clearly the raw thinking process followed to arrive to the desired automata and machines.

2.1 Simulations (1pt)

1. Select input sequences to show the different behaviours that the system can exhibit.
2. Simulate your Mealy or Moore machine on the selected inputs.
3. Discuss the results.

3 Hybrid System (3pt=2+1)

Consider now that the masses (solid black circles) of the pendulum can collide with each other and with the ground (horizontal stripped box). Furthermore, assume that such impacts are inelastic (introducing energy losses).

Additionally, assume that there are angular velocity sensors, as the one in the previous section, detecting unsafe link speeds. The sensors produce output values at 100 bits per second. Design a simple safety system that disables the voltage on the cart's motor for 2 seconds whenever an alarm is signalled.

1. Incorporate the effect of impacts in your model.
2. Incorporate the presence of the safety system with the low-quality sensor in your model.
3. Construct a complete hybrid model and present it in the form of a hybrid automaton or a jump-flow system.

3.1 Simulations (1 pt)

1. Select input sequences to show the different behaviours that the system can exhibit.
2. Simulate your resulting system on the selected inputs validating your model.

Alternative projects

Section 1

- You may consider any other robotic configuration or mechatronic system of similar complexity.
- You can consider a thermo-fluid network, with pumps and electric heaters for the fluid.
- Another, more concrete idea, is given by the system of Figure 2, illustrating a reservoir of water from which water flows into a turbine. The turbine recharges a battery that a pump can use to bring water up the reservoir again from a lower reservoir (assumed of unlimited capacity). The battery output voltage can be regulated by means of a DC-DC converter to apply the desired voltage to the pump. Initially, consider that $h = 0$.

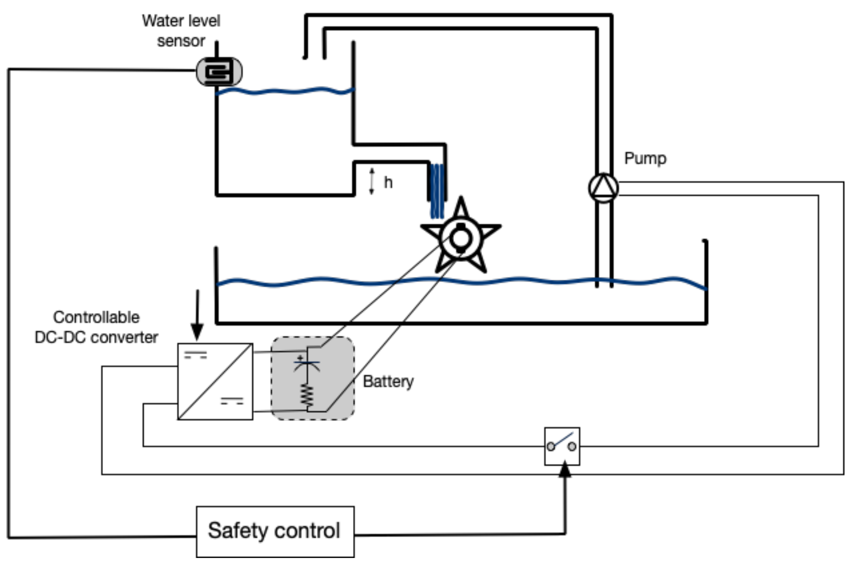

Figure 2: Water reservoir system.

Section 2

- You can consider any other sort of unreliable sensor, with a reasonably complex specification.
- Alternatively you can consider constraints on the availability of different actuators forcing certain ordering on the application of the actuators, which you could model with a Petri Net. E.g. you may consider that some actuator is only activated after two other actuators have injected on the system: one 3 units of energy, and the other has injected 1 unit of energy.

Section 3

- You may consider any other sensor or actuator with faults placed anywhere along your system, being used to trigger some action on the system.
- If you chose a system in part 1 that can exhibit impacts, incorporate them in the model.
- In the system of figure 2, consider $h \neq 0$, and model the resulting hybrid dynamics from this change.