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THESIS FOR THE BACHELOR OF MATHEMATICS

High Dimensional Regression Models

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Chapter 1

The first chapter

Lemma 1.1 (Basic Inequality).

$$\frac{\left\|X\left(\hat{\beta}-\beta^0\right)\right\|_2^2}{n}+\lambda\left\|\hat{\beta}\right\|_1 \leq 2 \varepsilon^T \frac{X\left(\hat{\beta}-\beta^0\right)}{n}+\lambda\left\|\beta^0\right\|_1$$

Proof.

$$\begin{aligned} & \frac{\left\|Y-X \hat{\beta}\right\|_2^2}{n}+\lambda\left\|\hat{\beta}\right\|_1 \leq \frac{\left\|Y-X \beta^0\right\|_2^2}{n}+\lambda\left\|\beta^0\right\|_1 \\ \Rightarrow & \frac{\left\|(X \beta^0+\varepsilon)-X \hat{\beta}\right\|_2^2}{n}+\lambda\left\|\hat{\beta}\right\|_1 \leq \frac{\left\|(X \beta^0+\varepsilon)-X \beta^0\right\|_2^2}{n}+\lambda\left\|\beta^0\right\|_1 \\ \Rightarrow & \frac{\left\|X\left(\beta^0-\hat{\beta}\right)\right\|_2^2+\left\|\varepsilon\right\|_2^2+\left\langle X\left(\beta^0-\hat{\beta}\right), \varepsilon\right\rangle}{n}+\lambda\left\|\hat{\beta}\right\|_1 \leq \frac{\left\|(X \beta^0+\varepsilon)-X \beta^0\right\|_2^2}{n}+\lambda\left\|\beta^0\right\|_1 \end{aligned}$$

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