## Decision tree

Original presentation from Jeff Howbert

## Example of a decision tree

nominal nominal ratio class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

splitting nodes Refund Yes NO MarSt Single, Divorced Married TaxInc NO < 80k >= 80K YES classification nodes

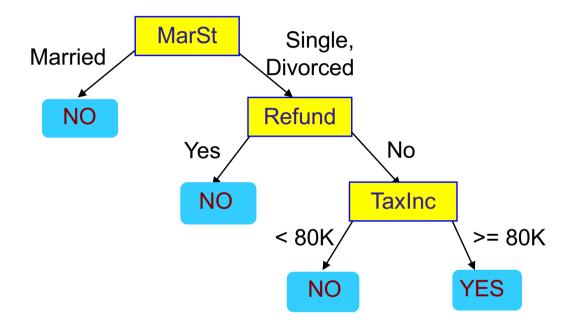
training data

model: decision tree

## Another example of decision tree

nominal nominal ratio class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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8	No	Single	85K	Yes
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10	No	Single	90K	Yes



There may be more than one tree that matches the same data!

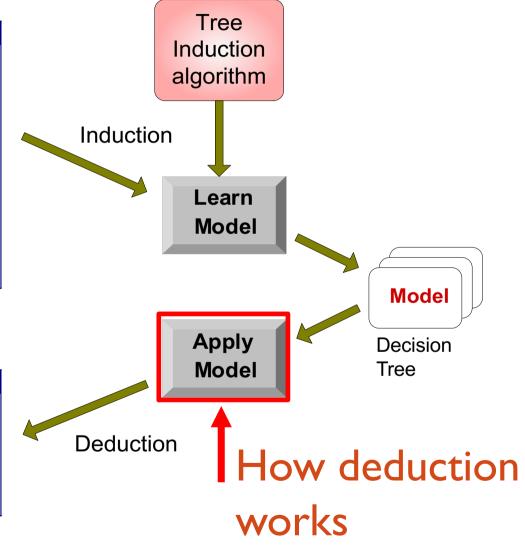
### Decision tree classification task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	No
2	No	Medium	100K	No
3	No	Small	70K	No
4	Yes	Medium	120K	No
5	No	Large	95K	Yes
6	No	Medium	60K	No
7	Yes	Large	220K	No
8	No	Small	85K	Yes
9	No	Medium	75K	No
10	No	Small	90K	Yes

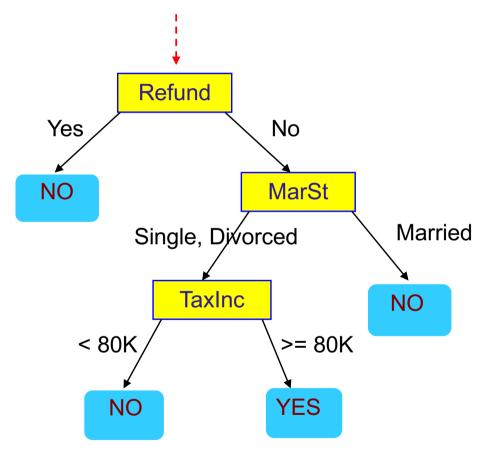
**Training Set** 

Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	?
12	Yes	Medium	80K	?
13	Yes	Large	110K	?
14	No	Small	95K	?
15	No	Large	67K	?

Test Set



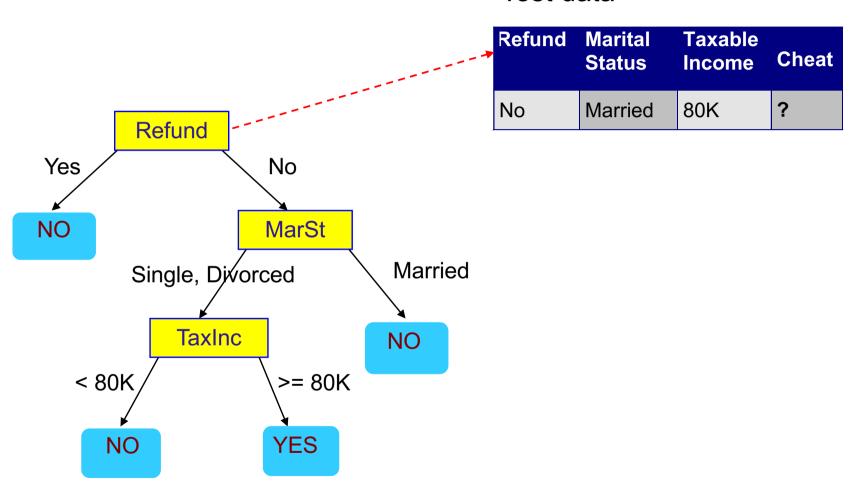
Start from the root of tree.



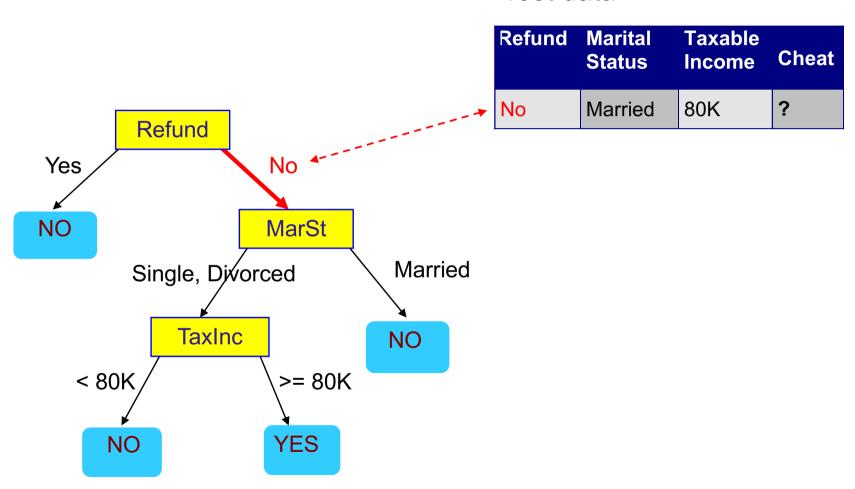
#### Test data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

#### Test data



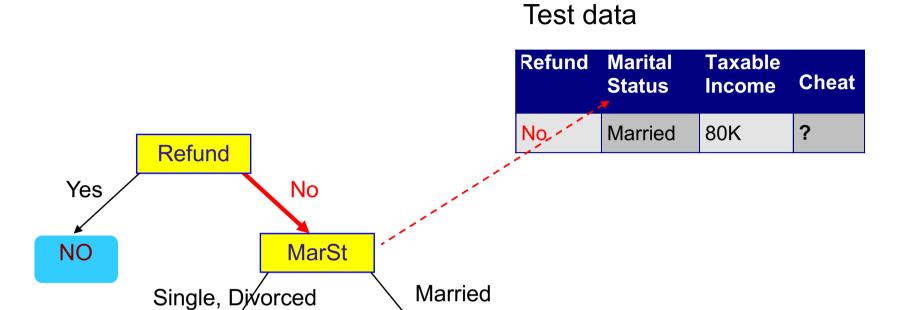
#### Test data



TaxInc

< 80K

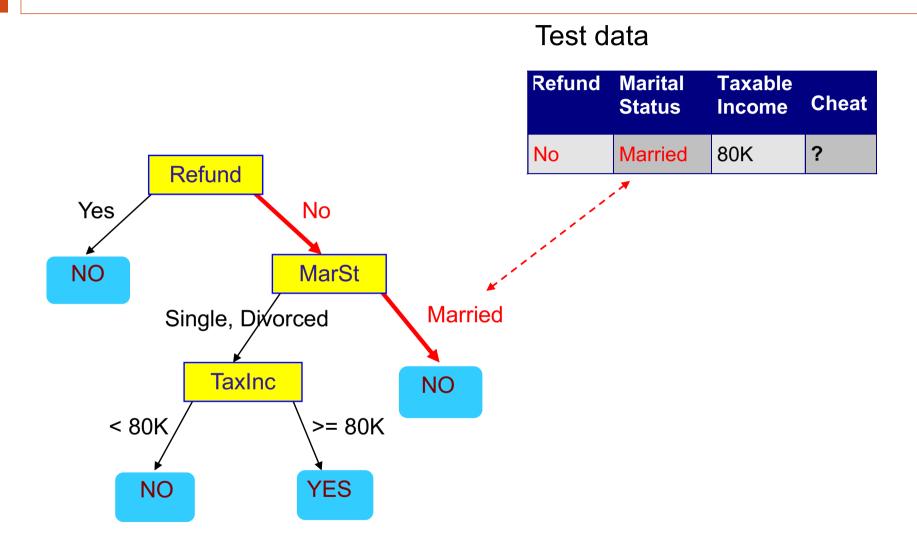
NO

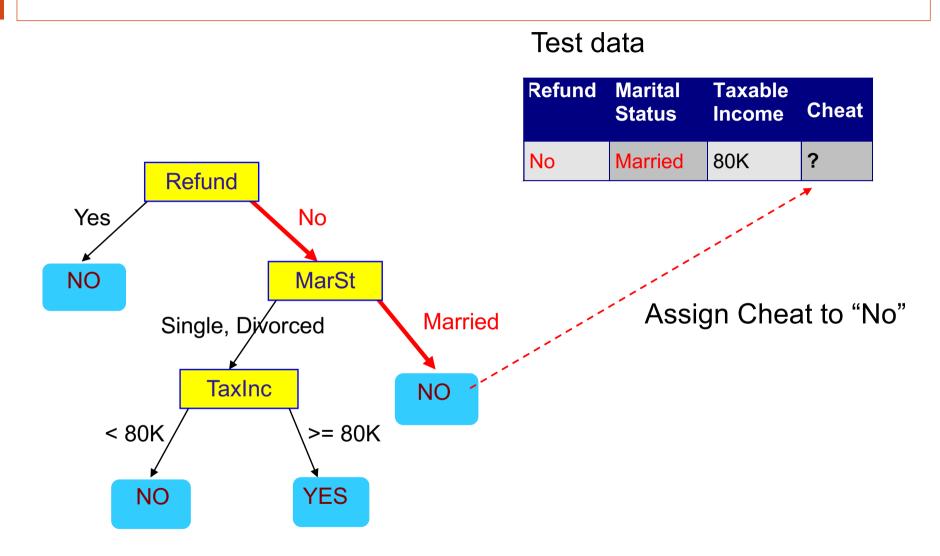


NO

>= 80K

YES





### **Decision tree induction**

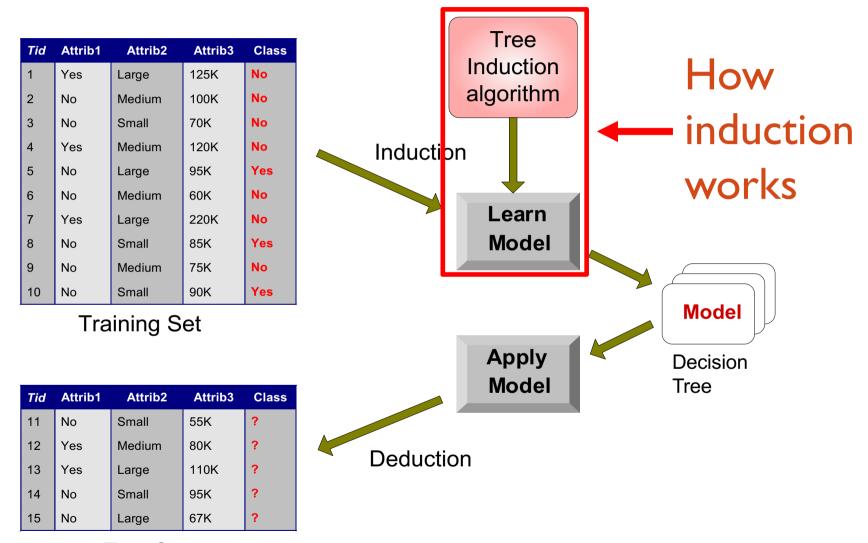
#### Deduction

- Very easy to execute
- Readable / Explainable

#### Induction

- How to build a decision tree
- Many algorithms:
  - Hunt's algorithm (one of the earliest)
  - ▶ CART
  - ▶ ID3, C4.5
  - ▶ SLIQ, SPRINT
- As our goal is not to build a competitor at sklearn, we're just going to look at a few principles of Hunt's algorithm

### Decision tree classification task

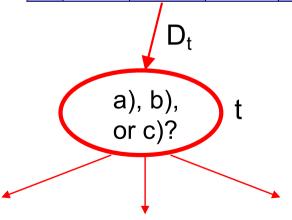


**Test Set** 

## General structure of Hunt's algorithm

- Hunt's algorithm is recursive.
- General procedure:
  - Let D<sub>t</sub> be the set of training records that reach a node t.
  - I. If all records in  $D_t$  belong to the same class  $y_t$ , then t is a leaf node labeled as  $y_t$ .
  - 2. If  $D_t$  is an empty set, then t is a leaf node labeled by the default class as  $y_d$ .
  - 3. If D<sub>t</sub> contains records that belong to more than one class, use an attribute test to **split** the data into smaller subsets, then apply the procedure to each subset.

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Select the majority classe Affect all node to this class

If the node is pure or empty

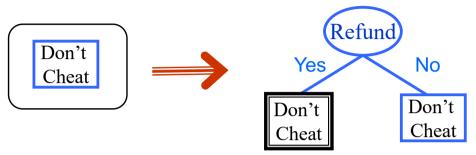
→ It's finished

Else select an attribute a

split the node

Black box = pure node (identical class labels for all samples)

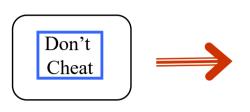
Blue box = impure node

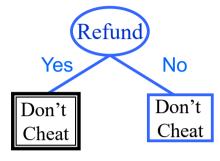


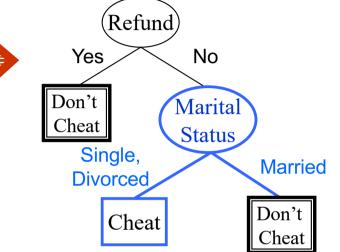
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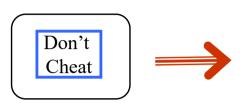




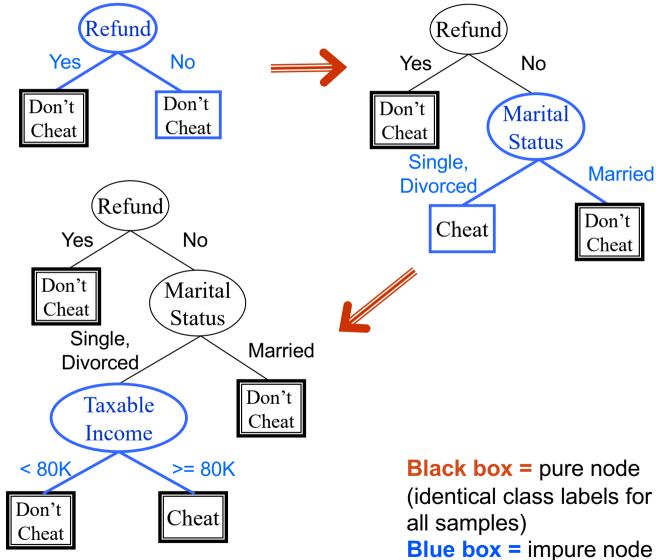
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- Step 1: establish majority (aka default) class for root node
  - ▶ 7/10 samples have label = No
- Step 2: choose Refund as split criterion for 10 samples
  - ▶ Refund = yes  $\rightarrow$  3 samples, all with label = No
  - ▶ Refund = no  $\rightarrow$  7 samples, 4 with label = No, 3 with label = Yes
- Step 3: choose Marital Status as split criterion for 7 samples have to decide how to group attribute values during split
  - Marital Status = single/divorced → 4 samples, 3 with label = Yes, 1 with label = No
  - Marital Status = married → 3 samples, all with label = No
- Step 4: choose Taxable Income as split criterion for 4 samples –
   have to decide value on which to split attribute
  - ► Taxable Income < 80K → 3 samples, all with label = No
  - Taxable Income ≥ 80K → 1 sample, with label = Yes

## Tree induction

#### Greedy strategy

> Split the records at each node based on an attribute test that optimizes some chosen criterion.

#### Issues

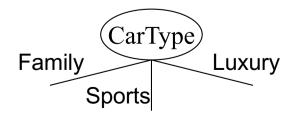
- Determine how to split the records
  - How to specify the test condition?
  - ▶ How to determine the best split?
- Determine when to stop splitting

## Specifying test condition

- Depends on attribute type
  - Nominal single, married
  - Ordinal small, medium, large
  - Continuous (interval or ratio)
- Depends on number of ways to split
  - ▶ Binary (two-way) split



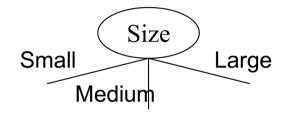
Multi-way split



## Splitting based on ordinal attributes

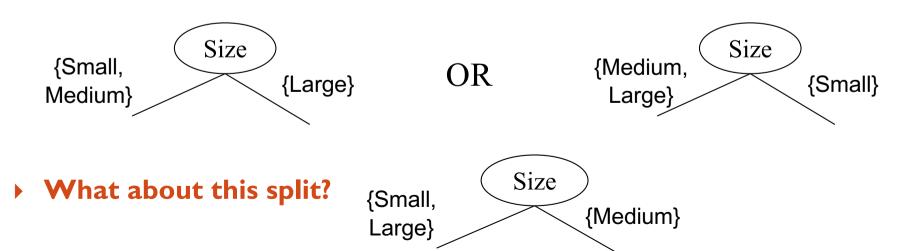
#### Multi-way split:

Use as many partitions as distinct values.



#### Binary split:

- Divides values into two subsets.
- Need to find optimal partitioning.

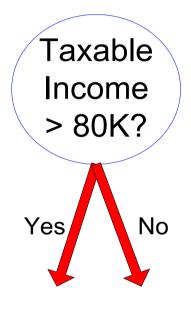


# Splitting based on continuous attributes

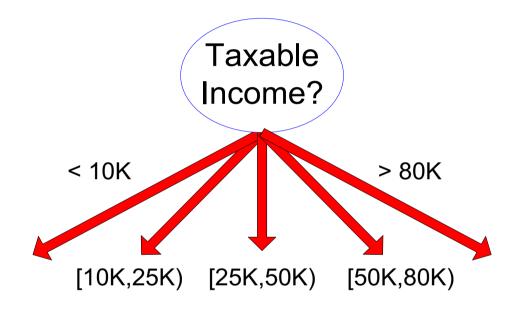
- Different ways of handling
  - Discretization to form an ordinal attribute
    - static
      - □ discretize once at the beginning
    - dynamic
      - □ ranges can be found by equal interval bucketing, equal frequency bucketing (percentiles), or clustering.
  - ▶ Threshold decision: (A < v) or  $(A \ge v)$ 
    - consider all possible split points v and find the one that gives the best split
    - can be more compute intensive

# Splitting based on continuous attributes

Splitting based on threshold decision



(i) Binary split



(ii) Multi-way split

## Tree induction

#### Greedy strategy

▶ Split the records at each node based on an attribute test that optimizes some chosen criterion.

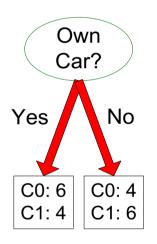
#### Issues

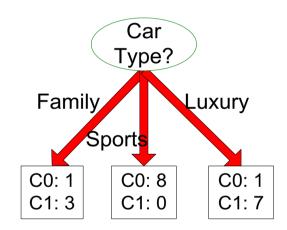
- Determine how to split the records
  - ▶ How to specify the test condition?
  - ▶ How to determine the best split?
- Determine when to stop splitting

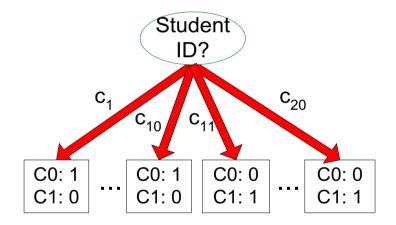
## Determining the best split

Before splitting: 10 records of class 0

10 records of class 1







Which attribute gives the best split?

## Determining the best split

- Greedy approach:
  - Nodes with homogeneous class distribution are preferred.
- Need a measure of node impurity:

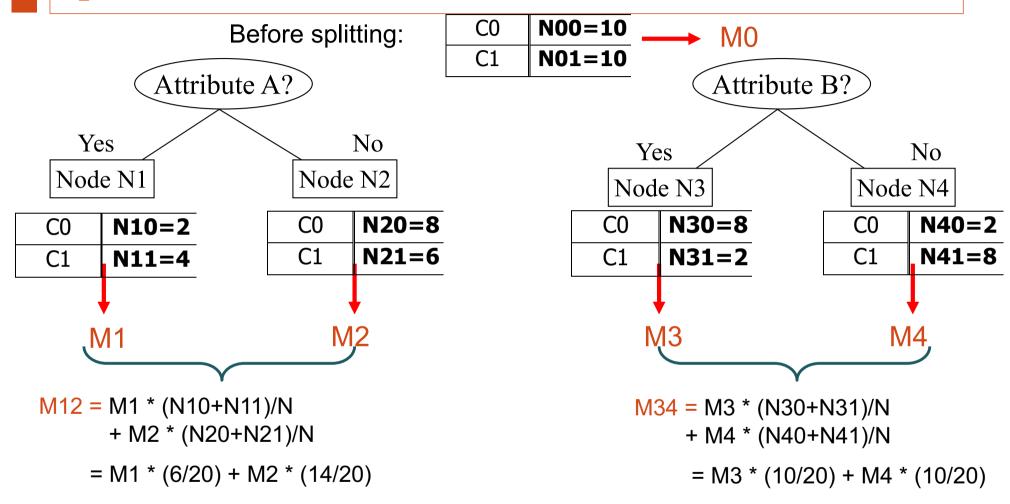
Non-homogeneous, high degree of impurity

C0: 9 C1: 1

Homogeneous, low degree of impurity

- Measures of node impurity
  - Gini index  $G(t) = 1 \sum_{i=0}^{c-1} [p(i|t)]^2$
  - Entropy  $E(t) = -\sum_{i=0}^{c-1} p(i|t) \log_2 p(i|t)$
  - Classification error =  $1 \max_{i}[p(i|t)]$
  - Where
    - p(i|t) denote the fraction of records belonging to class i at a given node t.
    - c is the number of class
    - ▶  $0 \log_2 0 = 0$  in entropy calculation

# Using a measure of impurity to determine best split



- Choose splitting attribute that maximizes gain : Gain = M0 M12 vs. M0 M34
  - M0: Measure of impurity before splitting
  - M1, M2, M3, M4: Measure of impurity for each node after splitting

# Measure of impurity: Gini index Example

Gini index for a given node t:

$$GINI(t) = 1 - \sum [p(j|t)]^2$$

- p(j | t) is the relative frequency of class j at node t
- Maximum = I I / c c = number of classes

- when records are equally distributed among all classes, implying least amount of information
- $\blacktriangleright$  Minimum = 0.0
  - when all records belong to one class, implying most amount of information.

C1	0
C2	6
Gini=	0.000

Gini=	0.278
C2	5
C1	1

C1	2	
C2	4	
Gini=0.444		

$$p(C1) = 2/6$$
  $p(C2) = 4/6$   
Gini = 1 - (2/6)<sup>2</sup> - (4/6)<sup>2</sup> = 0.444

$$p(C1) = 1/6$$
  $p(C2) = 5/6$ 

Gini = 
$$1 - (1/6)^2 - (5/6)^2 = 0.278$$

$$p(C1) = 0/6 = 0$$
  $p(C2) = 6/6 = 1$ 

Gini = 
$$1 - p(C1)^2 - p(C2)^2 = 1 - 0 - 1 = 0$$

## Tree induction

#### Greedy strategy

> Split the records at each node based on an attribute test that optimizes some chosen criterion.

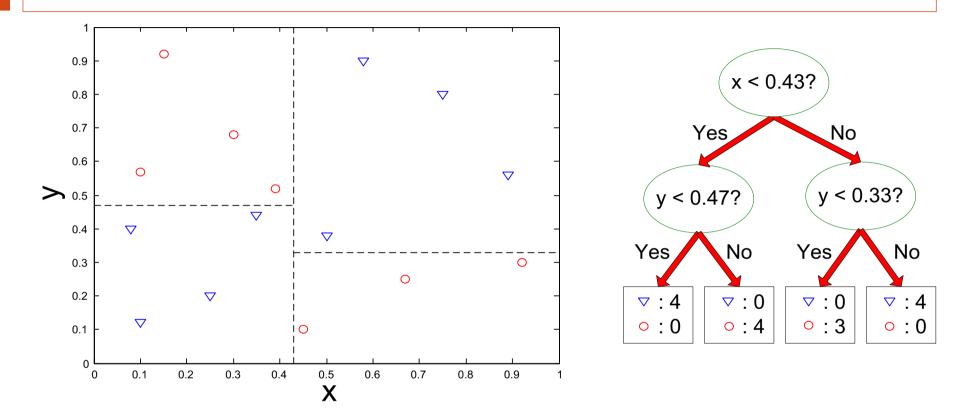
#### Issues

- Determine how to split the records
  - How to specify structure of split?
  - What is best attribute / attribute value for splitting?
- Determine when to stop splitting

## Stopping criteria for tree induction

- Stop expanding a node when all the records belong to the same class
- Stop expanding a node when all the records have identical (or very similar) attribute values
  - No remaining basis for splitting
- Early termination (also known as pruning)
  - Pre-pruning or Post-pruning.

## Decision trees: decision boundary



- Border between two neighboring regions of different classes is known as decision boundary.
- In decision trees, decision boundary segments are always parallel to attribute axes, because test condition involves one attribute at a time.

## Decision trees: addressing overfitting

- ▶ Pre-pruning (early stopping rules) before constructing new leafs
  - Stop the algorithm before it becomes a fully-grown tree
    - ▶ At each stage of splitting the tree, we check the cross-validation error
    - If the error does not decrease significantly enough then we stop
  - General stopping conditions for a node:
    - Stop if all instances belong to the same class
    - Stop if all the attribute values are the same
  - Early stopping conditions (more restrictive):
    - Stop if number of instances is less than some user-specified threshold
    - Stop if class distribution of instances are independent of the available features (e.g., using  $\chi$  2 test)
    - Stop if expanding the current node does not improve impurity measures (e.g., Gini or information gain).

## Decision trees: addressing overfitting

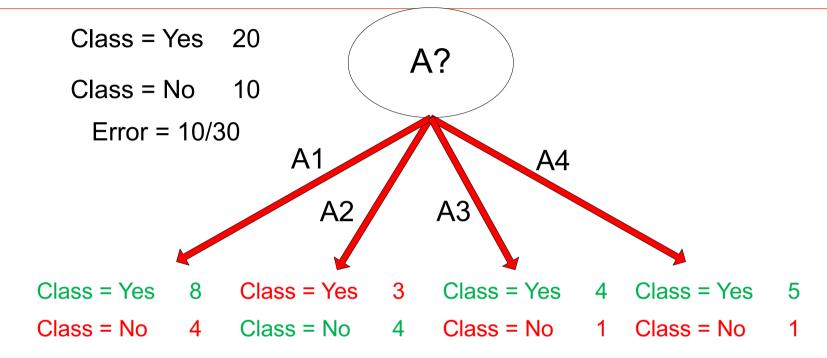
#### ▶ Post-pruning — after constructing new leafs

- As the name implies, pruning involves cutting back the tree.
- After a tree has been built (and in the absence of early stopping discussed below) it may be overfitted.
  - The final subsets (known as the *leaves* of the tree) each consist of only one or a few data points.
  - The tree has learned the data exactly, but a new data point that differs very slightly might not be predicted well.
- Pruning strategies,
  - Minimum error. The tree is pruned back to the point where the cross-validated error is a minimum.
    - Cross-validation is the process of building a tree with most of the data and then using the remaining part of the data to test the accuracy of the decision tree.
  - ▶ Smallest tree. The tree is pruned back slightly further than the minimum error.
    - Prune if the estimated generalization error is bigger than the error on the test set (optimistic or pessimistic approach)

## **Estimating Generalization Errors**

- Frror on training (Σ e(t))
- Generalization errors: error on testing  $(\Sigma e'(t))$
- Methods for estimating generalization errors:
  - Optimistic approach: e'(t) = e(t)
  - Pessimistic approach:
    - For each leaf node: e'(t) = (e(t)+0.5)
    - ▶ Total errors:  $e'(T) = e(T) + N \times 0.5$  (N: number of leaf nodes)
  - Example: For a tree with 30 leaf nodes and 10 errors on training (out of 1000 instances):
    - ▶ Training error = 10/1000 = 1%
    - ▶ Optimistic generalization error = 1% (the same)
    - ▶ Optimistic generalization error =  $(10 + 30 \times 0.5)/1000 = 2.5\%$

## Example of post-pruning



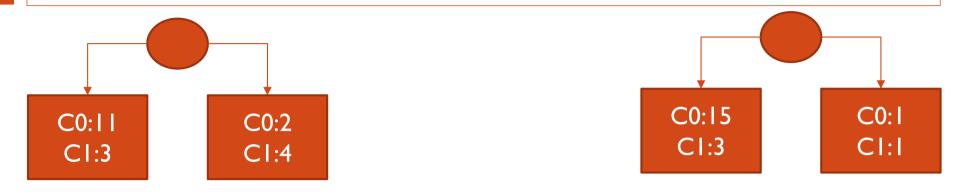
#### Before splitting

- Training error (before splitting) = 10/30
- Pessimistic error (before splitting) = (10+0.5)/30

#### After training

- Training error (after splitting) = (4+3+1+1)/30 = 9/30
- Pessimistic error (after splitting) = (9+4\*0.5)/30 = 11/30

# Exemple of post-pruning



	Case I	Case 2
Error train (before splitting)	7/20=0,35	4/20=0.2
Pessimistic error (before training)	(7+1*0.5)/20=0.375	(4+0.5)/20=0.225
Error train (after splitting)	(3+2)/20=0.25	(3+1)/20=0.2
Pessimistic error (after splitting)	(5+2*0.5)/20=0.3	(4+2*0.5)/20=0.25
Optimistic error	No prune	No prune
Pessimistic error	No prune	Prune

# Classification with decision trees

#### Advantages:

- Inexpensive to construct
- Extremely fast at classifying unknown records
- Easy to interpret for small-sized trees
- Can be combined with other decision techniques.
- Accuracy comparable to other classification techniques for many simple data sets

#### Disadvantages:

- They are unstable, meaning that a small change in the data can lead to a large change in the structure of the optimal decision tree.
- They are often relatively inaccurate. Many other predictors perform better with similar data. This can be remedied by replacing a single decision tree with a <u>random forest</u> of decision trees, but a random forest is not as easy to interpret as a single decision tree.
- Decision boundary restricted to being parallel to attribute axes
- Easy to overfit

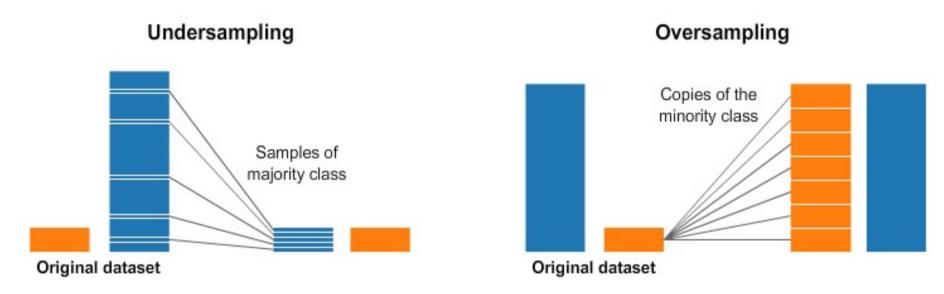
# **Decision tree in Python**

- https://scikit-learn.org/stable/modules/tree.html
- DecisionTreeClassifier()
  - ► To perform multi-class classification
- DecisionTreeRegressor ()
  - To resolve regression problems
- Some tips
  - Performing dimension reduction (PCA, ICA) before to construct trees,
    - gives a better chance of finding features that are discriminative.
  - Use max\_depth to control the size of the tree to prevent overfitting.
  - Use min\_samples\_split or min\_samples\_leaf to ensure that multiple samples inform every decision in the tree, by controlling which splits will be considered
    - A very small number will usually mean the tree will overfit, whereas a large number will prevent the tree from learning the data.
    - ▶ For classification with few classes, min\_samples\_leaf= I is often the best choice.
  - **Balance your dataset before training** to prevent the tree from being biased toward the classes that are dominant.

# Balance your dataset before training

#### Two methods:

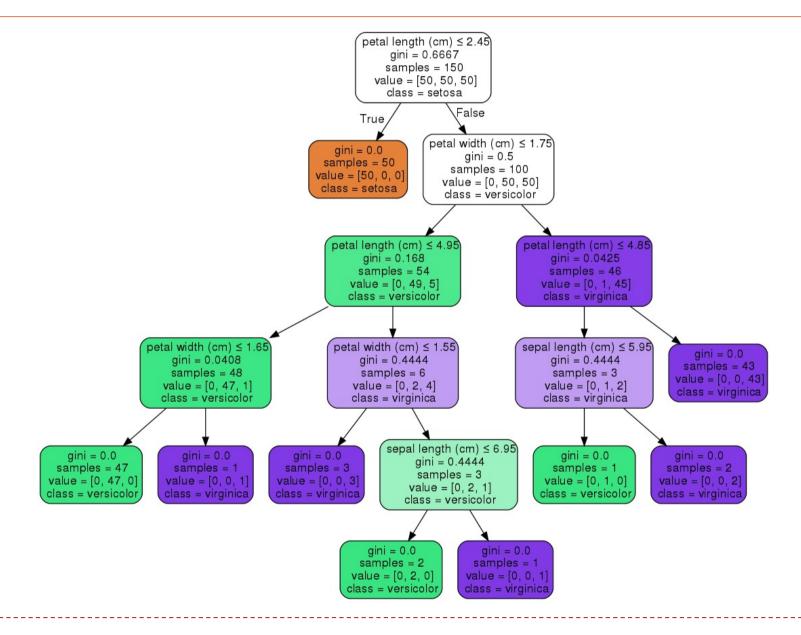
- Undersampling: select only some of the data from the majority class
- Oversampling: create copies of our minority class in order to have the same number of examples as the majority class has



# Visualize your tree

- # Initialize our decision tree object
- from sklearn import tree
- classification tree = tree.DecisionTreeClassifier()
- #Train our decision tree (tree induction + pruning)
- classification\_tree = classification\_tree.fit(X, y)
- # plot the tree
- import graphviz
- dot\_data = tree.export\_graphviz(classification\_tree, feature\_names=feature\_names, class\_names=target\_names)
- graph = graphviz.Source(dot\_data)
- graph.render("iris")

# Visualize your tree



### PROs decision tree

#### Easy to understand and interpret.

- Not require any statistical knowledge to read and interpret them.
- Its graphical representation is very intuitive and users can easily relate their hypothesis.
- Require very little data preparation.
  - All that remains to be done is to adjust a few hyperparameters such as the depth of the tree
  - It is not influenced by outliers and missing values to a fair degree.
- The cost of using the tree for inference is logarithmic in the number of data points used to train the tree.
  - This is a very great advantage because if we add data, the learning time changes little.
- Data type is not a constraint:
  - It can handle both numerical and categorical variables.
- Useful in Data exploration:
  - One of the fastest way to identify most significant variables and relation between two or more variables.
  - With the help of decision trees, we can create new variables / features that has better power to predict target variable (cf. <u>Trick to enhance power of regression model</u>)
- Non Parametric Method:
  - Decision tree is considered to be a non-parametric method. This means that decision trees have no assumptions about the space distribution and the classifier structure.

# **CONs** decision tree

#### Overfitting is quite common

- ▶ The reduction of dimensionality (via PCA) allows to circumvent this problem.
- Do pruning
- Use random forest

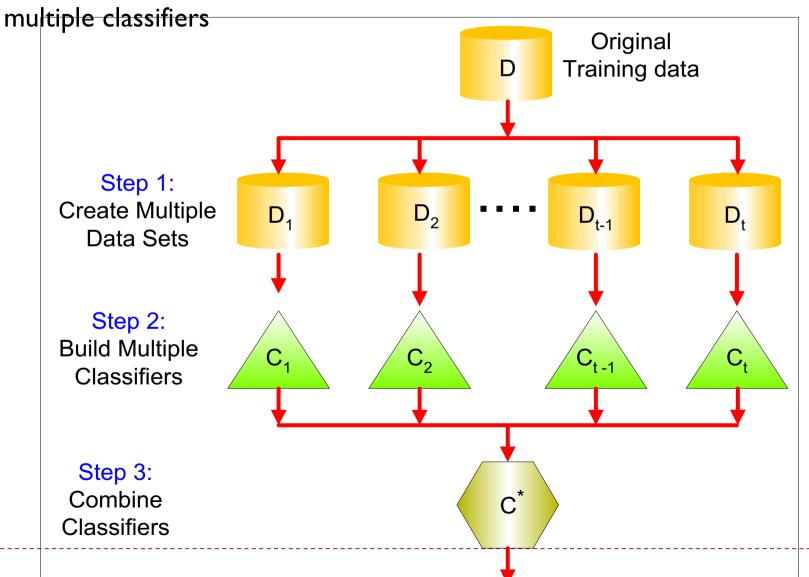
#### Not fit for continuous variables:

- While working with continuous numerical variables, decision tree looses information when it categorizes variables in different categories.
- Works poorly with unbalanced dataset
  - Always balance the classes if necessary.

# Ensemble methods

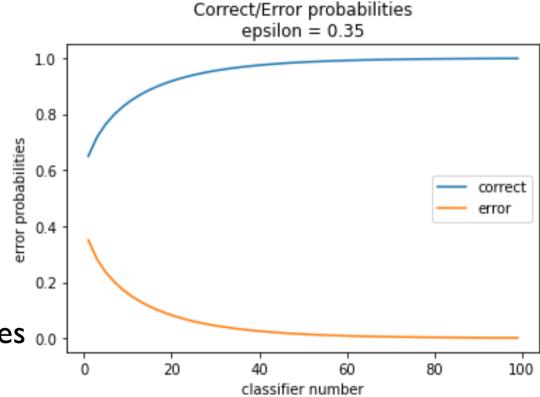
#### **Ensemble Methods**

Predict class label of test records by combining the predictions made by



# Why Ensemble Methods work?

- Suppose there are 25 base classifiers
  - Each classifier has error rate,  $\varepsilon = 0.35$
  - Assume errors made by classifiers are uncorrelated
  - Vote for the result
- Probability that the ensemble classifier makes 0.0 a wrong prediction:



$$P(X \ge 13) = \sum_{i=13}^{25} {25 \choose i} \varepsilon^{i} (1 - \varepsilon)^{25 - i} = 0.06$$

Or 94 % gives a correct prediction with a correct rate of 0.65

#### **Ensemble methods**

- Useful for classification or regression
  - For classification, aggregate predictions by voting.
  - For regression, aggregate predictions by averaging.
- Model types can be:
  - Heterogeneous
    - ▶ Example: neural net combined with SVM combined with decision tree combined with ...
  - ▶ Homogeneous most common in practice
    - Individual models referred to as base classifiers (or regressors)
    - ► Example: ensemble of 1000 decision trees
- Base classifiers: important properties
  - Computationally fast: usually need to compute large numbers of classifiers
  - Accuracy: error rate of each base classifier better than random
  - Diversity (lack of correlation)

# Base classifiers: important properties

#### Diversity

- Predictions vary significantly between classifiers
- Usually attained by using unstable classifier
  - small change in training data (or initial model weights) produces large change in model structure
- Examples of unstable classifiers:
  - decision trees
  - neural nets
  - rule-based
- Examples of stable classifiers:
  - ▶ Linear models: logistic Regression
  - ▶ Linear discriminant

#### How to create diverse base classifiers

- Random initialization model parameters
  - Network weights with neural networks
- Use random projection of the dataset on a lower-dimentional space
- Resample / subsample training data
  - Sample instances
    - Disjoint partitions
    - Randomly without replacement
    - Randomly with replacement (e.g. bagging)
  - Sample features (random subspace approach)
    - Randomly prior to training
    - Randomly during training (e.g. random forest)

### **Random Forest**

Original presentation from Jeff Howbert

#### From decision tree to random forest

- Decision trees are greedy
  - They choose which variable to split on using a greedy algorithm that minimizes error
  - sensitivity of single trees to the order of predictors,
  - Overfitting, it's easy
- Combining predictions from multiple trees should work better.
- Random forest changes the algorithm for the way that the sub-trees are learned so that the resulting predictions from all of the subtrees have less correlation
- Principle: the decision tree forest algorithm learns about multiple decision trees driven by slightly different subsets of data.

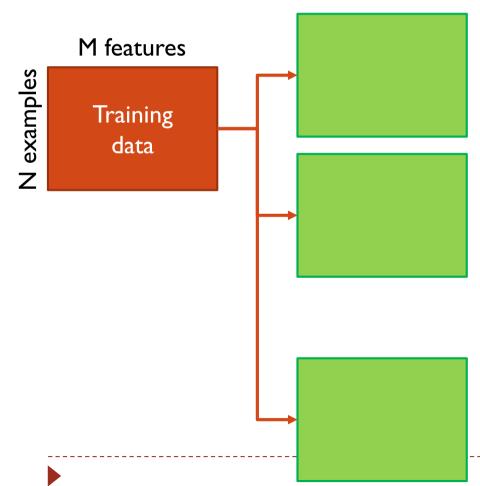
M features

N examples

Training data

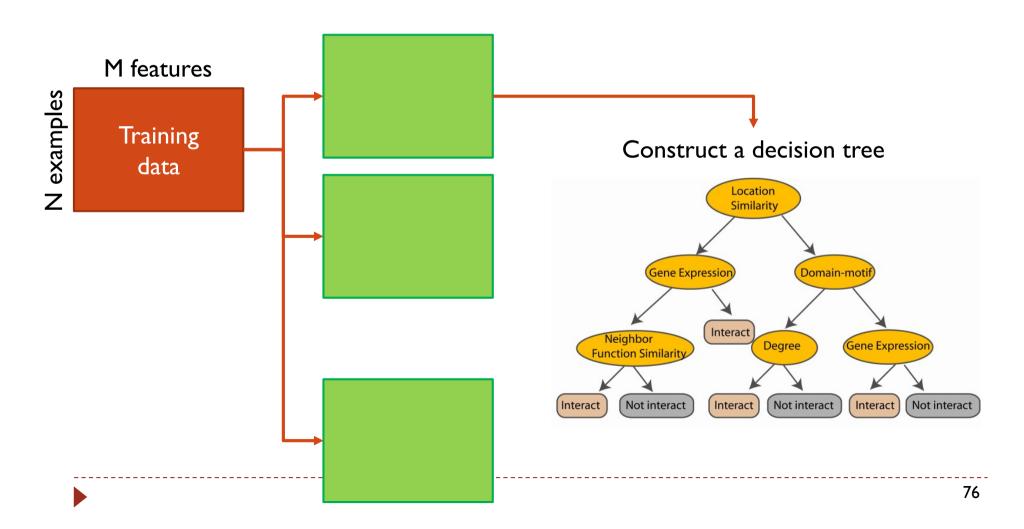
Create many (100) samples from the training data

- with a same number of observations M identical of the original data ((random sample with replacement. technique known as bootstrap)
- with m random features (generally  $m < \sqrt{M}$ ) I

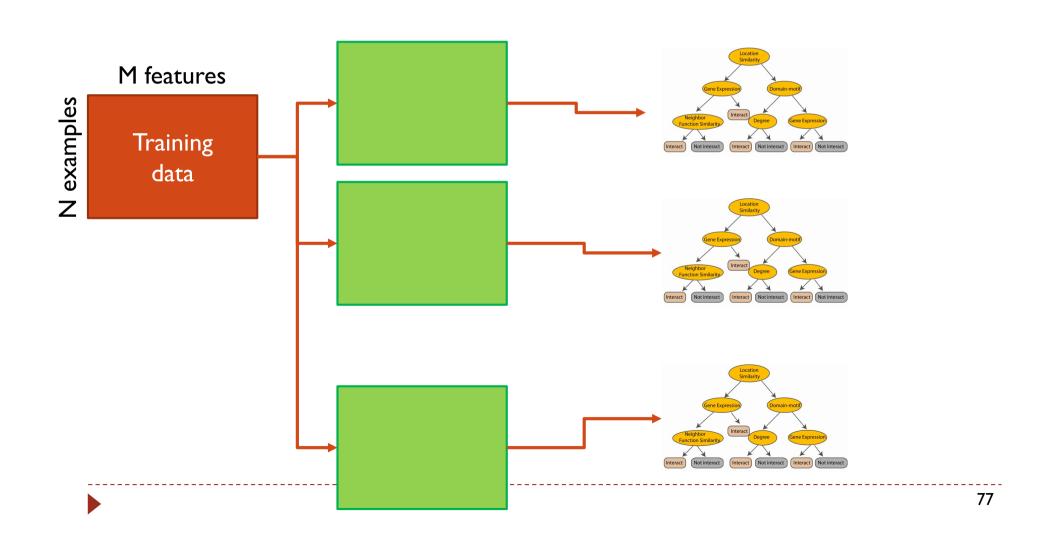


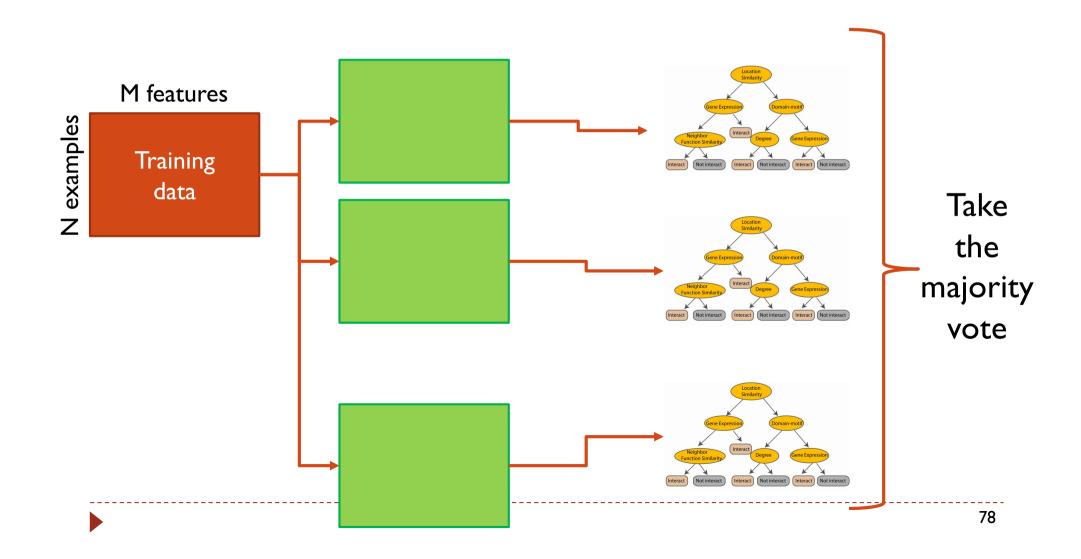
Create many (100) random sub-samples of the training data

- random sample with replacement technique known as bootstrap
- and randomly select for each training data only m features (generally  $m < \sqrt{M}$ ) I



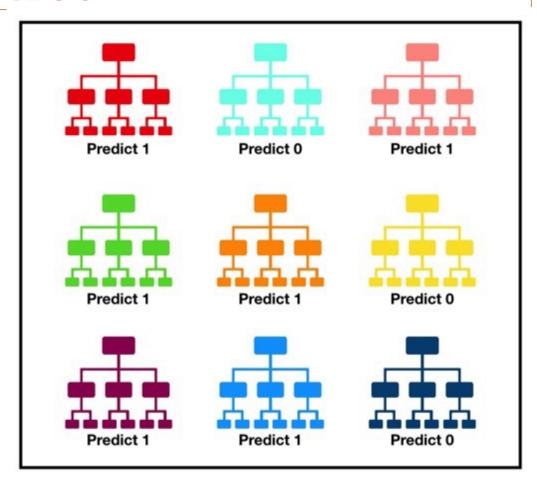
# Create decision tree from each bootstrap sample





# Why many trees give a more valuable result than one tree

- The reason for this wonderful effect is that the trees protect each other from their individual mistakes.
- While some trees may be wrong, many others will be right.
- Remember
  - With 25 classifier
  - $\blacktriangleright$  Error rate = 0.35
  - ▶ 94 % of correct prediction



Tally: Six 1s and Three 0s

**Prediction: 1** 

# **Random Forest in Python**

- Use RandomForestClassifier or RandomForestRegressor
- Main parameters
  - n\_estimators : integer, optional (default= 10)
    - ▶ The number of trees in the forest.
  - max\_depth : integer or None, optional (default=None)
    - ▶ The maximum depth of the tree.
    - If None, then nodes are expanded until all leaves are pure or until all leaves contain less than min\_samples\_split samples.
  - min\_samples\_split : int, float, optional (default=2)
    - ▶ The minimum number of samples required to split an internal node
  - min\_samples\_leaf : int, float, optional (default=1)
    - ▶ The minimum number of samples required to be at a leaf node.
    - A split point at any depth will only be considered if it leaves at least min\_samples\_leaf training samples in each of the left and right branches.

#### Random forest

#### Others parameters

- max\_features : int, float, string or None, optional (default="auto")
  - ▶ The number of features to consider when looking for the best split
- ▶ min\_impurity\_decrease : float, optional (default=0.)
  - A node will be split if this split induces a decrease of the impurity greater than or equal to this value.
- min\_impurity\_split : float, (default= l e-7)
  - ▶ Threshold for early stopping in tree growth.
  - ▶ A node will split if its impurity is above the threshold, otherwise it is a leaf.

# PROs of Random Forest Algorithm

- I. It can be used in classification and regression problems.
- 2. It solves the problem of overfitting as output is based on majority voting or averaging.
- 3. It performs well even if the data contains null/missing values.
- 4. Each decision tree created is independent of the other thus it shows the property of parallelization.
- 5. It is highly stable as the average answers given by a large number of trees are taken.
- 6. It maintains diversity as all the attributes are not considered while making each decision tree though it is not true in all cases.
- 7. It is immune to the curse of dimensionality. Since each tree does not consider all the attributes, feature space is reduced.
- 8. We don't have to segregate data into train and test as there will always be 30% of the data which is not seen by the decision tree made out of bootstrap.

# **CONs of Random Forest Algorithm**

- I. Random forest is highly complex when compared to decision trees where decisions can be made by following the path of the tree.
- 2. Training time is more compared to other models due to its complexity. Whenever it has to make a prediction each decision tree has to generate output for the given input data.