UNIVERSITÉ CÔTE D'AZUR

MSc. Data Science & Artificial Intelligence

Inverse Problems in Image Processing

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Assignment 1

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1 Exercise 1

Let f be given by:

$$f(x) = \frac{1}{2}\|Ax - y\|_2^2 \tag{1}$$

we want to compute the gradient of f. We have:

$$\begin{aligned}
\nabla_v f(x) &= \lim_{h\to 0} \frac{f(x+hv) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{\|A(x+hv) - y\|^2 - \|Ax - y\|^2}{2h} \\
&= \lim_{h\to 0} \frac{\|Ax - y\|^2 + 2h < Ax - y, Av > +h^2\|Av\|^2 - \|Ax - y\|^2}{2h} \\
&= \lim_{h\to 0} (Ax - y)^T Av + \frac{h}{2}\|Av\|^2 \\
&= \left\langle A^T(Ax - y), v \right\rangle,
\end{aligned}$$

2 Exercise 2

3 Exercise 3

4 Exercise 4