Probabilities refreshers: Homework for lecture 2

Joris LIMONIER

September 20, 2021

1 Exercise 1

We take 5 cards among 32.

$$X = \begin{cases} 50 & \text{4 cards have the same level} \\ 20 & \text{full} \\ 10 & \text{3 cards are the same, the 4th one is different and not a full} \\ 0 & \text{otherwise} \end{cases}$$

1.1 Determine the distribution

The universe is as follows: $\Omega = \{0, 10, 20, 50\}$. Now let us determine the PMF.

$$\begin{aligned} \mathbb{P}(X = 50) &= 4 \cdot \binom{5}{4}\binom{28}{1} / \binom{32}{5} \\ &= 4 \cdot \frac{5!}{4!1!} 28 \frac{5!27!}{32!} \\ &= \frac{4 \cdot 5 \cdot 5!}{29 \cdot 30 \cdot 31 \cdot 32} \\ &= \frac{4 \cdot 5}{8 \cdot 29 \cdot 31} \\ &= \frac{5}{1798} \end{aligned}$$

$$\begin{aligned}
\mathbb{P}(X=20) &= 8 \cdot \binom{4}{3} \cdot 7 \cdot \binom{4}{2} / \binom{32}{5} \\
&= 2^3 \cdot \frac{4!}{3!} \cdot 7 \cdot \frac{4!}{2 \cdot 2} \frac{27!5!}{32!} \\
&= 2^5 \cdot 7 \cdot \frac{5!}{28 \cdot 29 \cdot 30 \cdot 31 \cdot 32} \\
&= \frac{4 \cdot 7}{28 \cdot 29 \cdot 31} \\
&= \frac{1}{899}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(X=10) &= 8 \cdot \binom{4}{3}\binom{28}{1}\binom{24}{1} / \binom{32}{5} \\
&= 8 \cdot 4 \cdot 28 \cdot 24 \frac{5!27!}{32!} \\
&= 24 \frac{5!}{29 \cdot 30 \cdot 31} \\
&= 24 \frac{4}{29 \cdot 31} \\
&= \frac{96}{899}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(X=0) &= 1 - \mathbb{P}(X=50) - \mathbb{P}(X=20) - \mathbb{P}(X=10) \\
&= 1 - \frac{5}{1798} - \frac{2}{1798} - \frac{192}{1798} \\
&= \frac{1599}{1798}
\end{aligned}$$

1.2 If the player pays 5€ to play, is the game favorable to the player ?

(remove 5€ from the final expectation)

$$\mathbb{E}$$

2 Exercise 2

(See picture with plot)

$$f(x) = \begin{cases} \frac{a}{2} & x \in [1,3] \\ \frac{-a}{2} & x \in [3,5] \end{cases}$$