Probabilities refreshers: Homework for lecture 2

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$$f_X(t) = \begin{cases} 2a & t \in [-1, 0] \\ a & t \in [1, 4] \\ 0 & else \end{cases}$$

1 Value of a ?

$$\begin{aligned} & \int_{-\infty}^{\infty} f(t)dt = 1 \\ \implies & \int_{-1}^{0} f(t)dt + \int_{1}^{4} f(t)dt = 1 \\ \implies & \int_{-1}^{0} 2adt + \int_{1}^{4} adt = 1 \\ \implies & 2a[t]_{-1}^{0} + a[t]_{1}^{4} = 1 \\ \implies & 2a + a(4-1) = 1 \\ \implies & a = 1/5 \end{aligned}$$

2 Distribtuion of X ?

$$F_X(t) = \begin{cases} 0 & t \in [-\infty, -1] \\ 2a(t+1) & t \in [-1, 0] \\ 2a & t \in [0, 1] \\ 2a + a(t-1) & t \in [1, 4] \\ 1 & t \in [4, +\infty] \end{cases}$$

3 $\mathbb{P}(X \in [-0.5, 2])$?

$$\begin{aligned}\mathbb{P}(X \in [-0.5, 2]) &= F_X(2) - F_X(-0.5) \\ &= 3a - a \\ &= 2a\end{aligned}$$

4 Compute if possible expectation and variance of X

Since the support of X is included in $[-1, 4]$ and $f(X)$ is a

$$\begin{aligned}\mathbb{E}[X] &= \int_{-1}^{0} tf(t)\ dt + \int_{1}^{4} tf(t)\ dt \\ &= \int_{-1}^{0} 2at\ dt + \int_{1}^{4} at\ dt \\ &= \left[at^2\right]_{-1}^{0} 2at + \left[\frac{at^2}{2}\right]_{1}^{4} \\ &= (0 - a) + (8a - a/2) \\ &= \frac{13a}{2}\end{aligned}$$

$$\begin{aligned}\mathbb{E}[X^2] &= \int_{-\infty}^{\infty} x^2 f(x)\ dx \\ &= \int_{-1}^{0} x^2 f(x)\ dx + \int_{1}^{4} x^2 f(x)\ dx \\ &= \int_{-1}^{0} x^2 2a\ dx + \int_{1}^{4} x^2 a\ dx \\ &= 2a/3\left[x^3\right]_{-1}^{0} + a/3\left[x^3\right]_{1}^{4} \\ &= 2a/3 + 63a/3 \\ &= 65a/3\end{aligned}$$

So $Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2 = 65a/3 - (13a/2)^2$