Statistical inference practice

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1 2021 October 28 lecture

1.1 Exercise 1 (Student t)

If T has 4 degrees of freedom, find t such that $\mathbb{P}(T \leq t) = 0.8$
Since $0.8 \geq 0.5$, we know that t is positive.

$$\begin{aligned} & \mathbb{P}(T \leq t) = 0.8 \\ \implies & \mathbb{P}(T > t) = 0.2 \\ & \qquad\qquad \textit{by symmetry} \\ \implies & \mathbb{P}(|T| > t) = 0.4 \\ & \qquad\qquad \textit{by the table} \\ \implies & t = 0.941 \end{aligned}$$

If T has 4 degrees of freedom, find t such that $\mathbb{P}(T \leq t) = 0.1$
Since $0.1 \leq 0.5$, we know that t is negative.

$$\begin{aligned} & \mathbb{P}(T \leq t) = 0.1 \\ \implies & \mathbb{P}(T > -t) = 0.1 \\ & \qquad\qquad \textit{by symmetry} \\ \implies & \mathbb{P}(|T| > -t) = 0.2 \\ & \qquad\qquad \textit{by the table} \\ \implies & -t = 1.533 \\ \implies & t = -1.533 \end{aligned}$$

If T has 4 degrees of freedom, find t such that $\mathbb{P}(T \geq t) = 0.8$

Since $0.8 \geq 0.5$, we know that t is negative.

$$\begin{aligned}
&\mathbb{P}(T \geq t) = 0.8 \\
\implies &\mathbb{P}(T < t) = 0.2 \\
\implies &\mathbb{P}(T > -t) = 0.2 \\
&\qquad\qquad\qquad\qquad\qquad\qquad \textit{by symmetry} \\
\implies &\mathbb{P}(|T| > -t) = 0.4 \\
&\qquad\qquad\qquad\qquad\qquad\qquad \textit{by the table} \\
\implies &-t = 0.941 \\
\implies &t = -0.941
\end{aligned}$$

If T has 4 degrees of freedom, find t such that $\mathbb{P}(-t \leq T \leq t) = 0.9$

$$\begin{aligned}
&\mathbb{P}(-t \leq T \leq t) = 0.9 \\
\implies &\mathbb{P}(T \geq t) = 0.1 \\
&\qquad\qquad\qquad\qquad\qquad\qquad \textit{by the table} \\
\implies &t = 2.132
\end{aligned}$$

If T has 4 degrees of freedom, find the probability of $\mathbb{P}(T \leq -0.134)$

$$\begin{aligned}
\mathbb{P}(T \leq -0.134) &= \frac{1}{2}\mathbb{P}(|T| \geq 0.134) \\
&= \frac{0.9}{2} \\
&= 0.45
\end{aligned}$$

If T has 4 degrees of freedom, find the probability of $\mathbb{P}(T \geq -0.271)$

$$\begin{aligned}
\mathbb{P}(T \geq -0.271) &= 1 - \mathbb{P}(T \leq -0.271) \\
&= 1 - \mathbb{P}(T \geq 0.271) \\
&= 1 - \frac{1}{2}\mathbb{P}(|T| \geq 0.271) \\
&= 1 - \frac{0.8}{2} \\
&= 0.6
\end{aligned}$$

If T has 4 degrees of freedom, find the probability of $\mathbb{P}(T \leq 0.941)$

$$\begin{aligned}
\mathbb{P}(T \leq 0.941) &= 1 - \mathbb{P}(T \geq 0.941) \\
&= 1 - \frac{1}{2}\mathbb{P}(|T| \geq 0.941) \\
&= 1 - \frac{0.4}{2} \\
&= 0.8
\end{aligned}$$

If T has 4 degrees of freedom, find the probability of $\mathbb{P}(-0.414 \leq T \leq 1.533)$

$$\begin{aligned}
\mathbb{P}(-0.414 \leq T \leq 1.533) &= \mathbb{P}(T \leq 1.533) - \mathbb{P}(T \leq -0.414) \\
&= 1 - \mathbb{P}(T \geq 1.533) - \mathbb{P}(T \leq -0.414) \\
&= 1 - \frac{0.2}{2} - \frac{0.7}{2} \\
&= 0.55
\end{aligned}$$