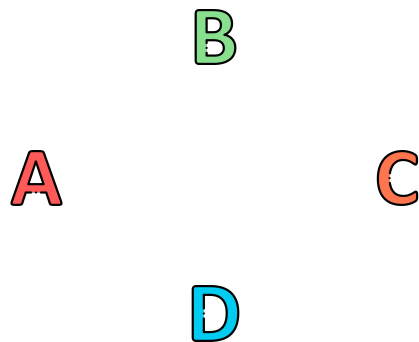




Taki Eddine MEKHALFA  
Taki-Eddine.MEKHALFA@inria.fr



Random variable: **A** smokes or not



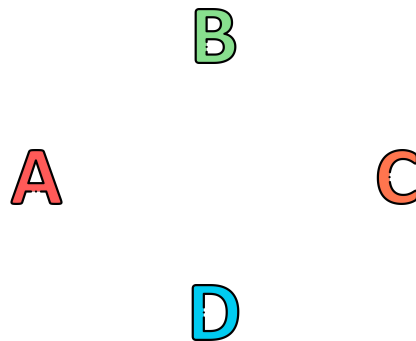
**A**

**B**

**C**

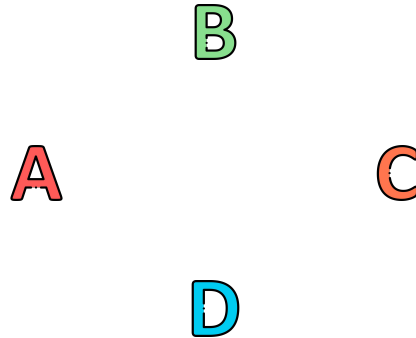
**D**

Suppose you know that **A** and **B** are friends! What does this tell you?



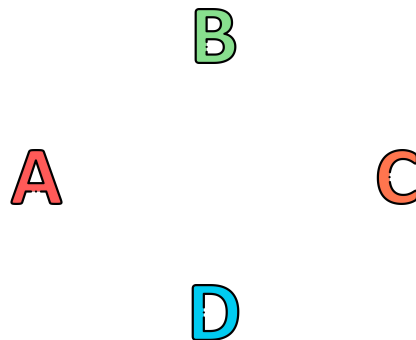
Suppose you know that **A** and **B** are friends! What does this tell you?

=> They might have similar smoking habits!



Suppose you know that **A** and **B** are friends! What does this tell you?

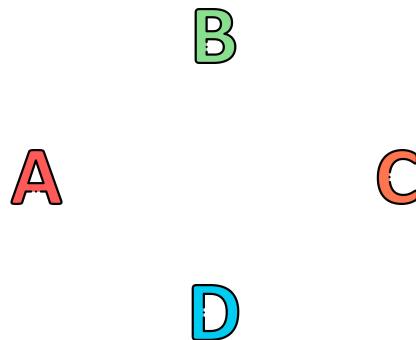
=> They might have similar smoking habits! Is this useful to model smoking behaviors?



Suppose you know that **A** and **B** are friends! What does this tell you?

=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

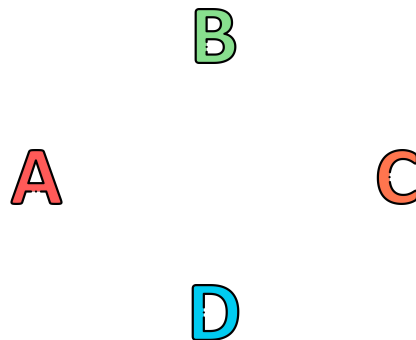


Suppose you know that **A** and **B** are friends! What does this tell you?

=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!





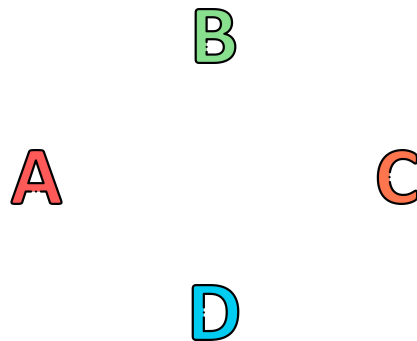
Suppose you know that **A** and **B** are friends! What does this tell you?

=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

=> Use Graphs!



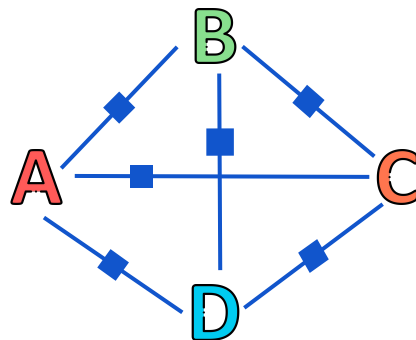
Suppose you know that **A** and **B** are friends! What does this tell you?

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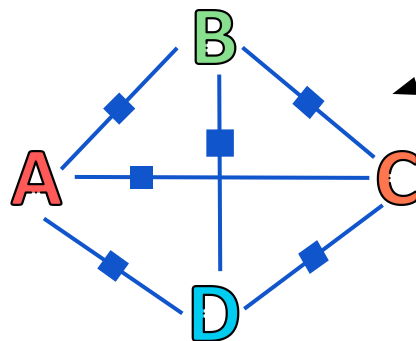
Suppose you know that **A** and **B** are friends! What does this tell you?

=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

=> Use Graphs!



What's the problem?

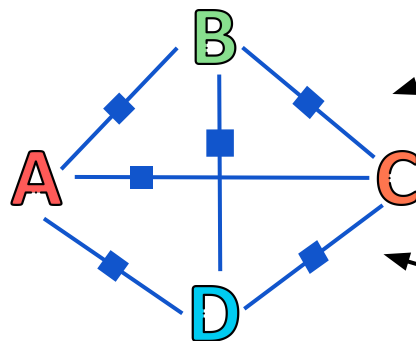
Suppose you know that **A** and **B** are friends! What does this tell you?

=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

=> Use Graphs!



What's the problem?

Every variable depends on every other variable!

Suppose you know that **A** and **B** are friends! What does this tell you?

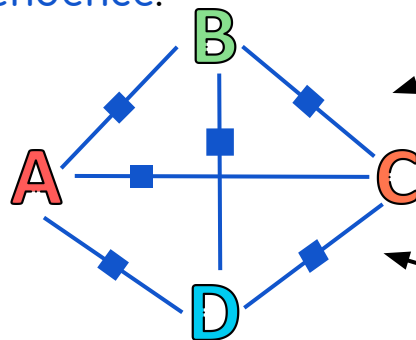
=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

=> Use Graphs!

=> Need a way to represent independence!



What's the problem?

Every variable depends on every other variable!

Suppose you know that **A** and **B** are friends! What does this tell you?

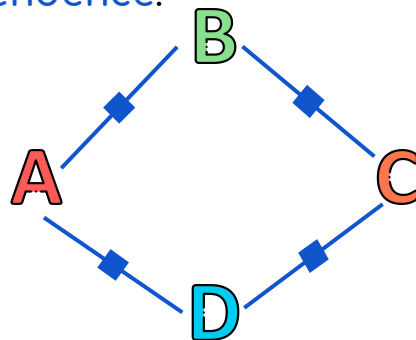
=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

=> Use Graphs!

=> Need a way to represent independence!



Suppose you know that **A** and **B** are friends! What does this tell you?

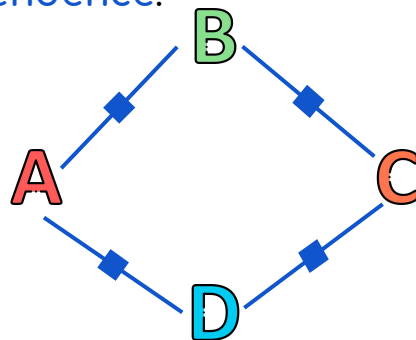
=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

=> Use Graphs!

=> Need a way to represent independence!



Does this mean **B** and **D** that are totally independent?

Suppose you know that **A** and **B** are friends! What does this tell you?

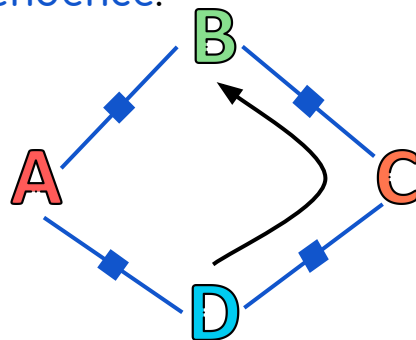
=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

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Does this mean **B** and **D** that are totally independent?



Suppose you know that **A** and **B** are friends! What does this tell you?

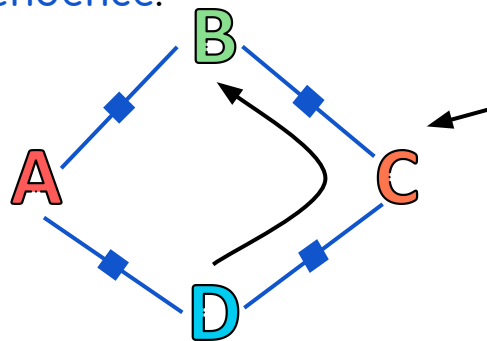
=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

=> Use Graphs!

=> Need a way to represent independence!



Does this mean **B** and **D** that are totally independent?

**B** is independent from **D**  
only when we know **A** and **C**

Suppose you know that **A** and **B** are friends! What does this tell you?

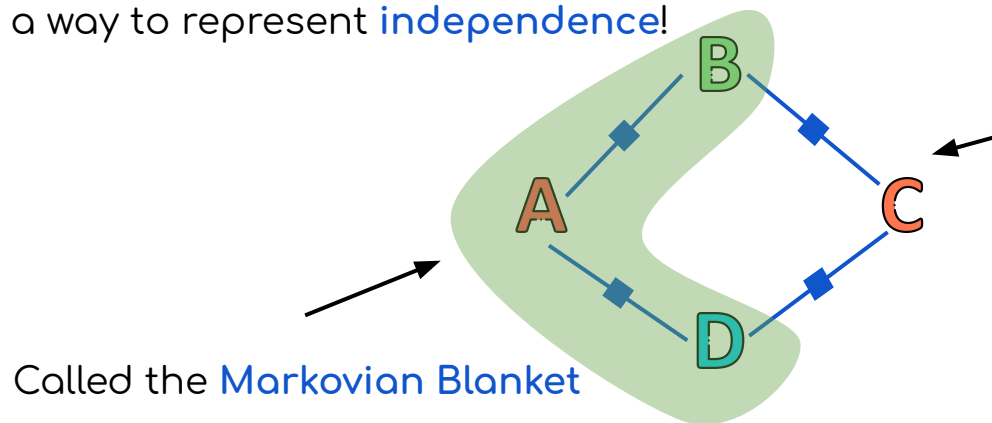
=> They might have similar smoking habits! Is this useful to model smoking behaviors?

=> Call it a feature then!

=> Need a way to represent interactions!

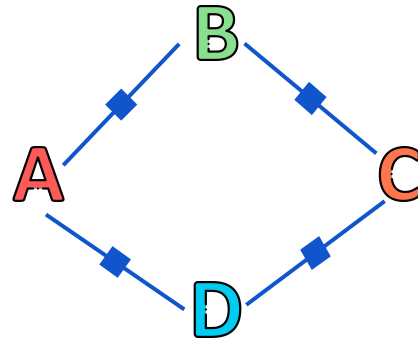
=> Use Graphs!

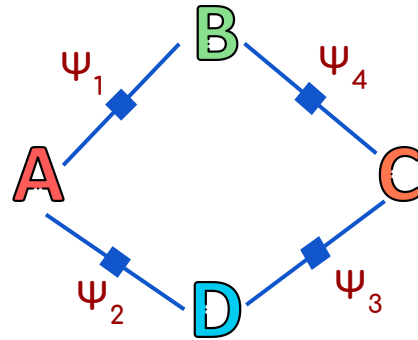
=> Need a way to represent independence!



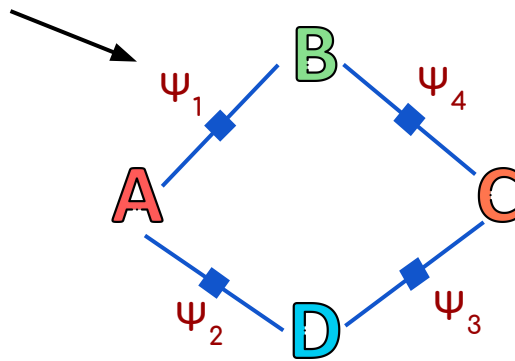
Does this mean **B** and **D** that are totally independent?

**B** is independent from **D**  
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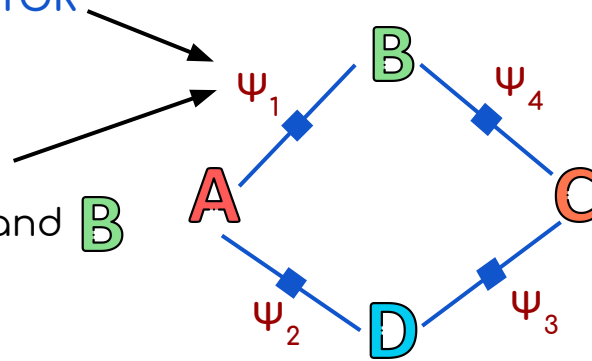




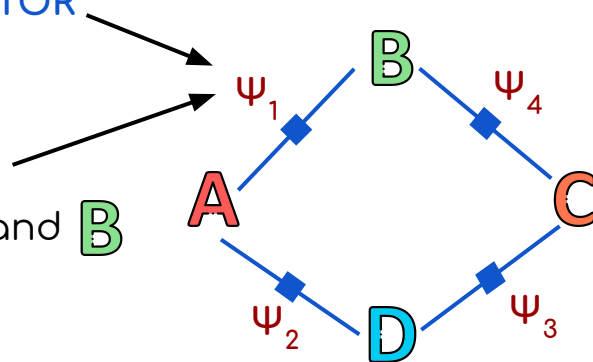
Call it a **FACTOR**



Call it a **FACTOR**



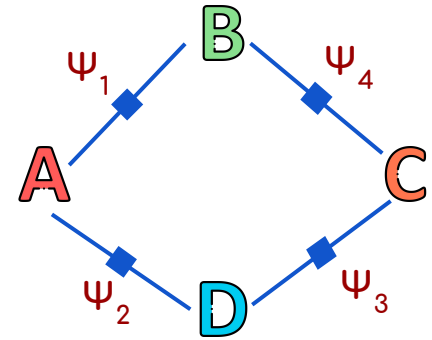
Call it a **FACTOR**



Compatibility between **A** and **B**

**ASSUMPTION:** A given state of variables (we say a possible world) is **more likely iff** the **overall compatibility is higher**

How to define compatibility in our case?

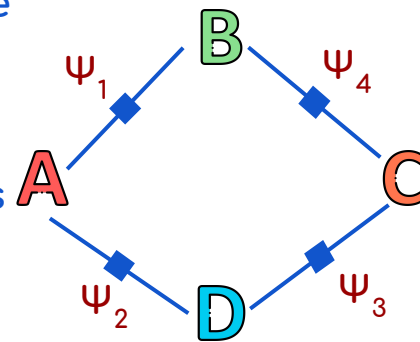




How to define compatibility in our case?

$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
- 0.5 if  $\mathbf{A}$  and  $\mathbf{B}$  are are not friends
- 0 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and don't have same smoking habits



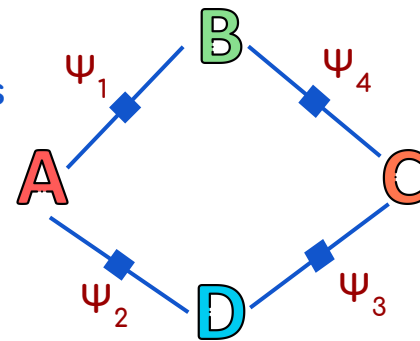
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- 0 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and don't have same smoking habits

Suppose that:

$\mathbf{A}$  is friends with  $\mathbf{B}$ .  $\mathbf{C}$  is friends with  $\mathbf{D}$

What world is most likely between the following two?:



$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

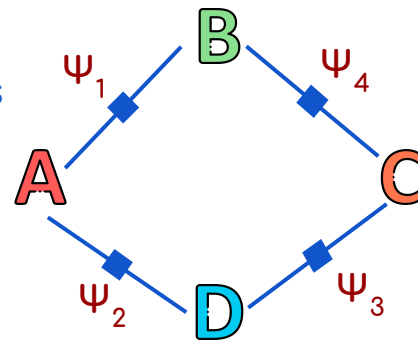
- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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Suppose that:

$\mathbf{A}$  is friends with  $\mathbf{B}$ .  $\mathbf{C}$  is friends with  $\mathbf{D}$

What world is most likely between the following two?:

1. Smokes{  $\mathbf{A}$ ,  $\mathbf{B}$  } and Does not Smoke {  $\mathbf{C}$ ,  $\mathbf{D}$  }
2. Smokes{  $\mathbf{A}$ ,  $\mathbf{C}$  } and Does not Smoke {  $\mathbf{B}$ ,  $\mathbf{D}$  }



$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

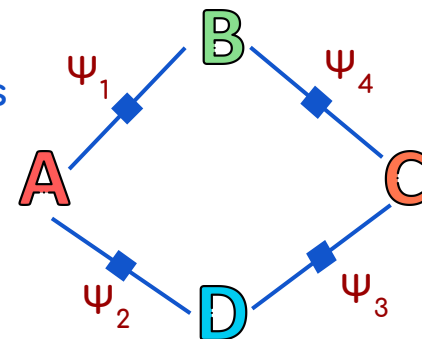
- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
- 0.5 if  $\mathbf{A}$  and  $\mathbf{B}$  are are not friends
- 0 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and don't have same smoking habits

Suppose that:

$\mathbf{A}$  is friends with  $\mathbf{B}$ .  $\mathbf{C}$  is friends with  $\mathbf{D}$

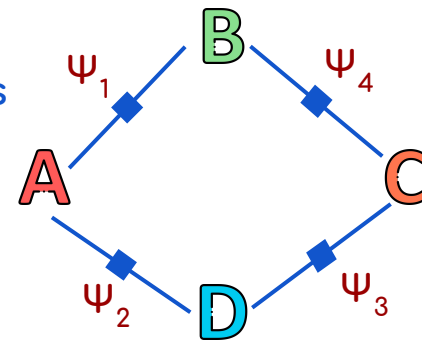
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2. Smokes{  $\mathbf{A}$ ,  $\mathbf{C}$  } and Does not Smoke {  $\mathbf{B}$ ,  $\mathbf{D}$  }



$\psi_1(\mathbf{A}, \mathbf{B}) =$ 

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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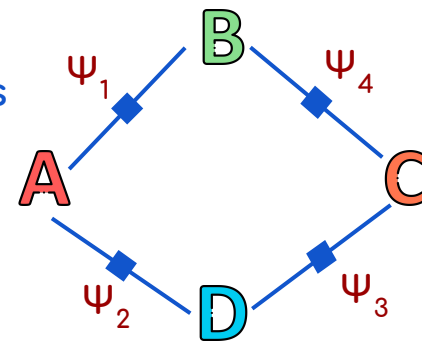
 $P(\text{Smokes}\{\mathbf{A}, \mathbf{B}\} \text{ and Does not Smoke } \{\mathbf{C}, \mathbf{D}\})$ 

$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
- 0.5 if  $\mathbf{A}$  and  $\mathbf{B}$  are not friends
- 0 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and don't have same smoking habits

$P(\text{Smokes}\{\mathbf{A}, \mathbf{B}\} \text{ and Does not Smoke } \{\mathbf{C}, \mathbf{D}\})$

$\propto \psi(\text{Smokes}\{\mathbf{A}, \mathbf{B}\} \text{ and Does not Smoke } \{\mathbf{C}, \mathbf{D}\})$



$$\psi_1(A, B) =$$

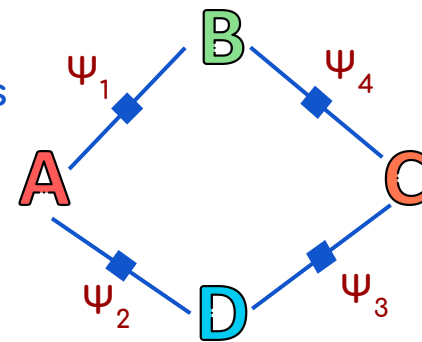
- 1 if  $A$  and  $B$  are friends and both smoke or both don't smoke
- 0.5 if  $A$  and  $B$  are not friends
- 0 if  $A$  and  $B$  are friends and don't have same smoking habits

$P(\text{Smokes}\{A, B\} \text{ and Does not Smoke } \{C, D\})$

$\propto \psi(\text{Smokes}\{A, B\} \text{ and Does not Smoke } \{C, D\})$

$= \exp(\psi_1(A, B)) \times \exp(\psi_2(A, D)) \exp(\psi_3(B, C)) \exp(\psi_4(C, D))$

=?



$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

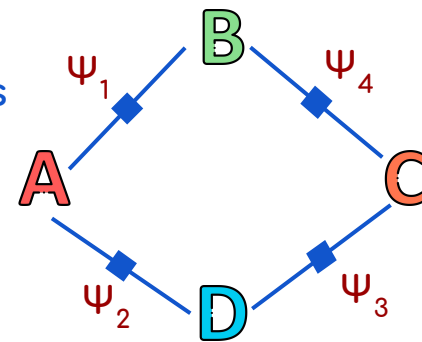
- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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- 0 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and don't have same smoking habits

$P(\text{Smokes}\{\mathbf{A}, \mathbf{B}\} \text{ and Does not Smoke } \{\mathbf{C}, \mathbf{D}\})$

$\propto \psi(\text{Smokes}\{\mathbf{A}, \mathbf{B}\} \text{ and Does not Smoke } \{\mathbf{C}, \mathbf{D}\})$

$= \exp(\psi_1(\mathbf{A}, \mathbf{B})) \times \exp(\psi_2(\mathbf{A}, \mathbf{D})) \exp(\psi_3(\mathbf{B}, \mathbf{C})) \exp(\psi_4(\mathbf{C}, \mathbf{D}))$

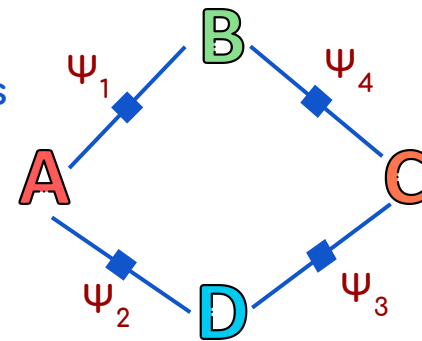
$= \exp(1) \times \exp(0.5) \times \exp(0.5) \times \exp(1) = \exp(3)$





$\psi_1(\mathbf{A}, \mathbf{B}) =$ 

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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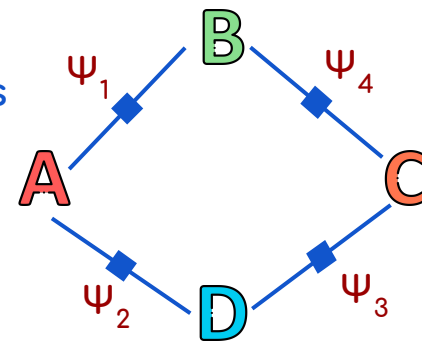
 $P(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$ 

$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

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$P(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$

$\propto \psi(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$



$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

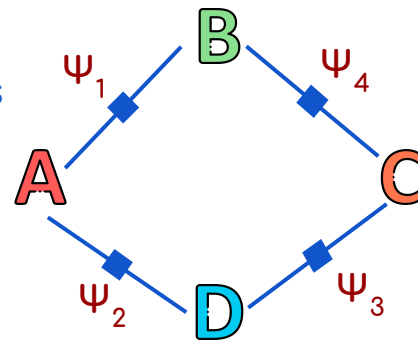
- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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$P(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$

$\propto \Psi(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$

$= \exp(\psi_1(\mathbf{A}, \mathbf{B})) \times \exp(\psi_2(\mathbf{A}, \mathbf{D})) \exp(\psi_3(\mathbf{B}, \mathbf{C})) \exp(\psi_4(\mathbf{C}, \mathbf{D}))$

$= ?$



$$\psi_1(\mathbf{A}, \mathbf{B}) =$$

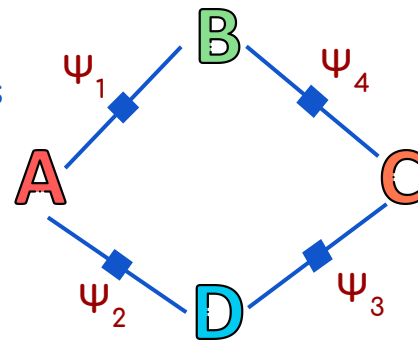
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$P(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$

$\propto \Psi(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$

$= \exp(\psi_1(\mathbf{A}, \mathbf{B})) \times \exp(\psi_2(\mathbf{A}, \mathbf{D})) \exp(\psi_3(\mathbf{B}, \mathbf{C})) \exp(\psi_4(\mathbf{C}, \mathbf{D}))$

$= \exp(0) \times \exp(0.5) \times \exp(0.5) \times \exp(0) = \exp(1)$



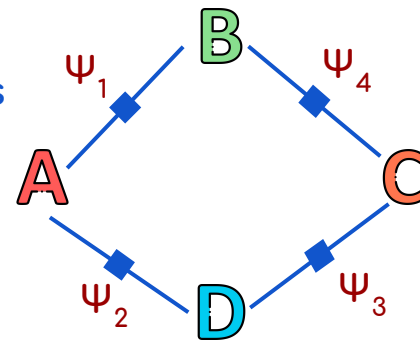
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$P(\text{Smokes}\{\mathbf{A}, \mathbf{B}\} \text{ and Does not Smoke } \{\mathbf{C}, \mathbf{D}\})$

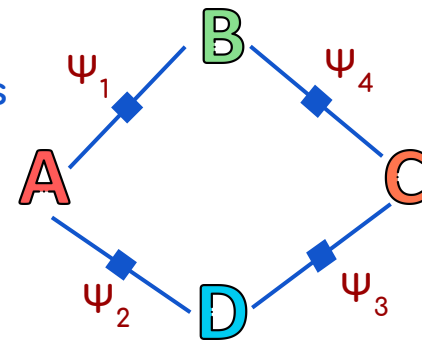
IS ~7.4 times more likely ( $\exp(2) = \exp(3) / \exp(1)$ ) than

$P(\text{Smokes}\{\mathbf{A}, \mathbf{C}\} \text{ and Does not Smoke } \{\mathbf{B}, \mathbf{D}\})$



$$\psi_{1, \text{friendship}}(\mathbf{A}, \mathbf{B}) =$$

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
- 0.5 if  $\mathbf{A}$  and  $\mathbf{B}$  are not friends
- 0 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and don't have same smoking habits

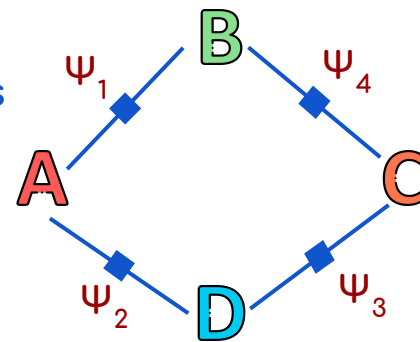


$$\psi_{1, \text{friendship}}(\mathbf{A}, \mathbf{B}) =$$

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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$$\psi_{1, \text{couple}}(\mathbf{A}, \mathbf{B}) =$$

- 5 if  $\mathbf{A}$  and  $\mathbf{B}$  are couples and both smoke
- 0.5 if  $\mathbf{A}$  and  $\mathbf{B}$  are not couples
- 0 if  $\mathbf{A}$  and  $\mathbf{B}$  are couples and don't have same smoking habits



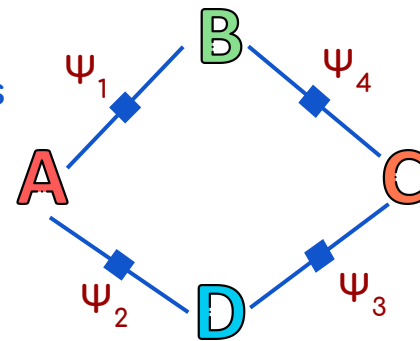
$$\psi_{1, \text{friendship}}(\mathbf{A}, \mathbf{B}) =$$

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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$$\psi_1(\mathbf{A}, \mathbf{B}) = \psi_{1, \text{friendship}}(\mathbf{A}, \mathbf{B}) + \psi_{1, \text{couple}}(\mathbf{A}, \mathbf{B})$$





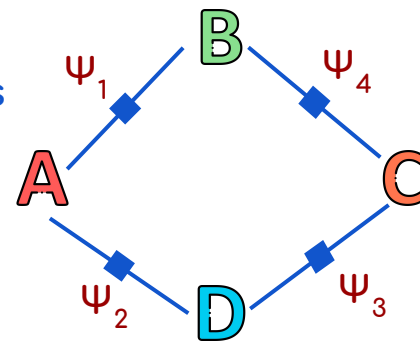
$$\psi_{1, \text{friendship}}(\mathbf{A}, \mathbf{B}) =$$

- 1 if  $\mathbf{A}$  and  $\mathbf{B}$  are friends and both smoke or both don't smoke
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$$\psi_1(\mathbf{A}, \mathbf{B}) = \psi_{1, \text{friendship}}(\mathbf{A}, \mathbf{B}) + \psi_{1, \text{couple}}(\mathbf{A}, \mathbf{B})$$



Suppose I show you a biased coin:  with parameter  $\theta$  where  $P(\text{Head}) = \theta$

I flip the coin 100 times in front of you, finally we get 30 heads and 70 tails.

Suppose I show you a biased coin:



with parameter  $\theta$  where  $P(\text{Head}) = \theta$

I flip the coin 100 times in front of you, finally we get 30 heads and 70 tails.

Intuitively, what is an estimate for  $\theta$ ?

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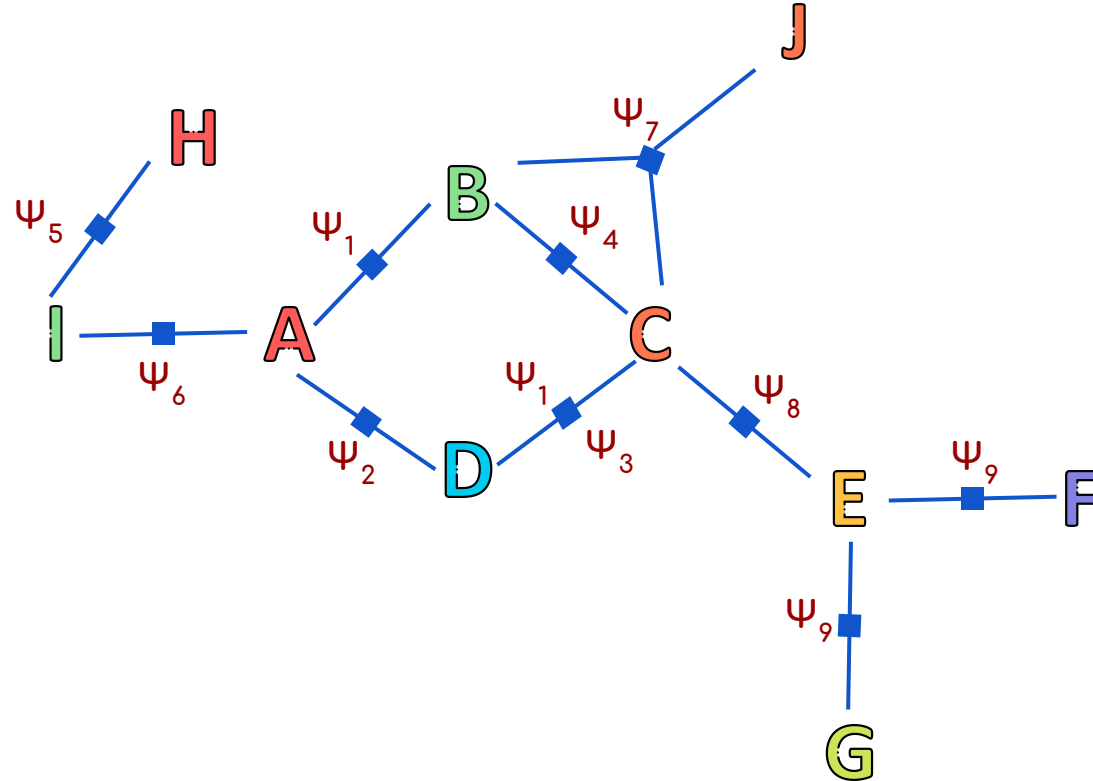
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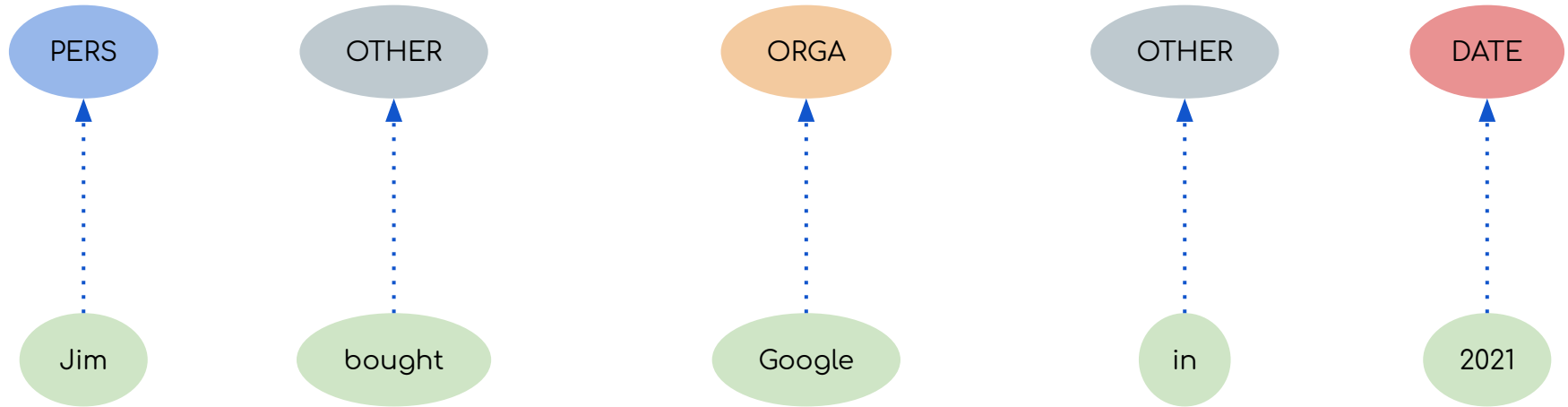
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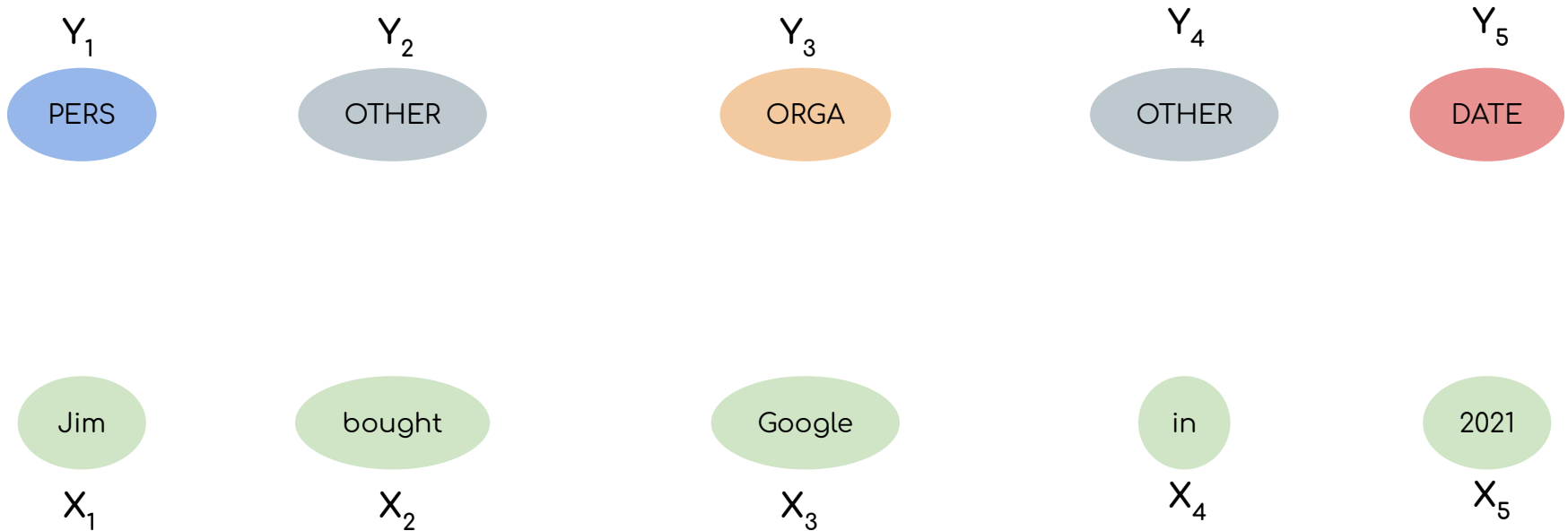
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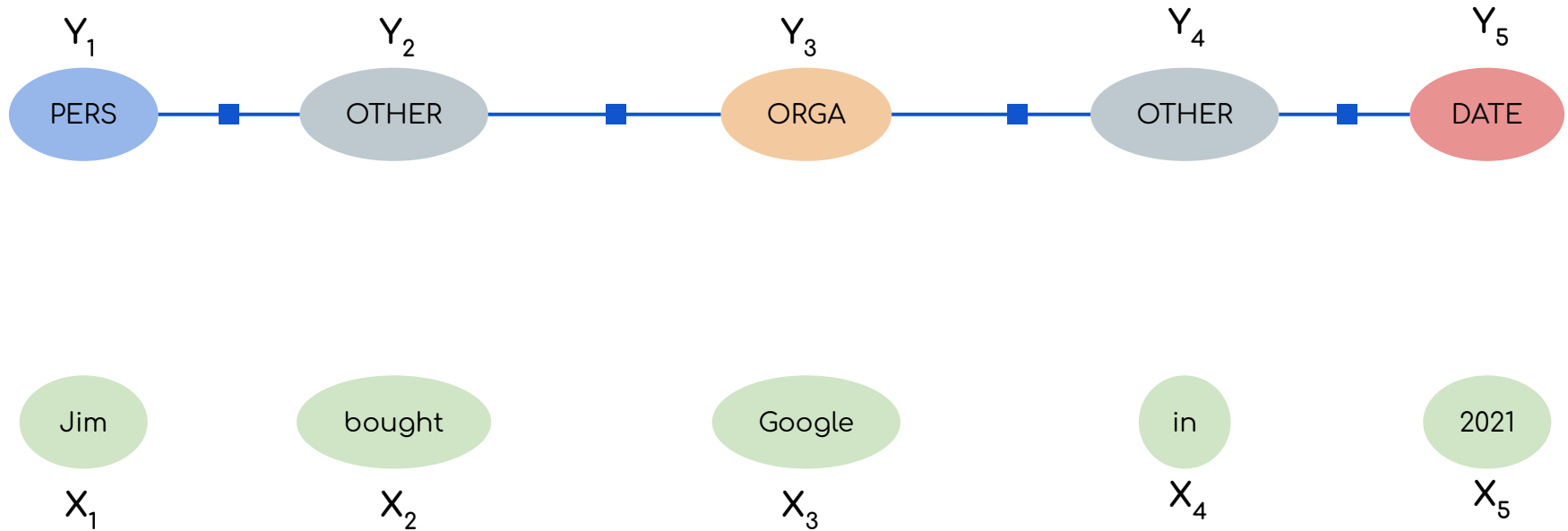
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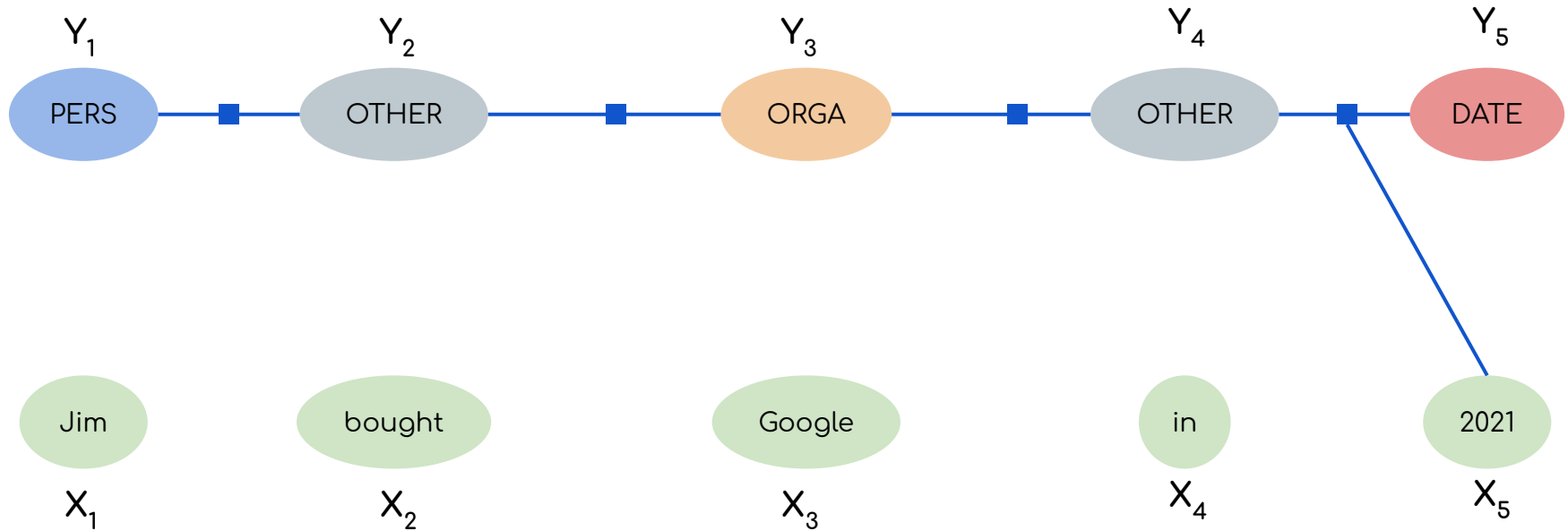
Derivative  $\Rightarrow 30/\theta - 70/(1-\theta) \Rightarrow$  it hits zero when  $\theta = 30/100$

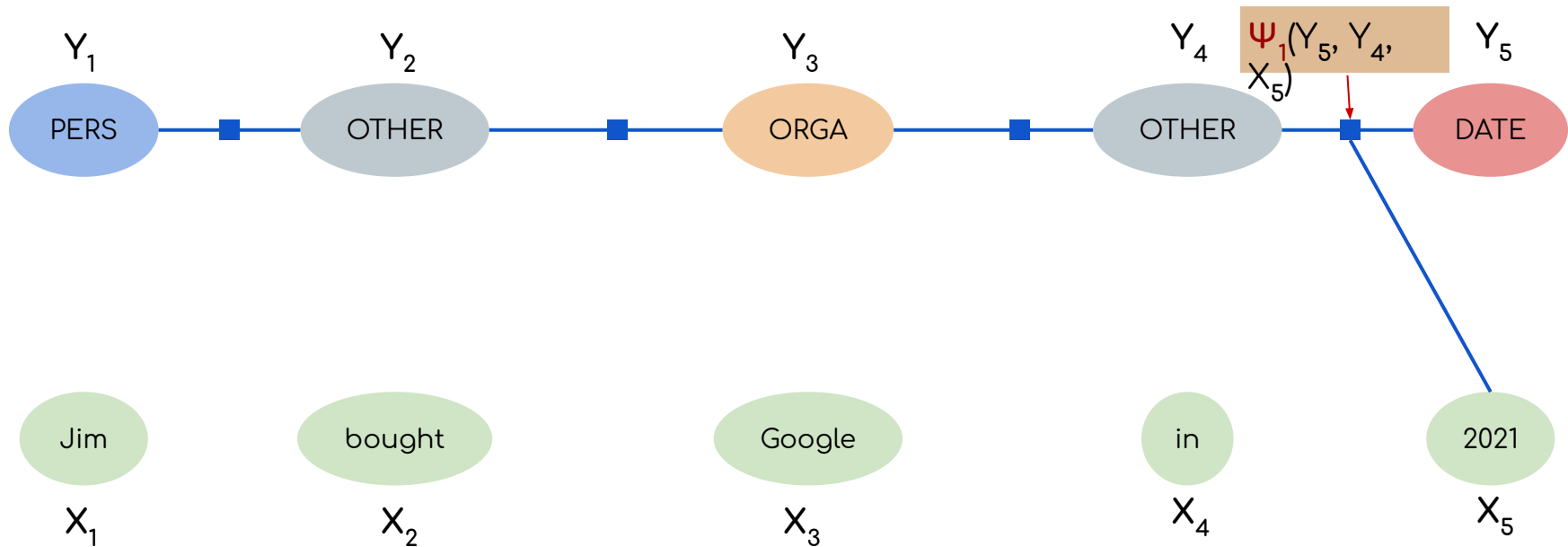




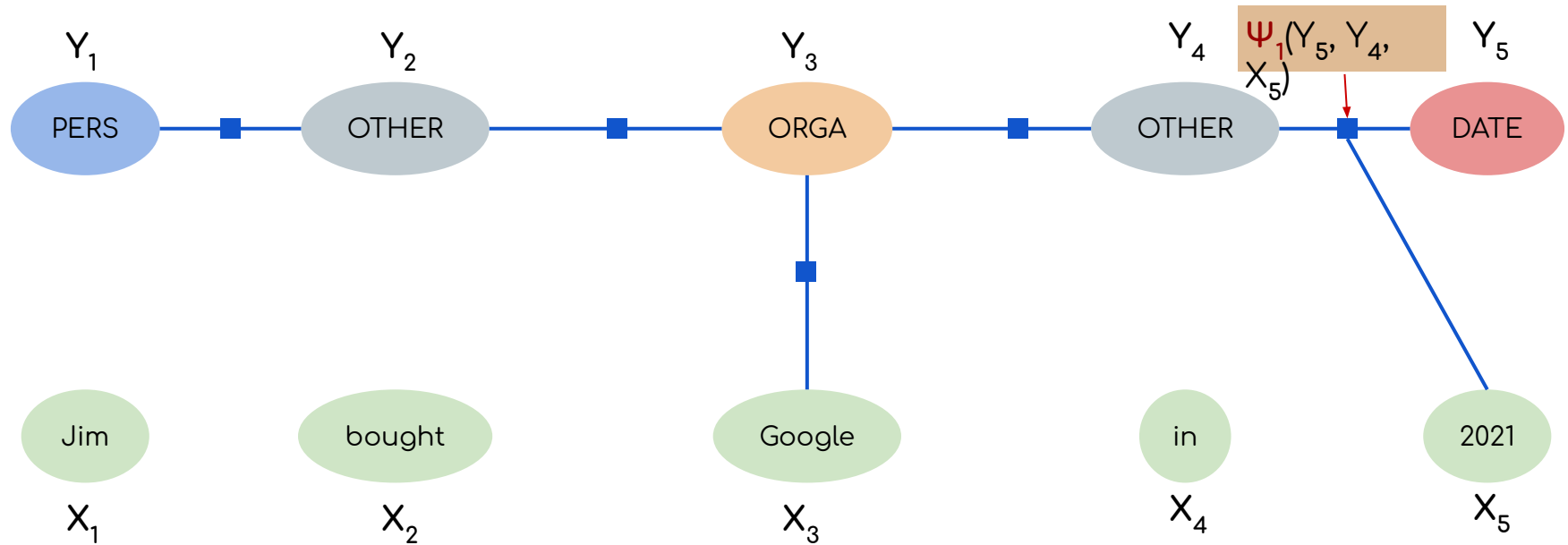




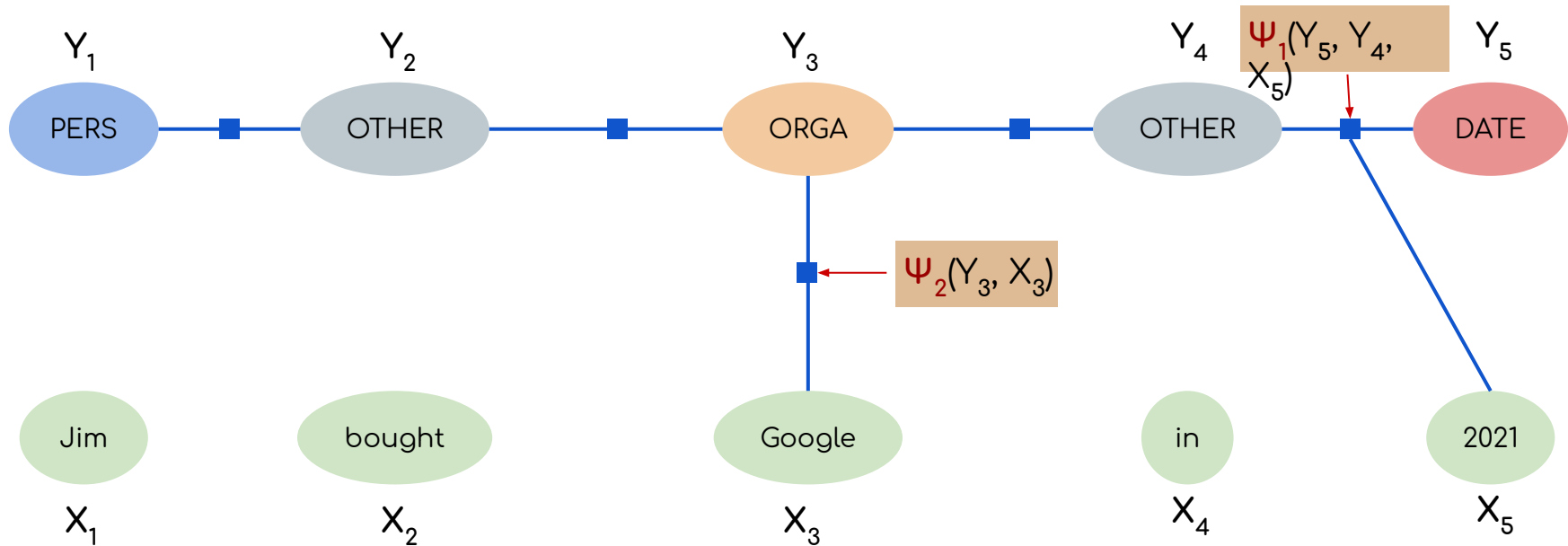




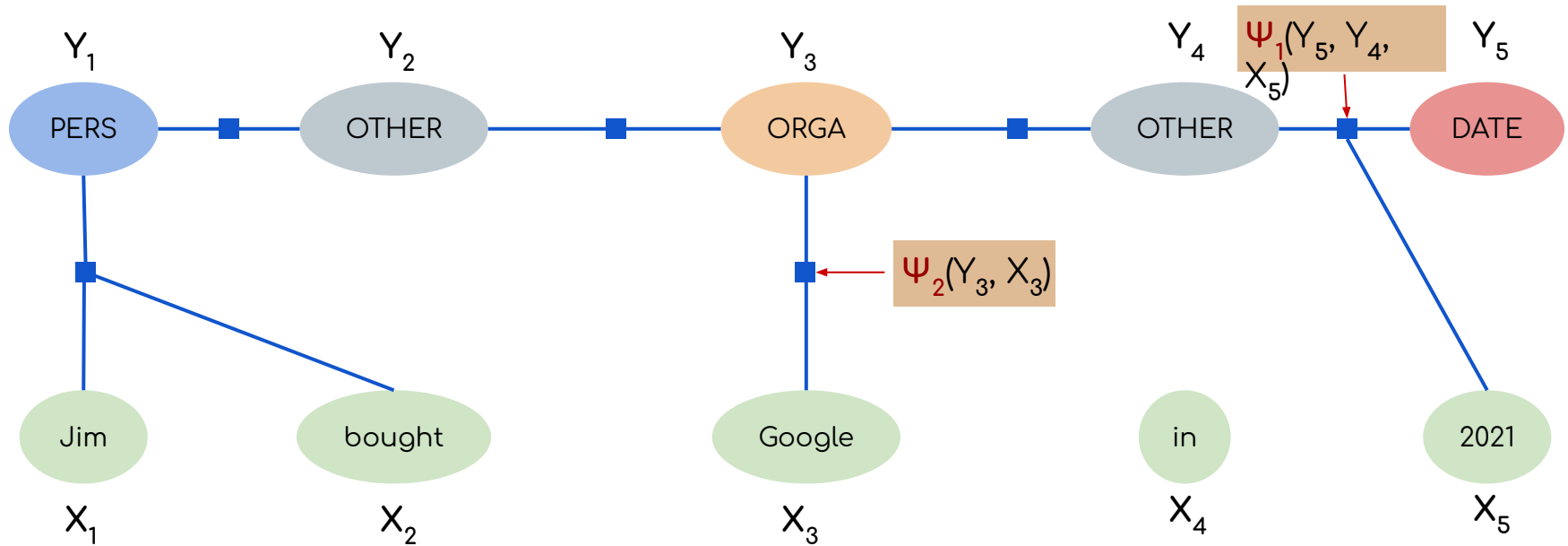
$\psi_1(Y_5, Y_4, X_5) = \theta_1$  if  $Y_5 = \text{'DATE'}$  and  $Y_4 = \text{'OTHER'}$  and  $X_5$  is a number else 0.

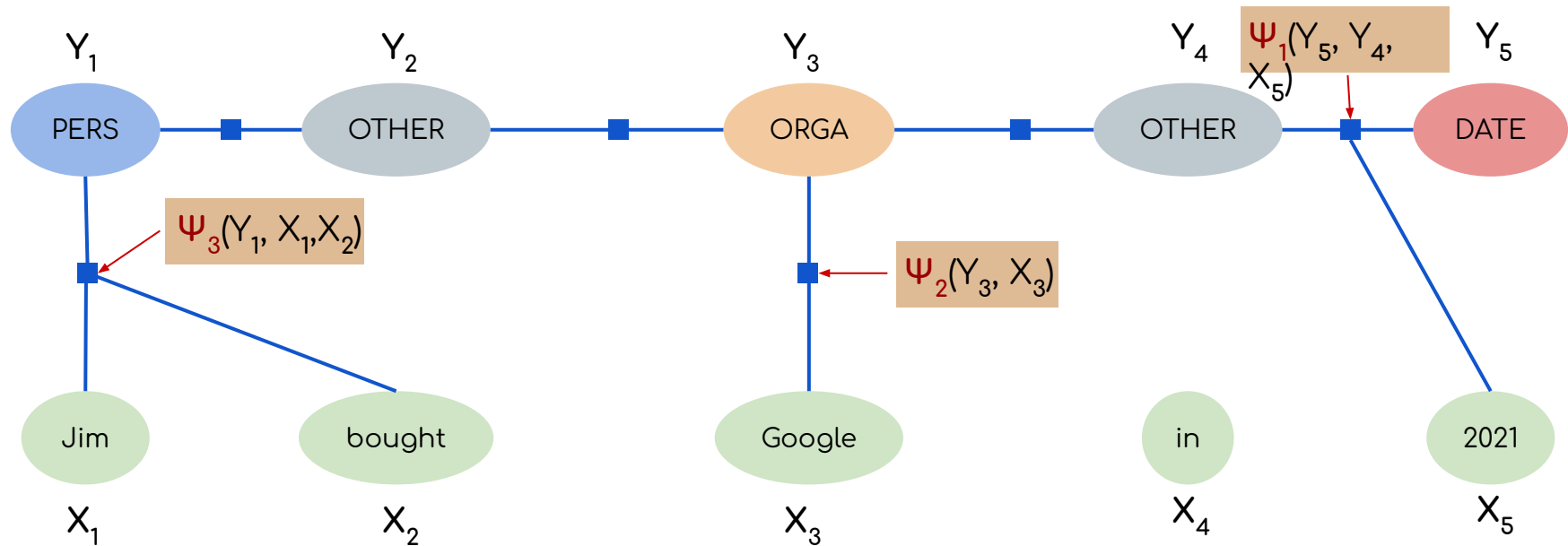




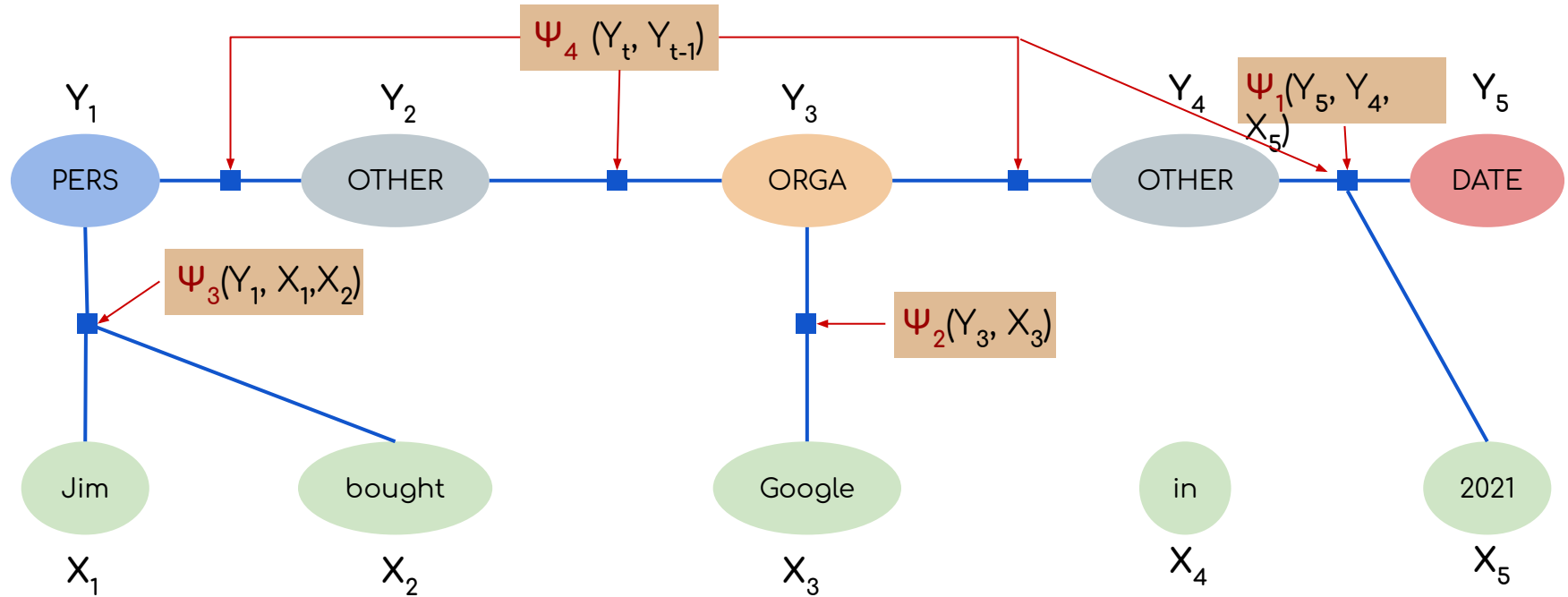


$\Psi_2(Y_3, X_3) = \theta_2$  if  $Y_3 = \text{'ORGA'}$  and  $X_3$  starts with a capital letter, else 0.



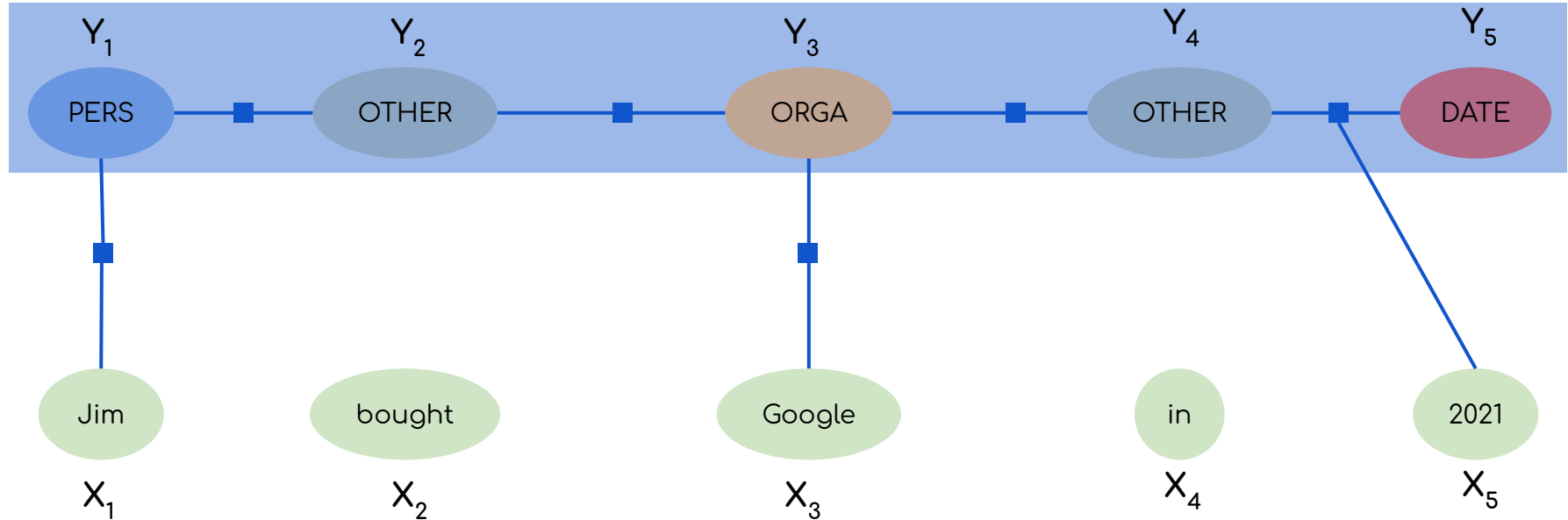


$\Psi_3(Y_1, X_1, X_2) = \theta_3$  if  $Y_1 = \text{'PERS'}$  and  $X_1$  belongs to a dictionary of names and  $X_2$  is a verb



$$\psi_4(Y_3, Y_2) = \theta_4 \text{ if } Y_2 = \text{'OTHER'} \text{ and } Y_3 = \text{'ORGA'}$$

We are interested in modeling  $P(Y | X)$  and not  $P(Y,X) \Rightarrow$  **CONDITIONAL**



$$\Psi_4(Y_3, Y_2) = \theta_4 \text{ if } Y_2 = \text{'OTHER'} \text{ and } Y_3 = \text{'ORGA'}$$

## Take away points:

- Factor models are a pattern recognition machine learning model for **structured prediction**
- CRF's model  $P(Y | X)$  where  $X$  is always observable and does not care about  $P(X)$  or  $P(X | Y)$  => A **discriminative model**, can use **complex features  $X$**  and **more efficient**, **BUT** always needs complete features  $X$  => **does not handle incomplete data** very well
- We use **linear chain CRF**'s (Every  $Y_t$  depends only on  $Y_{t-1}$  and  $Y_{t+1}$ ) for sequential tasks such as NER, Speech to text, ...

