Advanced Deep Learning

Joris LIMONIER

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1 Proposed exercise lecture 02

Prove equivalence of formulations for implicit and explicit methods. The implicit Euler method is given by:

$$w^{k+1} = w^k - \tau \nabla \phi(w^{k+1}) \tag{1}$$

The explicit Euler method is given by:

$$w^{k+1} = w^k - \tau \nabla \phi(w^k) \tag{2}$$

2 Proposed exercise lecture 03

Find a closed form for the exponential moving average. The recursive expression of the exponential moving average is given by:

$$\begin{cases} \langle a \rangle_0^\alpha = (1-\alpha) a_0 \\ \langle a \rangle_{n+1}^\alpha = \alpha \langle a \rangle_n^\alpha + (1-\alpha) a_{n+1} \end{cases} \tag{3}$$

$$
\begin{aligned}
&\langle a\rangle_{n+1}^{\alpha} = \alpha\langle a\rangle_{n}^{\alpha} + (1-\alpha)a_{n+1} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = \alpha\left[\alpha\langle a\rangle_{n-1}^{\alpha} + (1-\alpha)a_{n}\right] + (1-\alpha)a_{n+1} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = \alpha^{2}\langle a\rangle_{n-1}^{\alpha} + \alpha(1-\alpha)a_{n} + (1-\alpha)a_{n+1} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = \alpha^{2}\left[\alpha\langle a\rangle_{n-2}^{\alpha} + (1-\alpha)a_{n-1}\right] + \alpha(1-\alpha)a_{n} + (1-\alpha)a_{n+1} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = \alpha^{3}\langle a\rangle_{n-2}^{\alpha} + \alpha^{2}(1-\alpha)a_{n-1} + \alpha(1-\alpha)a_{n} + (1-\alpha)a_{n+1} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = \alpha^{3}\langle a\rangle_{n-2}^{\alpha} + (1-\alpha)\sum_{k=0}^{2}\alpha^{k}a_{n+1-k} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = \alpha^{n+1}\langle a\rangle_{0}^{\alpha} + (1-\alpha)\sum_{k=0}^{n}\alpha^{k}a_{n+1-k} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = \alpha^{n+1}(1-\alpha)a_{0} + (1-\alpha)\sum_{k=0}^{n}\alpha^{k}a_{n+1-k} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = (1-\alpha)\sum_{k=0}^{n+1}\alpha^{k}a_{n+1-k} \\
\implies &\langle a\rangle_{n+1}^{\alpha} = (1-\alpha)\sum_{k=0}^{n+1}\alpha^{n+1-k}a_{k}
\end{aligned}
$$