Practical report
Filters & z-transform

Joris Stork: 6185320, Jayke Meijer: 6049885

January 11, 2011

1 Introduction

2 Questions

2.1 Opgave 1

2.1.1

$\Omega = 2\pi f T_s = 2\pi * 20 * \frac{1}{1000} = \frac{4\pi}{100} \Rightarrow N = 50.$

2.1.2

$2 * \frac{50\pi}{3200} < \frac{128\pi}{3200} < 3 * \frac{50\pi}{3200} \Rightarrow$ The sine lies between the second and third multiples.

2.1.3

f	Ω met $T_s = 0.001$	Periode N	?de harm $< \Omega <$?de harm
$20Hz$	$\frac{4\pi}{100}$	50	2de ... 3de
$30Hz$	$\frac{6\pi}{100}$	$33\frac{1}{3}$	3de ... 4de

2.1.4

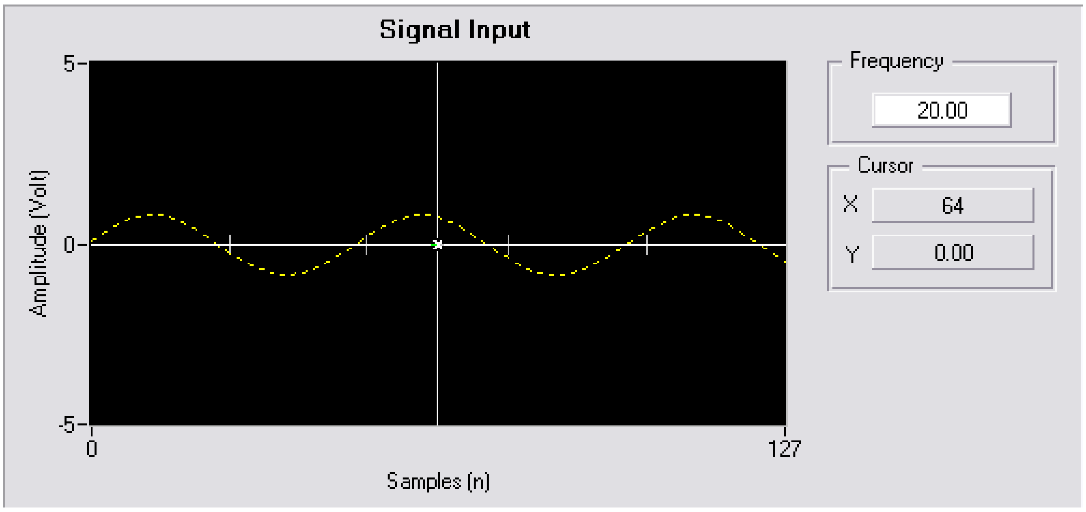

Figure 1: Input sine: $f = 20Hz$

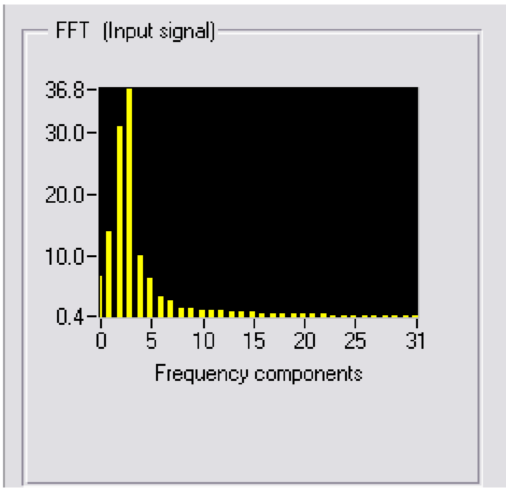

Figure 2: FFT: $f = 20Hz$

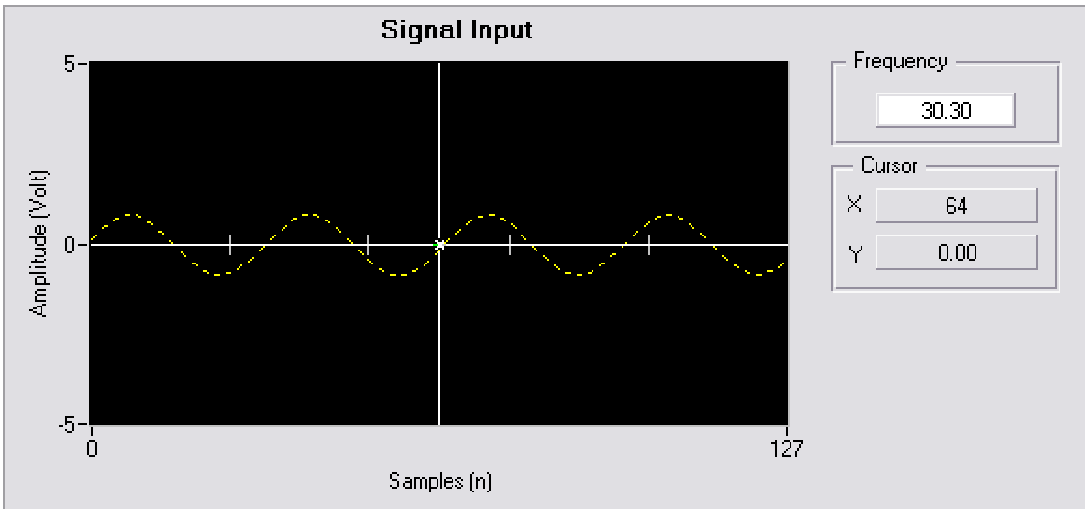

Figure 3: Input sine: $f = 30Hz$

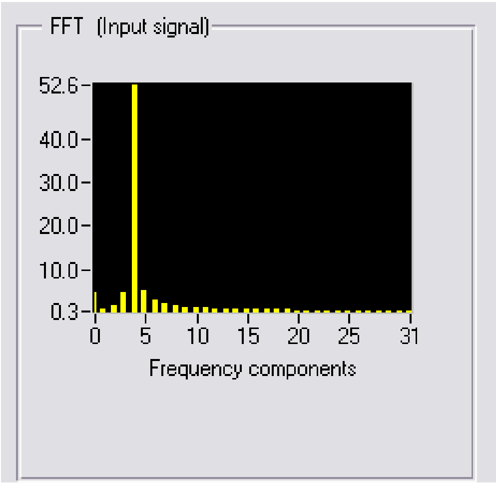

Figure 4: FFT: $f = 30Hz$

2.2 Opgave 2

2.2.1

$\Omega = 2\pi f T_s = 2\pi * 20 * \frac{1}{500} = \frac{4\pi}{50} \Rightarrow N = 25.$

2.2.2

f	Ω met $T_s = 0.001$	Periode N	?de harm $< \Omega <$?de harm
$20Hz$	$\frac{4\pi}{50}$	25	5de . . . 6de
$30Hz$	$\frac{6\pi}{50}$	$16\frac{2}{3}$	7de . . . 8ste

2.2.3

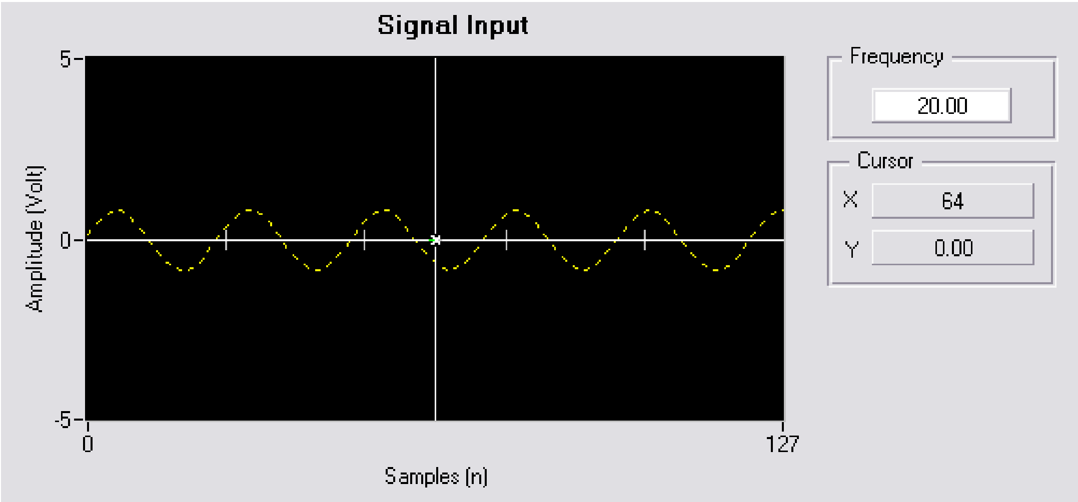

Figure 5: Input sine: $f = 20Hz$

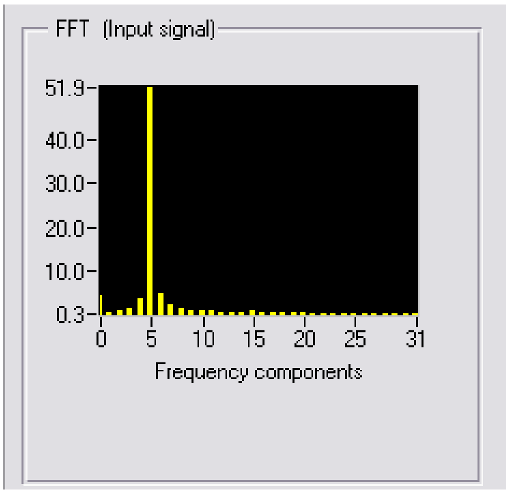

Figure 6: FFT: $f = 20Hz$

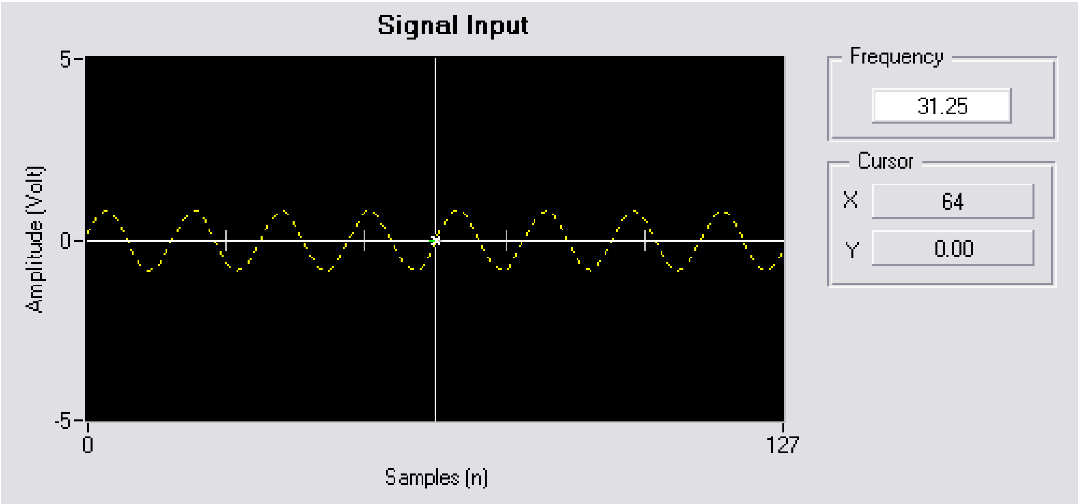

Figure 7: Input sine: $f = 30Hz$

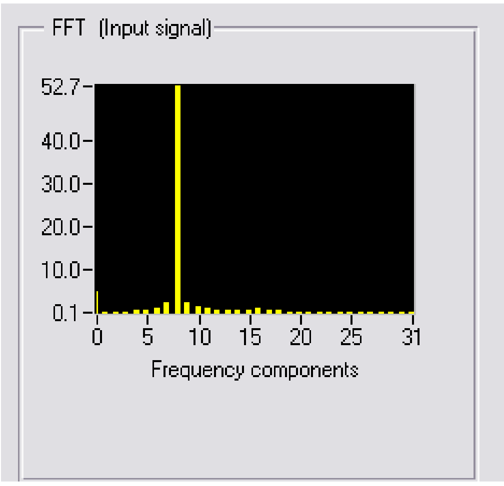

Figure 8: FFT: $f = 30Hz$

2.3 Opgave 3

2.3.1

$\Omega = 2\pi f T_s = 2\pi * 20 * \frac{1}{1000} = \frac{4\pi}{100} \Rightarrow N = 50.$

2.3.2

$2 * \frac{50\pi}{3200} < \frac{128\pi}{3200} < 3 * \frac{50\pi}{3200} \Rightarrow$ The sine lies between the second and third multiples.

2.3.3

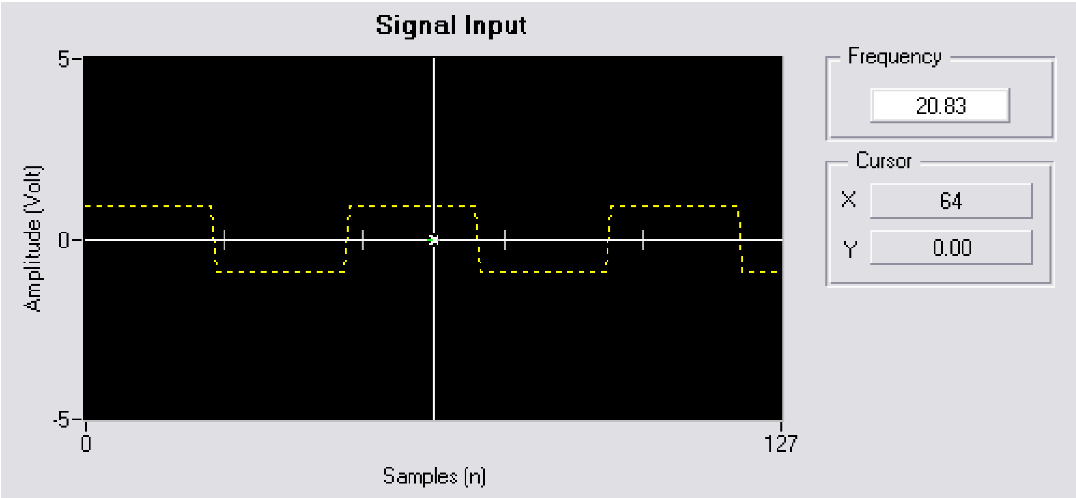

Figure 9: Input sine: $f = 20Hz$

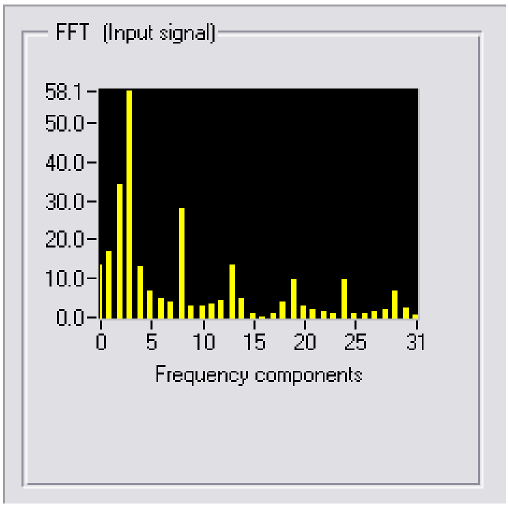

Figure 10: Input FFT

The third, eighth, 13th, 18th, 23rd, 28th, i.e. 3+(5m)th. This is because they have to be uneven to sum into an approximation of the block-signal.

2.3.4

The transfer function $H(z)$ is defined by $H(z) = K\frac{z-0}{z-0.9} = \frac{0.5z}{z-0.9}$

2.3.5

$y[n] - 0.9y[n-1] = 0.5x[n] \Leftrightarrow y[n] = 0.9y[n-1] + 0.5x[n]$

2.3.6

Maximum amplitude of $|Hz|$ occurs at $z = 1$: $|Hz|_{max} = \frac{0.5}{1-0.9} = 5$.

2.3.7

In the following figure we see that the maximum frequency contribution after our low pass filter is around 165, whereas in fig. 10 the value was around 58.

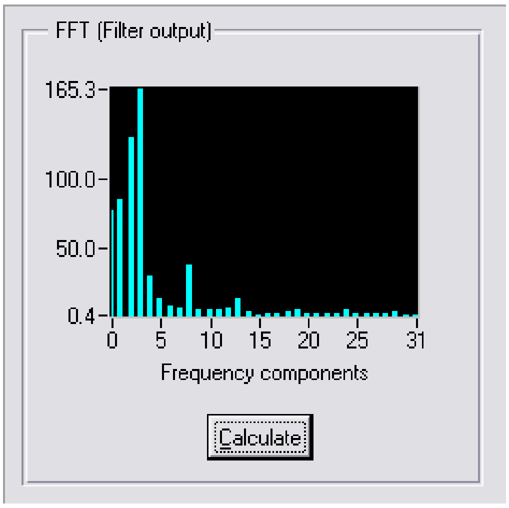

Figure 11: Output FFT

2.4 Opgave 4

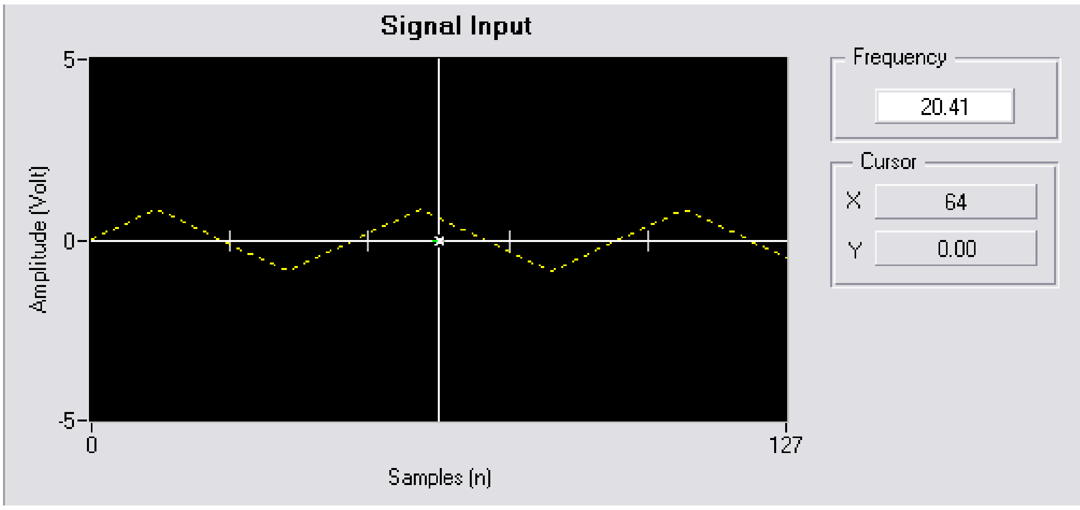

Figure 12: Output FFT

2.4.1

$\Omega = 2\pi f T_s = 2\pi * 20 * \frac{1}{1000} = \frac{4\pi}{100} \Rightarrow N = 50.$

2.4.2

The strongest neighbouring harmonics are as in Opgave 1, question 2. The ramp form does not affect this. The following figure confirms this:

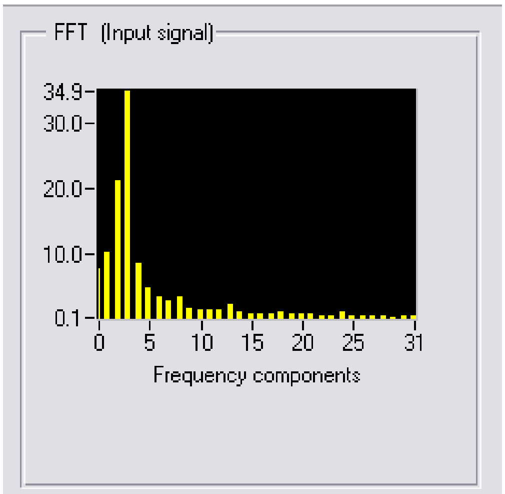

Figure 13: Output FFT

2.4.3

The strongest harmonics are at the lowest frequencies (harmonics 1 through 4), with relative peaks at 3, 8, 13 etc. This is similar to the pattern in Opgave 3, question 3, but with smaller peaks. The smaller peaks can be explained by the ramp form of the input signal, which requires less of the higher-frequency harmonic peaks to approximate than the square input signal in Opgave 3.

2.4.4

The second harmonic corresponds to $\frac{4\pi}{128} = \frac{\pi}{32}$, and the third corresponds to $\frac{6\pi}{128} = \frac{3\pi}{64}$. From this we can derive $z_1 = \cos\frac{\pi}{32} + j\sin\frac{\pi}{32} = e^{j\frac{\pi}{32}}$ and $z_2 = \cos\frac{3\pi}{64} + j\sin\frac{3\pi}{64} = e^{j\frac{3\pi}{64}}$.

2.4.5

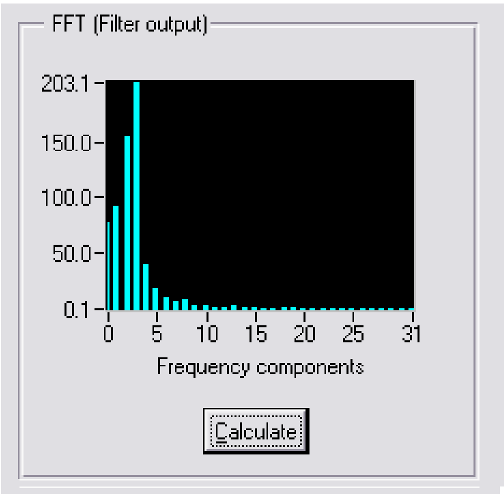

Figure 14: Output FFT

$H(z) = \frac{z}{z-0.9}$

2.4.6

$|H(z_1)| = |\frac{e^{j\frac{\pi}{32}}}{e^{j\frac{\pi}{32}}-0.9}| = 7.3$ and $|H(z_2)| = |\frac{e^{j\frac{3\pi}{64}}}{e^{j\frac{3\pi}{64}}-0.9}| = 5.8$

2.5 Opgave 5

2.5.1

$H(z) = \frac{(z+e^{j\frac{\pi}{10}})(z+e^{-j\frac{\pi}{10}})}{z^2} = \frac{z^2+e^{j\frac{\pi}{10}}e^{-j\frac{\pi}{10}}-ze^{j\frac{\pi}{10}}-ze^{-j\frac{\pi}{10}}}{z^2} = \frac{z^2-ze^{j\frac{\pi}{10}}-ze^{-j\frac{\pi}{10}}+1}{z^2} = \frac{z^2-2z\cos\frac{\pi}{10}+1}{z^2}$

$\Rightarrow b_0 = 1, b_1 = -2\cos\frac{pi}{10} = -1.902113, b_2 = 1, a_0 = 1, a_1 = a_2 = 0$
$\Rightarrow y[n] - a_1 y[n-1] - a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$
$y[n] = x[n] - 2\cos\frac{\pi}{10}x[n-1] + x[n-2].$

2.5.2

f	Ω with $T_s = 0.001$	$\lvert H(e^{j\Omega})\rvert$
$10Hz$	$2\pi \times 10 \times 0.001 = \frac{\pi}{50}$	$\lvert H(e^{j\frac{\pi}{50}})\rvert = 0.093941$
$20Hz$	$\frac{4\pi}{100} = \frac{\pi}{25}$	0.082116
$30Hz$	$\frac{6\pi}{100} = \frac{3\pi}{50}$	0.062462
$40Hz$	$\frac{8\pi}{100} = \frac{2\pi}{25}$	0.03505
$50Hz$	$\frac{10\pi}{100} = \frac{\pi}{10}$	0
$60Hz$	$\frac{12\pi}{100} = \frac{6\pi}{50}$	0.04256

$\Omega 2\pi \times f \times T_s = 2\pi \times 10 \times \frac{1}{1000} = \frac{20\pi}{1000} = \frac{2\pi}{100}$
$\Omega 2\pi \times 20 \times T_s = 2\pi \times 20 \times \frac{1}{1000} = \frac{40\pi}{1000} = \frac{4\pi}{100}$
$\Omega 2\pi \times 30 \times T_s = 2\pi \times 10 \times \frac{1}{1000} = \frac{60\pi}{1000} = \frac{6\pi}{100}$
$|H(z)| = \frac{z^2-1.90211321}{z^2} = \frac{(e^{j\frac{\pi}{50}})^2-1.902113\times e^{j\frac{\pi}{50}}+1}{(e^{j\frac{\pi}{50}})^2}$

2.5.3

Our experiment confirmed that the $50Hz$ frequency was filtered out, as derived and listed in the table above. We can conclude that the filter we have created is a band-stop filter that attenuates frequencies in the region of $50Hz$.