Sampelen van een analoog signaal

Gegeven continu signaal: $\sin$ met $f = 100Hz$

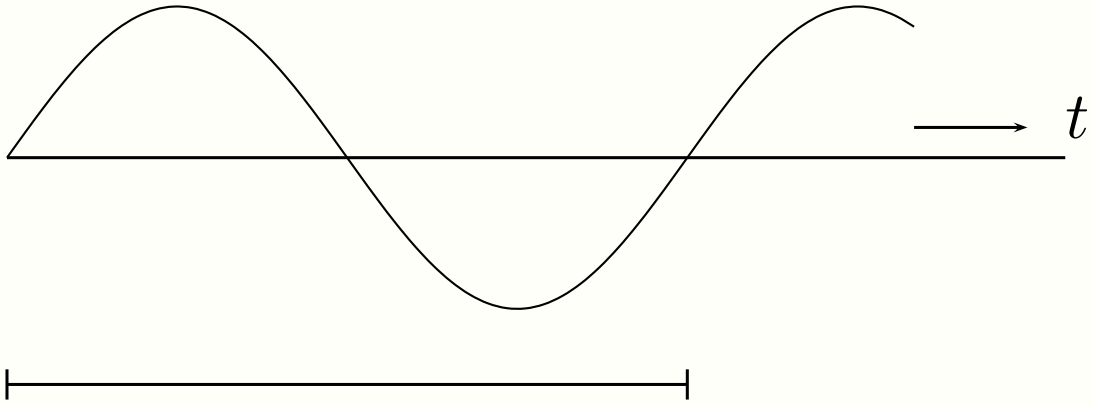
$x(t) = \sin \omega t = \sin 2\pi f t = \sin 200\pi t$

$\omega = 2\pi f$ met ω hoekfrequentie en f frequentie in Hz

$T = \frac{2\pi}{\omega} = \frac{1}{f} = \frac{2\pi}{200\pi} = 0.01$s

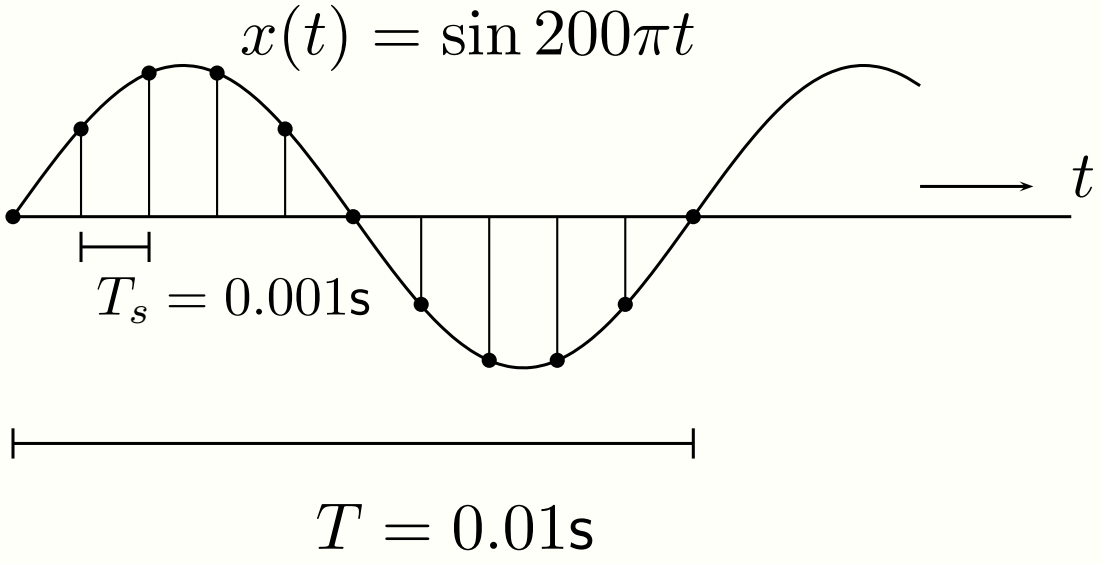
Sampelen van een analoog signaal

$x(t) = \sin 200\pi t$ met $T = \frac{2\pi}{\omega} = \frac{2\pi}{200\pi} = 0.01\text{s}$



Sampelen van een analoog signaal

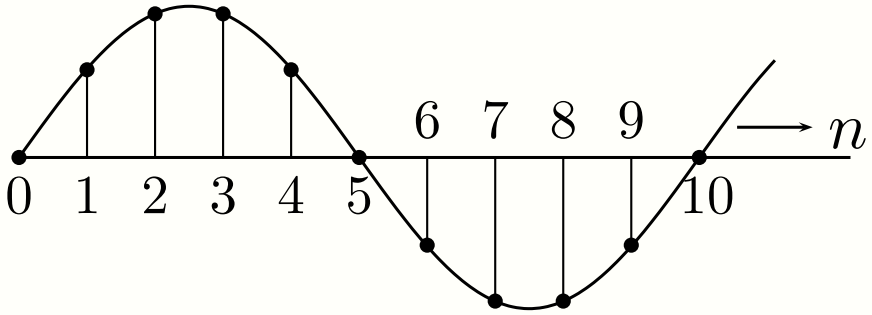
Sampelen met $f_s = 1000Hz$, dus $T_s = \frac{1}{1000Hz} = 0.001$s



Sampelen van een analoog signaal

$x(t) = \sin 200\pi t$ invullen $t = n \cdot T_s$:

$x[n] = \sin(200\pi \cdot 0.001 \cdot n) = \sin \frac{2\pi}{10} n$



$$x[0] = 0$$

$$x[1] = \sin \frac{2\pi}{10} = 0.59$$

$$x[2] = \sin 2\frac{2\pi}{10} = 0.95$$

$$x[3] = \sin 3\frac{2\pi}{10} = 0.95$$

$$x[4] = \sin 4\frac{2\pi}{10} = 0.59$$

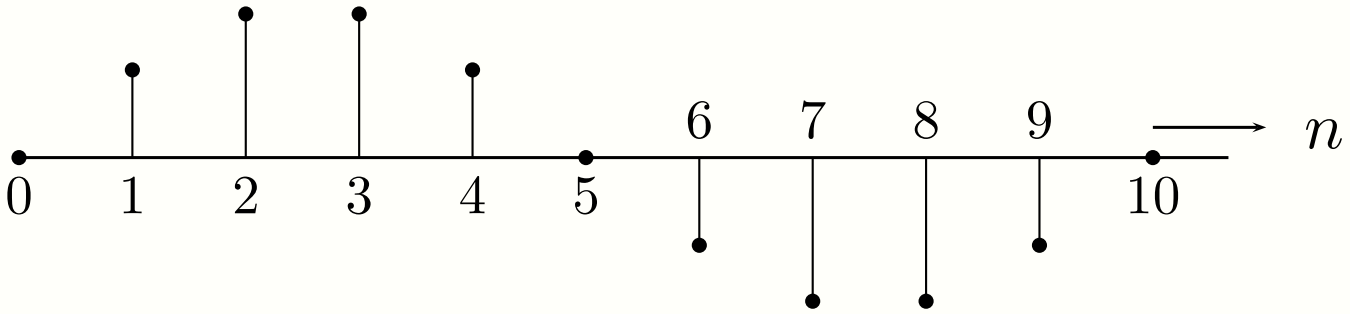
etc.

Sampelen van een analoog signaal

Gesampelde signaal (beginnend bij $n = 0$):

$$(x[n]) : 0 \quad 0.59 \quad 0.95 \quad 0.95 \quad 0.59 \quad 0 \quad -0.59 \quad -0.95 \quad \ldots$$

$x[n] = \sin \frac{2\pi}{10} n$ gestand. frequentie $\Omega = \frac{2\pi}{10}$



Dus hoe van ω naar Ω? $\Omega = \omega \cdot T_s$

Als gegeven een sinus met analoge frequentie ω:

$x(t) = \sin \omega \cdot t$

Sampelen met sampelinterval T_s: $t = n \cdot T_s$ invullen:

$\sin(\omega \cdot T_s \cdot n) = \sin(\Omega \cdot n)$

Dan ontstaat sinus met gestandaardiseerde frequentie Ω:

$x[n] = \sin(\Omega \cdot n)$

Kan je van Ω naar ω?

$\Omega = \omega \cdot T_s \Rightarrow \omega = \Omega / T_s$

Dat betekent dat je naast de frequentie Ω ook het sampelinterval T_s moet weten om de analoge frequentie ω te achterhalen

Voorbeeld:
$\Omega = \frac{2\pi}{10}$ en $T_s = 0.01 \Rightarrow \omega = \frac{2\pi/10}{0.01} = 20\pi$

$\Omega = \frac{2\pi}{100}$ en $T_s = 0.001 \Rightarrow \omega = \frac{2\pi/100}{0.001} = 20\pi$

Frequentie en periode digitaal signaal

$x[n] = \sin \Omega n$ frequentie Ω

Voorbeeld

$x[n] = \sin \frac{2\pi}{10} n$ frequentie $\Omega = \frac{2\pi}{10}$

periode $N = 10$

$x[n] = \sin \frac{2\pi}{9} n$ frequentie $\Omega = \frac{2\pi}{9}$

periode $N = 9$

$N = \frac{2\pi}{\Omega} \times k$ (Waarom k wordt zo duidelijk)

Opgave 1

$x(t) = \sin(2\pi \cdot 100 \cdot t) = \sin(200\pi t)$, dus $f = 100Hz$

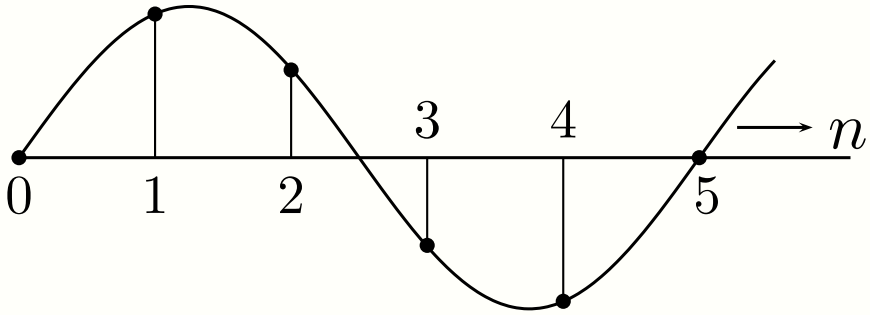
Sampel met $T_s = 0.002$, dus $f_s = 500Hz$

Bereken de eerste vijf waarden van $x[n]$

Wat is de frequentie Ω en wat is de periode N?

Uitwerking opgave 1

$$t = n \cdot T_s \Rightarrow x[n] = \sin(200\pi \cdot 0.002 \cdot n) = \sin(\tfrac{2\pi}{5}n)$$



$$\Omega = \tfrac{2\pi}{5} \quad N = 5$$

$$x[0] = 0$$

$$x[1] = \sin\tfrac{2\pi}{5} = 0.95$$

$$x[2] = \sin 2\tfrac{2\pi}{5} = 0.59$$

$$x[3] = \sin 3\tfrac{2\pi}{5} = -0.59$$

$$x[4] = \sin 4\tfrac{2\pi}{5} = -0.95$$

$$x[5] = \sin 5\tfrac{2\pi}{5} = 0$$

Frequentie en periode digitaal signaal

analoog		digitaal
ω	$\xrightarrow[sampelen]{T_s}$	$\Omega = \omega \cdot T_s$

$$x(t) = \sin(\omega t) \qquad x[n] = \sin(\omega \cdot T_s \cdot n) = \sin(\Omega n)$$

$$T = \frac{2\pi}{\omega} \qquad N = \frac{2\pi}{\Omega} \times k$$

Opgave 2

Gegeven een sinus met frequentie $f = 30Hz$

Sampel met $f_s = 135Hz$, wat is ω, Ω en N?

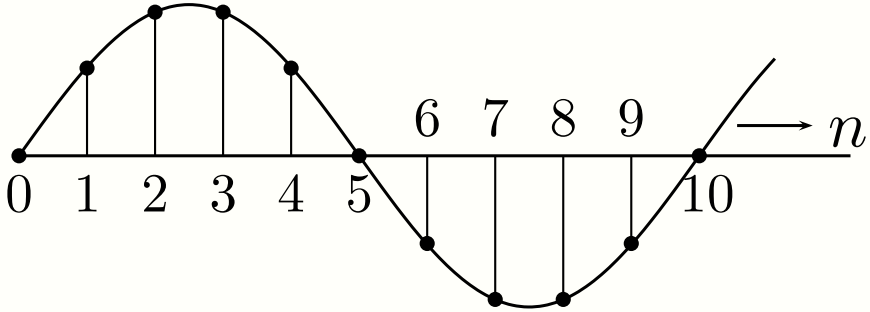
$\omega = 2\pi \cdot f = 60\pi \Rightarrow x(t) = \sin(60\pi t) \Rightarrow T = \frac{2\pi}{60\pi} = \frac{1}{30}s$

$x[n] = \sin(60\pi \cdot \frac{1}{135} \cdot n) = \sin(\frac{4\pi}{9}n)$

$\Omega = \frac{4\pi}{9} \Rightarrow$

$N = \frac{2\pi}{\Omega} \times 2 = \frac{2\pi}{4\pi/9} \times 2 = 2\pi \times \frac{9}{4\pi} \times 2 = 9$

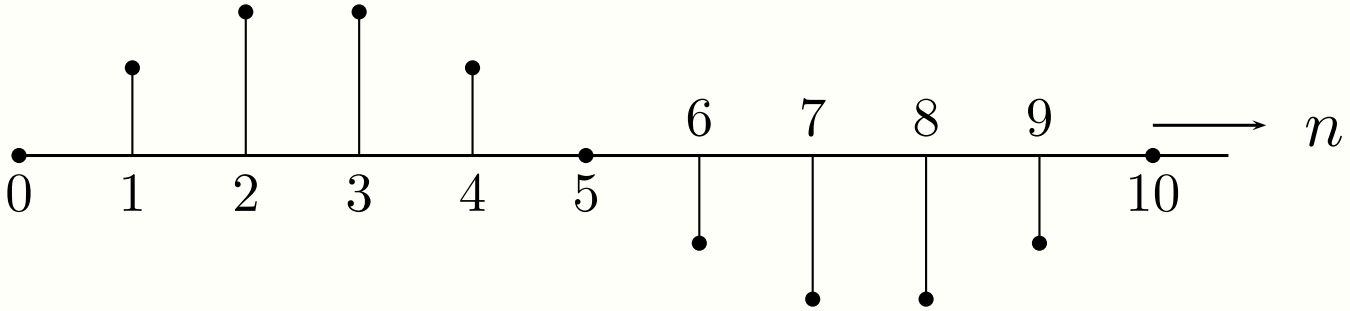
$x[n] = \sin \Omega n$



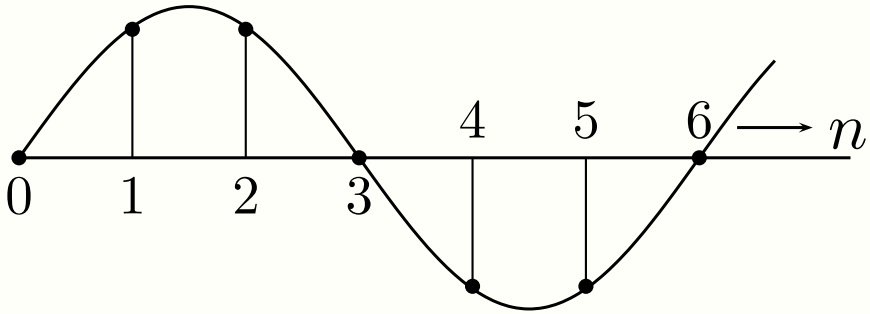
$$x[n] = \sin \tfrac{2\pi}{10} n$$

$$\Omega = \tfrac{2\pi}{10}$$

$$N = 10$$



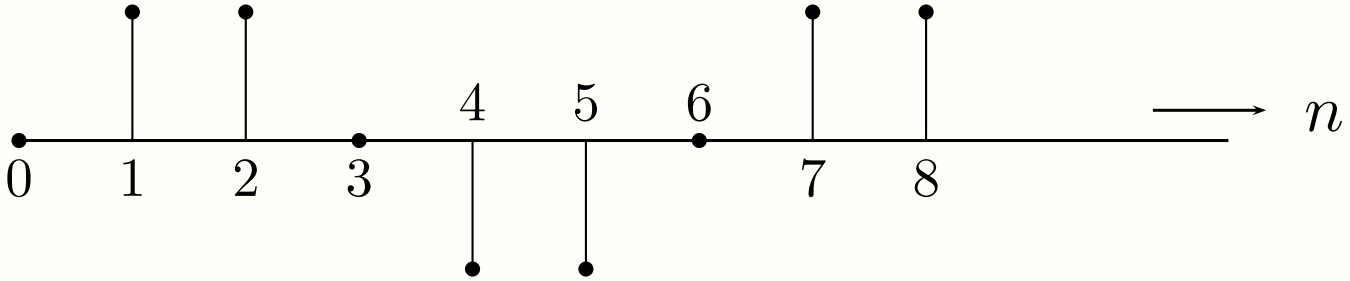
$x[n] = \sin \Omega n$



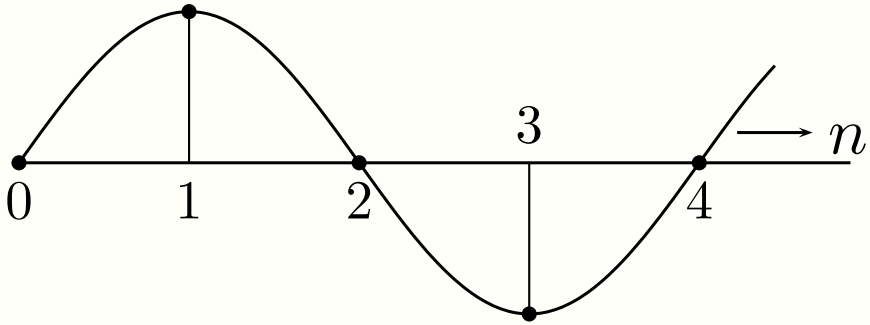
$$x[n] = \sin \frac{2\pi}{6} n$$

$$\Omega = \frac{2\pi}{6}$$

$$N = 6$$



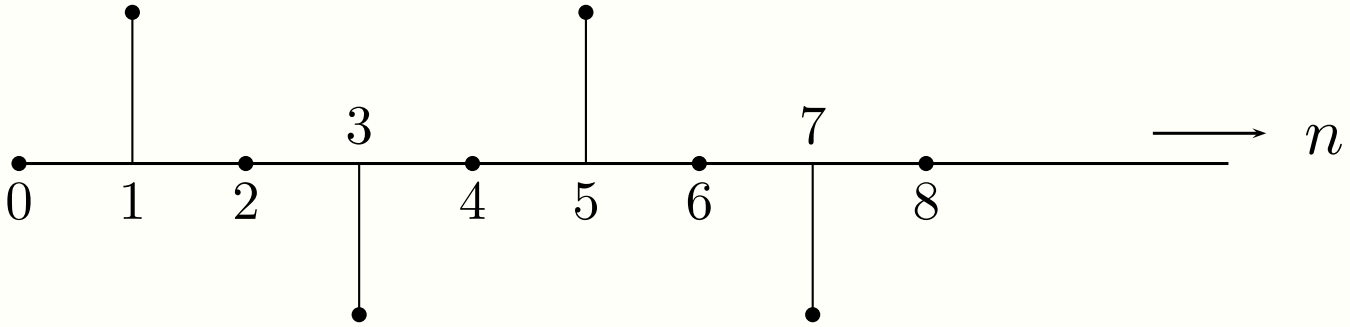
$x[n] = \sin \Omega n$



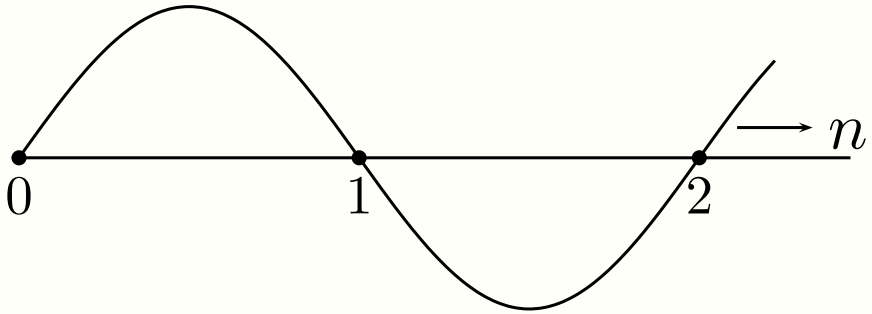
$$x[n] = \sin \tfrac{2\pi}{4} n$$

$$\Omega = \tfrac{2\pi}{4}$$

$$N = 4$$



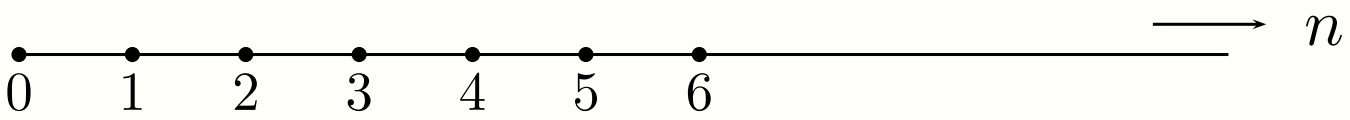
$x[n] = \sin \Omega n$



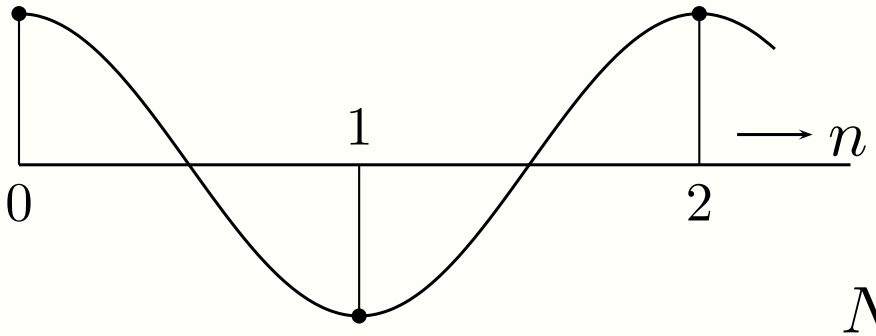
$$x[n] = \sin \frac{2\pi}{2} n$$

$$\Omega = \frac{2\pi}{2} = \pi$$

constant



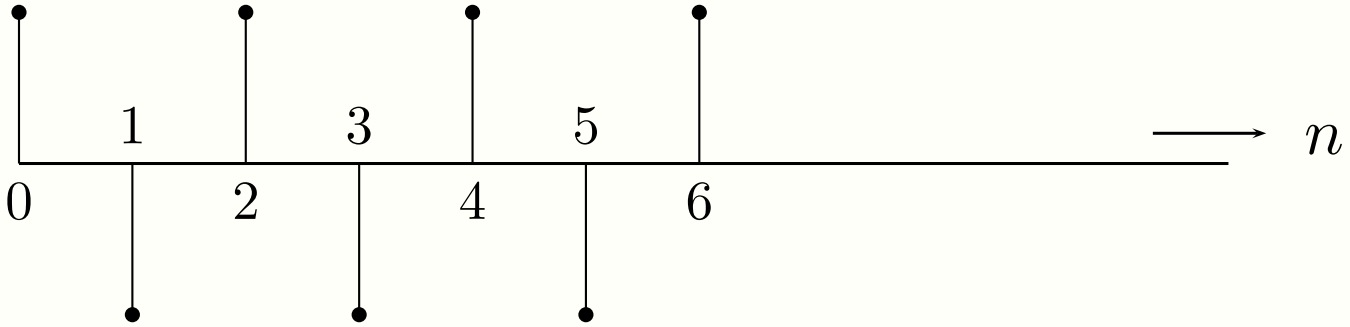
$x[n] = \cos \Omega n$



$$x[n] = \cos \tfrac{2\pi}{2} n$$

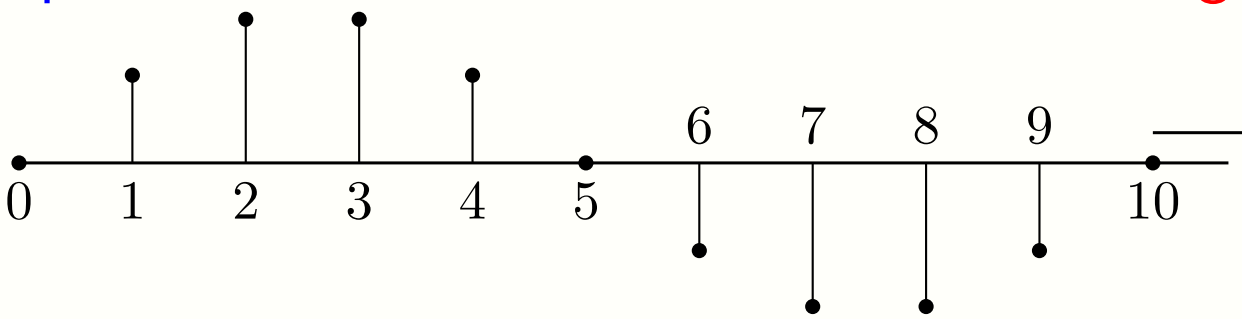
$$\Omega = \tfrac{2\pi}{2} = \pi$$

$N = 2$ is hoogste frequentie

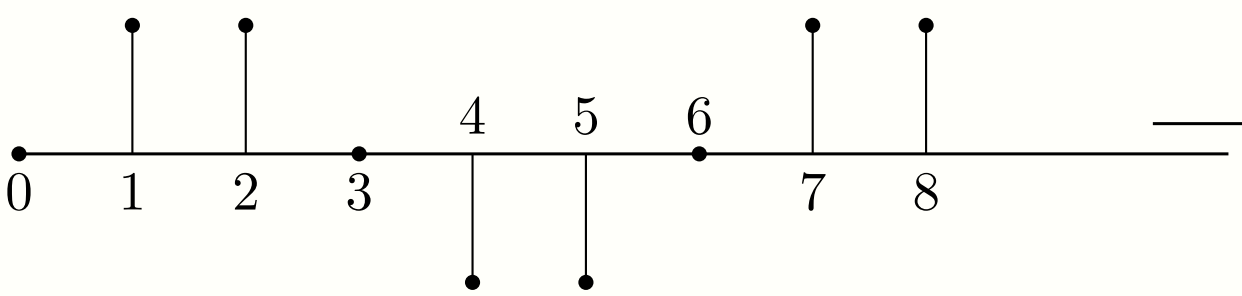


lage frequentie

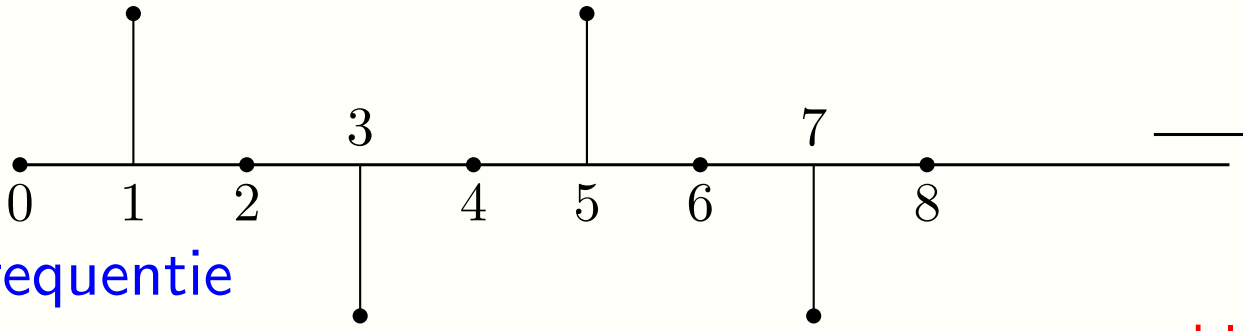
grote periode



$N = 10$



$N = 6$

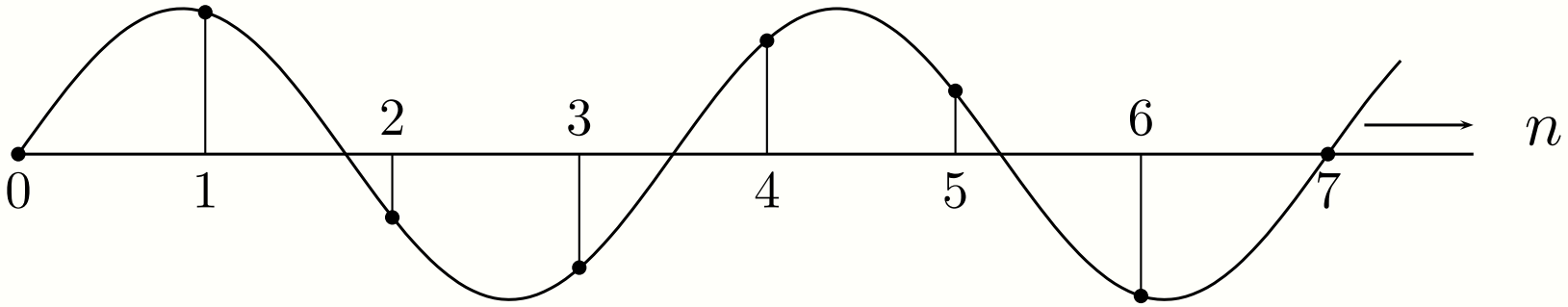


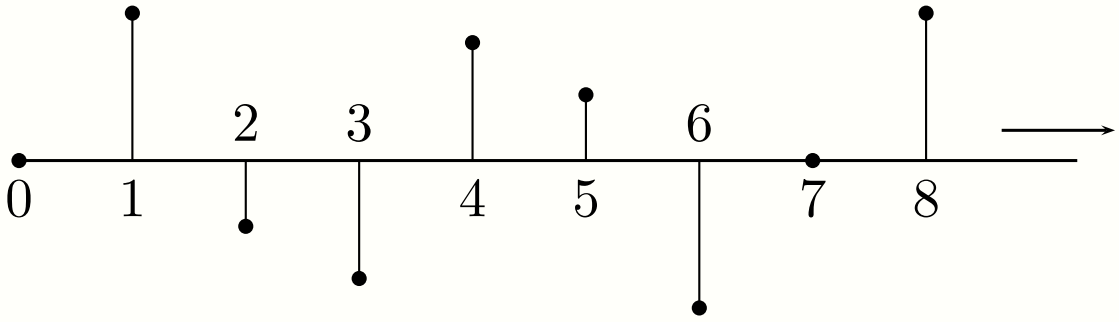
hoge frequentie

$N = 4$

kleine periode

$$x[n] = \sin \frac{4\pi}{7} n$$

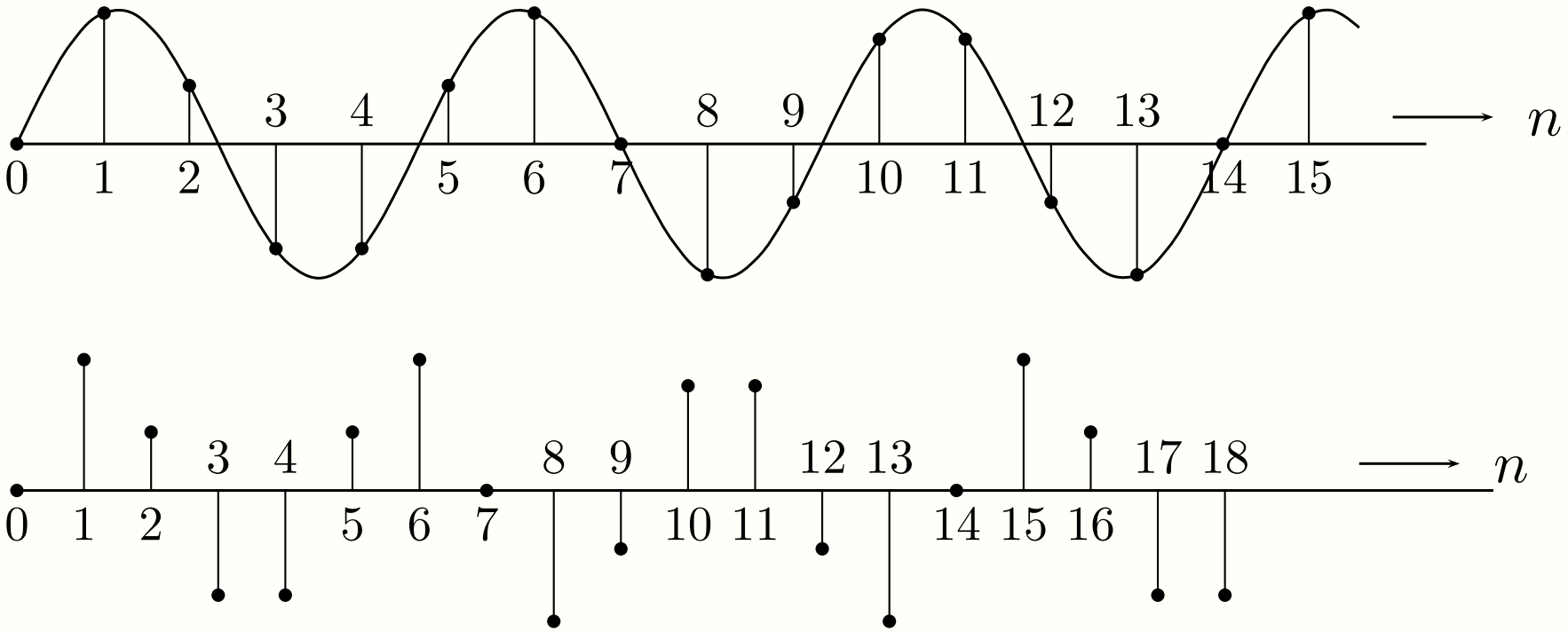




$$\Omega = \frac{4\pi}{7}$$

$$N = 7$$

$$x[n] = \sin \frac{3\pi}{7} n \Rightarrow N = 14$$



Opgaven

Wat is de periode N van de volgende signalen?

1. $\sin(\frac{\pi}{9}n)$
2. $\sin(\frac{4\pi}{9}n)$
3. $\sin(\frac{\pi}{5}n) + \sin(\frac{\pi}{10}n)$
4. $\sin(0.01\pi n)$
5. $\sin(\frac{\pi}{4}n) + \cos(\frac{\pi}{2}n)$
6. $\sin(n)$

1. $\Omega = \frac{\pi}{9} = \frac{2\pi}{18} \Rightarrow N = 18$

2. $\Omega = \frac{4\pi}{9} \Rightarrow N = 9$

3. $\Omega_1 = \frac{\pi}{5} = \frac{2\pi}{10} \Rightarrow N_1 = 10$, $\Omega_2 = \frac{\pi}{10} = \frac{2\pi}{20} \Rightarrow N_2 = 20$

$\Rightarrow N = 20$

4. $\Omega = \frac{\pi}{100} = \frac{2\pi}{200} \Rightarrow N = 200$

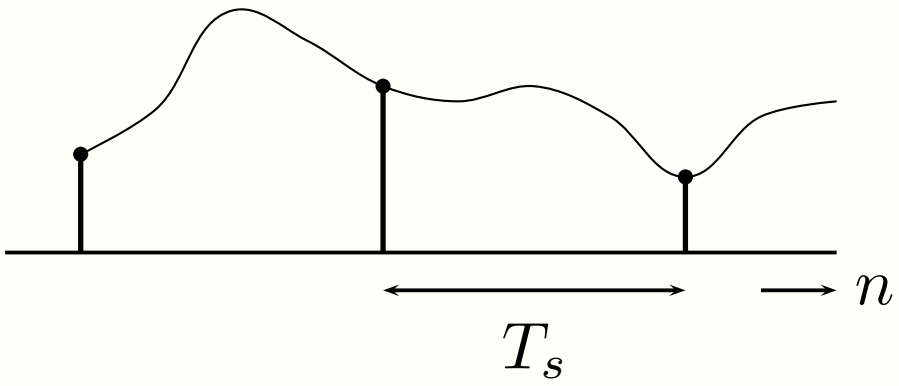
5. $\Omega_1 = \frac{\pi}{4} = \frac{2\pi}{8} \Rightarrow N_1 = 8$, $\Omega_2 = \frac{\pi}{2} = \frac{2\pi}{4} \Rightarrow N_2 = 4$

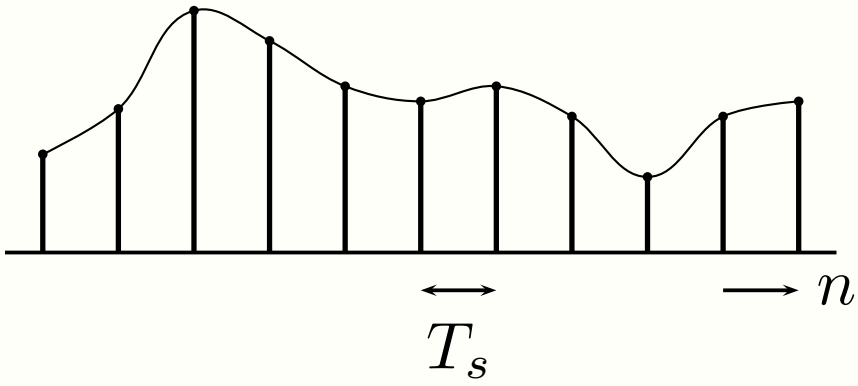
$\Rightarrow N = 8$

6. niet periodiek

Sampling frequentie?

Hoe vaak sampelen?





Sampling theorema van Shannon

Als maximale frequentie $f_{max}Hz \Rightarrow f_s = 2f_{max}$

Dus als $f_{max} = 3kHz \Rightarrow$ 6000 keer per seconde

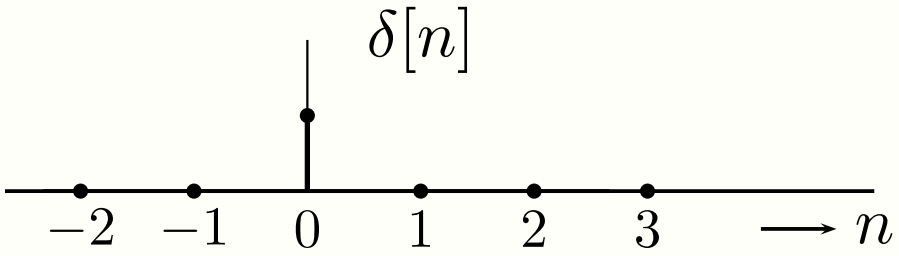
2 samples per periode van hoogste frequentie

Basissignalen

eenheidsimpuls $\delta[n]$:

$$\delta[n] = 0 \text{ als } n \neq 0$$

$$\delta[n] = 1 \text{ als } n = 0$$



Opgave

Wat is $\delta[n-1]$?

$$\delta[n-1] = 0 \text{ als } n-1 \neq 0 \Rightarrow n \neq 1$$

$$\delta[n-1] = 1 \text{ als } n-1 = 0 \Rightarrow n = 1$$