

Concepts of programming languages

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A reversible programming language.

Not turing complete!



Reversibility

Every statement can be reverted. No history is stored.

```
x += y * 3
```



Reversibility

Every statement can be reverted. No history is stored.

`x += y * 3`

`x -= y * 3`



Injective functions

Reversible languages can only compute injective functions.

$$\forall x, y : f(x) = f(y) \implies x = y \quad (1)$$

Every output has only a single input.



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$$h(x) = (x, g(x)) \quad (2)$$



Turing completeness

Turing machines can compute non-injective functions.

Reversible languages are not turing complete.

Reversible Turing complete.



Turing machines

Infinite tape of memory

Finite set of states

Transition function

- ▶ Current state
- ▶ Current symbol on tape
- ▶ Write symbol
- ▶ Move tape pointer
- ▶ Next state



Turing machines

Forward deterministic: given any state and tape, there is at most one transition *from* that state.

Backward deterministic: given any state and tape, there is at most one transition *to* that state.

P is the class of forward deterministic turing machines, NP of non-deterministic turing machines.

Reversible Turing complete: a language that can simulate forward and backward deterministic turing machines.



What do reversible languages compute

Given a forward deterministic turing machine that computes $f(x)$,

There exists a reversible turing machine that computes $x \rightarrow (x, f(x))$.

More memory.



Variables

- ▶ All global variables
- ▶ Default value
- ▶ Modification operators
- ▶ Only support for +=, -= and ^=



Limitations

- ▶ There is no `*=` and `/=`
- ▶ A variable that occurs on the left can not occur on the right in the same statement
 - ▶ `x-=x` is forbidden

a b c

procedure main

a += 3

b -= a + 4

c += a - b



Procedures

- ▶ No parameters
- ▶ There exists version with parameters
- ▶ Pass by reference

a

```
procedure main
```

```
    call f
```

```
    uncall g
```

```
procedure f
```

```
    a += 3
```

```
procedure g
```

```
    a -= 5
```

```
    a += 1
```



Loop

```
from e1 do
  s1
loop
  s2
until e2
```

- ▶ e1 is true only the first iteration, false every other iteration
- ▶ s1 is executed after e1 on every iteration
- ▶ e2 is false until the last run
- ▶ s2 is executed if e2 is true, continue to e1



Example

fib: calculates $(n+1)$ -th and $(n+2)$ -th Fibonacci number.

```
procedure fib
```

```
    if n = 0 then
```

```
        x1 += 1    ; -- 1st Fib nr is 1.
```

```
        x2 += 1    ; -- 2nd Fib nr is 1.
```

```
    else
```

```
        n -= 1
```

```
        call fib
```

```
        x1 += x2
```

```
        x1 <=> x2
```

```
    fi x1 = x2
```



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```

```
    call fib
```

```
    x1 += x2
```

```
    x1 <=> x2
```

```
  fi x1 = x2    ; -- Used for inverting the if-statement.
```

Q: How do we calculate the inverse?

$$\mathcal{I}[\text{if } e_1 \text{ then } s_1 \text{ else } s_2 \text{ fi } e_2] = \text{if } e_2 \text{ then } \mathcal{I}[s_1] \text{ else } \mathcal{I}[s_2] \text{ fi } e_1$$


Example

fib: calculates $(n+1)$ -th and $(n+2)$ -th Fibonacci number.

```
procedure fibInverse
  if x1 = x2 then
    x2 -= 1          ; -- 2nd Fib nr is 1.
    x1 -= 1          ; -- 1st Fib nr is 1.
  else
    x1 <=> x2
    x1 -= x2
    call fibInverse
    n += 1
  fi n = 0
```



Example

fib: calculates $(n+1)$ -th and $(n+2)$ -th Fibonacci number.

► Q: What does the inverse of fib do?

```
procedure fibInverse
```

```
  if x1 = x2 then
```

```
    x2 -= 1           ; -- 2nd Fib nr is 1.
```

```
    x1 -= 1           ; -- 1st Fib nr is 1.
```

```
  else
```

```
    x1 <=> x2
```

```
    x1 -= x2
```

```
    call fibInverse
```

```
    n += 1
```

```
  fi n = 0
```



This information is not truly lost, however.

...

Entropy of the system must increase or remain equal. In this case, **heat** is dissipated into the environment.



Landauer's principle

In computers, information about past states is often lost (or erased) as computations are carried out.

...

However, the second law of thermodynamics still applies.



This means that circuits *must* dissipate some amount of heat as information gets destroyed.

...

Commonly referred to as **Landauer's principle**.



Reversible computing

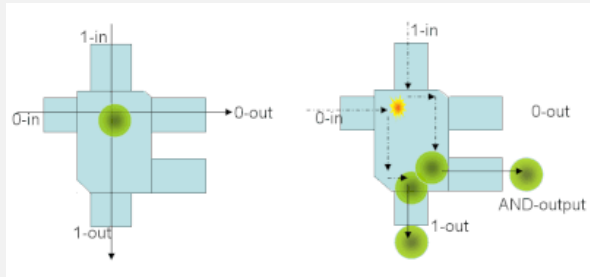


Figure 1: Billiard ball AND-gate

This is also referred to as a **Toffoli gate**.



Toffoli gates are mainly theoretical

...

But circuits with energy dissipation below the von Neumann-Landauer limit have been built.



Quantum Computing

How does all this apply to quantum physics?

The underlying physical processes of quantum computing are actually fundamentally reversible.



Similar to the frictionless billiard ball gate, information cannot leave a quantum circuit in the form of heat.

The system is said to be **logically reversible**.



Logical reversibility

Since all logical information is preserved in such systems, it is impossible to carry out certain computations

Specifically, it is impossible to carry out computations that reach a logical state that can also be reached through other paths of computation.

...

Notice the similarity with reversible Turing machines!



Relational Programming

Injective Programming

r -Turing Complete

backwards deterministic

restricted language constructs

Relational Programming

Turing Complete

backwards non-deterministic

search procedure (aka *resolution*)



Prolog basics

A logic program consists of *facts* and *rules*.

```
parent(alice, joe).  
parent(bob, joe).  
parent(joe, mary).  
parent( gloria, mary).
```

```
ancestor(X, Y) :- parent(X, Y).  
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
```

```
descendant(X, Y) :- ancestor(Y, X).
```



The user can then *query* the runtime system, as such:

```
?- ancestor(alice, mary).  
true.
```

```
?- parent(X, mary).  
X = joe;  
X = gloria.
```

```
?- ancestor(X, mary), not parent(X, mary).  
X = alice;  
X = bob.
```



Demonstration - Type Predicate

Assume a type predicate, relating expressions with types:

`type(expr, t) :-`



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```
type(expr, t) :- ... .
```

You would normally use it to perform *type-checking*:

```
?- type(1 + 1, int).
```

```
true.
```

```
?- type(1 + 1, string).
```

```
false.
```



But you can also perform *type-inference*:

```
?- type(1 + 1, Type).  
Type = int.  
?- type("hello world", Type).  
Type = string.  
?- type( $\lambda x$ :int -> x, Type).  
Type = int -> int.  
?- type( $\lambda x$  -> x, int -> Type).  
Type = int.
```



Going in the reverse direction, you can generate programs:

```
?- type(Expr, int).  
Expr = 1;  
Expr = 2;  
...  
Expr = 1 + 1;  
Expr = 1 + 2;  
...  
Expr = if true then 1 else 1;  
...
```

Of course, this does not make much sense without a sufficiently expressive type system.



Demonstration - Relational Interpreter

Assume you have implemented a *relational interpreter*:

```
eval(program, result) :- ... .
```

```
?- eval(map (+ 1) [1 2 3], Result).
```

```
Result = [2 3 4].
```



Demonstration - Relational Interpreter

Assume you have implemented a *relational interpreter*:

```
eval(program, result) :- ... .
```

```
?- eval(map (+ 1) [1 2 3], Result).
```

```
Result = [2 3 4].
```

Non-deterministic constructs are also natural:

```
?- eval(amb [a, b, c], Result).
```

```
Result = a;
```

```
Result = b;
```

```
Result = c.
```



But you can also perform *program synthesis* by-example:

```
?- eval(F 1, 2),...,eval(map F [1 2 3], [2 3 4]).
```

```
...
```

```
F = λx -> x + 1;
```

```
...
```

```
F = λx -> x - 10 + 10 + 1;
```

```
...
```



But you can also perform *program synthesis* by-example:

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?- eval(F 1, 2),...,eval(map F [1 2 3], [2 3 4]).
```

```
...
```

```
F = λx -> x + 1;
```

```
...
```

```
F = λx -> x - 10 + 10 + 1;
```

```
...
```

Quine generation is straightforward:

```
?- eval(Quine, Quine).
```

```
...
```

```
Quine = (λa -> a ++ show a) "(λa -> a ++ show a) ";
```

```
...
```



In practice, bi-directionality breaks with the usage of *extra-logical* features:

- ▶ **Variable projection:** inspecting values at runtime
- ▶ **Cut (!):** disables backtracking in certain places
- ▶ **Assert/Retract:** dynamically insert/remove facts



Logic Programming IRL

In practice, bi-directionality breaks with the usage of *extra-logical* features:

- ▶ **Variable projection:** inspecting values at runtime
- ▶ **Cut (!):** disables backtracking in certain places
- ▶ **Assert/Retract:** dynamically insert/remove facts

MiniKanren is a more recent logic programming language, which avoids extra-logical features (as much as possible).



Higher abstraction

- ▶ Relational programming, as well as functional programming, both belong to the *declarative* paradigm.
- ▶ They both focus on *what* a program does, rather than *how*.



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- ▶ Relational programming, as well as functional programming, both belong to the *declarative* paradigm.
- ▶ They both focus on *what* a program does, rather than *how*.

Question

How can we combine them, to get the best of both worlds?



Hanus: Janus embedded in Haskell

In our research project, we use *TemplateHaskell* and *QuasiQuotation* to embed Janus in Haskell:

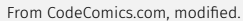
```
[hanus| procedure encode(im :: Image, ret :: [Byte]) {  
    -- Janus commands with antiquotation  
}|]  
  
encode :: Image -> [Byte]  
encode = call encode  
  
decode :: [Byte] -> Image  
decode = uncall encode
```



Come and check out our poster in de Vagant!



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References

Axelsen, Holger Bock, and Robert Glück. “What do reversible programs compute?” FoSSaCS. 2011. Yokoyama, Tetsuo, and Robert Glück. “A reversible programming language and its invertible self-interpreter.” Proceedings of the 2007 ACM SIGPLAN symposium on Partial evaluation and semantics-based program manipulation. ACM, 2007.

