

Concepts of programming languages

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A reversible programming language.

Not turing complete!



Reversibility

Every statement can be reverted. No history is stored.

`x += y * 3`



Reversibility

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$x \ += \ y \ * \ 3$

$x \ -= \ y \ * \ 3$



Injective functions

Reversible languages can only compute injective functions.

$$\forall x, y : f(x) = f(y) \implies x = y \quad (1)$$

Every output has only a single input.



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$$h(x) = (x, g(x)) \quad (2)$$



Turing completeness

Turing machines can compute non-injective functions.

Reversible languages are not turing complete.

Reversible Turing complete.



Turing machines

Infinite tape of memory

Finite set of states

Transition function

- ▶ Current state
- ▶ Current symbol on tape
- ▶ Write symbol
- ▶ Move tape pointer
- ▶ Next state



Turing machines

Forward deterministic: given any state and tape, there is at most one transition *from* that state.

Backward deterministic: given any state and tape, there is at most one transition *to* that state.

P is the class of forward deterministic turing machines, NP of non-deterministic turing machines.

Reversible Turing complete: a language that can simulate forward and backward deterministic turing machines.



What do reversible languages compute

Given a forward deterministic turing machine that computes $f(x)$,

There exists a reversible turing machine that computes $x \rightarrow (x, f(x))$.

More memory.



Example

fib: calculates $(n+1)$ -th and $(n+2)$ -th Fibonacci number.

```
procedure fib
  if n = 0 then
    x1 += 1      ; -- 1st Fib nr is 1.
    x2 += 1      ; -- 2nd Fib nr is 1.
  else
    n -= 1
    call fib
    x1 += x2
    x1 <=> x2
  fi x1 = x2
```



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```
    call fib
```

```
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```

```
    x1 <=> x2
```

```
  fi x1 = x2      ; -- Used for inverting the if-statement.
```

Q: How do we calculate the inverse?

$$\mathcal{I}[\text{if } e_1 \text{ then } s_1 \text{ else } s_2 \text{ fi } e_2] = \text{if } e_2 \text{ then } \mathcal{I}[s_1] \text{ else } \mathcal{I}[s_2] \text{ fi } e_1$$


Example

fib: calculates $(n+1)$ -th and $(n+2)$ -th Fibonacci number.

```
procedure fibInverse
  if x1 = x2 then
    x2 -= 1          ; -- 2nd Fib nr is 1.
    x1 -= 1          ; -- 1st Fib nr is 1.
  else
    x1 <=> x2
    x1 -= x2
    call fibInverse
    n += 1
  fi n = 0
```



Example

fib: calculates $(n+1)$ -th and $(n+2)$ -th Fibonacci number.

- Q: What does the inverse of fib do?

```
procedure fibInverse
```

```
  if x1 = x2 then
```

```
    x2 -= 1           ; -- 2nd Fib nr is 1.
```

```
    x1 -= 1           ; -- 1st Fib nr is 1.
```

```
  else
```

```
    x1 <=> x2
```

```
    x1 -= x2
```

```
    call fibInverse
```

```
    n += 1
```

```
  fi n = 0
```



Relational Programming

Injective Programming

r -Turing Complete

backwards deterministic

restricted language constructs

Relational Programming

Turing Complete

backwards non-deterministic

search procedure (aka *resolution*)



Prolog basics

A logic program consists of *facts* and *rules*.

```
parent(alice, joe).  
parent(bob, joe).  
parent(joe, mary).  
parent( gloria, mary).
```

```
ancestor(X, Y) :- parent(X, Y).  
ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
```

```
descendant(X, Y) :- ancestor(Y, X).
```



The user can then *query* the runtime system, as such:

```
?- ancestor(alice, mary).  
true.
```

```
?- parent(X, mary).  
X = joe;  
X = gloria.
```

```
?- ancestor(X, mary), not parent(X, mary).  
X = alice;  
X = bob.
```



Demonstration - Type Predicate

Assume a type predicate, relating expressions with types:

```
type(expr, t) :- ... .
```



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You would normally use it to perform *type-checking*:

```
?- type(1 + 1, int).
```

```
true.
```

```
?- type(1 + 1, string).
```

```
false.
```



But you can also perform *type-inference*:

```
?- type(1 + 1, Type).  
Type = int.  
?- type("hello world", Type).  
Type = string.  
?- type( $\lambda x$ :int -> x, Type).  
Type = int -> int.  
?- type( $\lambda x$  -> x, int -> Type).  
Type = int.
```



Going in the reverse direction, you can generate programs:

```
?- type(Expr, int).  
Expr = 1;  
Expr = 2;  
...  
Expr = 1 + 1;  
Expr = 1 + 2;  
...  
Expr = if true then 1 else 1;  
...
```

Of course, this does not make much sense without a sufficiently expressive type system.



Demonstration - Relational Interpreter

Assume you have implemented a *relational interpreter*:

```
eval(program, result) :- ... .
```

```
?- eval(map (+ 1) [1 2 3], Result).
```

```
Result = [2 3 4].
```



Demonstration - Relational Interpreter

Assume you have implemented a *relational interpreter*:

```
eval(program, result) :- ... .
```

```
?- eval(map (+ 1) [1 2 3], Result).
```

```
Result = [2 3 4].
```

Non-deterministic constructs are also natural:

```
?- eval(amb [a, b, c], Result).
```

```
Result = a;
```

```
Result = b;
```

```
Result = c.
```



But you can also perform *program synthesis* by-example:

```
?- eval(F 1, 2),...,eval(map F [1 2 3], [2 3 4]).
```

```
...
```

```
F = λx -> x + 1;
```

```
...
```

```
F = λx -> x - 10 + 10 + 1;
```

```
...
```



But you can also perform *program synthesis* by-example:

```
?- eval(F 1, 2),...,eval(map F [1 2 3], [2 3 4]).
```

```
...
```

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F = λx -> x + 1;
```

```
...
```

```
F = λx -> x - 10 + 10 + 1;
```

```
...
```

Quine generation is straightforward:

```
?- eval(Quine, Quine).
```

```
...
```

```
Quine = (λa -> a ++ show a) "(λa -> a ++ show a) ";
```

```
...
```



Logic Programming IRL

In practice, bi-directionality breaks with the usage of *extra-logical* features:

- ▶ **Variable projection:** inspecting values at runtime
- ▶ **Cut (!):** disables backtracking in certain places
- ▶ **Assert/Retract:** dynamically insert/remove facts



Logic Programming IRL

In practice, bi-directionality breaks with the usage of *extra-logical* features:

- ▶ **Variable projection:** inspecting values at runtime
- ▶ **Cut (!):** disables backtracking in certain places
- ▶ **Assert/Retract:** dynamically insert/remove facts

MiniKanren is a more recent logic programming language, which avoids extra-logical features (as much as possible).



Higher abstraction

- ▶ Relational programming, as well as functional programming, both belong to the *declarative* paradigm.
- ▶ They both focus on *what* a program does, rather than *how*.



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Question

How can we combine them, to get the best of both worlds?



Hanus: Janus embedded in Haskell

In our research project, we use *TemplateHaskell* and *QuasiQuotation* to embed Janus in Haskell:

```
[hanus| procedure encode(im :: Image, ret :: [Byte]) {  
    -- Janus commands with antiquotation  
}|]  
  
encode :: Image -> [Byte]  
encode = call encode  
  
decode :: [Byte] -> Image  
decode = uncall encode
```



Come and check out our poster in de Vagant!



Thanks! Questions?



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References

Axelsen, Holger Bock, and Robert Glück. “What do reversible programs compute?” FoSSaCS. 2011. Yokoyama, Tetsuo, and Robert Glück. “A reversible programming language and its invertible self-interpreter.” Proceedings of the 2007 ACM SIGPLAN symposium on Partial evaluation and semantics-based program manipulation. ACM, 2007.

