

#### Universidad de Cádiz

FACULTAD DE CIENCIAS

# GEOMETRIC INVARIANTS AND INNER STRUCTURE

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#### GEOMETRIC INVARIANTS AND INNER STRUCTURE

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Firma del Doctorando

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Cádiz, Septiembre 2023

## Acknowledgements

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### **Abstract**

First thing, a summary of this dissertation in both English and Spanish will be presented.

#### 1.1 Abstract

#### 1.2 Resumen

### Introduction

### **Materials and Methods**

## Chapter 1

## Chapter 2

## Chapter 3

APPENDIX

### **Conclusions**

APPENDIX B

### **Notation**

Next, we will proceed to explain the main notation followed in this dissertation:

$B_X$	the closed unit ball in $X$
$U_X$	the open unit ball in $X$
$S_X$	$\dots$ the unit sphere in $X$
$B_X(x,r)$	the closed ball of center $x$ and radius $r$ in $X$
$U_X(x,r)$	$\dots$ the open ball of center $x$ and radius $r$ in $X$
$S_X(x,r)$	the sphere of center $x$ and radius $r$ in $X$
int( <i>M</i> )	the topological interior of $M$
$\operatorname{int}_A(M)$	the topological interior of $M$ relative to $A$
cl(M)	the topological closure of $M$
cl <sub>A</sub> (M)	the topological closure of $M$ relative to $A$
bd( <i>M</i> )	the topological boundary of $M$

#### **B. NOTATION**

$\mathrm{bd}_A(M)$	$\dots$ the topological boundary of $M$ relative to $A$
ext( <i>M</i> )	the set of extreme points $F$
X*	$\dots$ the topological dual of $X$
X**	$\dots$ the topological bidual of $X$
F(f,A) th	e supporting hyperplane relative to $f \in X^*$ in $A$
F(f)	. the supporting hyperplane relative to $f$ in $B_X$
$\exp(B_X)$	the set of exposed points of $B_X$
E(f) is	the edge of the unit ball with respect to $f \in S_{X^*}$
$rot(B_X)$	the set of rotund points of $B_X$
$pexp(B_X)$	the set of proper exposed points of $B_X$
$\mathscr{C}_X$	the set of facets of $B_X$
$\operatorname{st}(x,B_X)$	$\dots$ the starlike set of center $x$
$smo(B_X)$	the set of the smooth points of $B_X$
$\mathscr{P}(\mathscr{X})$	the power set of ${\mathcal X}$
ν	the spherical image map from $S_X$ to $S_{X^*}$
$frm(B_X)$	the frame of the unit ball
inn( <i>M</i> )	$\dots$ the set of inner points of $M$
span(M)	$\dots$ the linear span of $M$
span(M)	$\dots$ the closed linear span of $M$
co( <i>M</i> )	$\dots$ the convex hull of $M$
<del>co</del> ( <i>M</i> )	the closed convex hull of $M$
inter( <i>M</i> )	the set of the internal points of $M$
$\operatorname{adj}(M)$	$\dots$ the set of the adjacent elements of $M$
sadj( <i>M</i> )	the set of the strongly adjacent elements of $M$
suppv( <i>T</i> )	the set of supporting vectors of the operator $T$
$suppv_1(x^*)$ the	set of 1-supporting vectors of the functional $x^*$

nsupp(M)the set of non-support points of M $\mu_A$ the Minkowski functional of AMUpthe Mazur-Ulman propertyPpthe P-property or property PIpthe I-property or inner propertyFpthe F-property or flat property

# APPENDIX

## Appendix