

# COM303: Digital Signal Processing

## Lecture 21: Image Compression

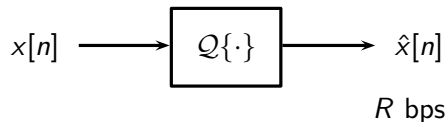
- ▶ introduction to quantization
- ▶ the problem of image compression
- ▶ the JPEG standard

quantization: the basics

# Quantization

- ▶ digital devices can only deal with integers ( $R$  bits per sample)
- ▶ we need to map the range of a signal onto a finite set of values
- ▶ e.g. pixel sensor level is represented with 8 bits
- ▶ irreversible loss of information  $\rightarrow$  quantization noise

# Uniform scalar quantization

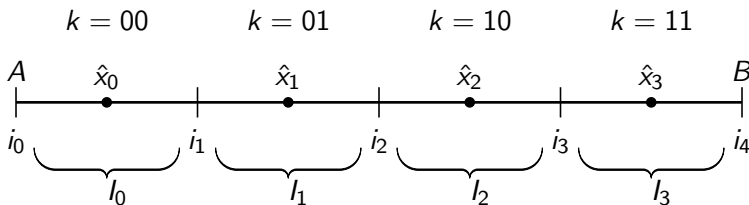


The simplest quantizer:

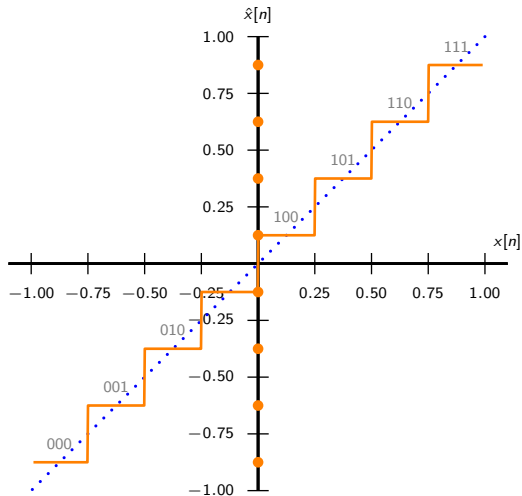
- ▶ each sample is encoded individually (hence *scalar*) using  $R$  bits
- ▶ each sample is quantized independently (memoryless quantization)
- ▶ the input range is divided into  $2^R$  equal-size intervals

# Uniform scalar quantization

- ▶ input assumed uniformly distributed over  $[A, B]$
- ▶ range is split into  $2^R$  *equal* intervals of width  $\Delta = (B - A)2^{-R}$
- ▶ quantized value is interval's midpoint



# Uniform 3-Bit quantization function



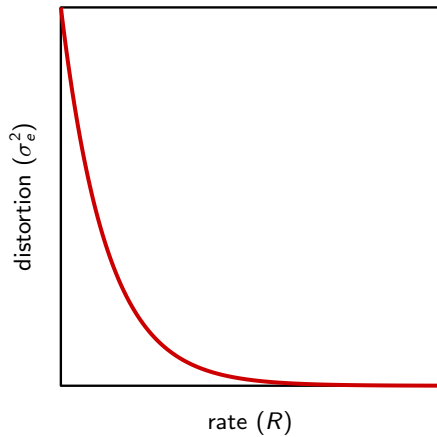
# Quantization Error

$$e[n] = \mathcal{Q}\{x[n]\} - x[n] = \hat{x}[n] - x[n]$$

- ▶ model  $x[n]$  as a stochastic process
- ▶ model error as a white noise sequence:
  - error samples are uncorrelated
  - all error samples have the same distribution



# Rate/Distortion Curve



# Dynamic range after quantization

Convenient rule of thumb:

► signal to noise ratio:

$$\text{SNR} = 2^{2R}$$

► in dBs:

$$\text{SNR}_{\text{dB}} = 10 \log_{10} 2^{2R} \approx 6R \text{ dB}$$

# The “6dB/bit” rule of thumb

- ▶ a compact disk has 16 bits/sample:

$$\max \text{SNR} = 96\text{dB}$$

- ▶ a DVD has 24 bits/sample:

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## image compression fundamentals

## Remember our thought experiment...

- ▶ consider all possible  $256 \times 256$ , 8bpp images
- ▶ each image is 524,288 bits
- ▶ total number of possible images:  $2^{524,288} \approx 10^{157,826}$
- ▶ number of atoms in the universe:  $10^{82}$

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# How many bits per image?

## Another thought experiment

- ▶ take *all* images in the world and list them in an “encyclopedia of images”
- ▶ to indicate an image, simply give its number
- ▶ on the Internet:  $M = 50$  billion
- ▶ raw encoding: 524,288 bits per image
- ▶ enumeration-based encoding:  $\log_2 M \approx 36$  bits per image
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# Compression

Another approach:

- ▶ exploit “physical” redundancy
- ▶ allocate bits for things that matter (e.g. edges)
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- ▶ some information is discarded: *lossy* compression



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- ▶ compressing at block level
- ▶ using a suitable transform (i.e., a change of basis)
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- ▶ compress remote regions independently

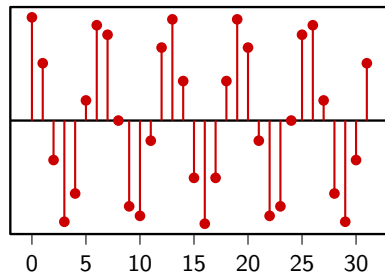
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# Transform coding

A simple example:

- ▶ take a DT signal, assume  $R$  bits per sample
- ▶ storing the signal requires  $NR$  bits
- ▶ now you take the DFT and it looks like this
- ▶ in theory, we can just code the two nonzero DFT coefficients!

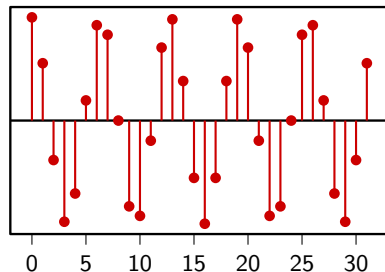




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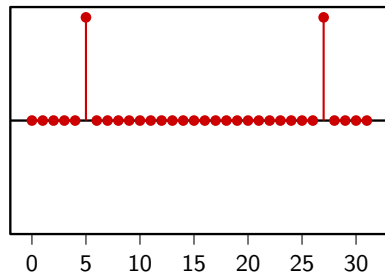
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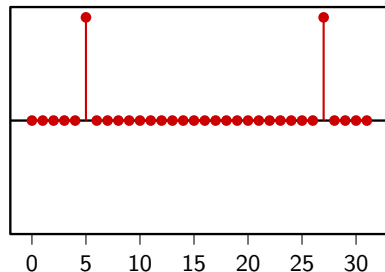
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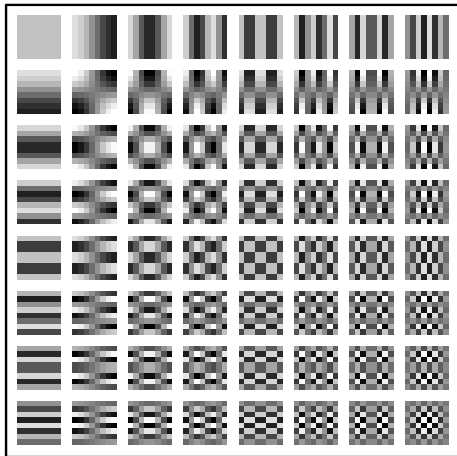
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## 2D-DCT

$$C[k_1, k_2] = \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} x[n_1, n_2] \cos \left[ \frac{\pi}{N} \left( n_1 + \frac{1}{2} \right) k_1 \right] \cos \left[ \frac{\pi}{N} \left( n_2 + \frac{1}{2} \right) k_2 \right]$$

$$C[k_1, k_2] \in \mathbb{R}$$

## DCT basis vectors for an $8 \times 8$ image





# Smart quantization of the DCT coefficients

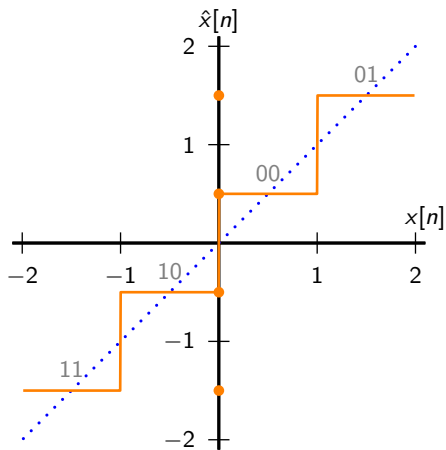
- ▶ deadzone
- ▶ variable step (fine to coarse)

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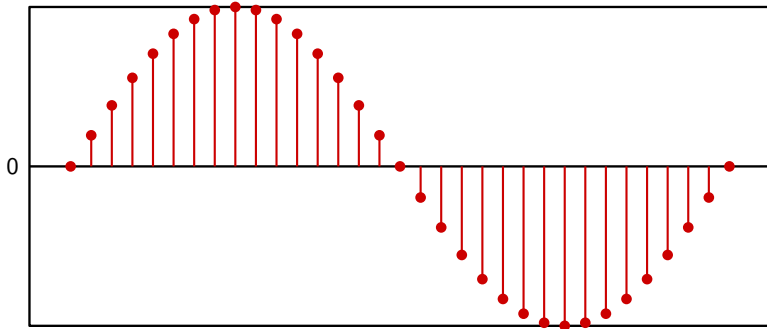
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# Standard Uniform Quantization

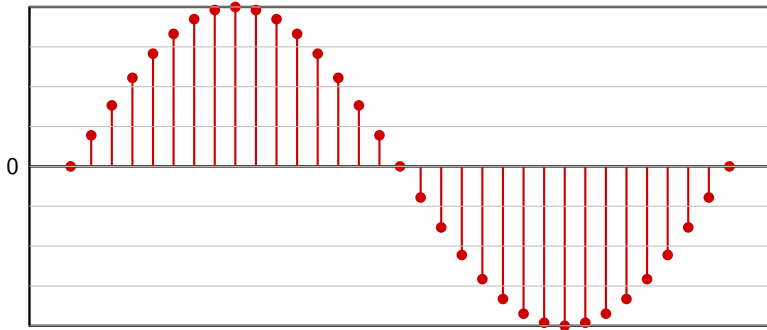
$$\hat{x} = \text{floor}(x) + 0.5$$



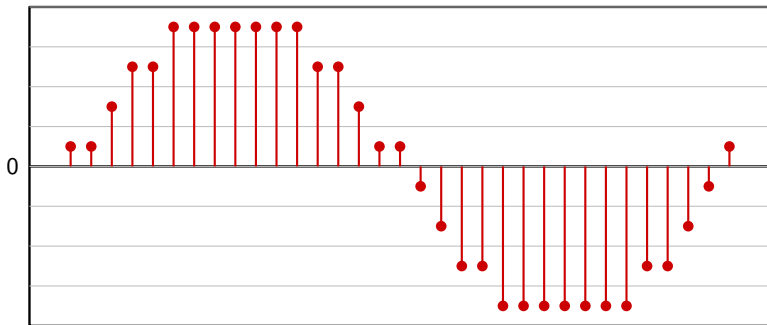
## Quantizing a full-range signal



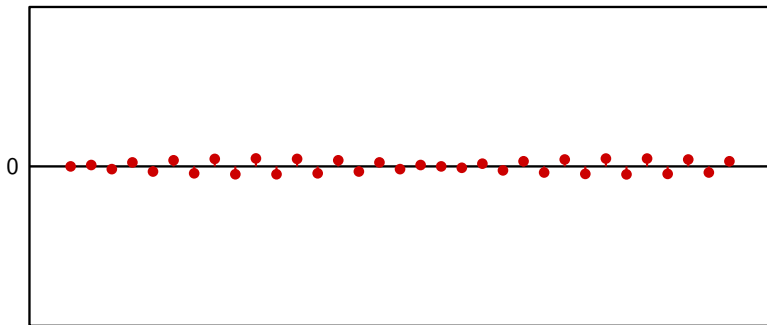
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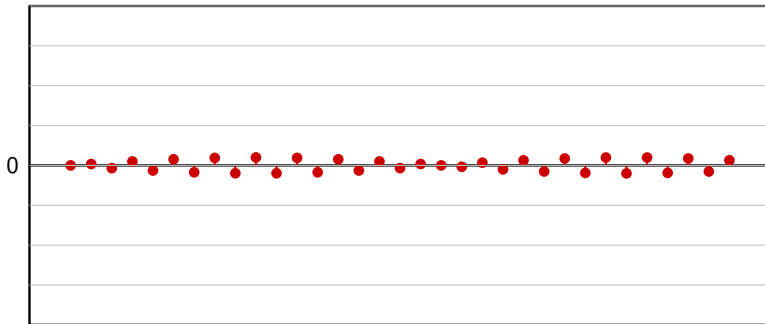
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## Quantizing a small, noise-like signal

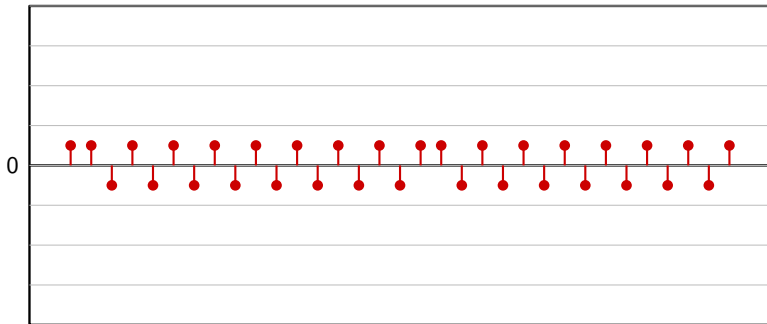


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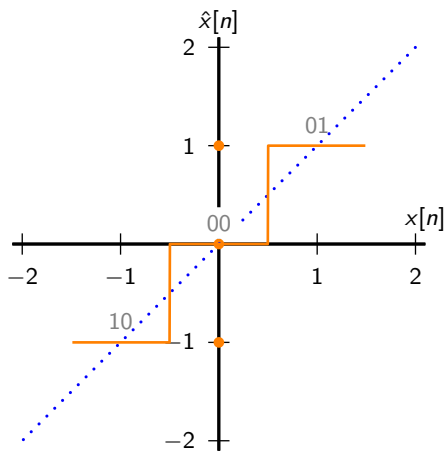


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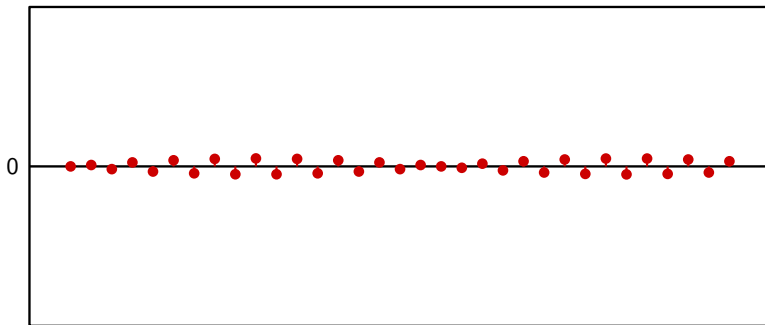


# Deadzone Quantization

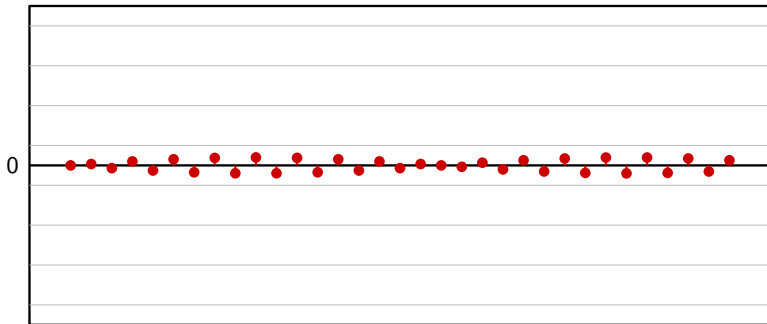
$$\hat{x} = \text{round}(x)$$



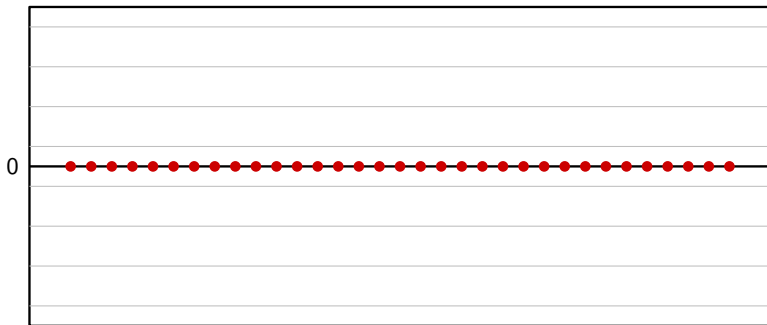
## Quantizing a small, noise-like signal, take 2



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# Entropy coding

- ▶ minimize the effort to encode a certain amount of information
- ▶ associate short symbols to frequent values and vice-versa
- ▶ if it sounds familiar it's because it is...

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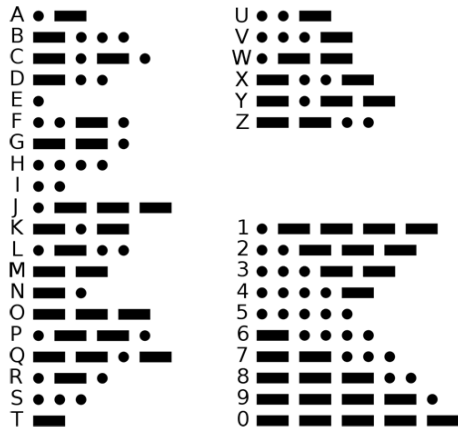
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# Entropy coding



the JPEG standard

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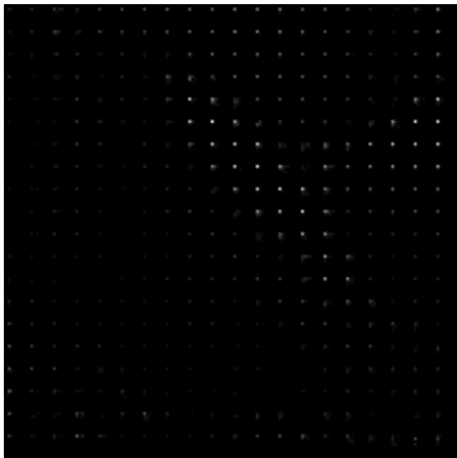
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- ▶ run-length encoding and Huffman coding

## DCT coefficients of image blocks (detail)





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# Smart quantization

- ▶ most coefficients are negligible → captured by the deadzone
- ▶ some coefficients have a higher visual impact
- ▶ find out the critical coefficients by experimentation
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## Psychovisually-tuned quantization table

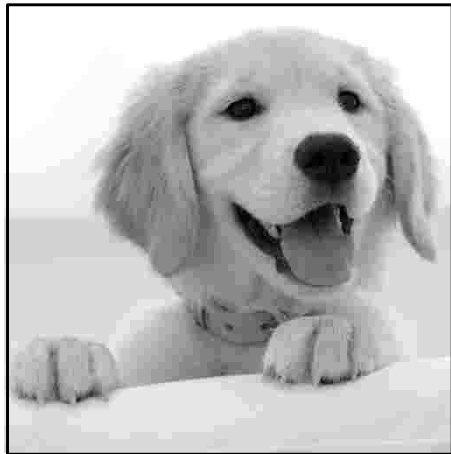
$$\hat{c}[k_1, k_2] = \text{round}(c[k_1, k_2]/Q[k_1, k_2])$$

$$Q = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

## Advantages of tuned quantization intervals at 0.2bpp



uniform



tuned

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# Efficient coding

- ▶ most coefficients are small, decreasing with index
- ▶ use zigzag scan to maximize ordering
- ▶ quantization will create long series of zeros

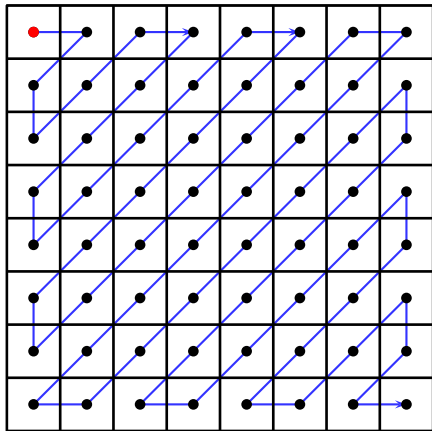
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## Zigzag scan



## Example

$$\begin{bmatrix} 100 & -60 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

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[illegible]

# Runlength encoding

- ▶ the DC value is encoded differentially wrt previous block
- ▶ each nonzero AC value is encoded as the triple

$$[(r, s), c]$$

- $r$  is the *runlength* i.e. the number of zeros before the current value
- $s$  is the *category* i.e. the number of bits needed to encode the value
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$[100], [(0, 6), -60], [(4, 3), 6], [(3, 4), 13], [(8, 1), -1], [(0, 0)]$

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$[100]$ ,  $[(0, 6), -60]$ ,  $[(4, 3), 6]$ ,  $[(3, 4), 13]$ ,  $[(8, 1), -1]$ ,  $[(0, 0)]$

# The runlength-size pairs

- ▶ by design,  $(r, s) \in \mathcal{A}$  with  $|\mathcal{A}| = 256$
- ▶ in theory, 8 bits per pair
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- ▶ by design,  $(r, s) \in \mathcal{A}$  with  $|\mathcal{A}| = 256$
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# Variable-length encoding

great idea: shorter binary sequences for common symbols

A	• ■	U	• • ■
B	■ ■ • •	V	• • • ■
C	■ ■ • ■ •	W	• ■ ■ ■
D	■ ■ • •	X	■ ■ • • ■
E	•	Y	■ ■ • ■ ■
F	• • ■ ■ •	Z	■ ■ ■ • •
G	■ ■ ■ ■ •		
H	• • • •		
I	• •		
J	• ■ ■ ■ ■		
K	■ ■ • ■ ■	1	• ■ ■ ■ ■ ■
L	• ■ ■ • •	2	• • ■ ■ ■ ■
M	■ ■ ■ ■	3	• • • ■ ■ ■
N	■ ■ •	4	• • • • ■ ■
O	■ ■ ■ ■ ■	5	• • • • •
P	• ■ ■ ■ ■ •	6	■ ■ • • •
Q	■ ■ ■ ■ • ■	7	■ ■ ■ • • •
R	• ■ ■ •	8	■ ■ ■ ■ • •
S	• • •	9	■ ■ ■ ■ ■ •
T	■	0	■ ■ ■ ■ ■ ■

# Variable-length encoding

however: if symbols have different lengths, we must know how to parse them!

- ▶ in English, spaces separate words → extra symbol (wasteful)
- ▶ in Morse code, pauses separate letters and words (wasteful)
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# Prefix-free codes

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- ▶ can parse a bitstream sequentially with no look-ahead
- ▶ extremely easy to understand graphically...

# Prefix-free codes

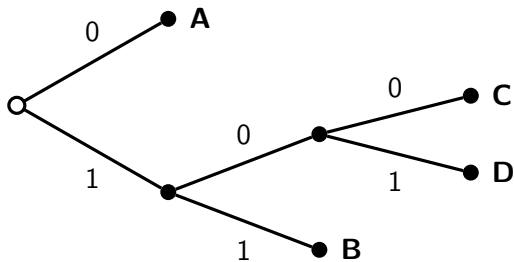
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## Prefix-free codes

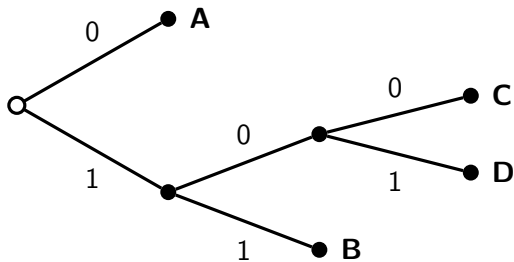
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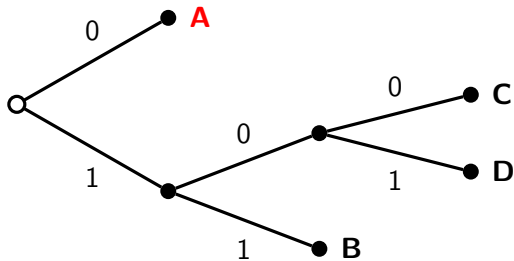
001100110101100

## Prefix-free code



001100110101100

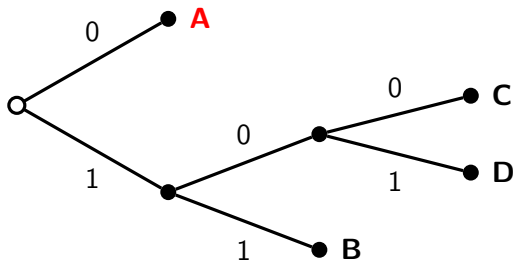
## Prefix-free code



001100110101100

A

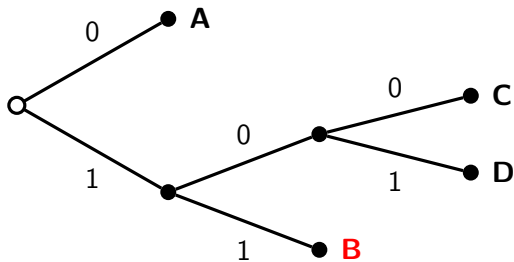
## Prefix-free code



001100110101100

AA

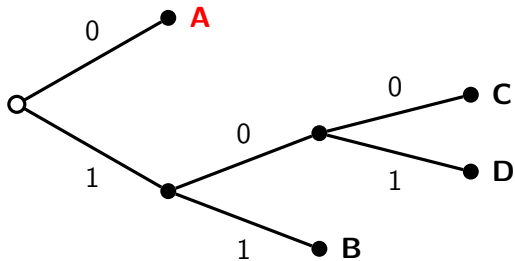
## Prefix-free code



00**11**00110101100

AAB

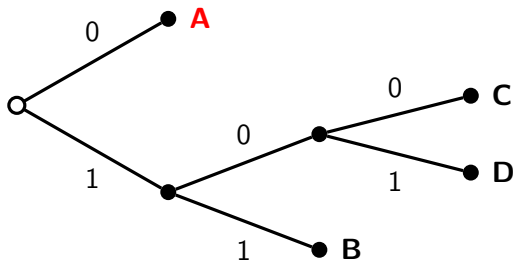
## Prefix-free code



001100110101100

AABA

## Prefix-free code

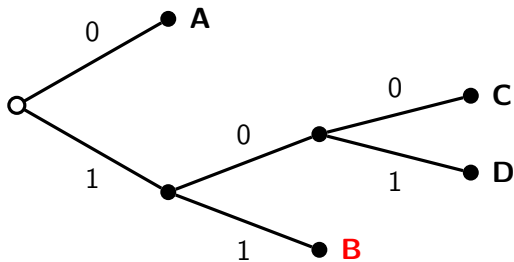


001100110101100

AABAA



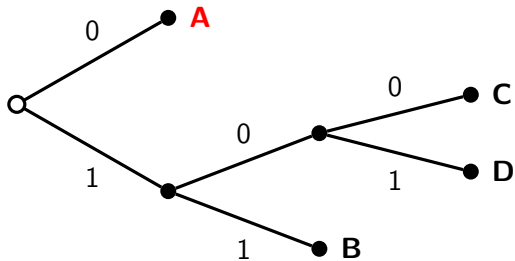
## Prefix-free code



001100**1**10101100

AABAAB

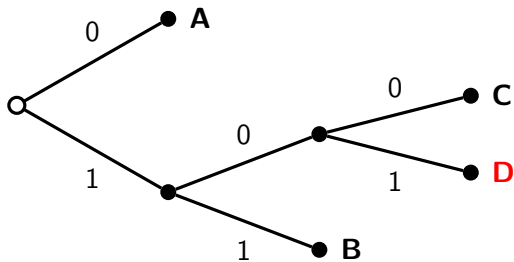
## Prefix-free code



001100110101100

AABAABA

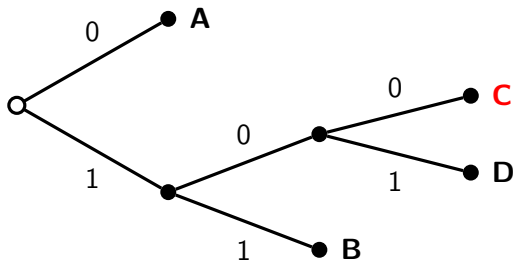
## Prefix-free code



001100110101100

AABAABAD

## Prefix-free code



001100110101100

AABAABADC

# Entropy coding

goal: minimize message length

- ▶ assign short sequences to more frequent symbols
- ▶ the Huffman algorithm builds the optimal code for a set of symbol probabilities
- ▶ in JPEG, you can use a “general-purpose” Huffman code or build your own (but then you pay a “side-information” price)

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## Example

► four symbols: A, B, C, D

► probability table:

$$p(A) = 0.38$$

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$$p(D) = 0.2$$



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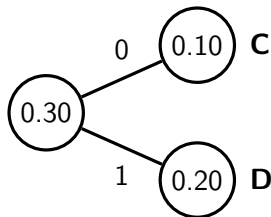
$$p(A) = 0.38$$

$$p(B) = 0.32$$

$$p(C) = 0.1$$

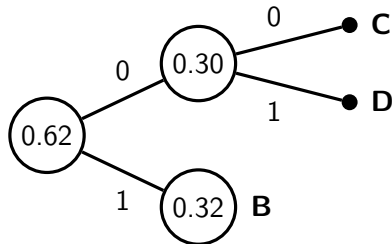
$$p(D) = 0.2$$

## Building the Huffman code



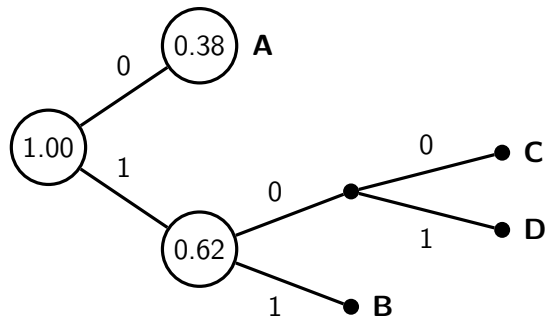
$$p(A) = 0.38 \quad p(B) = 0.32 \quad p(C) = 0.1 \quad p(D) = 0.2$$

## Building the Huffman code



$$p(A) = 0.38 \quad p(B) = 0.32 \quad p(C + D) = 0.3$$

# Huffman Coding



$$p(A) = 0.38 \quad p(B + C + D) = 0.62$$

# Conclusions

- ▶ JPEG is a very complex and comprehensive standard:
  - lossless, lossy
  - color, B&W
  - progressive encoding
  - HDR (12bpp) for medical imaging
- ▶ JPEG is VERY good:
  - compression factor of 10:1 virtually indistinguishable
  - rates of 1bpp for RGB images acceptable (25:1 compression ratio)
- ▶ other important compression schemes:
  - TIFF, JPEG2000
  - MPEG (MP3)