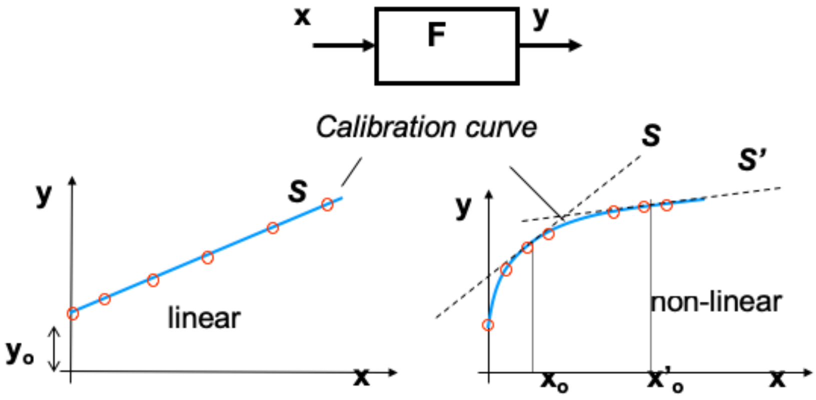
CHAPTER 2

NOISE IN BIOMEDICAL INSTRUMENTATION

Static characteristics of a measurement system (reminder)



- Offset

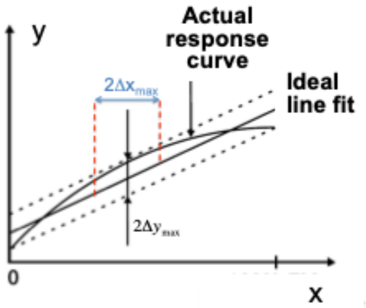
$$y = S \cdot x + y_o$$

- Sensitivity

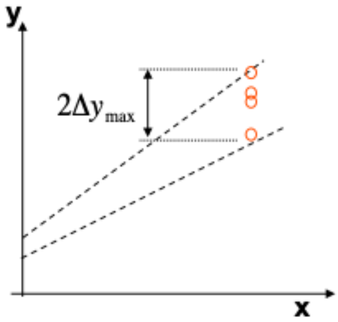
$$S = \left(\frac{\Delta y}{\Delta x} \right)_{x=x_o}$$

Static characteristics (reminder)

- Linearity



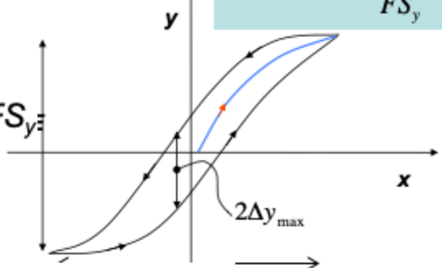
- Repeatability



- Hysteresis

$$Error\ (\%) = \frac{\pm\Delta y_{max}}{FS_y} = \frac{\pm\Delta x_{max}}{FS}$$

Full Scale: FS_y



Full Scale: FS

- **Stability:** conserving same output when input is constant over time
- **Drift:** low change over time

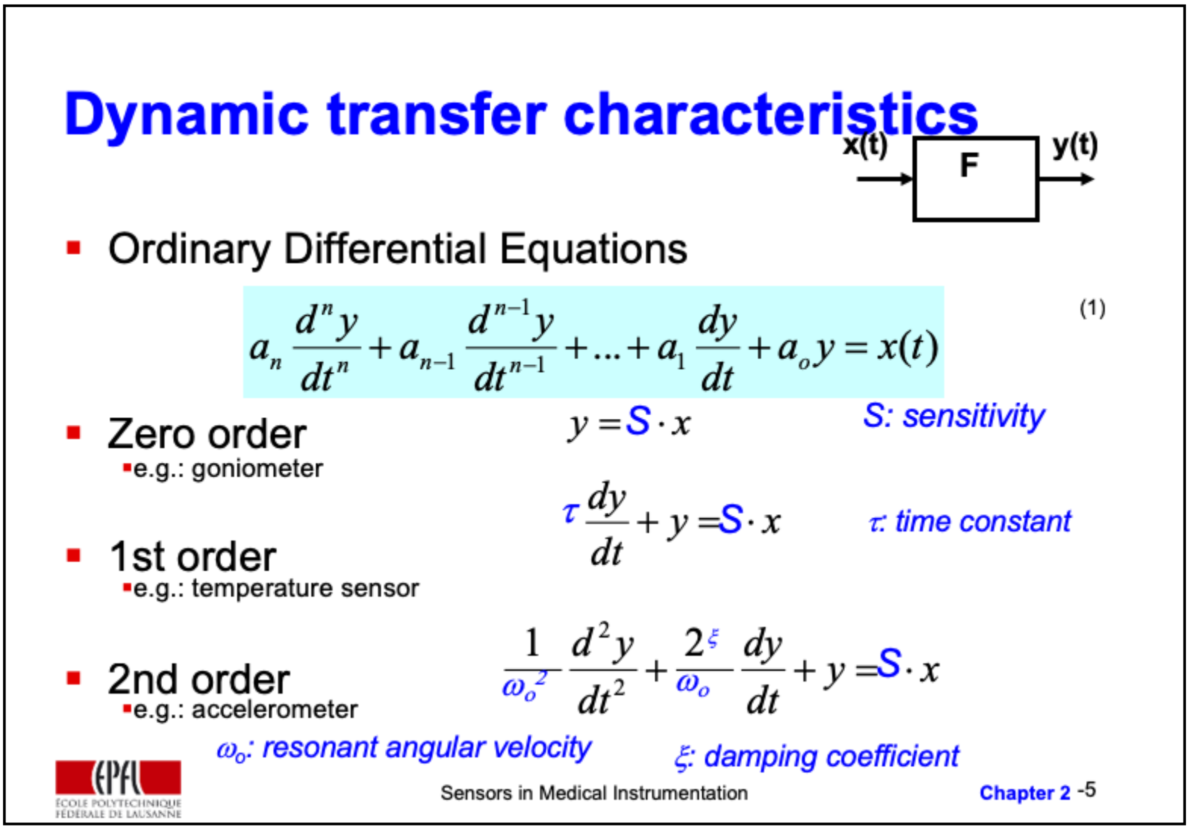
Example: error calculation of a pressure sensor

Pressure range	FS	0-200	kPa
Offset	Voff	±1	mV
Sensitivity	ΔV/ΔP	0.2	mV/kPa
Linearity		±0.5%	FS
Hysteresis		±0.5%	FS
Temperature effect on FS (0 to 50°C, Tref=25°C)	T_FS	±2%	FS
Temperature effect on Offset (0 to 50°C, Tref=25°C)	T_OFF	±1	mV
Offset Stability		±0.5%	FS

$$Error_{max} = \pm \sum_i |\Delta x_i|$$

$$Error_{probable} = \pm \sqrt{\Delta x_i^2}$$

Error $_{OFF}$=±5kPa, Error$_{lin}$=±1kPa, Eror$_{Hyst}$=±1kPa
Error$_{T,_FS}$=±4kPa, Error$_{T,_OFF}$=±5kPa, Error $_{Stab}$=±1kPa
Error maximum=±(5+1+1+4+5+1)=±17kPa
Error probable=±$\sqrt{(5^2+1^2+1^2+4^2+5^2+1^2)}$=±8.3kPa

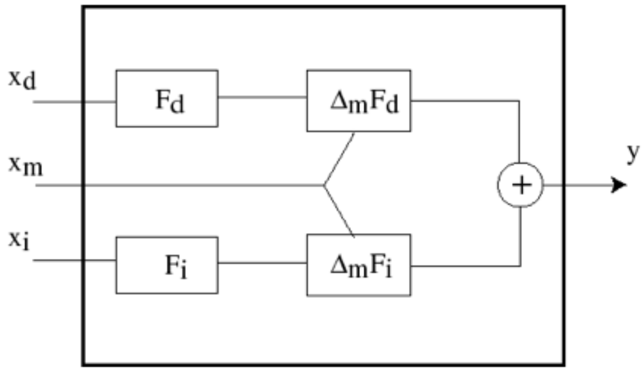


D’une manière générale, la variation en fonction du temps des valeurs de la grandeur d’entrée x génère des variations des valeurs de la grandeur de sortie. On peut modéliser cette correspondance par l’équation différentielle (1) ou n désigne l’ordre du système de mesure dynamique. En général, on limite la modélisation aux système d’ordre 0, 1 et 2.

Les coefficients a_n sont déterminés théoriquement par modélisation, ou expérimentalement par l’analyse du comportement du système soumis à des sollicitations appropriées. Ces sollicitations sont les formes typiques du signal x: impulsion, saut unitaire, sinusoïdal.

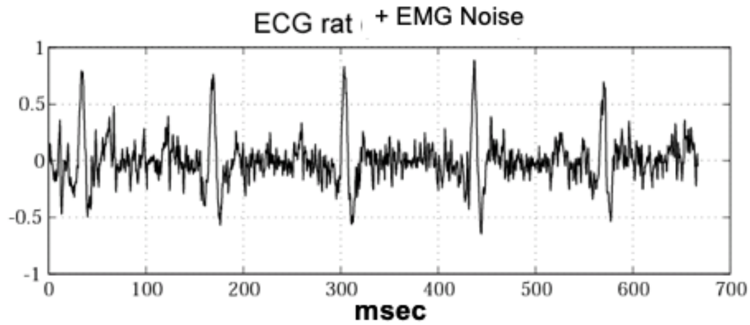
General model

- x_d : desired value, measurand, with transfer function F_d
- x_m : modifying value
- x_i : interfering value, with transfer function F_i



Examples (M: modifying, I: Interfering)

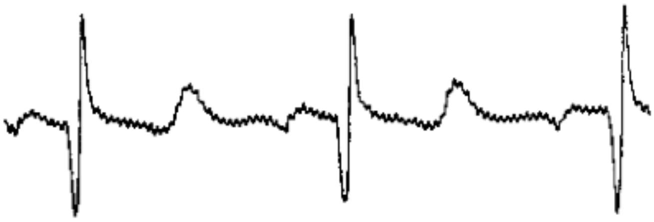
- Power line interference, 50Hz, white noise **I**
- Electrode movement **M**
- Signals emanating from other devices nearby: **I** electrosurgery
- EMG noise when measuring the ECG. **I**



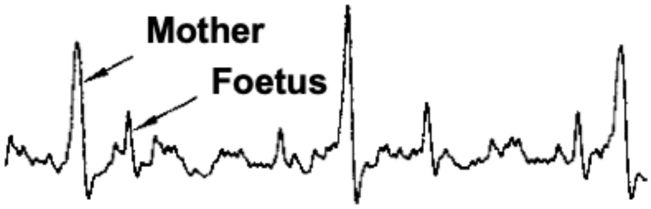
Examples of interfering value

- The heart rate of the mother can overshadow that of the fœtus

a)



b)



Noise characteristics

- Amplitude
  - Ratio of signal to noise
    Signal to **N**oise **R**atio: **SNR**

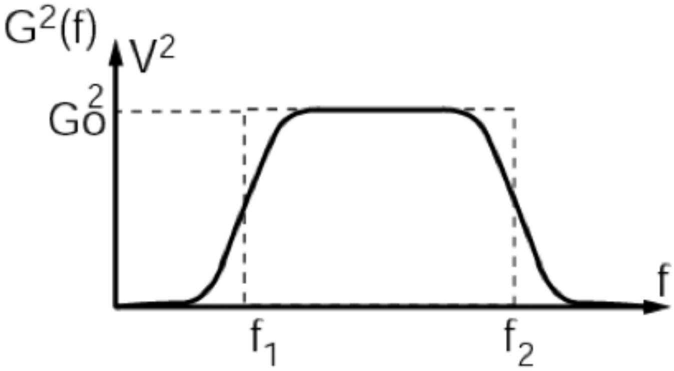
$$SNR = \frac{s}{n} = \frac{(\text{desired value}, V)^2}{(\text{deviating value}, V)^2} = \frac{\text{Signal Power}}{\text{Noise Power}}$$

  - While performing a measurement, we aim to obtain a high SNR by increasing the signal amplitude and decreasing the noise amplitude.
  - The SNR is expressed in dB.

$$SNR_{dB} = 10 \log \frac{s}{n}$$

Noise characteristics

- Bandwidth of noise B_N



$$B_N = \frac{1}{G_o^2}\int_0^\infty G^2(f)\,df$$

Miscellaneous types of intrinsic noise

- **Thermal noise (Johnson)**

In a conductor: the random movements of the atoms are transmitted to the conducting electrons

- **Shot noise**

Random fluctuation of current associated with crossing an electric potential barrier

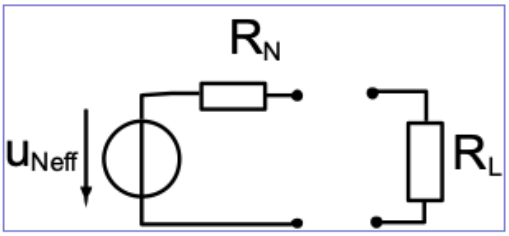
- **1/f noise**

Instability and drift phenomena, non-homogeneities of materials, contacts between conductors

Intrinsic noise source

- Thermal noise
  In a conductor: the random movements of the atoms are transmitted to the conducting electrons

$$U_{Neff}^{2} = 4KTR_N B_N$$



U^2_{Neff} : noise strength

T : conductor temperature in ° K

K : Boltzmann constant =1.38x10^{-23}J/K

$$U_{Neff} / \sqrt{B_N} = 0.13\sqrt{R_N} \ \ (\eta\mathrm{V}/\sqrt{\mathrm{Hz}}) \quad (20^{o}\mathrm{C})$$

Spectral density of noise

- The **psd**: **p**ower **s**pectral **d**ensity, Φ_N is the energy of the noise produced by a conductor at each cycle

$$\phi_N = \frac{P_N}{B_N}$$

W/Hz (joule/cycle)

Example: amplifier INA118 (BB)

Parameter	Conditions	Type INA118PB	Units
Input noise voltage	G = 1000		
f = 10 Hz		11	$nV/\sqrt{Hz}$
f = 100 Hz		10	$nV/\sqrt{Hz}$
f = 1000 Hz		10	$nV/\sqrt{Hz}$
f_B = 0.1 Hz to 10 Hz		0.28	$\mu Vp-p$
Noise current			
f = 10 Hz		2.0	$pA/\sqrt{Hz}$
f = 1000 Hz		0.3	$pA/\sqrt{Hz}$
f_B = 0.1 Hz to 10 Hz		80	$pAp-p$

- Example: for a bandwidth of 1kHz, the input noise is

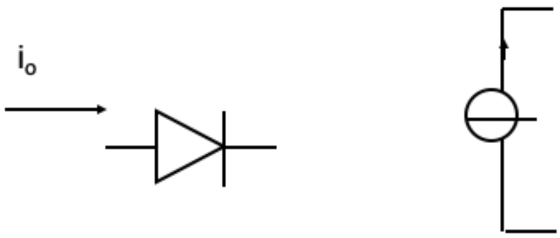
  $10 \cdot 10^{-9} \cdot \sqrt{1000} = 0.32\mu V$.

Shot noise or Schottky noise

- White noise, normal distribution

$$I_{b,eff} = \sqrt{2ei_oB_N}$$

$e=1.6x10^{-19}C$



Additionnal 1/f noise

- $$\phi_N = \frac{\lambda}{f}$$

  λ = constant depending on current amplitude

- Power of noise generated in the frequency bandwidth of $B_N = [f_1, f_2]$

$$P_N = \int_{f_1}^{f_2} \frac{\lambda}{f} df = \lambda \ln \frac{f_2}{f_1}$$

Extrinsic noise sources

- Interferences
  - 50 Hz from electrical power lines.

  - Devices
    - Electrosurgery 500 kHz – 3 MHz
    - Diathermy 500 kHz – 3000 MHz

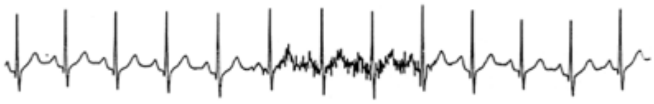
Very high frequencies compared to those of physiological signals, but …

Noise source: interferences

- At the input level of measurement devices, the interferences can be rectified and thus demodulated for erroneous DC (offset) components
- Noise is mainly decreased by **shielding**

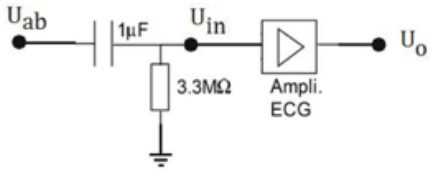
Artefact

- EMG while measuring the ECG

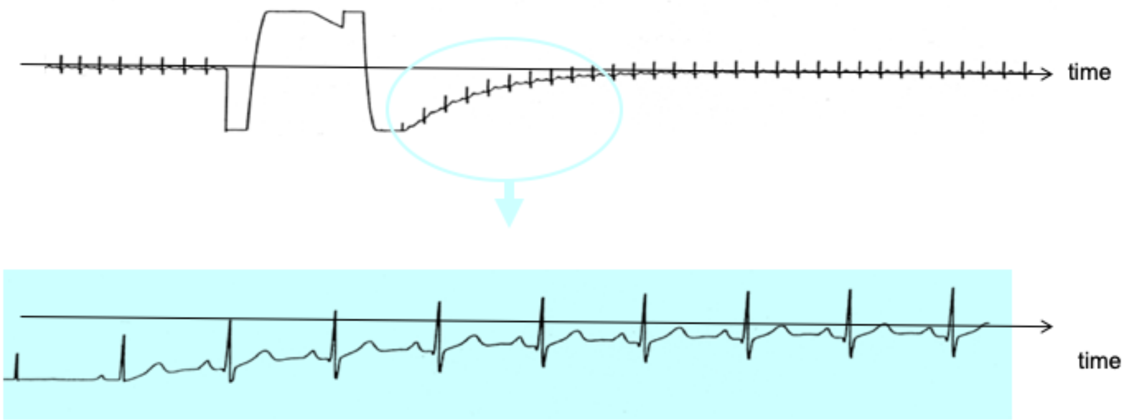


- Electrode movement : change in electrode-skin contact impedance (flexible cables, gel)
- Skin movement
- Mechanical movement : vibrations
- Voltage drift

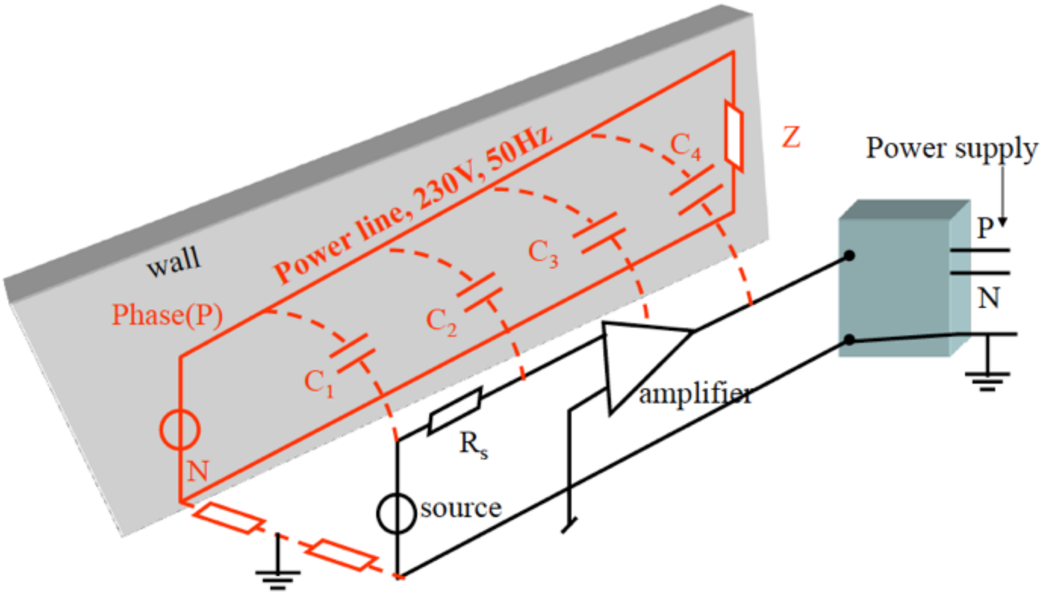
Example (see exercice)



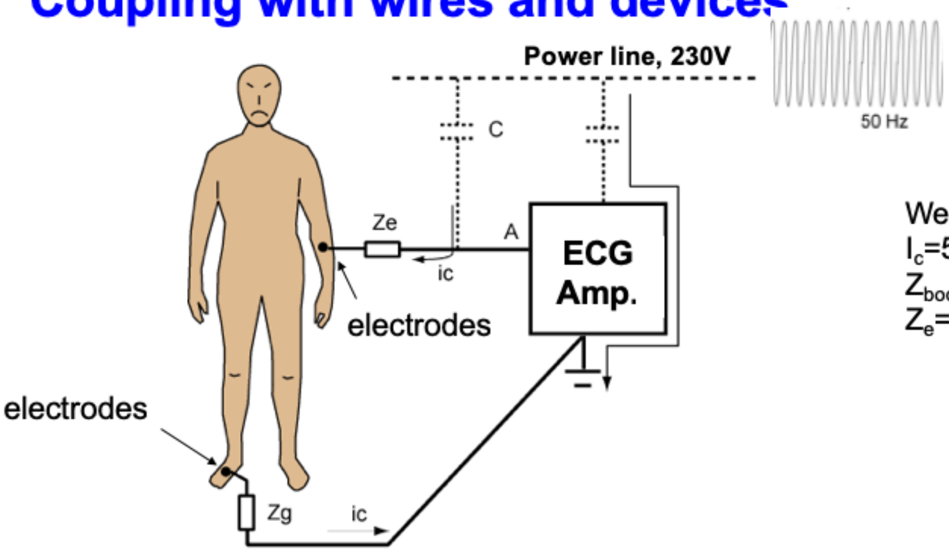
- Effect of a transition voltage(depend on RC value)



Noise due to capacitive coupling



Coupling with wires and devices



We assume:
I_c=5nA_{rms}
Z_{body}≈500Ω
Z_e=Z_g=50kΩ

$$U_A=(Z_e+Z_g+Z_{body})\cdot i_c = 2\cdot 50\cdot 10^3\cdot 5\cdot 10^{-9}=0.5mV_{rms}$$

Coupling with the subject

$U_{mc}=Z_g.i_c$

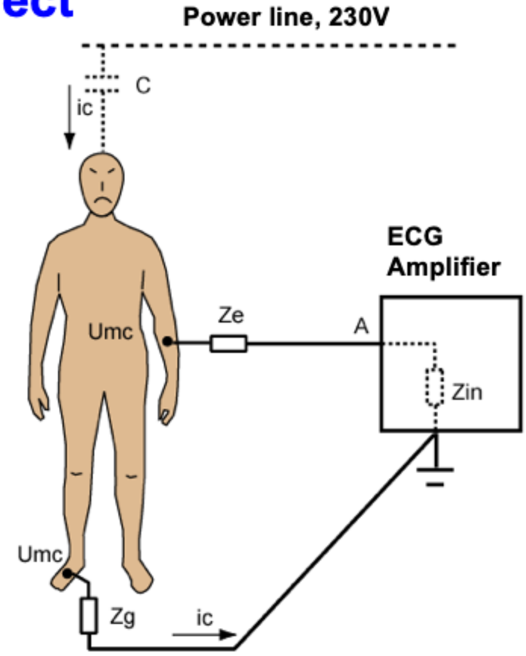
For $Z_g=50k\Omega$ and $i_c=0.2\mu A_{rms}$,

$U_{mc}=50\ .\ 10^3 \cdot 0.2\ .\ 10^{-6}=10mV_{rms}$

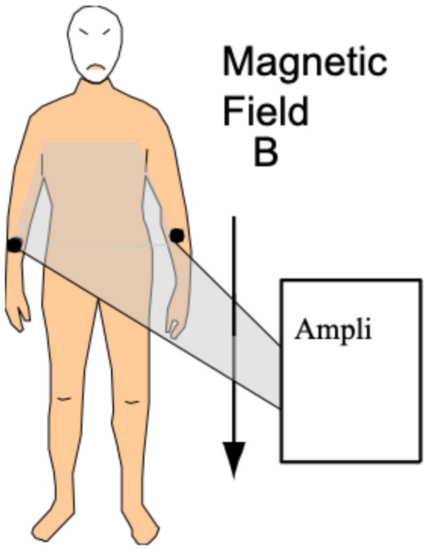
Noise at the input of the amplifier:

$(Z_{in}=10M\Omega,\ Z_e=50k\Omega)$

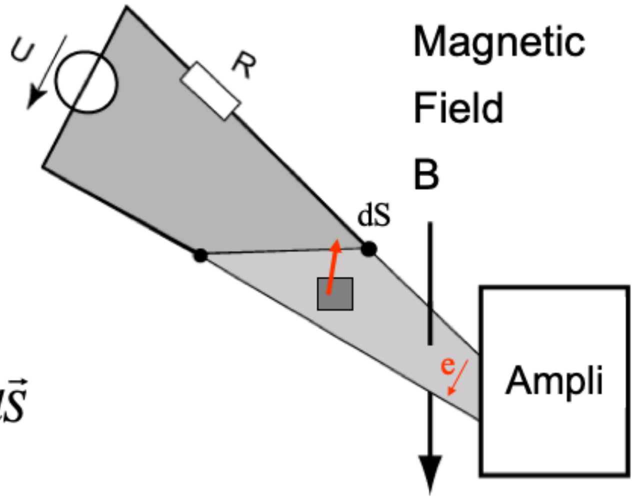
$$U_A = \frac{Z_{in}}{Z_{in}+Z_e} U_{mc} \cong 10mV_{rms}!!$$



Magnetic fields



Magnetic coupling



$$e = \int_S \frac{d\vec{B}}{dt} \cdot d\vec{S}$$

Decreasing noise: coupling with wires

- Differential amplifier

$$U_{AB} = U_A - U_B = Z_e \cdot i_{c2} - Z_e^{'} \cdot i_{c1}$$

If the two wires are close :

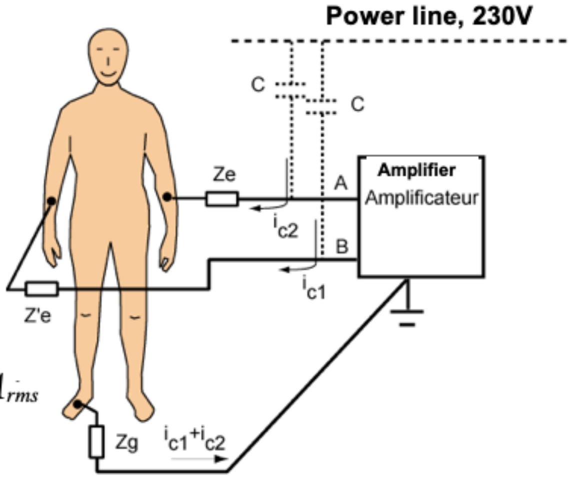
$$i_{c1} = i_{c2} = i_c$$

$$U_{AB} = U_A - U_B = (Z_e - Z_e^{'}) \cdot i_c$$

for $Z_e - Z_e^{'} = 25k\Omega$ and $i_c = 5nA_{rms}$

$$U_{AB} = 125\mu V_{rms}$$

Without differential amplifier : $U_{AB} = 0.5mV_{rms}$



Decreasing noise : coupling with the subject

- Common mode voltage of the source

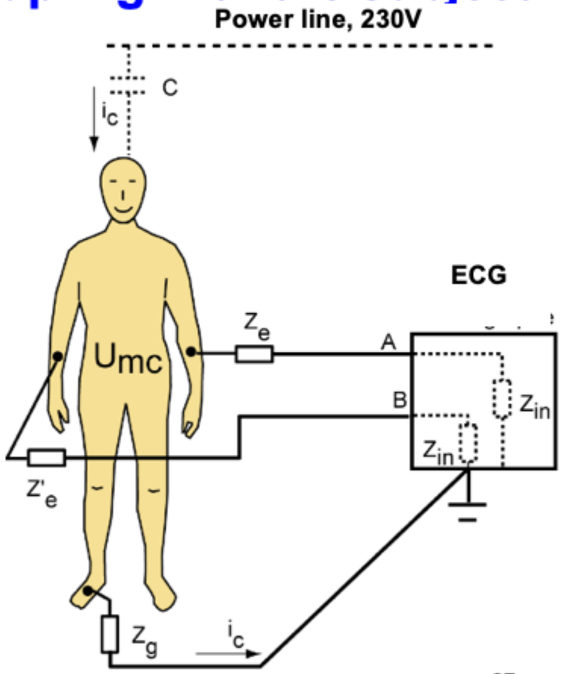
$$U_{mc} = i_c \cdot Z_g$$

For $Z_g = 50k\Omega$

and $i_c = 0.2\mu A_{rms}$

$$U_{mc} = 10mV_{rms}$$

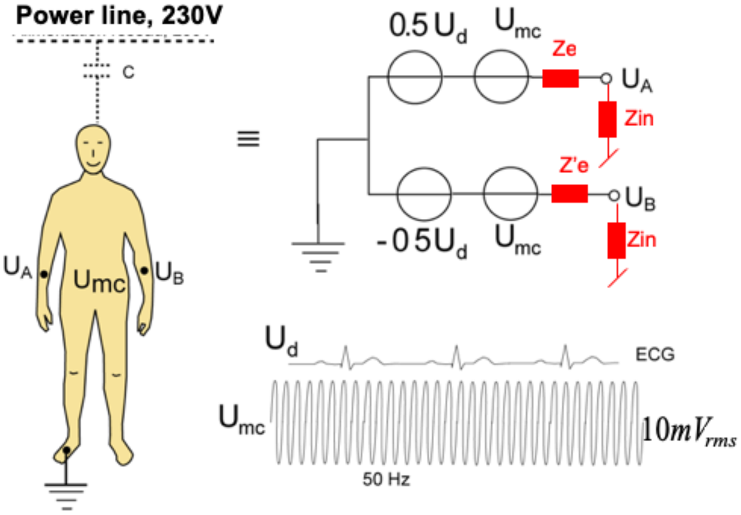
Higher in amplitude than ECG voltage to measure!



Ic=0.2 mA ?

Example

- For $Z_{in} >> Z_e$ and $Z_{in} >> Z'_e$
  ($U_d = 0$) :
  $U_{AB} = U_A - U_B$
  $= U_{mc}\left(\frac{Z_{in}}{Z_{in}+Z_e} - \frac{Z_{in}}{Z_{in}+Z'_e}\right)$
  $= \frac{Z'_e - Z_e}{Z_{in}} U_{mc}$
  $U_{AB} = 10mV \cdot \frac{20k\Omega}{5M\Omega} = 40\mu V_{rms}$



Common mode voltage : definition

U_{mc}

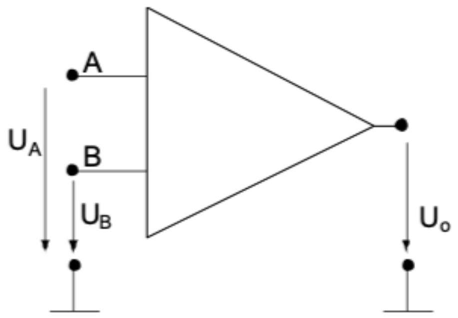
- U_{mc}: Voltage common to U_A and U_B , does not provide any information

$$U_{mc} = \frac{U_A + U_B}{2}$$

$$U_{AB} = U_A - U_B$$

$$U_A = U_{mc} + \frac{U_{AB}}{2}$$

$$U_B = U_{mc} - \frac{U_{AB}}{2}$$



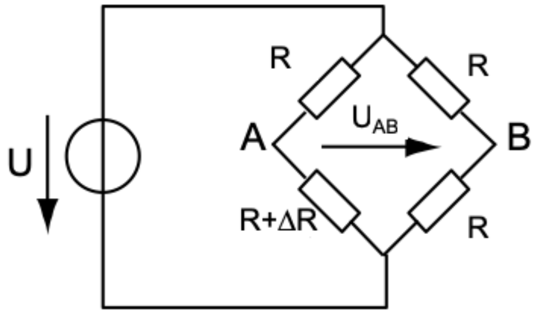
Common mode voltage : origins

- Common mode voltage due to power source

$$U_B = \frac{U}{2}$$

$$U_A = \frac{U}{2} + U_{AB} = \frac{U}{2} + \frac{\Delta R}{4R}U$$

$$U_{mc} \cong \frac{U}{2}$$



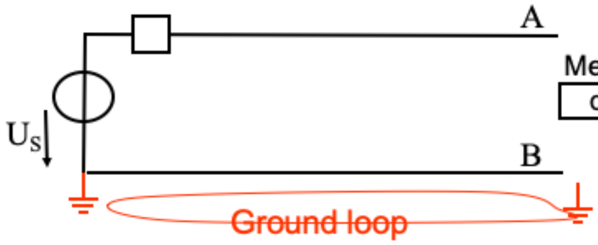
- For $U = 20V$ and $\frac{\Delta R}{R} = 0.01$:

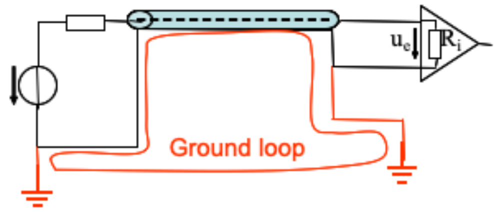
$U_{mc} = 10V$, $U_{AB} = 0.05V$

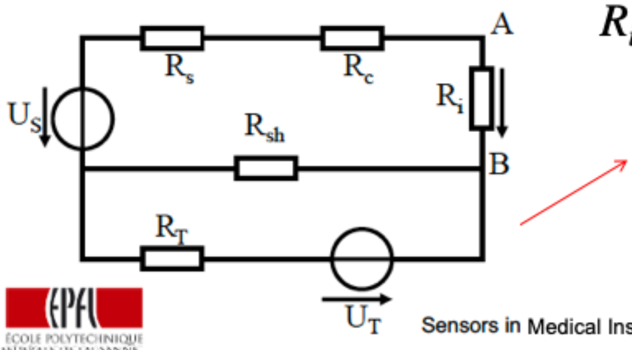
U_{mc} can be many times higher than U_{AB}

Common mode voltage : origins

- Common mode voltage due to ground







R_i very high (open circuit)

Voltage divider (R_{sh}, R_T)

$$U_{mc} = \frac{R_{sh}}{R_{sh} + R_T} U_T$$

$$R_{sh} \approx 10R_T \qquad U_{mc} \cong U_T$$

U_T could be much higher than U_S

Common mode voltage : origins

- Common mode voltage of the source

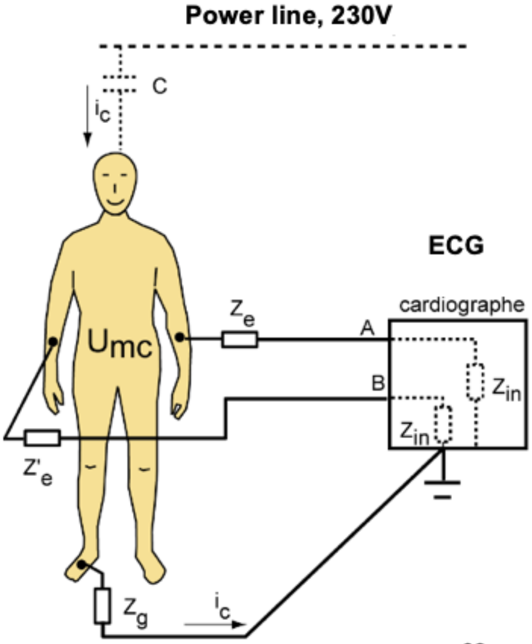
$$U_{mc} = i_c \cdot Z_g$$

For $Z_g = 50k\Omega$

and $i_c = 0.2\mu A_{rms}$

$$U_{mc} = 10mV_{rms}$$

Higher in amplitude than ECG voltage to measure!



Ic=0.2 mA ?

Model of a differential amplifier

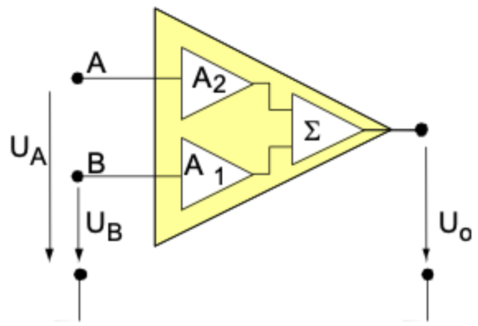
$$U_o = A_2 \cdot U_A - A_1 \cdot U_B$$

$$= A_2 \left(U_{mc} + \frac{U_{AB}}{2} \right) - A_1 \left(U_{mc} - \frac{U_{AB}}{2} \right)$$

$$= \underbrace{(A_2 - A_1)}_{A_c} U_{mc} + \underbrace{\frac{A_1 + A_2}{2}}_{A_d} U_{AB}$$

Common mode gain

Differential gain



- Differential Gain (A_d)
- Common Mode Rejection Ratio (CMRR)

$$CMRR = \frac{A_d}{A_c} \qquad U_o = A_d \left(U_{AB} + \frac{1}{CMRR} \right) U_{mc}$$

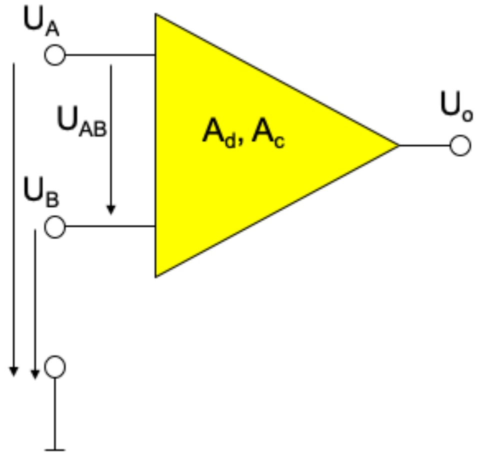
$$CMRR_{dB} = 20 \log_{10} \left| \frac{A_d}{A_c} \right|$$

Model of a differential amplifier

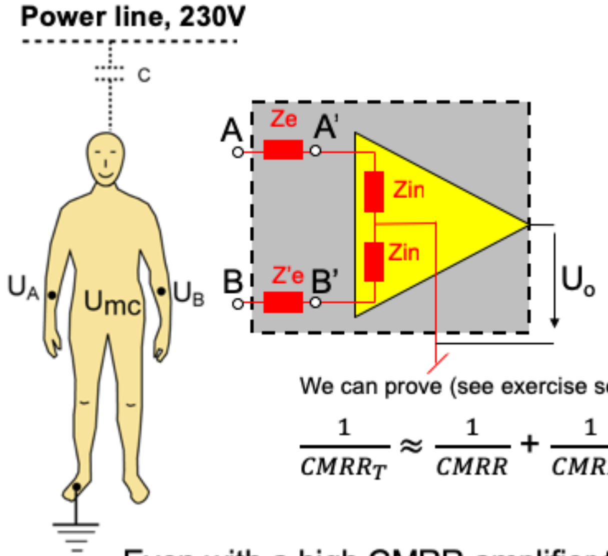
$$U_0 = A_d U_{AB} + A_c U_{mc} \qquad CMRR = \frac{A_d}{A_c}$$

$$U_A = U_{mc} + \frac{U_{AB}}{2}$$

$$U_A = U_{mc} - \frac{U_{AB}}{2}$$



Differential circuit with several stages



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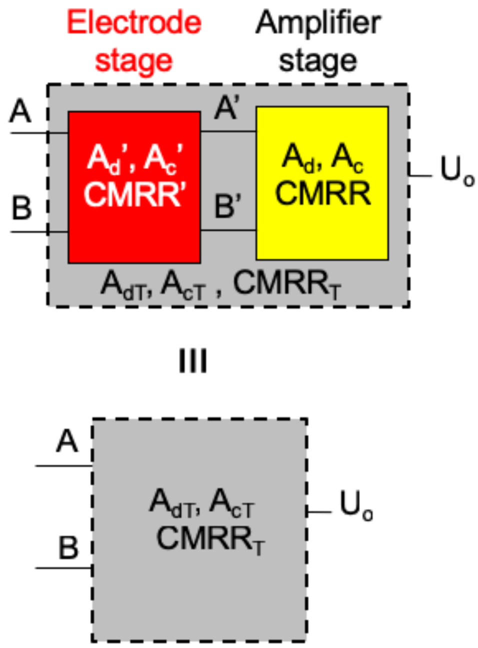


We can prove (see exercise session):

$$\frac{1}{CMRR_T} \approx \frac{1}{CMRR} + \frac{1}{CMRR'}$$

Even with a high CMRR amplifier the electrode asymmetry may degrade the performance→ decrease U_{mc}

Remarks

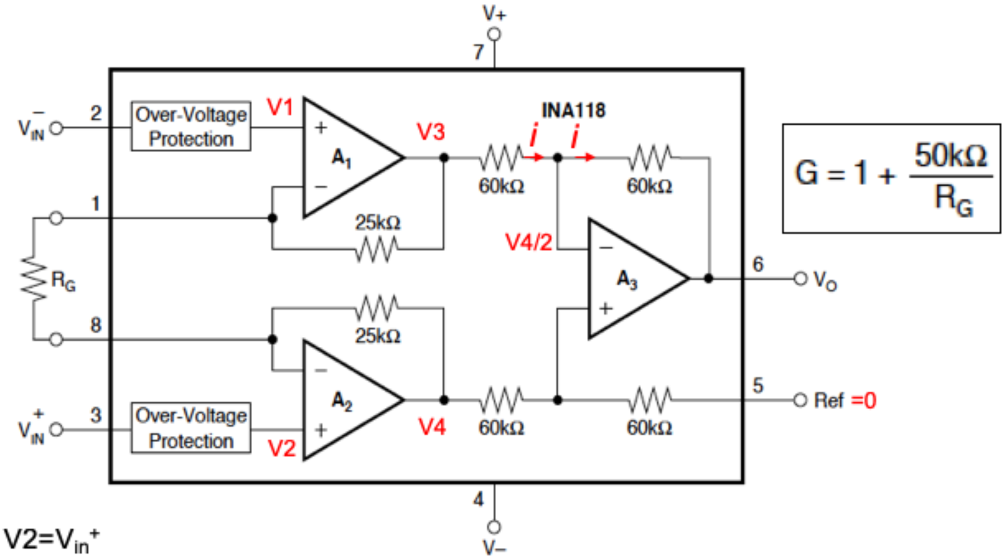
- The skin-electrode contact impedance should be considered in order to reduce the noise of a potential amplifier.

- Decreasing the effect of U_{mc} by minimizing the electrode-skin contact impedance or by increasing the value of Z_{in} and the **CMRR**.

Instrumentation amplifier

- Tunable differential gain from 1 to 10'000, up to 100 Hz (decreasing thereafter with frequency)
- **Input impedance** is very high: $10^{10}\Omega$ in parallel with a few pF
- **Output impedance** is very low (0.1Ω)
- Polarization current at input is very low (few pA to few nA)
- High thermal stability
- High **CMRR** (100 dB and more up to 50 Hz)

Example: INA 118 from Burr-brown CMRR=110dB for G=1000 and bandwidth of 7kHz.



$V1=V_{in}^-$, $V2=V_{in}^+$

In the voltage divider: 25k/RG/25k: V2-V1=(RG/(RG+50k)(V4-V3) → V4-V3=(1+50k/RG)(V2-V1)

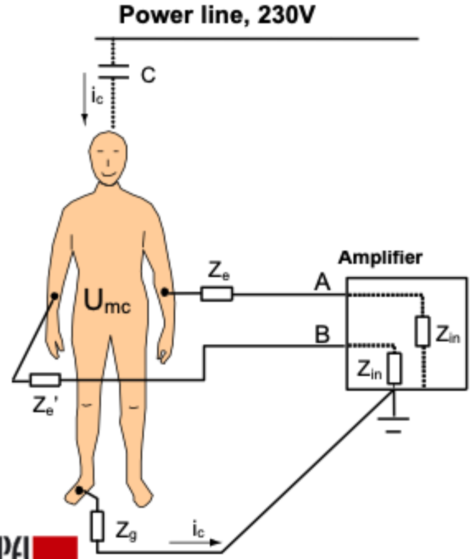
For amplifier A3: V3-V4/2=V4/2-Vo → Vo=V4-V3==(1+50k/RG)(V2-V1)=(1+50k/RG)(Vin^+-Vin^-)

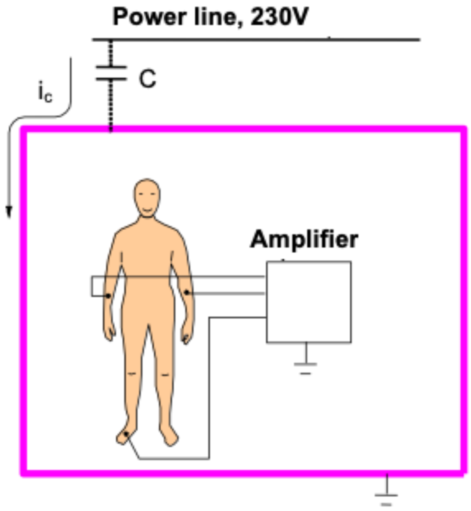
Vo=(1+50k/RG)(Vin^+-Vin^-)

Decreasing common mode voltage (1)

The measurement amplifier is wired to the ground

$$U_{mc} = f(i_c) = f(C) \Rightarrow \text{decrease}\, C \Rightarrow \text{electrostatic shielding}$$





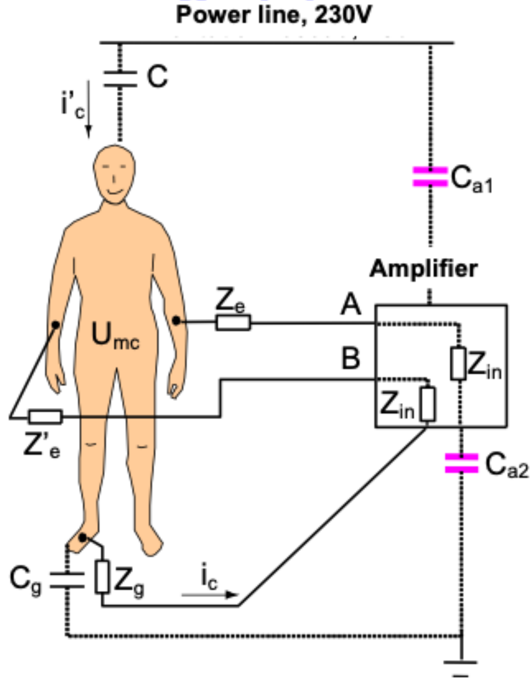
Decreasing common mode voltage (2)

The measurement amplifier is not wired to ground

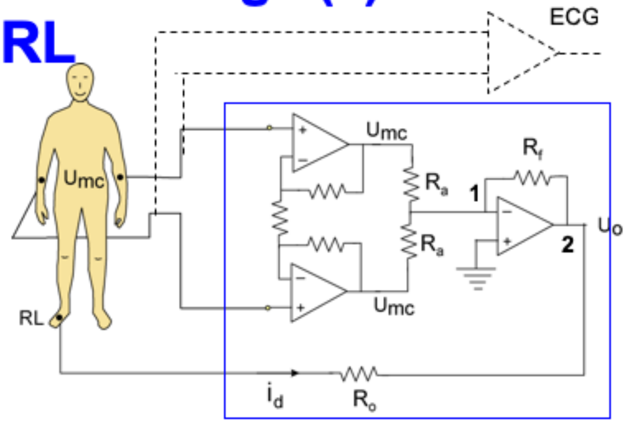
$$i_c = f(C, C_{a1}, C_{a2}, C_g)$$

$$\Rightarrow \text{reduce } C_{a1} \text{ et } C_{a2}$$

$$\Rightarrow \text{shielding of the amplifier}$$



Decreasing common mode voltage (3): "Driven Right Leg": DRL



DRL

Sum of currents in 1:

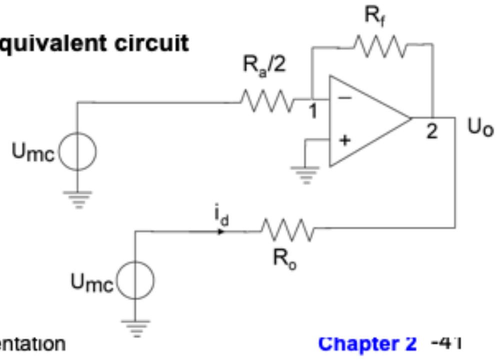
$$\frac{2U_{mc}}{R_a} + \frac{U_o}{R_f} = 0$$

$$U_o = -2U_{mc}\frac{R_f}{R_a}$$

in 2:

$$U_{mc} = R_o i_d + U_o \quad = R_o i_d - 2U_{mc}\frac{R_f}{R_a}$$

$$U_{mc} = \frac{R_o i_d}{1 + 2\frac{R_f}{R_a}}$$



DRL circuit

- $$U_{mc} = \frac{R_o i_d}{1 + 2\frac{R_f}{R_a}}$$

  For $R_f >> R_a$, U_{mc} can be considerably decreased.

  Ex.: $R_o = 100k$, $R_f = 5M$, $R_a = 25k$, $i_d = 0.2\,\mu A_{rms}$

  $U_{mc} = 50\,\mu V$ (instead of 10 mV in the previous case)

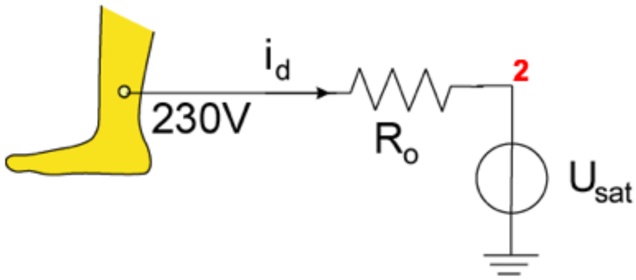
Protection against accidental power surge

- If the subject touches a conductor (230V) the output of the amplifier (2) saturates:

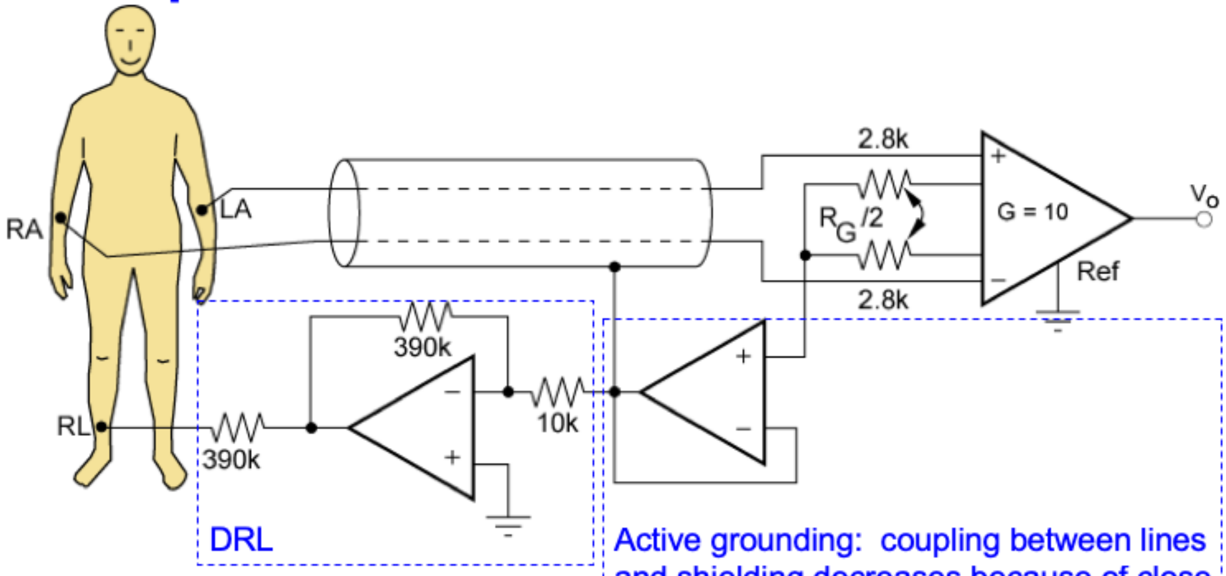
  $$U_o = U_{sat} = 15V$$

- For $R_o = 5M\Omega$, the current in the subject is:

  $$i_d = \frac{230-15}{5.10^6} = 43\mu A$$



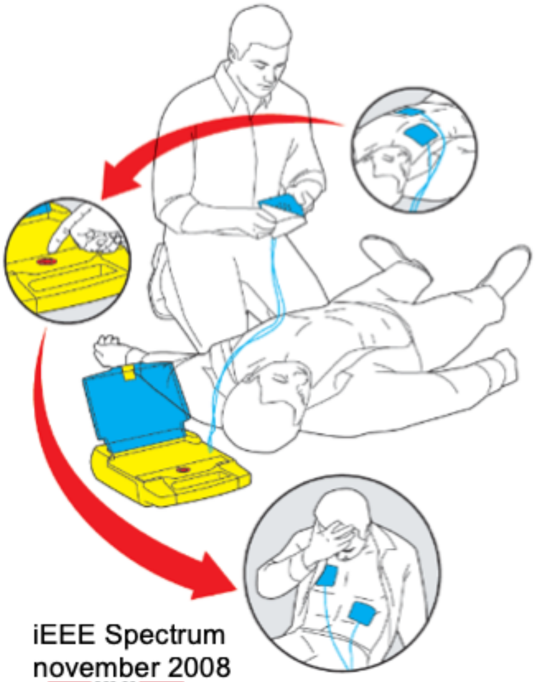
Example: DRL circuit

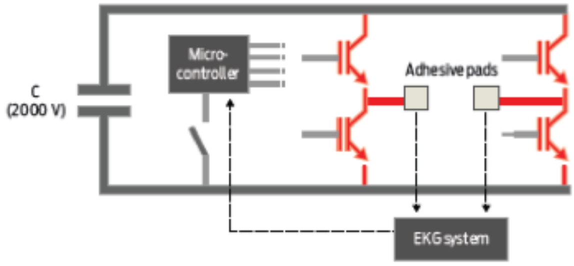


Active grounding: coupling between lines and shielding decreases because of close potential

What if only two electrodes are available? (e.g. Defibrillator)

Defibrillator





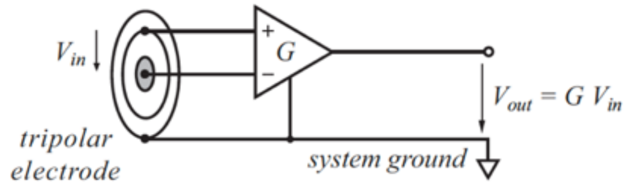
Two electrodes to measure the ECG

iEEE Spectrum
november 2008

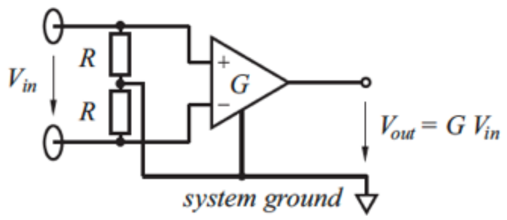
Use the average of the two electrode (U_{mc}) as reference

Example of reference electrode

- Localised bioelectric signal (EMG, ENG)



- ECG belt with two electrodes
  - Common-mode voltage derived by a voltage divider

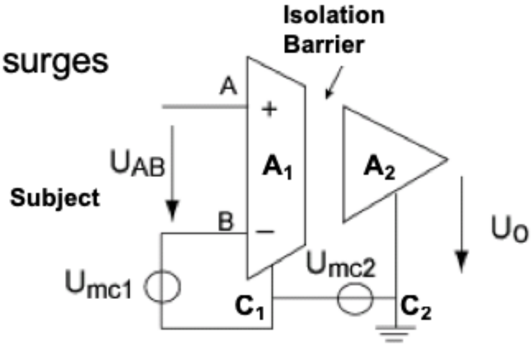


T. Werner Degen, ETHZ thesis 19640, 2011

Tripolar electrodes are used to measure well located differential signals as for example the electrical activity of a nerve at a particular spot

Isolation amplifier

- Decreases common mode voltage U_{mc} that is very high (disconnection in the ground loop)
- Protection of subjects against power surges
  U_{mc2} very high



Stage A_1: instrumentation amplifier

Stage A_2: amplifier of gain 1

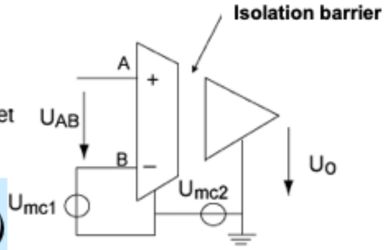
C_1: wired to the ground of the source

C_2: wired to earth or ground

Isolation barrier: total disconnection of ohmic connection

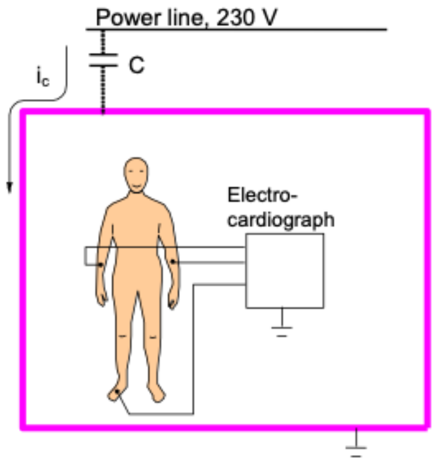
Between A_1et A_2, transfer of signal by electric coupling (magnetic, optic)

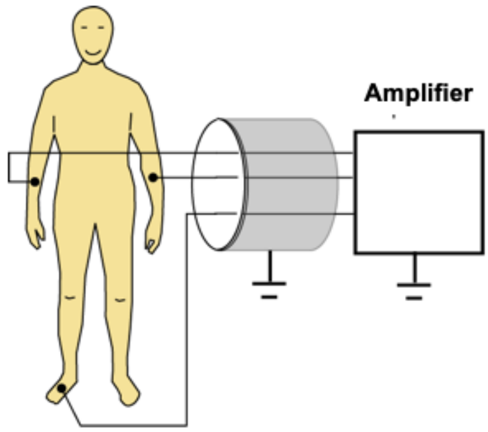
Isolation amplifier



- $$U_o = A_d\left(U_{AB} + \frac{1}{CMRR}U_{mc1} + \frac{1}{IMRR}U_{mc2}\right)$$
- CMRR : common mode rejection ratio (>100dB)
- IMRR : Isolation mode rejection ratio (> 140dB)
- A_d : differential gain
- U_{mc1} : common mode voltage of the subject (tens of volts)
- U_{mc2} : common mode voltage of the device (can reach hundreds of volts)

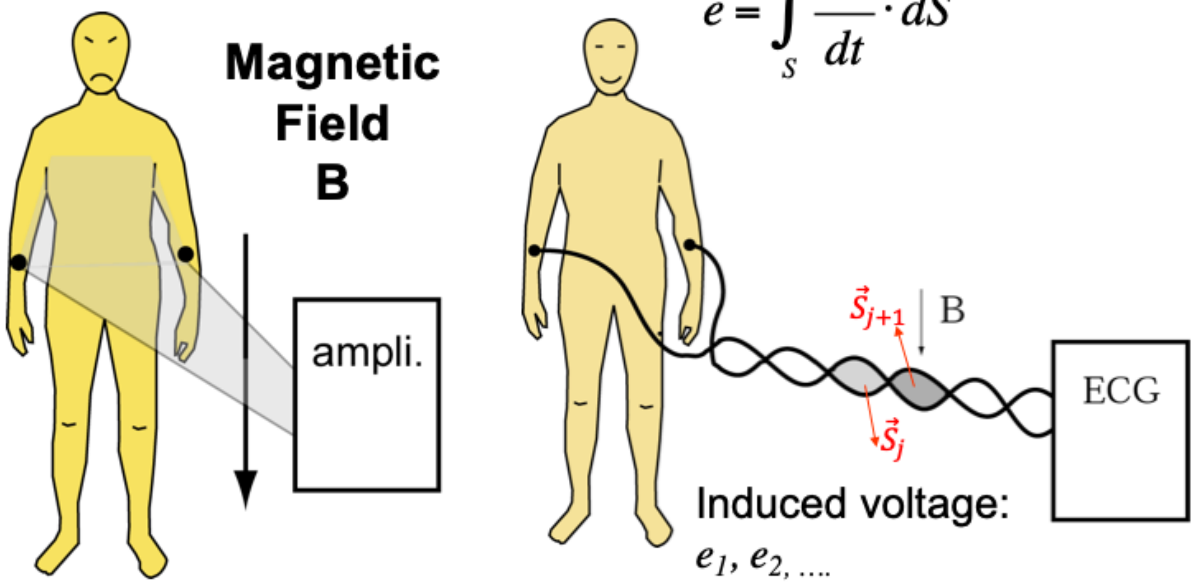
Electrostatic shielding and protection against magnetic field





Protection against magntic field

$$e = \int_S \frac{d\vec{B}}{dt} \cdot d\vec{S}$$



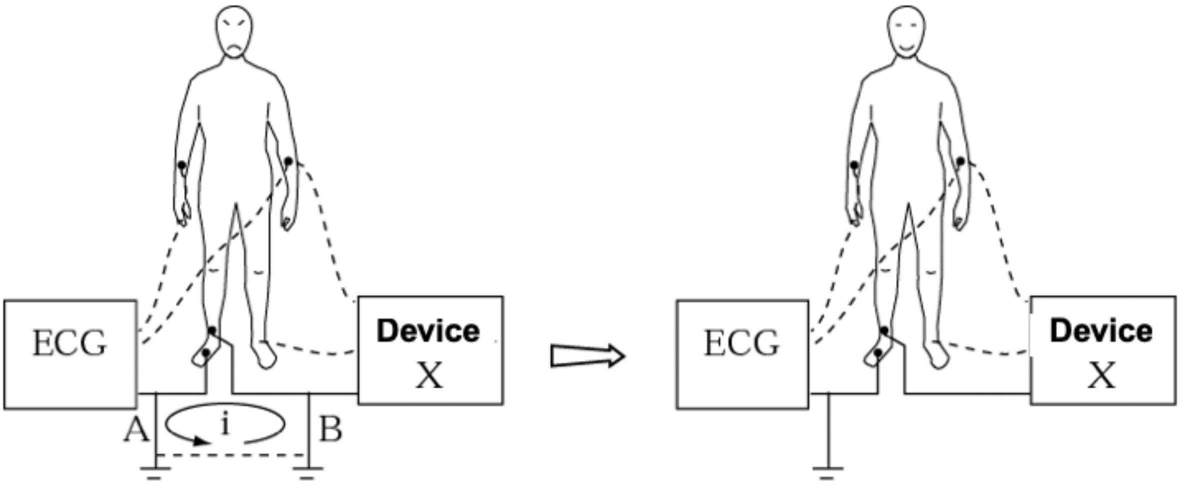
Induced voltage:

$e_1, e_2, \ldots$

$e_j \simeq - e_{j+1}$

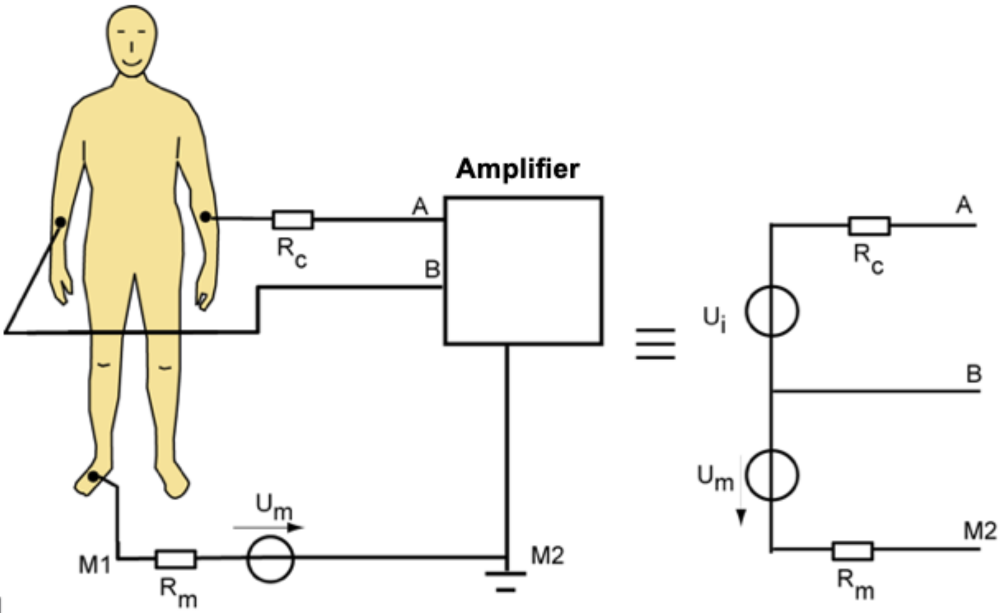
Example

- For $U_A > U_B$, $U_A < U_{mc} < U_B$

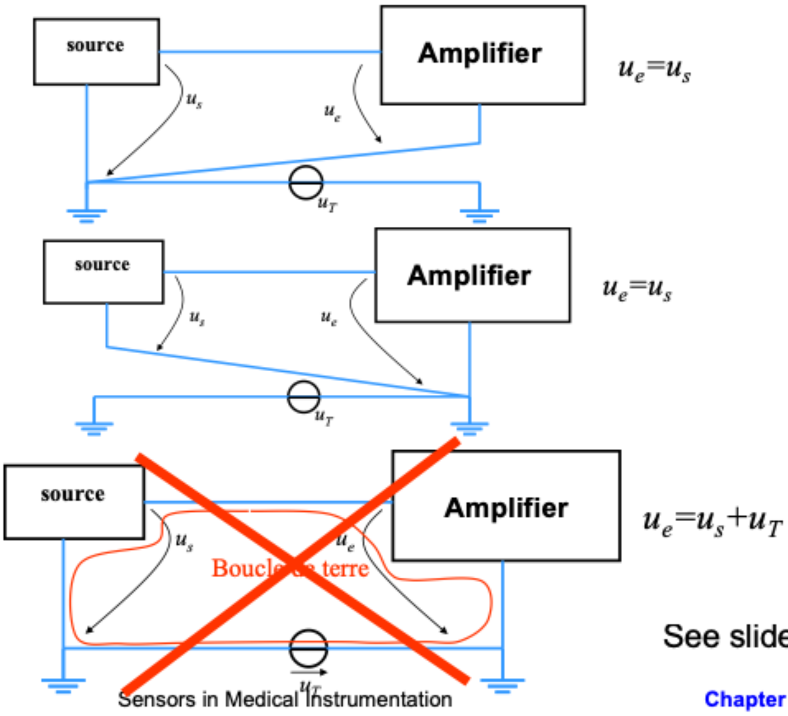


Grounding

- R_c<<R internal of the amplifier

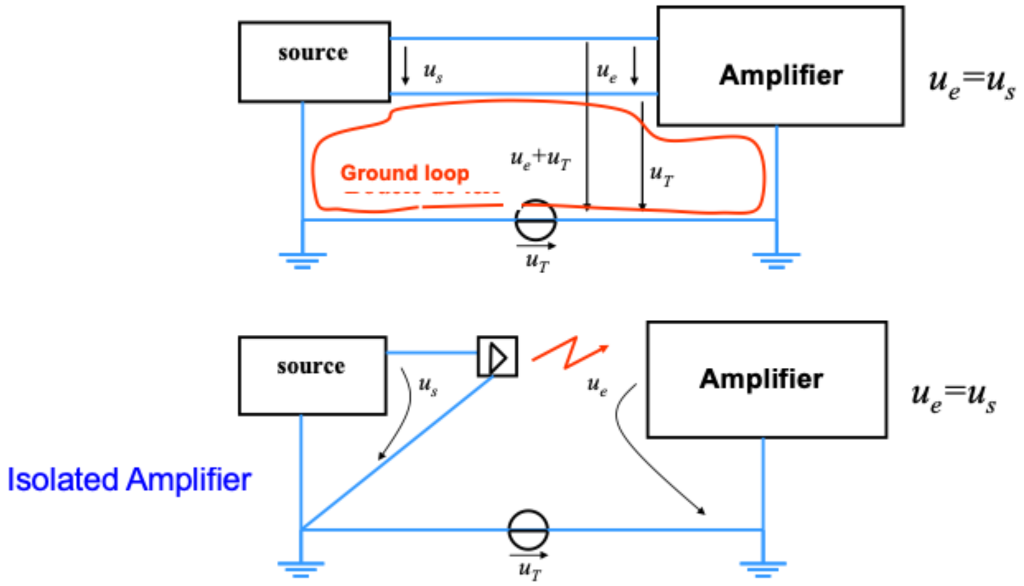


Wiring of asymmetrical amplifiers



See slide 31

Wiring of differential and insulating amplifiers



Electrical filtering

- Low-pass (anti-aliasing, sampling)
- High-pass (offset, drift)

- Non-stationary signals
  - Adaptative filtering
  - Wavelet

Next lecture

- Chapter 3:
  Biopotential measurement

Annex: thermal noise model

- Noise power provided at charge:

$$P_{N,L} = R_L I_N^2 = \frac{R_L}{(R_N + R_L)^2} U_{Neff}^2$$

- The **maximum noise power** that this source can provide exteriorly is when R_N=R_L,

$$P_N = \frac{1}{4R_N} U_{Neff}^2 = KTB_N \qquad U_{Neff}^2 = 4KTR_N B_N$$

- The noise from measurement devices (amplifier or voltmeter) is also expressed in V^2/Hz, or V/$\sqrt{Hz}$