COM303: Digital Signal Processing

Lecture 12: Filter design

Overview

- filter design: problem statement
- IIR design
- two more ideal filters

filter design

The filter design problem

You are given a set of requirements:

- frequency response: passband(s) and stopband(s)
- phase: overall delay, linearity
- some limit on computational resources and/or numerical precision

You must determine N, M, a_k's and b_k's in

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_{M-1} z^{-M}}{1 + a_1 z^{-1} + \ldots + a_{N-1} z^{-N}}$$

in order to best fulfill the requirements

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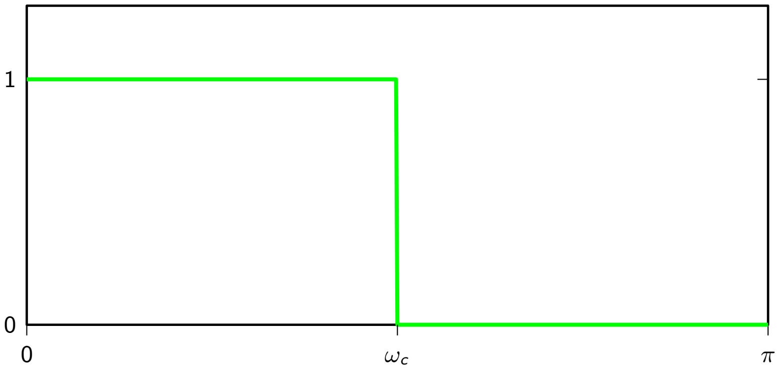
Example: lowpass specs



Practical limitations

- passband/stopband transitions cannot be infinitely sharp
  $\Rightarrow$ use *transition bands*
- magnitude response cannot be constant over an interval
  $\Rightarrow$ specify magnitude *tolerances* over bands
- in general:
  - smaller transition bands $\Rightarrow$ higher filter order
  - smaller error tolerances $\Rightarrow$ higher filter order
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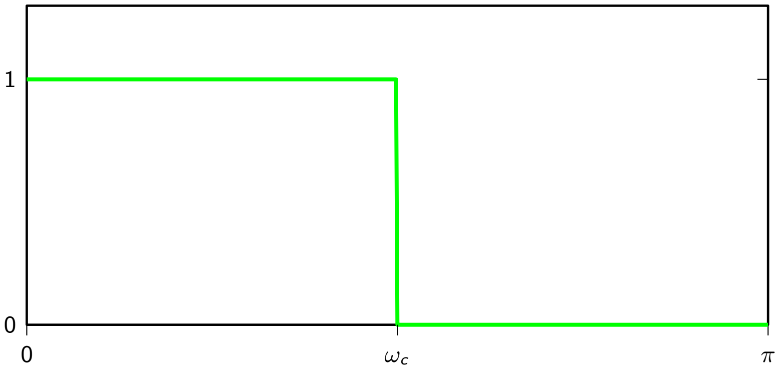
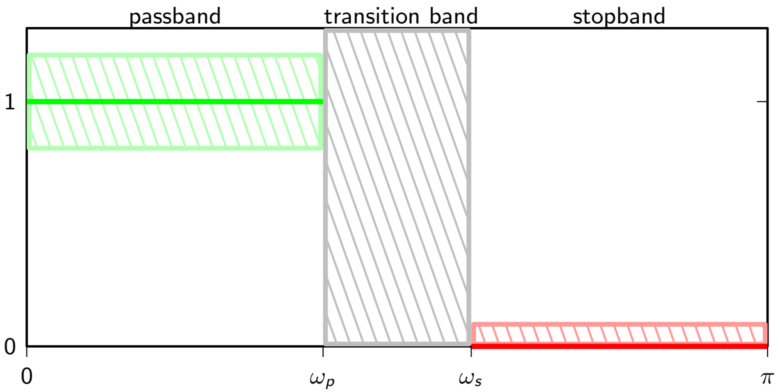


Realistic specs



Why we can't have a "vertical" transition

$$H(z) = \frac{B(z)}{A(z)} \quad \text{is a rational function with } A, B \in C^\infty$$

polynomial rational functions cannot have jump discontinuities

Why we can't have a flat response

$$H(z) = \frac{B(z)}{A(z)}, \qquad \text{with } A \text{ and } B \text{ polynomials}$$

$$\begin{aligned}
H(e^{j\omega}) = c \text{ over an interval} \quad &\Rightarrow \quad B(z) - cA(z) = 0 \text{ over an interval} \\
&\Rightarrow \quad B(z) - cA(z) \text{ has an infinite number of roots} \\
&\Rightarrow \quad B(z) - cA(z) = 0 \text{ for all values of } z \\
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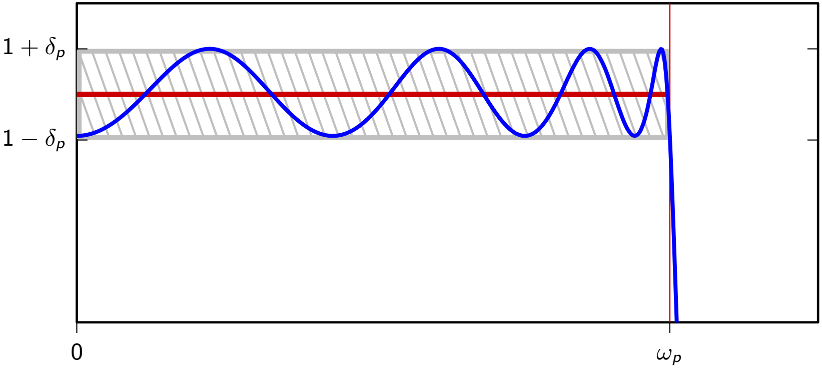
Important case: equiripple error



The big questions

- IIR or FIR?
- how to determine the coefficients?
- how to evaluate the performance?

IIRs: pros and cons

Pros:

- computationally efficient
- strong attenuation easy
- "natural sounding" in audio applications

Cons:

- stability and numerical precision issues
- limit cycles
- difficult to design for arbitrary response
- nonlinear phase

FIRs: pros and cons

Pros:

- can be designed with linear phase
- always stable
- numerically precise
- optimal design techniques exist
- efficient in multirate processing

Cons:

- computationally much more expensive
- because of length, significant delay (hard to use in live audio)

The design methods

- finding N, M, a_k's and b_k's from specs is a hard nonlinear problem
- established methods:
  - IIR: conversion of analog design
  - FIR: optimal minimax filter design

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IIR design

IIR: conversion of analog design

Filter design was an established art long before digital processing appeared

- lots of nice analog filters exist
- methods exist to "translate" the analog design into a rational transfer function
- most numerical packages (Matlab, Numpy, etc.) provide ready-made routines
- design involves specifying some parameters and testing that the specs are fulfilled

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Butterworth lowpass

Magnitude response:

- maximally flat
- monotonic over $[0, \pi]$

Design parameters:

- order N (N poles and N zeros)
- cutoff frequency

Design test criterion:

- width of transition band
- passband error

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Butterworth lowpass in NumPy

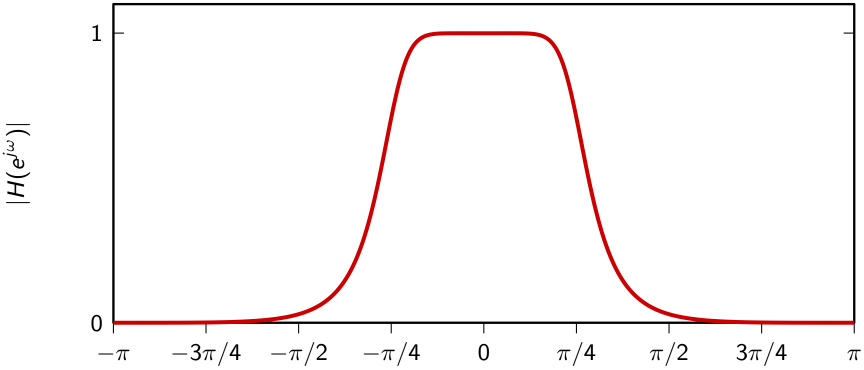
```
import scipy.signal as sp

b, a = sp.butter(4, 0.25)

wb, Hb = sp.freqz(b, a, 1024);
plt.plot(wb/np.pi, np.abs(Hb));
```

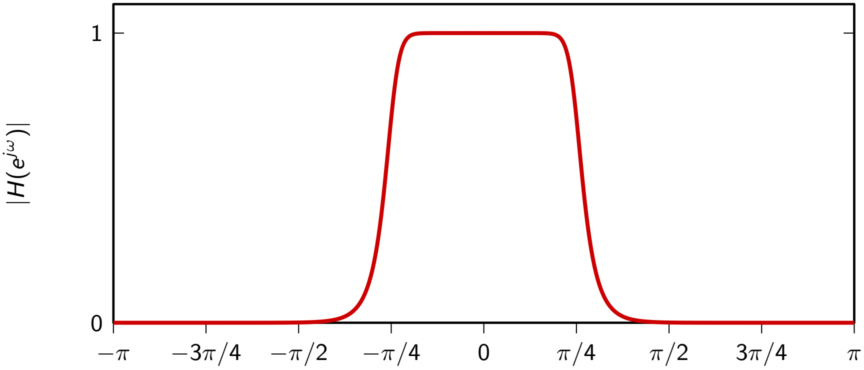
Butterworth lowpass example

$N = 4, \omega_c = \pi/4$



Butterworth lowpass example

$N = 8, \omega_c = \pi/4$



Chebyshev lowpass

Magnitude response:

- equiripple in passband
- monotonic in stopband
- (or vice-versa)

Design parameters:

- order N (N poles and N zeros)
- passband max error
- cutoff frequency

Design test criterion:

- width of transition band
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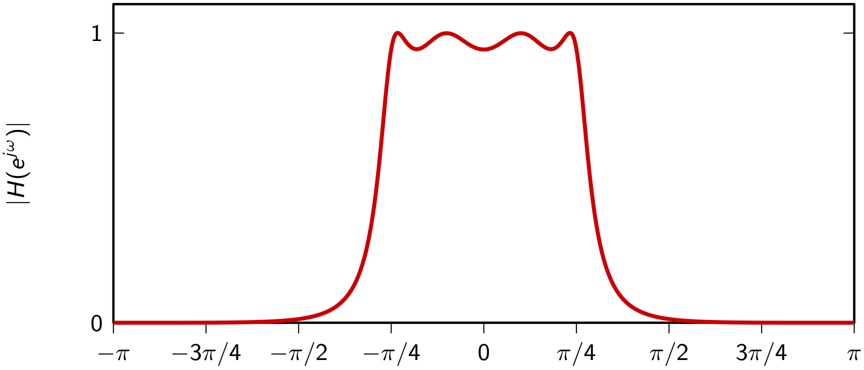
Chebyshev lowpass in NumPy

```
b, a = sp.cheby1(4, .12, 0.25)
```

Chebyshev lowpass example

$$N = 4, \omega_c = \pi/4, e_{\max} = 12\%$$



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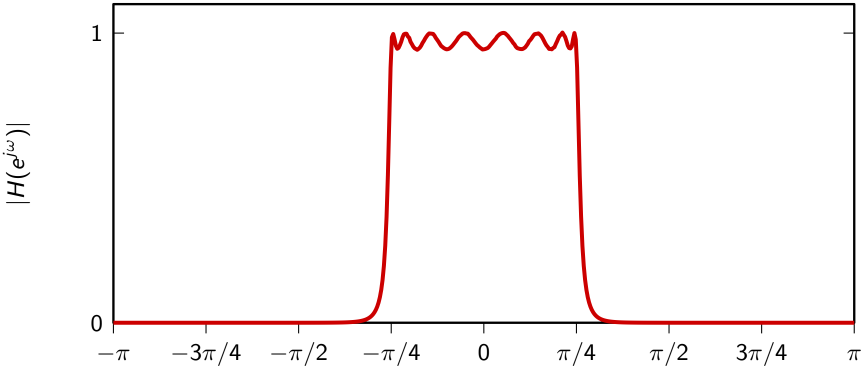


Elliptic lowpass

Magnitude response:

- equiripple in passband and stopband

Design parameters:

- order N
- cutoff frequency
- passband max error
- stopband min attenuation

Design test criterion:

- width of transition band

Elliptic lowpass

Magnitude response:

- equiripple in passband and stopband

Design parameters:

- order N
- cutoff frequency
- passband max error
- stopband min attenuation

Design test criterion:

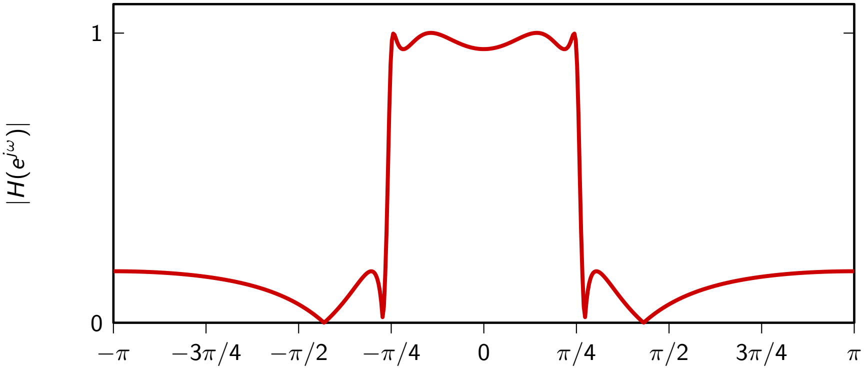
- width of transition band

Elliptic lowpass in NumPy

```
b, a = sp.ellip(4, .1, 50, 0.25)
```

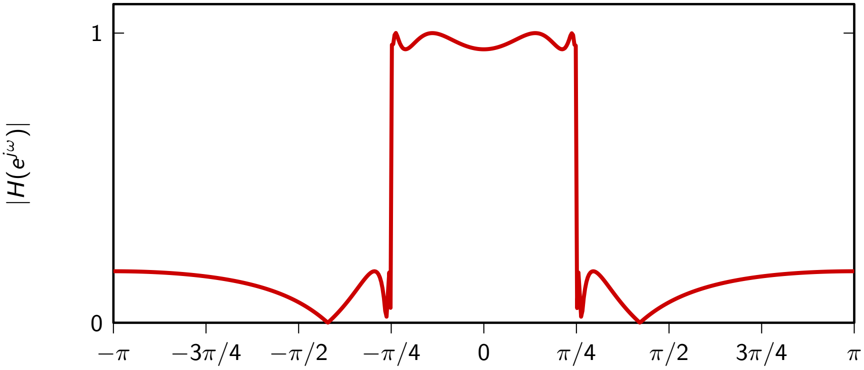
Elliptic lowpass example

$$N = 4, \omega_c = \pi/4, e_{\text{max}} = 12\%, \text{att}_{\text{min}} = 0.03$$



Elliptic lowpass example

$$N = 6, \omega_c = \pi/4, e_{\mathrm{max}} = 12\%, \mathrm{att}_{\mathrm{min}} = 0.03$$



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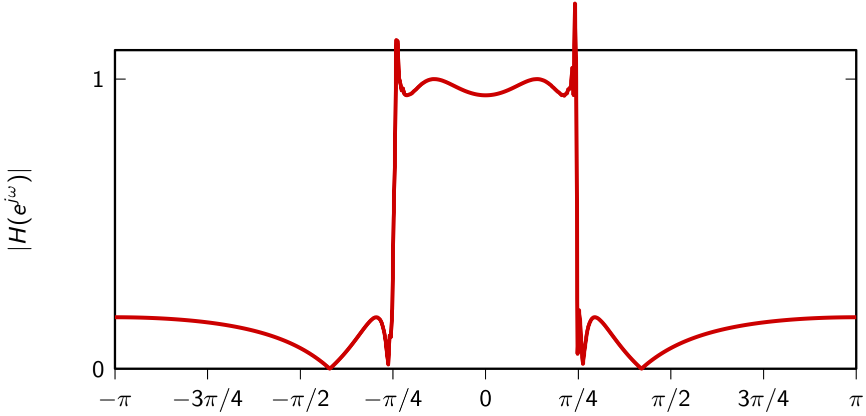


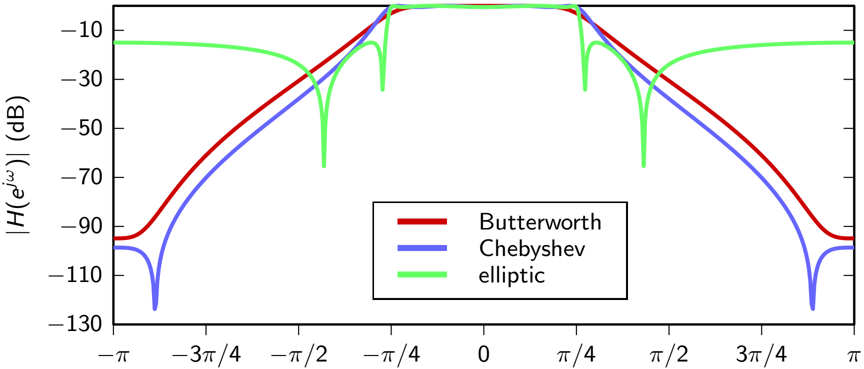
Magnitude response in decibels

- filter max passband magnitude G
- filter attenuation expressed in decibels as:

$$A_{\text{dB}} = 20 \log_{10}(|H(e^{j\omega})|/G)$$

- useful to compare attenuations between filters

4-th order lowpass comparison



Qualitative comparison

For a given order N

- sharpness of transition band: Elliptic $>$ Chebyshev $>$ Butterworth
- phase distortion: Butterworth $<$ Chebyshev $<$ Elliptic
- pasband ripples Butterworth $<$ Chebyshev $<$ Elliptic
- stopband attenuation: Elliptic $>$ Chebyshev $>$ Butterworth

a couple more ideal filters

Overview

- the fractional delay
- the Hilbert filter

consider a simple delay...

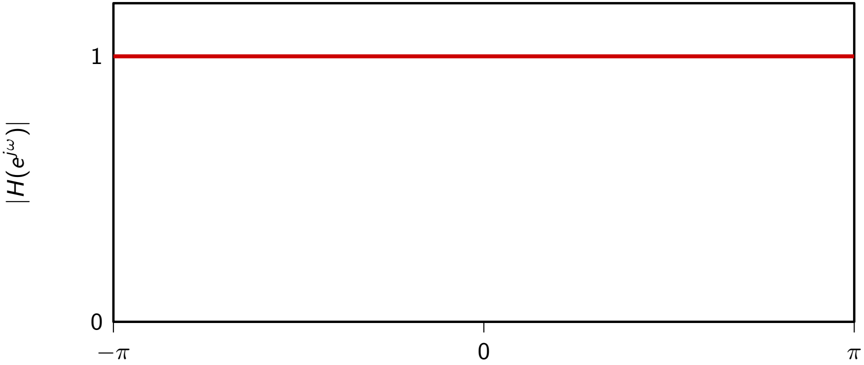
$$x[n] \longrightarrow \boxed{z^{-d}} \longrightarrow x[n-d]$$

$$H(e^{j\omega}) = e^{-j\omega d} \qquad d \in \mathbb{Z}$$

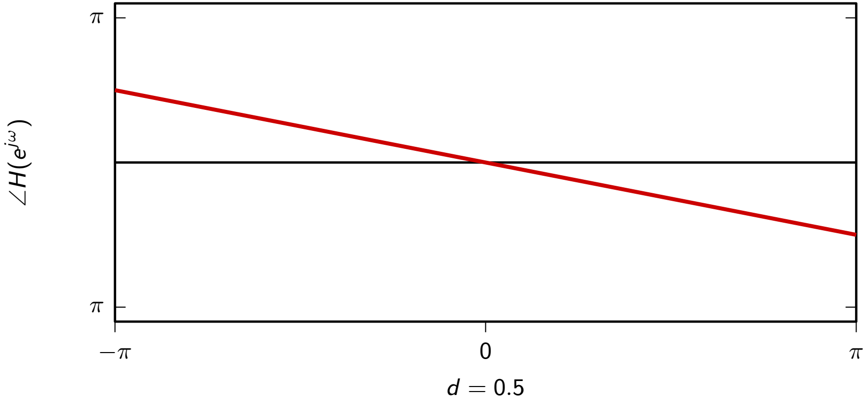
question

what happens if, in $H(e^{j\omega})$ we use a non-integer $d \in \mathbb{R}$?

Fractional delay: magnitude response



Fractional delay: phase response



impulse response

$$
\begin{aligned}
h[n] &= \text{IDTFT}\left\{e^{-j\omega d}\right\} \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j\omega d} e^{j\omega n}\, d\omega \\
&= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega (n-d)}\, d\omega \\
&= \frac{1}{\pi(n-d)} \frac{e^{j\pi(n-d)} - e^{-j\pi(n-d)}}{2j} \\
&= \frac{\sin \pi(n-d)}{\pi(n-d)} \\
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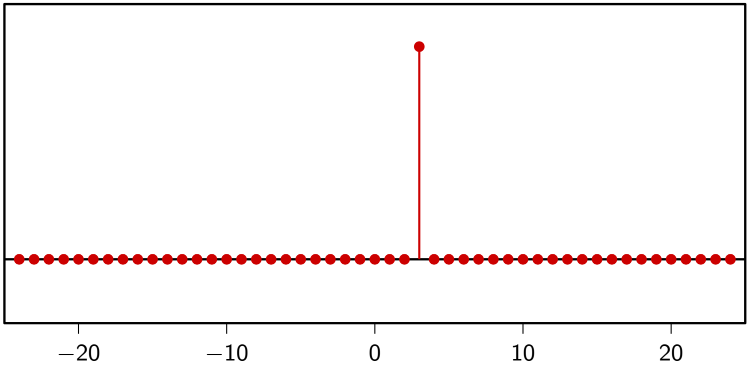
Fractional delay: impulse response

$d = 3$



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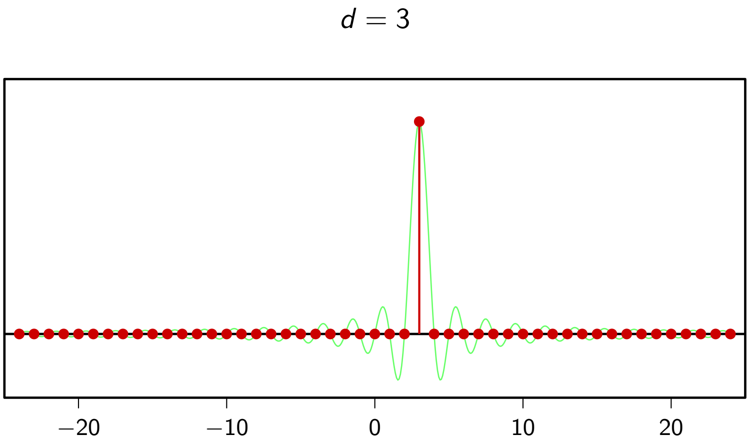

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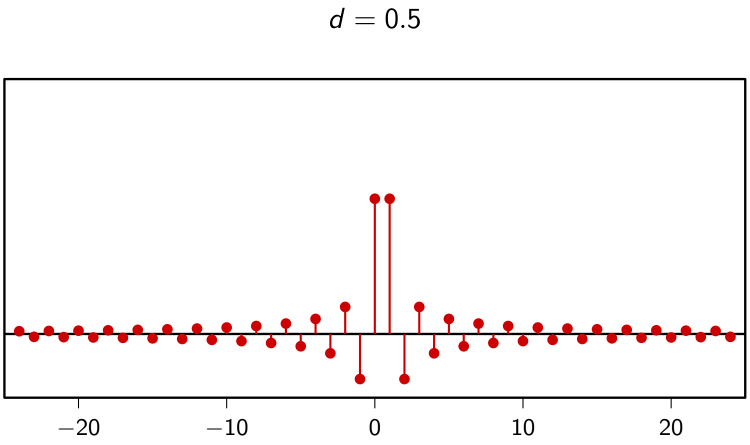
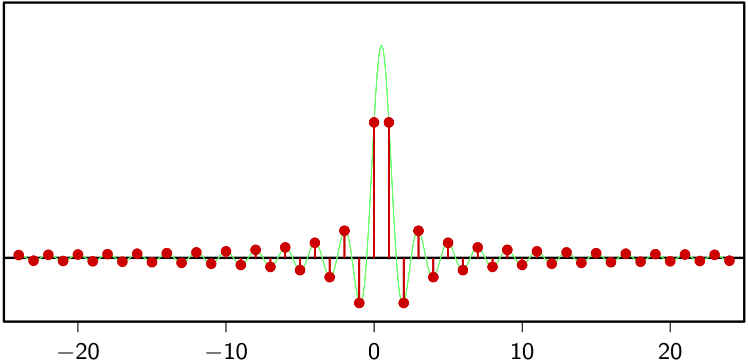


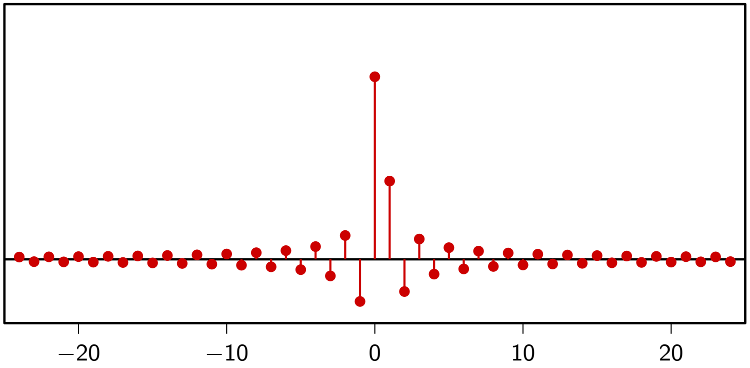
Fractional delay: impulse response

$d = 0.5$



Fractional delay: impulse response

$d = 0.3$



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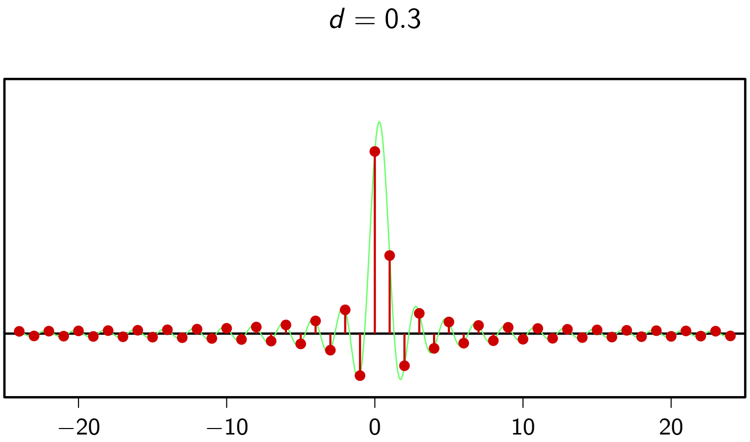

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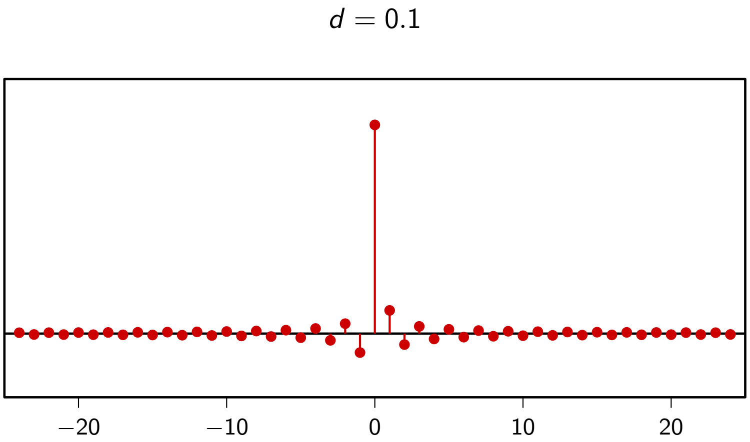

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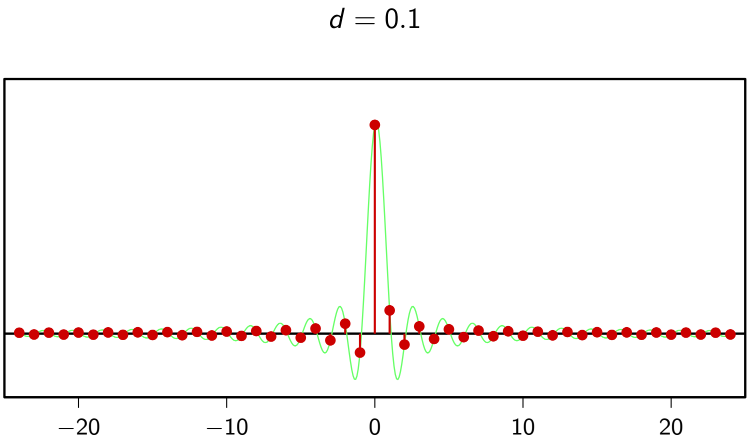

fractional delay

- fractional delay computes “in-between” values for samples
- it is an ideal filter!
- often approximated with local interpolation
- all will be clear when we study the sampling theorem

the Hilbert filter

a quirky machine

$$\cos(\omega_0 n) \longrightarrow \boxed{H(z)} \longrightarrow \sin(\omega_0 n)$$

can we build such a thing?

in the frequency domain

$$\text{DTFT}\left\{2\cos(\omega_0 n)\right\} = \tilde{\delta}(\omega - \omega_0) + \tilde{\delta}(\omega + \omega_0)$$

$$\text{DTFT}\left\{2\sin(\omega_0 n)\right\} = -j\tilde{\delta}(\omega - \omega_0) + j\tilde{\delta}(\omega + \omega_0)$$

$$H(e^{j\omega})[\tilde{\delta}(\omega - \omega_0) + \tilde{\delta}(\omega + \omega_0)] = -j\tilde{\delta}(\omega - \omega_0) + j\tilde{\delta}(\omega + \omega_0)$$

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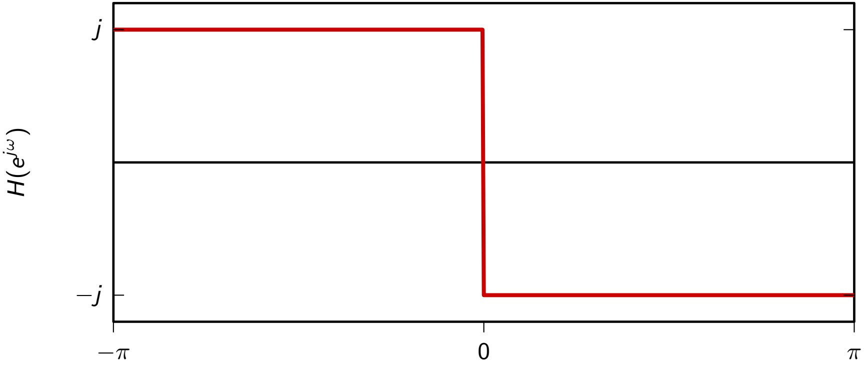
in the frequency domain

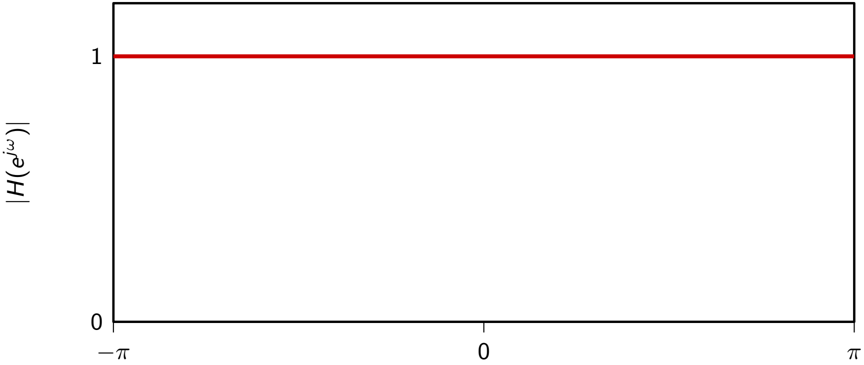
for the machine to work at all frequencies:

$$H(e^{j\omega_0}) = \begin{cases} -j & \text{for } 0 \leq \omega < \pi \\ +j & \text{for } -\pi \leq \omega < 0 \end{cases} \qquad \text{(}2\pi\text{-periodic)}$$

Hilbert filter



Hilbert filter is an allpass



impulse response

$$
\begin{aligned}
h[n] &= \frac{1}{2\pi}\int_{-\pi}^{0} je^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi} -je^{j\omega n}d\omega \\
&= \frac{1}{2\pi n}[1 - e^{-j\pi n} - (e^{j\pi n} - 1)] \\
&= \frac{1}{\pi n}[1 - \cos(\pi n)] \\
&= \begin{cases} \frac{2}{\pi n} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}
\end{aligned}
$$

impulse response

$$
\begin{aligned}
h[n] &= \frac{1}{2\pi}\int_{-\pi}^{0} je^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi} -je^{j\omega n}d\omega \\
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&= \begin{cases} \frac{2}{\pi n} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}
\end{aligned}
$$

impulse response

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{0} je^{j\omega n}d\omega + \frac{1}{2\pi}\int_{0}^{\pi} -je^{j\omega n}d\omega$$
$$= \frac{1}{2\pi n}[1 - e^{-j\pi n} - (e^{j\pi n} - 1)]$$
$$= \frac{1}{\pi n}[1 - \cos(\pi n)]$$
$$= \begin{cases} \frac{2}{\pi n} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

impulse response

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\begin{aligned}
h[n] &= \frac{1}{2\pi}\int_{-\pi}^{0} je^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{0}^{\pi} -je^{j\omega n}\,d\omega \\
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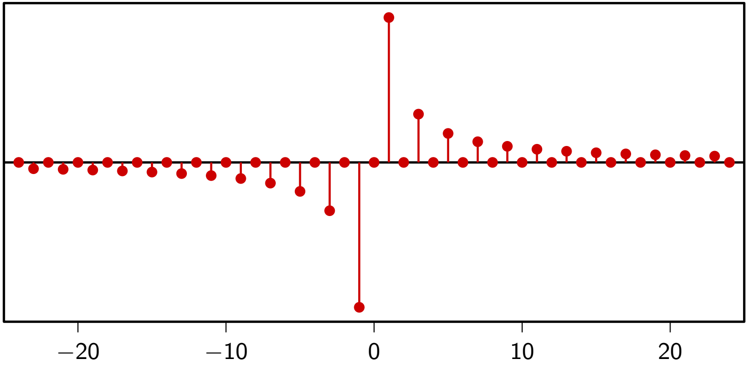
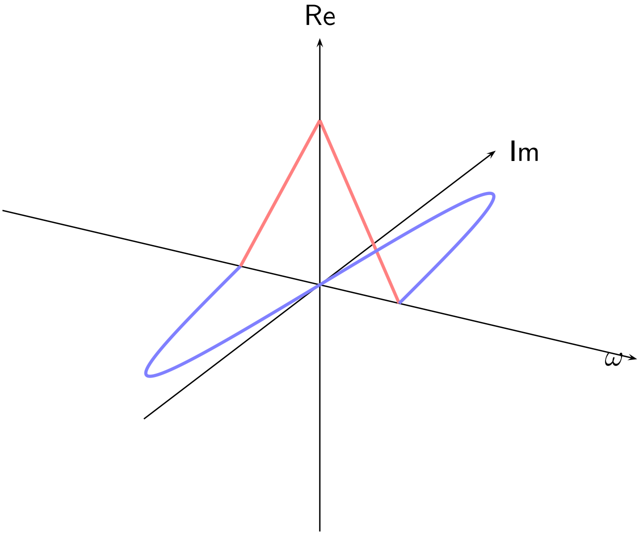
&= \begin{cases} \frac{2}{\pi n} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}
\end{aligned}
$$

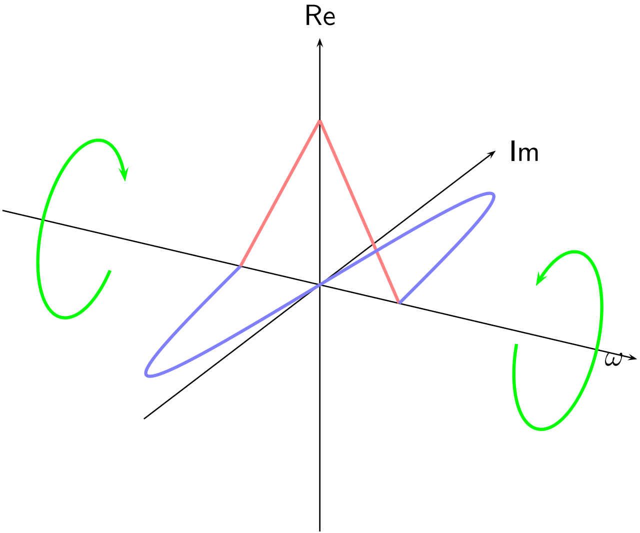
Hilbert filter



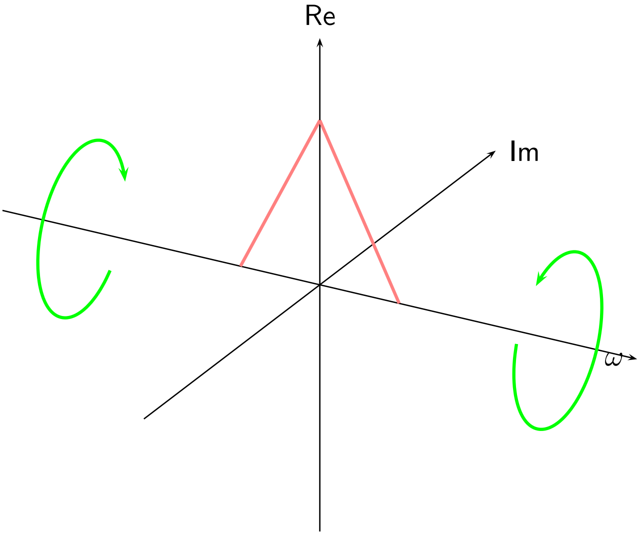
what does the Hilbert filter do?



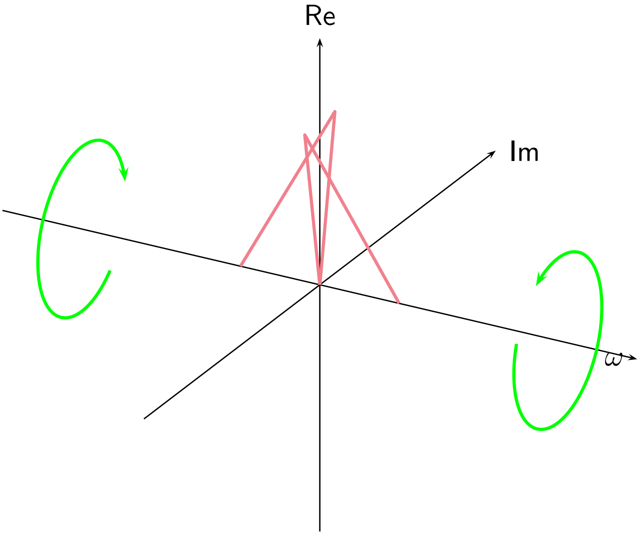
what does the Hilbert filter do?



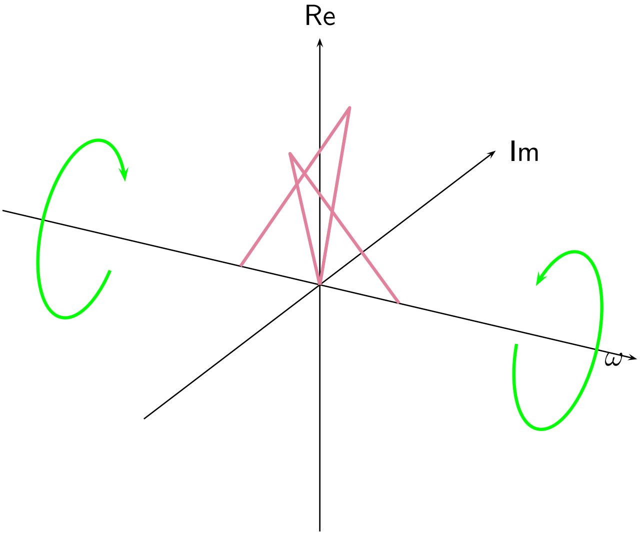
effect of the Hilbert filter (real part)



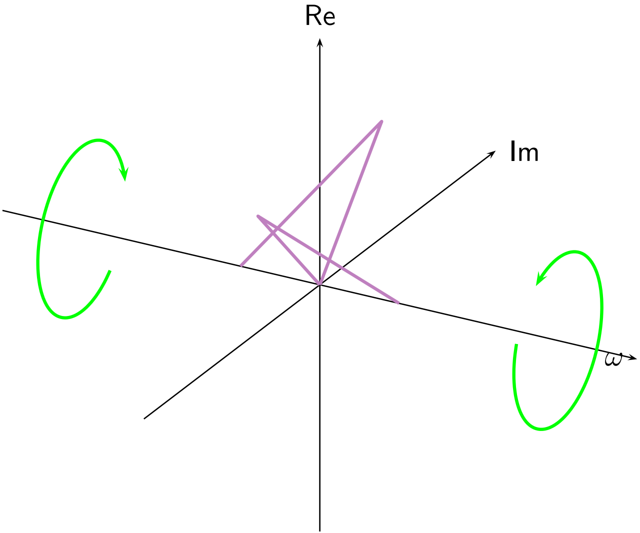
effect of the Hilbert filter (real part)



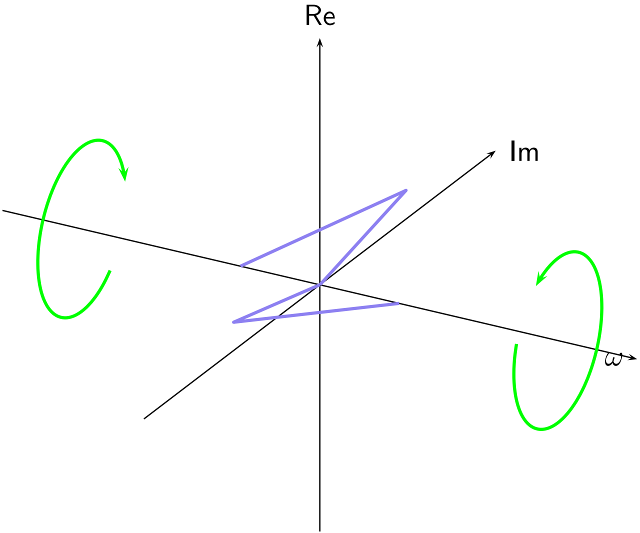
effect of the Hilbert filter (real part)



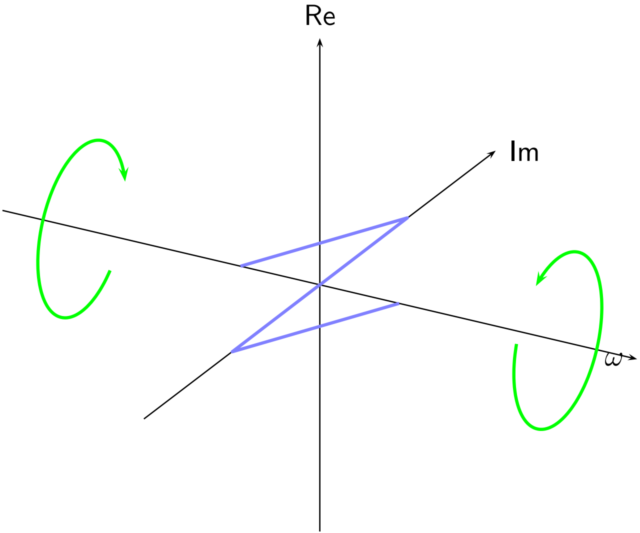
effect of the Hilbert filter (real part)



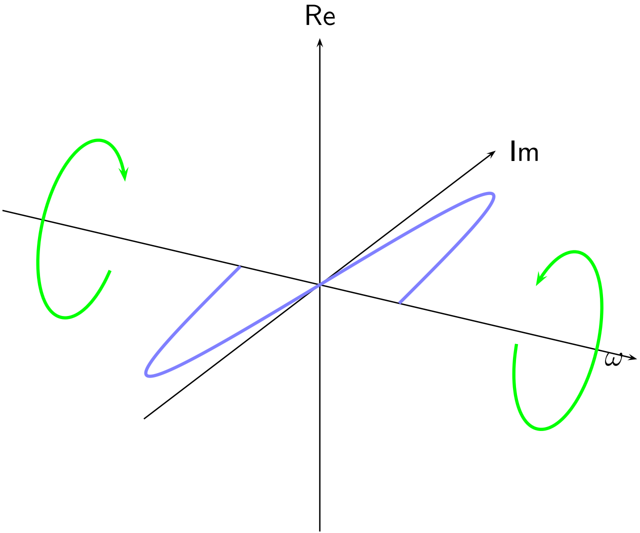
effect of the Hilbert filter (real part)



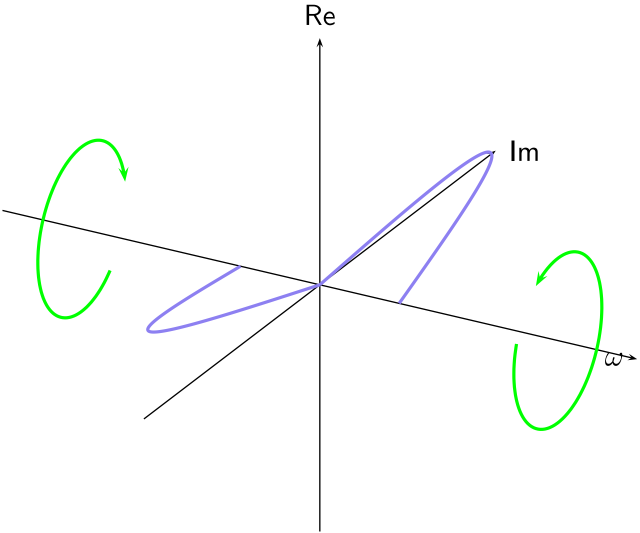
effect of the Hilbert filter (real part)



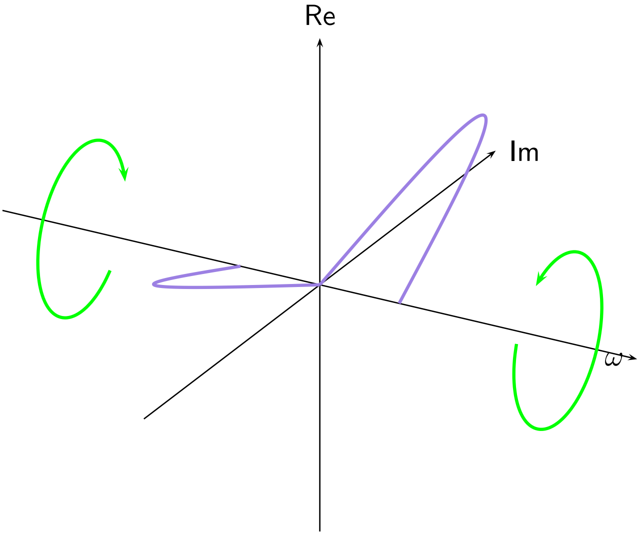
effect of the Hilbert filter (imaginary part)



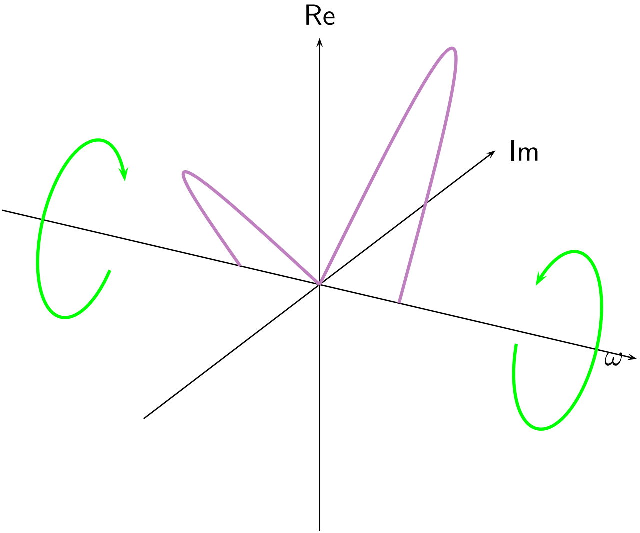
effect of the Hilbert filter (imaginary part)



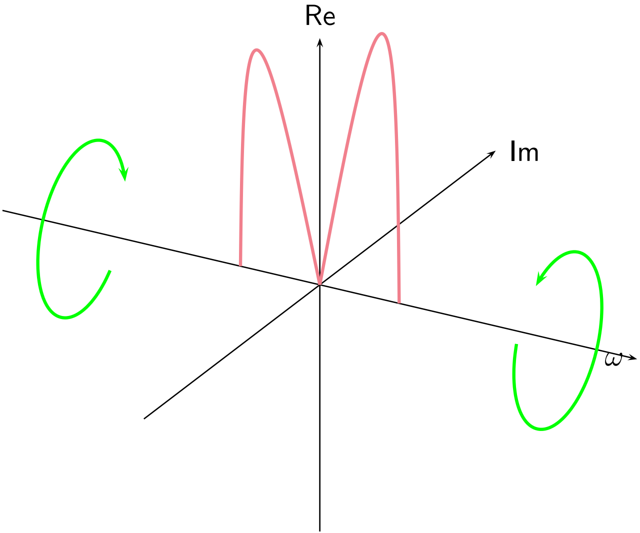
effect of the Hilbert filter (imaginary part)



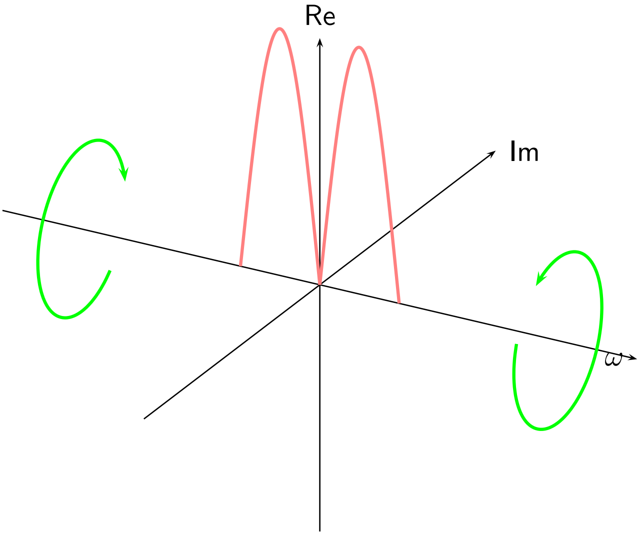
effect of the Hilbert filter (imaginary part)



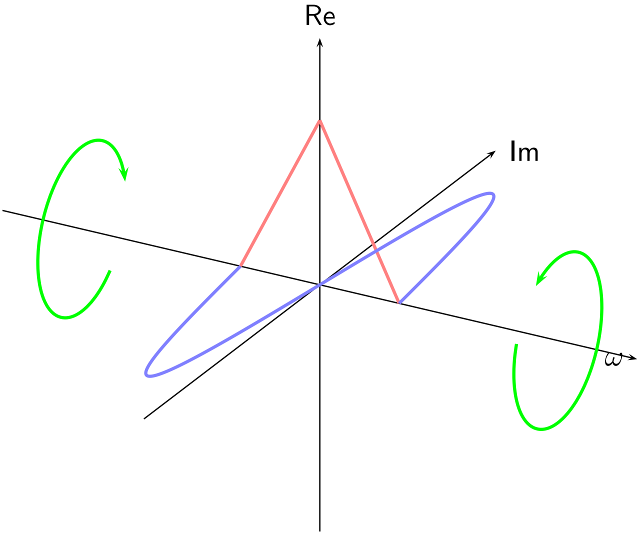
effect of the Hilbert filter (imaginary part)



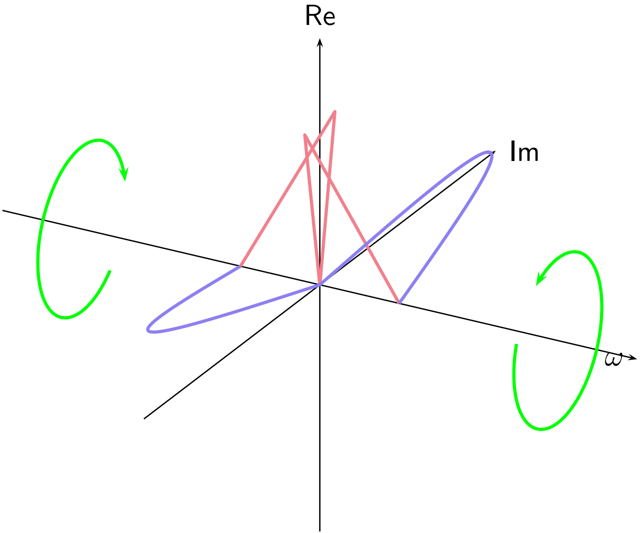
effect of the Hilbert filter (imaginary part)



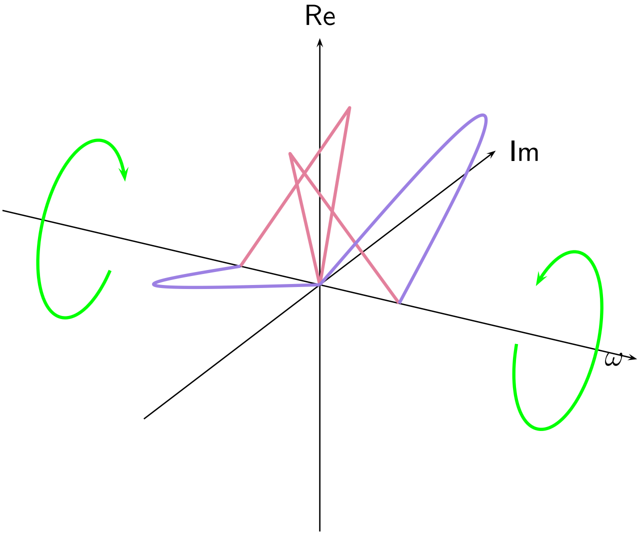
effect of the Hilbert filter



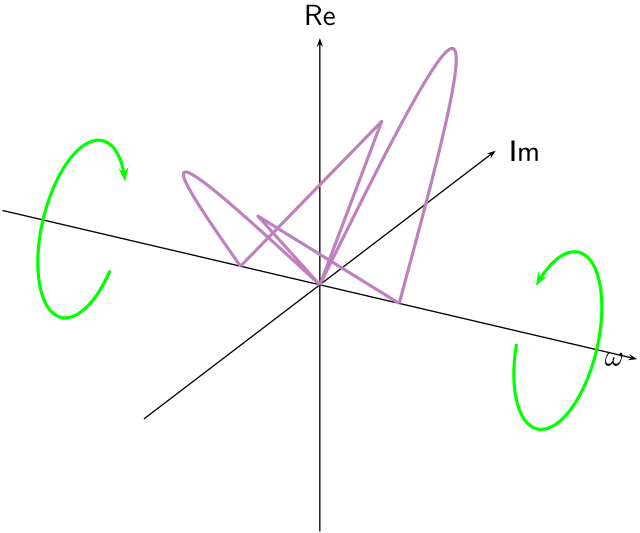
effect of the Hilbert filter



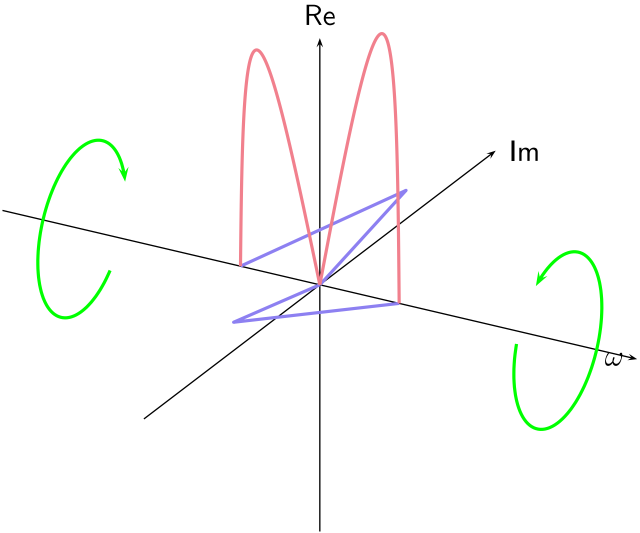
effect of the Hilbert filter



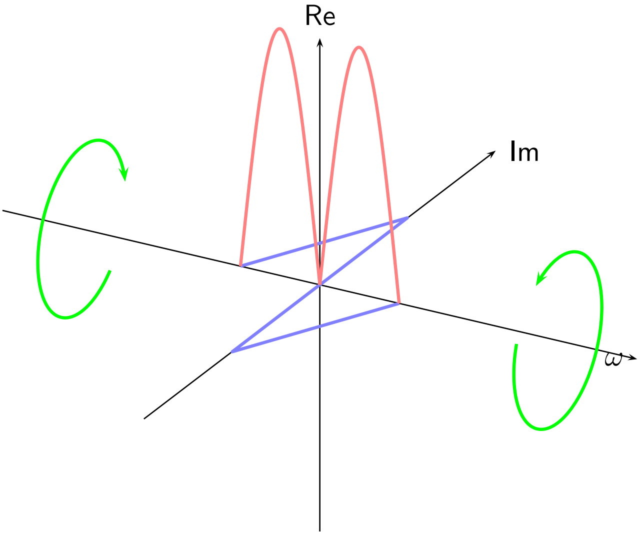
effect of the Hilbert filter



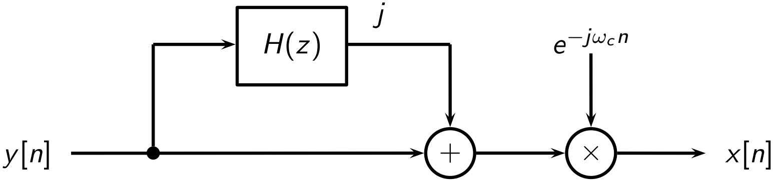
effect of the Hilbert filter



effect of the Hilbert filter

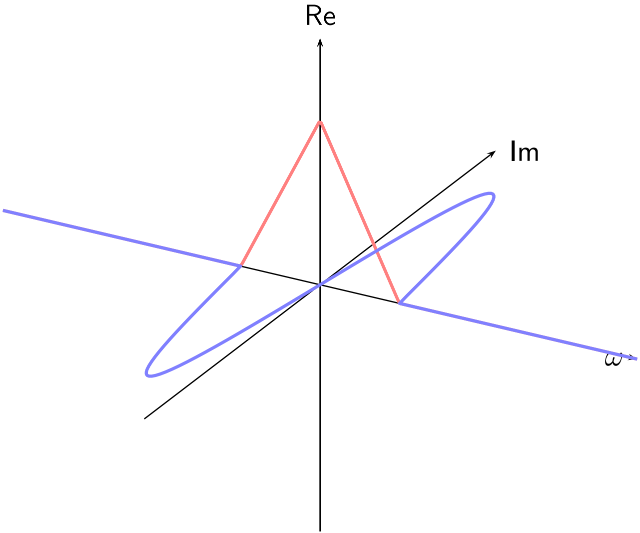


Hilbert demodulation



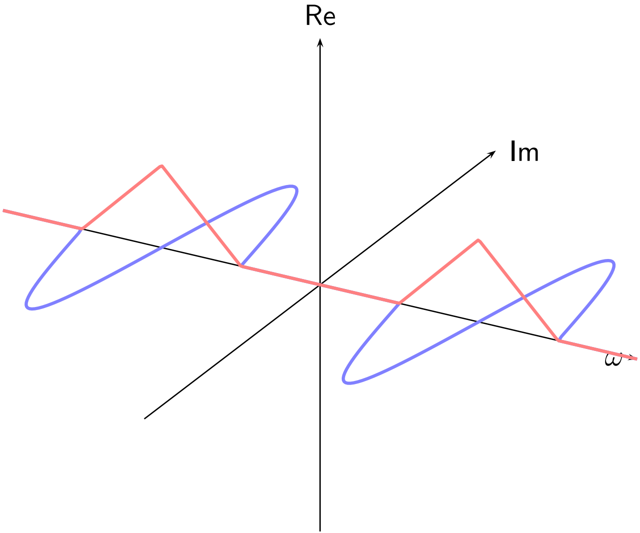
Hilbert demodulation

$x[n]$



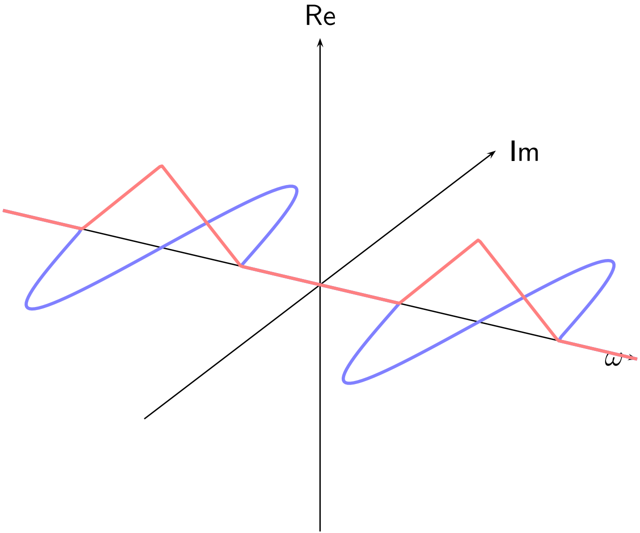
Hilbert demodulation

$x[n]\cos(\omega_0 n) = y[n]$



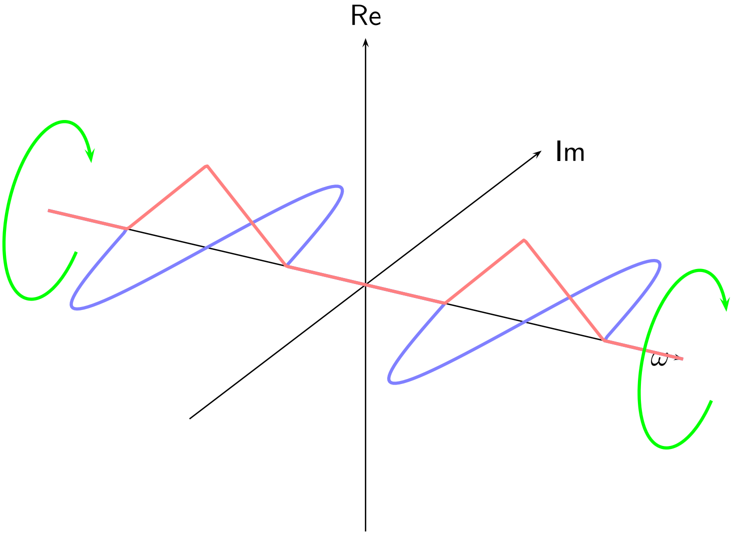
Hilbert demodulation: $jy[n] * h[n]$

$y[n]$



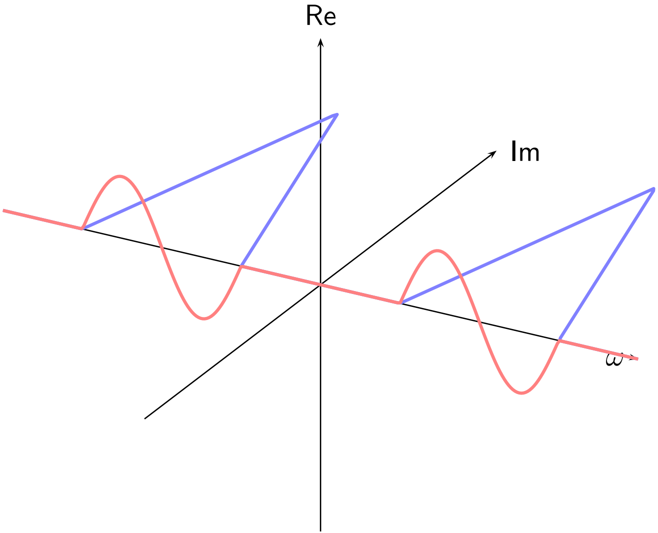
Hilbert demodulation: $jy[n] * h[n]$

$jy[n]$



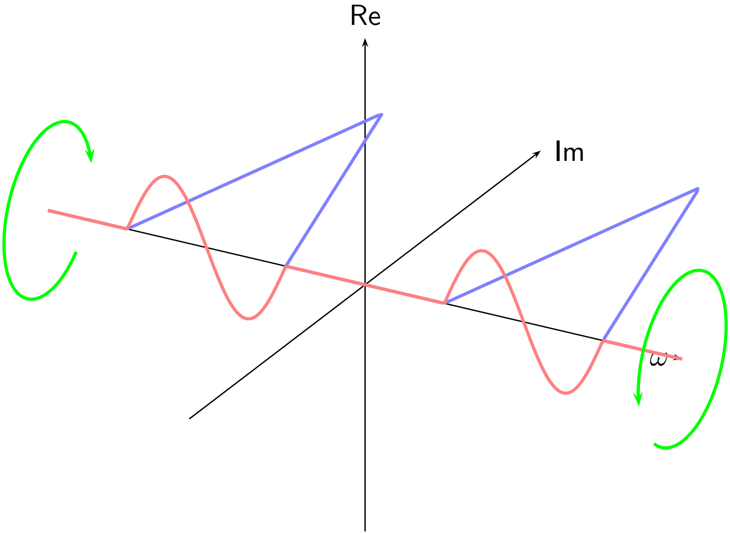
Hilbert demodulation: $jy[n] * h[n]$

$jy[n]$



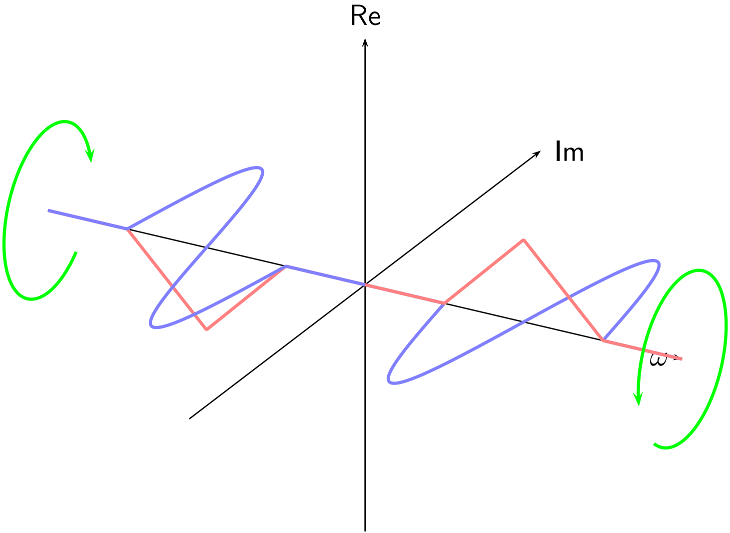
Hilbert demodulation: $jy[n] * h[n]$

$jy[n] * h[n]$

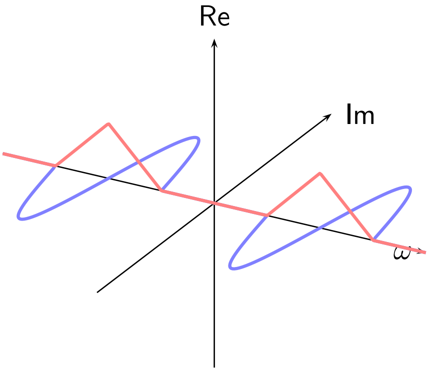


Hilbert demodulation: $jy[n] * h[n]$

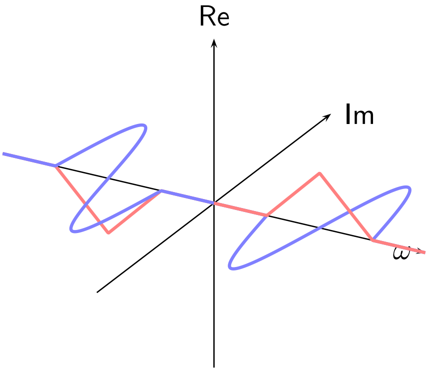
$jy[n] * h[n]$



Hilbert demodulation: $jy[n] * h[n] + y[n]$

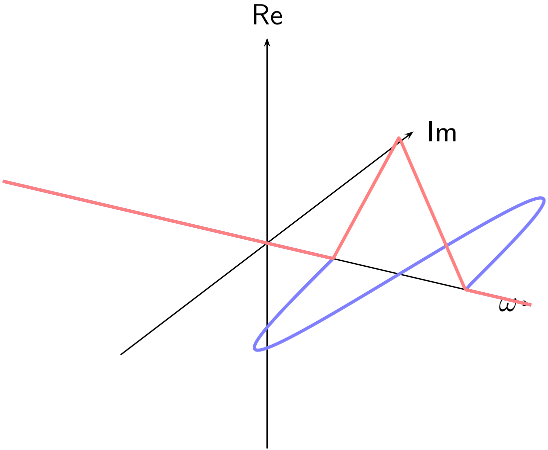


$y[n]$

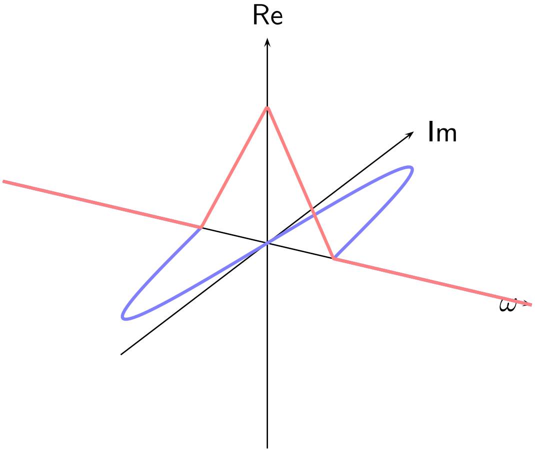


$jy[n] * h[n]$

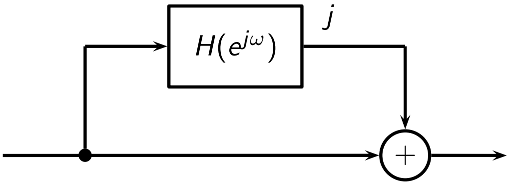
Hilbert demodulation: $jy[n] * h[n] + y[n] = x[n]e^{j\omega_0 n}$



Hilbert demodulation: $(jy[n] * h[n] + y[n])e^{-j\omega_0 n}$



Hilbert demodulator extracts the positive frequencies



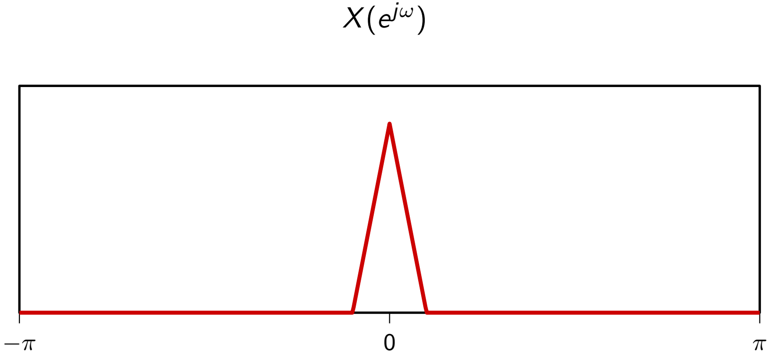
$$G(e^{j\omega}) = 1 + jH(e^{j\omega}) = \begin{cases} 2 & 0 \leq \omega < \pi \\ 0 & -\pi \leq \omega < 0 \end{cases}$$

Classic Demodulation

remember the classic demodulation scheme:

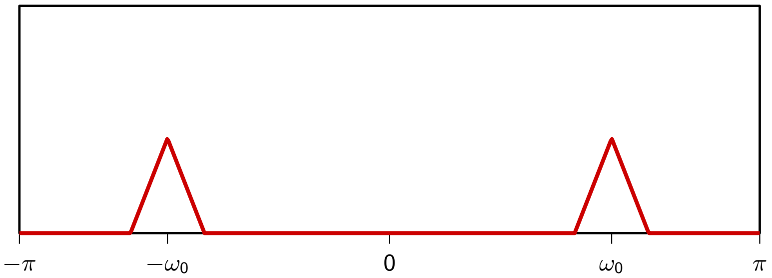
- apply sinusoidal modulation to $x[n]$: $y[n] = x[n]\cos\omega_0 n$
- demodulate by multiplying by the carrier $x'[n] = y[n]\cos\omega_0 n$
- remove unwanted high-frequency components via lowpass filtering

Classic Demodulation

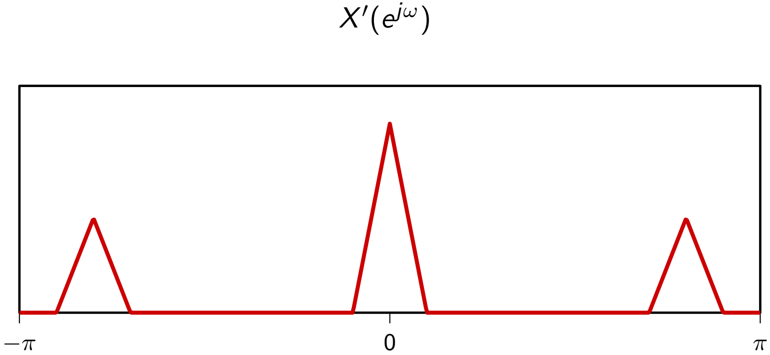


Classic Demodulation

$Y(e^{j\omega})$

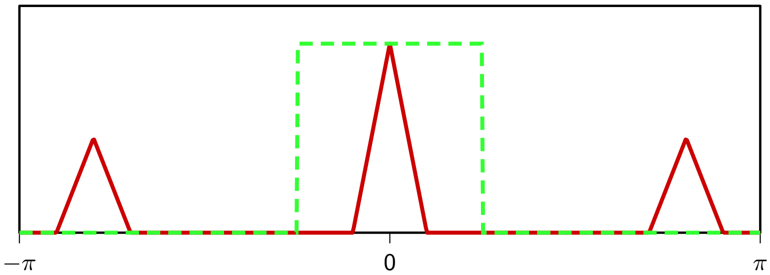


Classic Demodulation

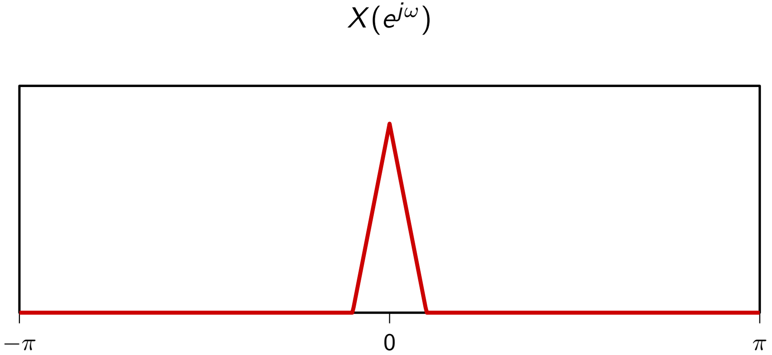


Classic Demodulation

$X'(e^{j\omega})$



Classic Demodulation



Hilbert vs Classic Demodulation

- no need to know the bandwidth of the signal
- same filter for all modulation frequencies
- good FIR approximations for the Hilbert filter exist!