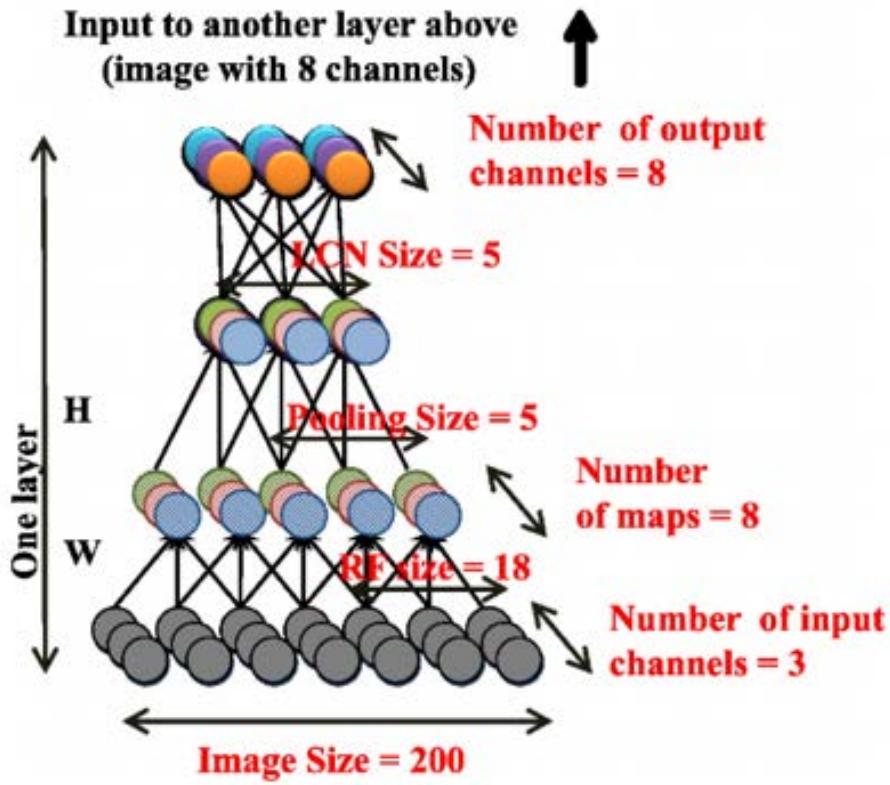
DEEP LEARNING CRASH COURSE

- Single Layer Perceptron
- Multiple Layer Perceptron
- Convolutional Neural Net



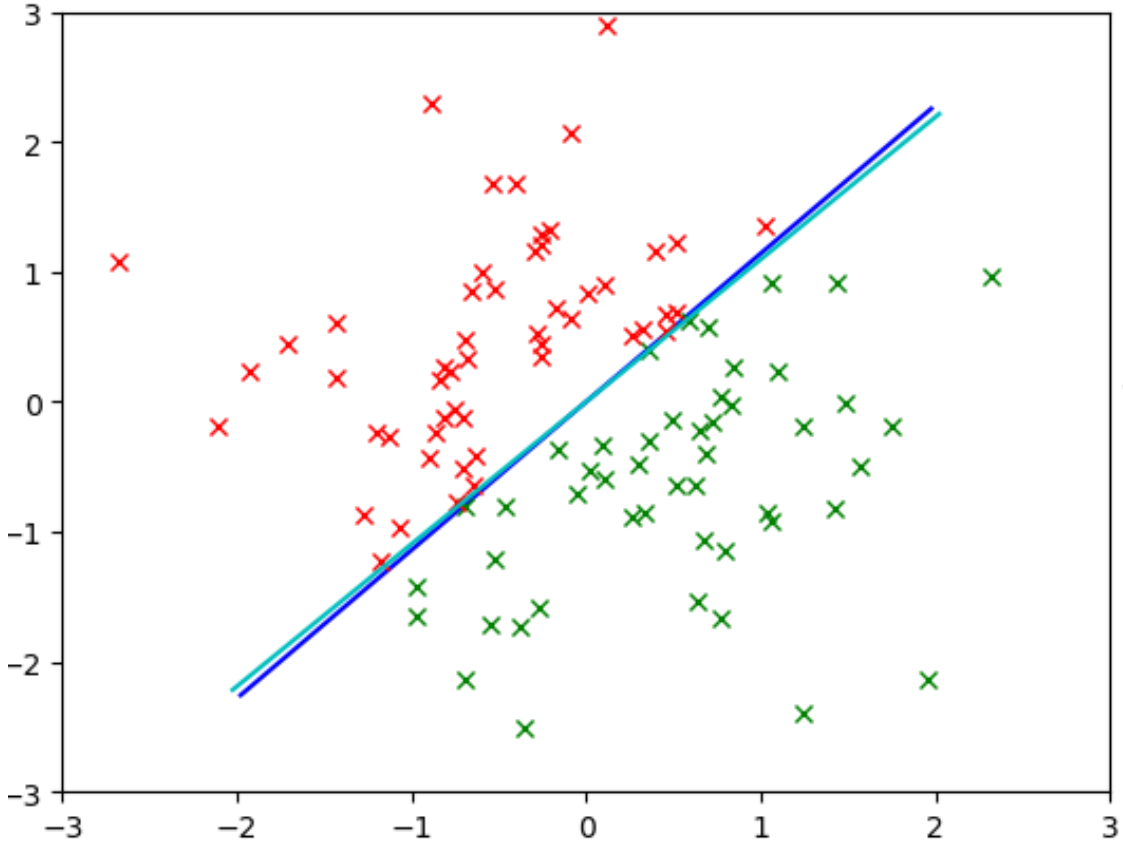
M.A. Nielsen. Neural Networks and Deep Learning, 2015
http://neuralnetworksanddeeplearning.com/

ARTIFICAL INTELLIGENCE

1997: Deep Blue beats chess World Champion

2016: AlphaGo beats go world champion

PERCEPTRON



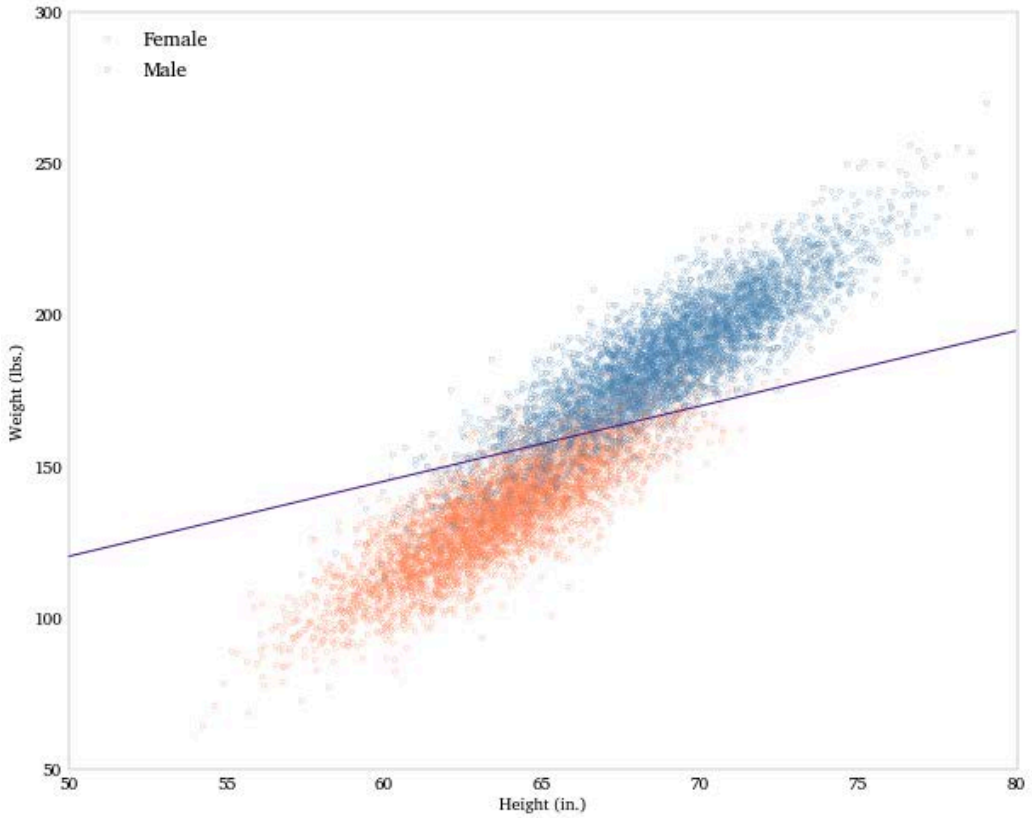
$$y(\mathbf{x};\tilde{\mathbf{w}}) = \begin{cases} 1 & \text{if} \quad \tilde{\mathbf{w}}^T\tilde{\mathbf{x}} \geq 0\,, \\ -1 & \text{otherwise.} \end{cases}$$

$$\tilde{\mathbf{x}} = [1, x_1, ..., x_n]$$

Choose the **w** that minimizes:

$$E(\tilde{\mathbf{w}}) = -\sum_{n=1}^{N}(\tilde{\mathbf{w}}^T\tilde{\mathbf{x}}_n)t_n$$

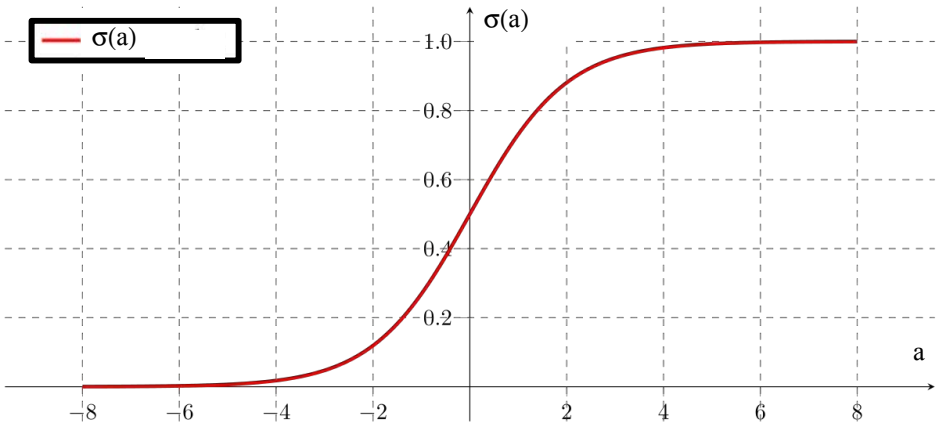
LOGISTIC REGRESSION



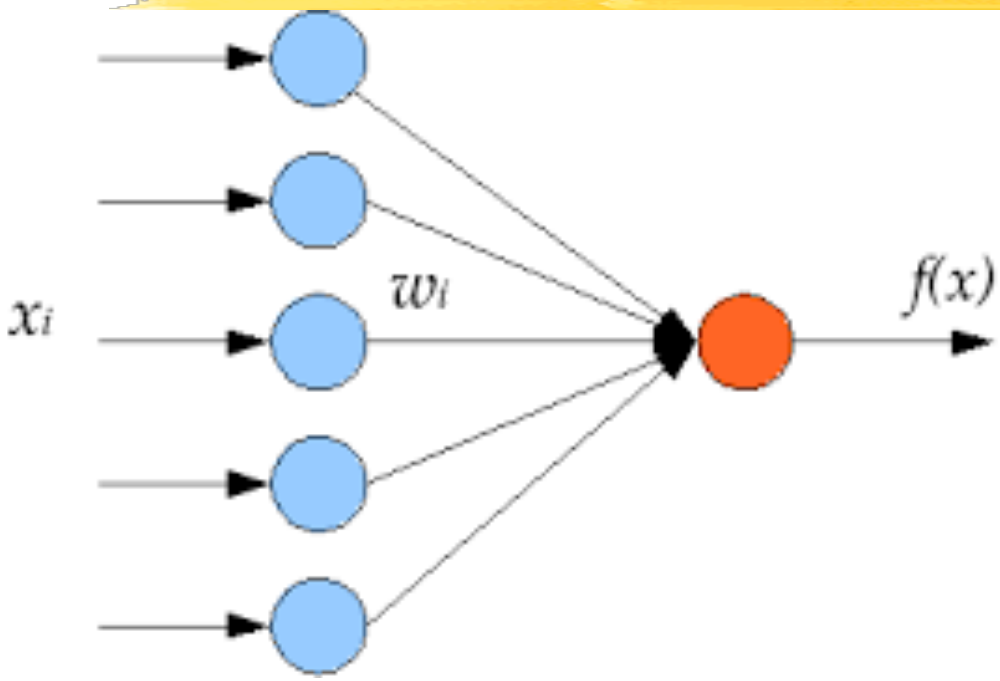
$$y(\mathbf{x}; \mathbf{w}) = \sigma(\mathbf{w}^T \mathbf{x} + w_0)$$

Choose the **w** that minimizes:

$$E(\mathbf{w}) = -\sum_n \{t_n \ln y_n + (1 - t_n) \ln(1 - y_n)\}$$

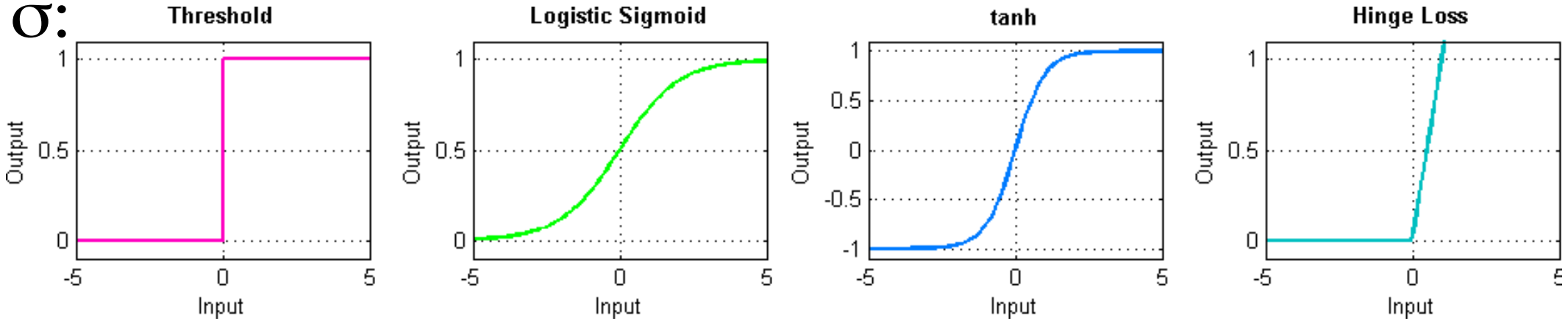


SINGLE LAYER PERCEPTRON

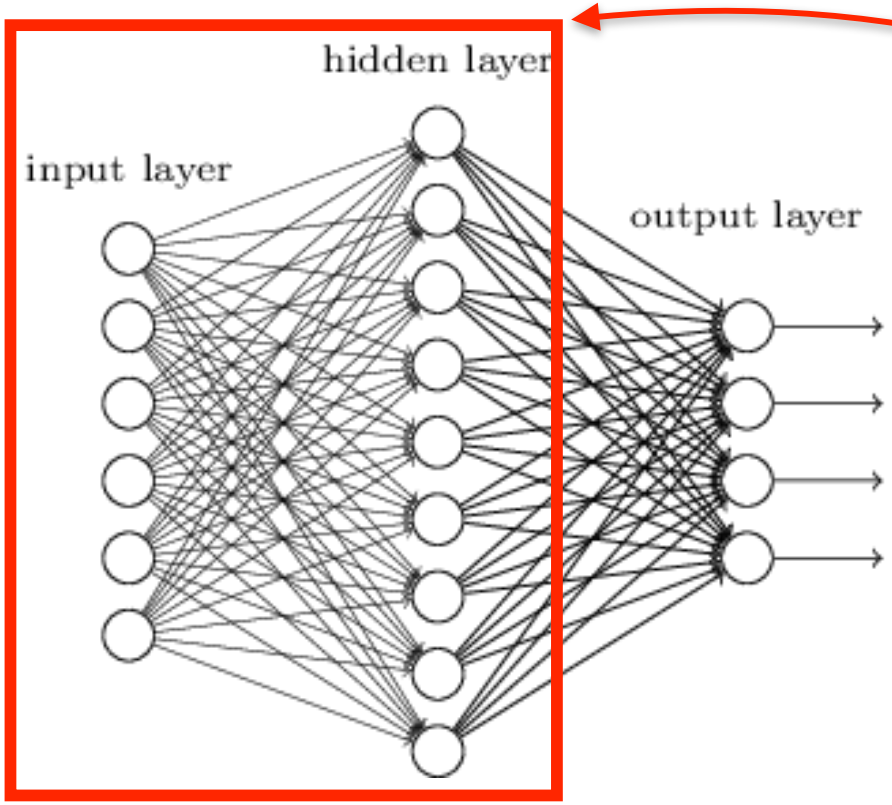


$$f(\mathbf{x}) = \sigma(\mathbf{w} \cdot \mathbf{x} + \mathbf{b})$$

σ:



MULTILAYER PERCEPTRON

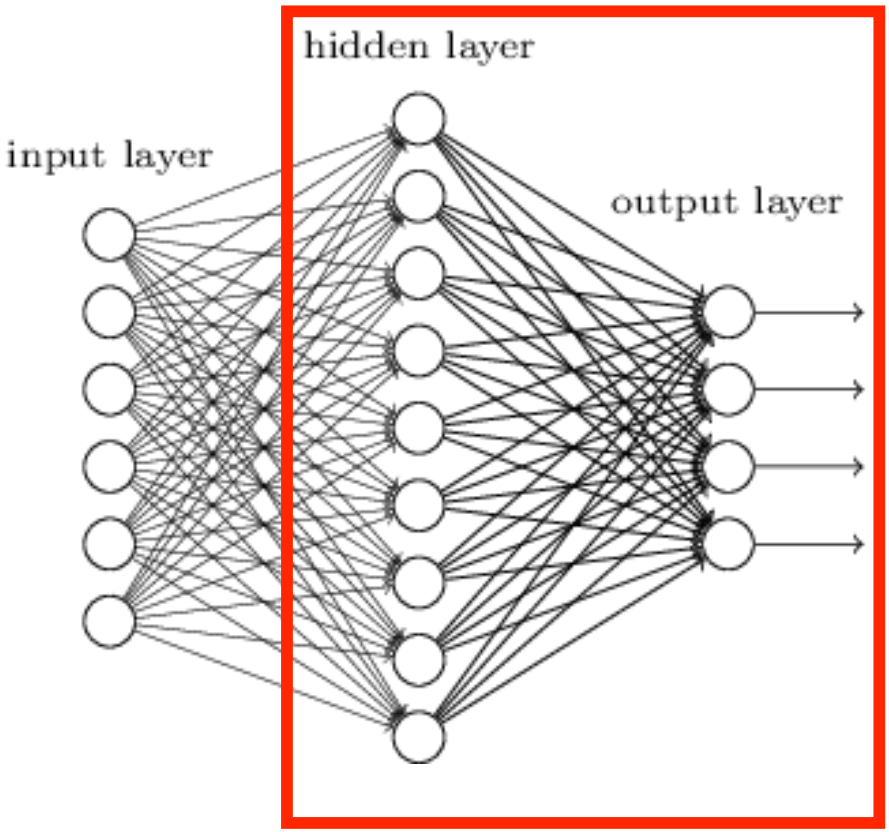


$$\mathbf{h} = \sigma_1(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$

$$\mathbf{y} = \sigma_2(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

- The process can be repeated several times to create a vector **h**.

MULTILAYER PERCEPTRON



$$\mathbf{h} = \sigma_1(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$

$$\mathbf{y} = \sigma_2(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

- The process can be repeated several times to create a vector **h**.
- It can then be done again to produce an output **y**.

- —> This output is a **differentiable** function of the weights.

BINARY CASE

Given a training set $\{\mathbf{x}_n, t_n\}_{1 \leq n \leq N}$ where $t_n \in \{0, 1\}$, minimize

$$\begin{aligned} E(\mathbf{W}, \mathbf{b}) &= -\frac{1}{N} \sum_{1}^{N} \left[t_i \log(y_i) + (1 - t_i) \log(1 - y_i) \right] , \\ y_i &= f(\mathbf{x}_i) \\ &= \sigma(\mathbf{W}_2(\sigma(\mathbf{W}_1 \mathbf{x}_i + \mathbf{b}_1)) + \mathbf{b}_2), \end{aligned}$$

that is, minimize the number of misclassified samples.

Since E is a differentiable function of **W** and **b**, this can be done using a gradient-based technique, also known as back propagation.

MULTI-CLASS CASE

In the multi-class case, the probability that input vector $\mathbf{x}$ belongs to class c is taken to be

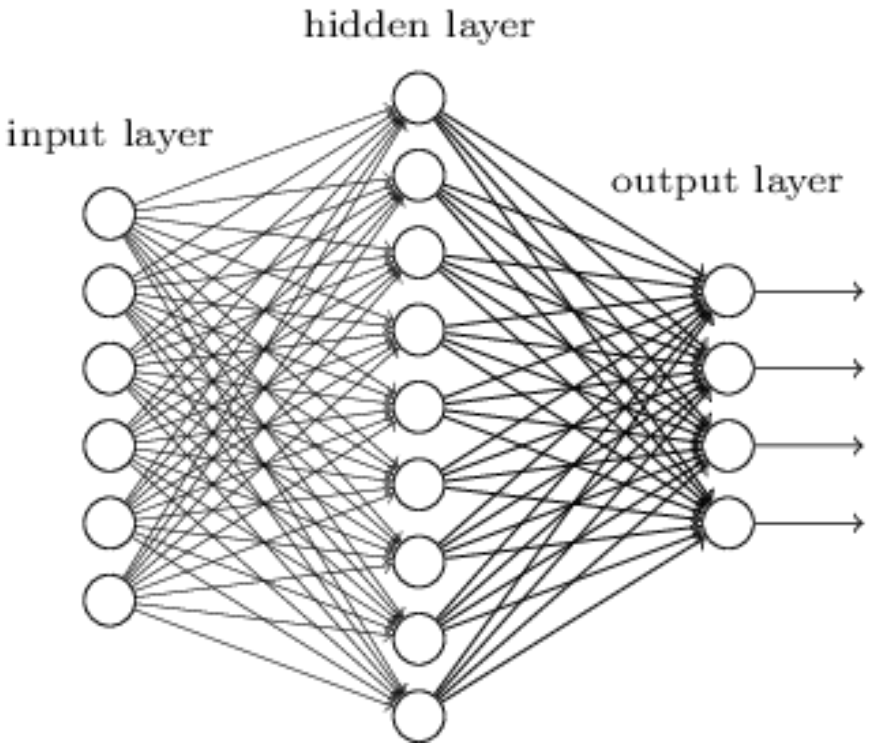
$$P(\mathbf{x} \in c|\mathbf{W}, \mathbf{b}) = \frac{y^c(\mathbf{x}; \mathbf{W}, \mathbf{b})}{\sum_k y^k(\mathbf{x}; \mathbf{W}, \mathbf{b})} .$$

Given a training set $\{\mathbf{x}_n, t_n^1, \ldots, t_n^C\}_{1 \leq n \leq N}$ where $t_n^c \in \{0, 1\}$, minimize

$$E(\mathbf{W}, \mathbf{b}) = -\sum_n \sum_c t_n^c \log(P(\mathbf{x}_n \in c|\mathbf{W}, \mathbf{b})) .$$

Since E remains a differentiable function of **W** and **b**, this can also be done using a gradient-based technique, also known as back propagation.

HINGE LOSS OR RELU

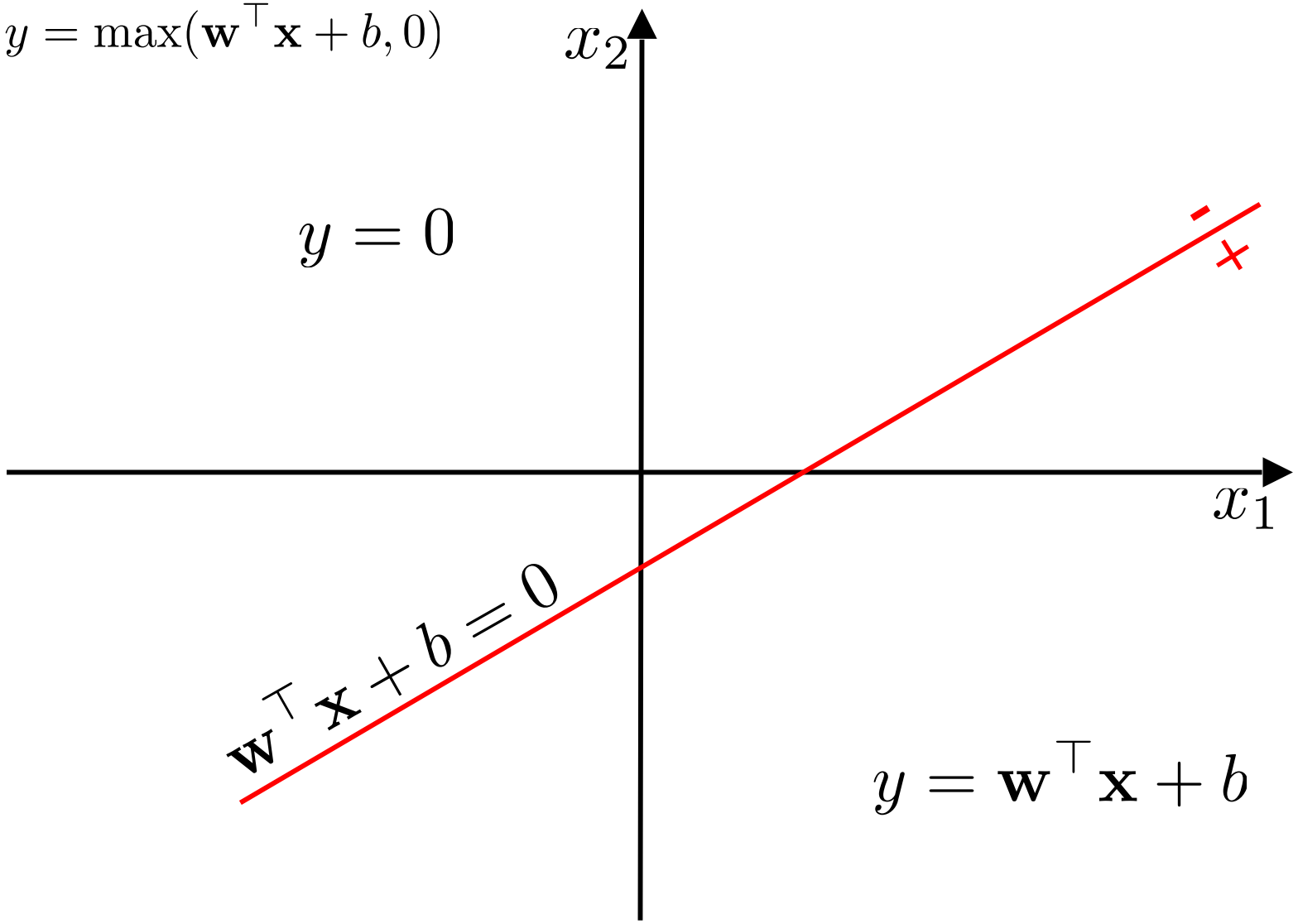


$$\mathbf{h} = \sigma(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$
$$\mathbf{y} = \sigma(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$

$\sigma(\mathbf{x}) = \max(0,\mathbf{x})$.

- Each node defines a hyperplane.
- The resulting function is piecewise linear affine and continuous.

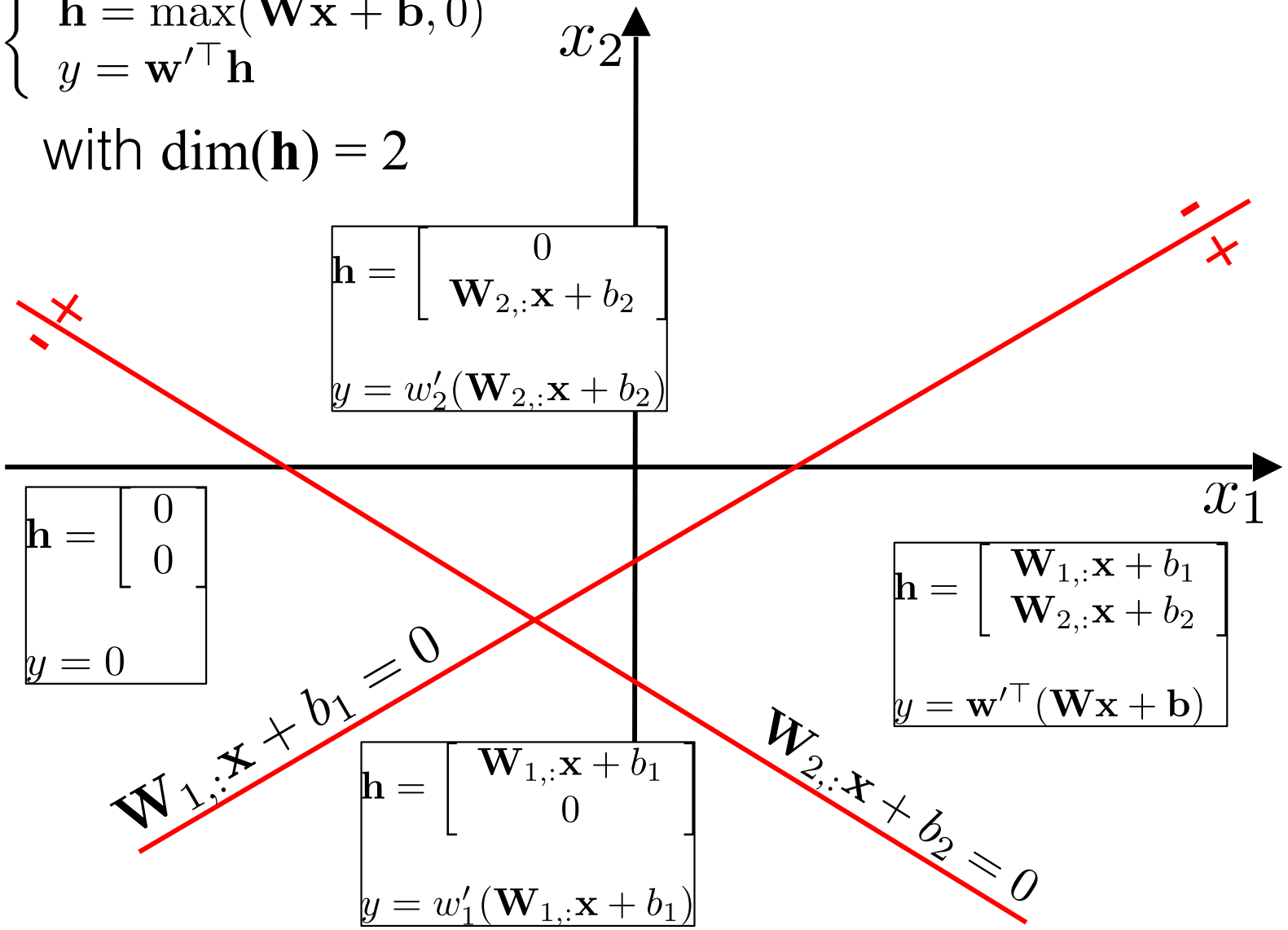
ONE SINGLE HYPERPLANE



TWO HYPERPLANES

$$\begin{cases} \mathbf{h} = \max(\mathbf{W}\mathbf{x} + \mathbf{b}, 0) \\ y = \mathbf{w}'^{\top}\mathbf{h} \end{cases}$$

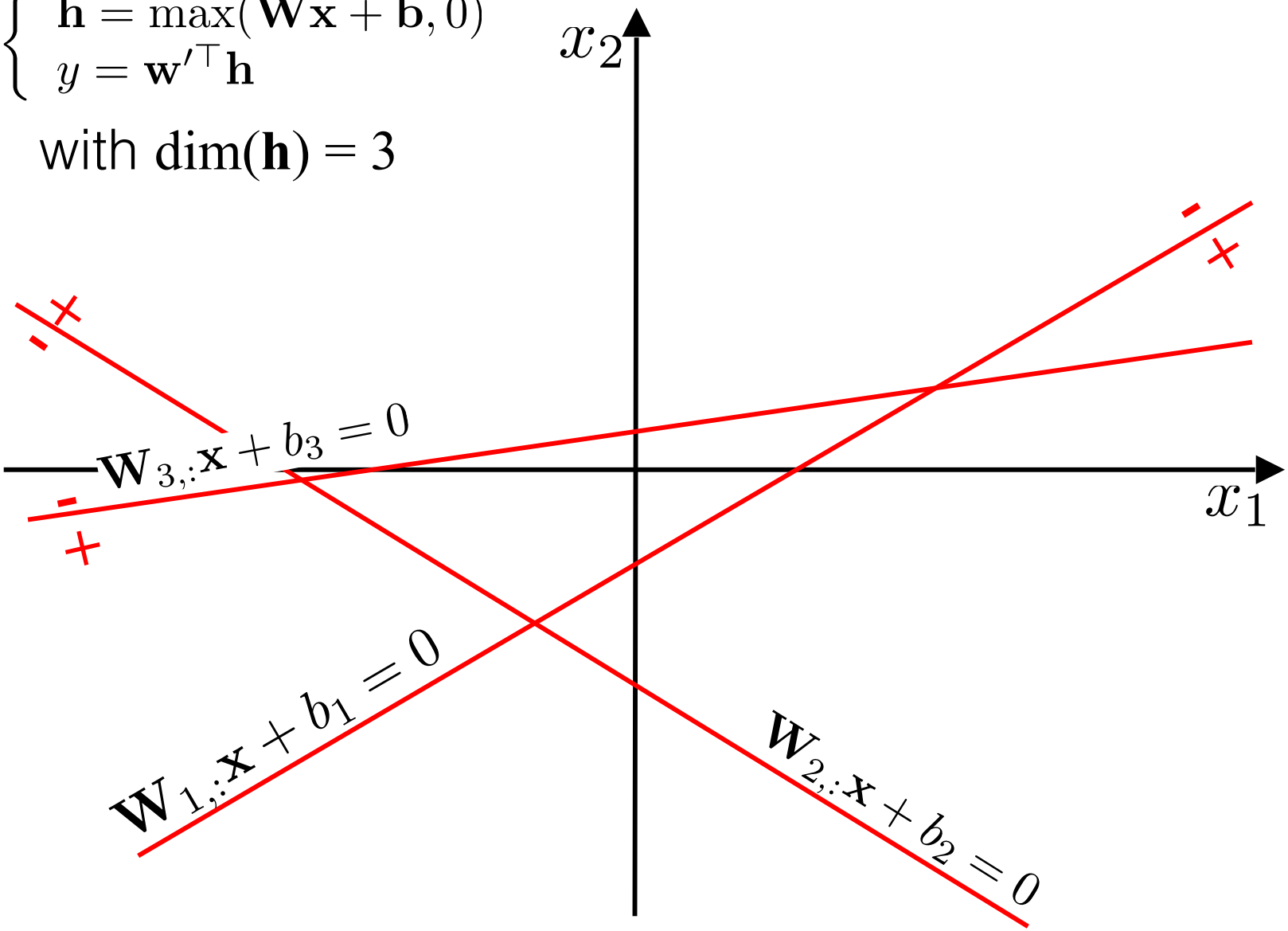
with $\dim(\mathbf{h}) = 2$



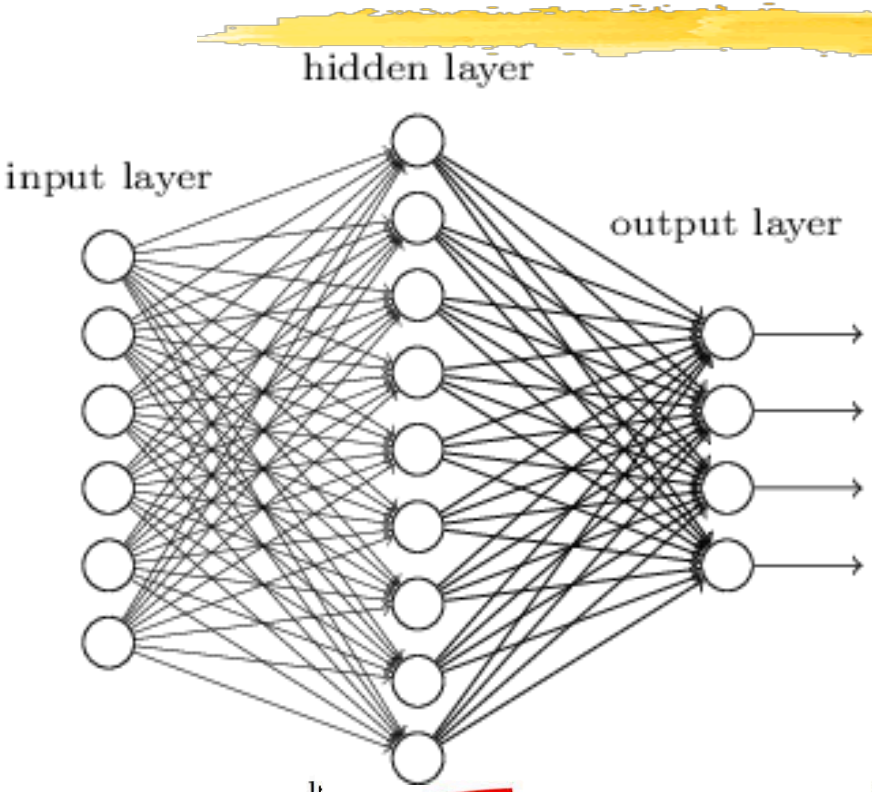
THREE HYPERPLANES

$$\begin{cases} \mathbf{h} = \max(\mathbf{W}\mathbf{x} + \mathbf{b}, 0) \\ y = \mathbf{w}'^{\top}\mathbf{h} \end{cases}$$

with $\dim(\mathbf{h}) = 3$

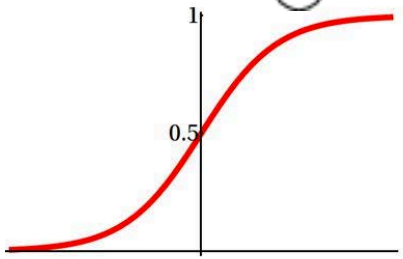


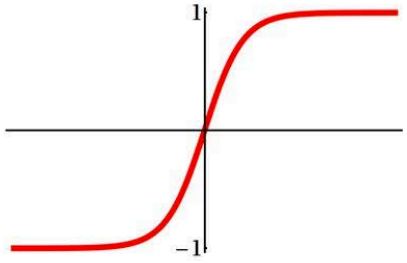
SIGMOID AND TANH



$$\mathbf{h} = \sigma(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1)$$

$$\mathbf{y} = \sigma(\mathbf{W}_2\mathbf{h} + \mathbf{b}_2)$$



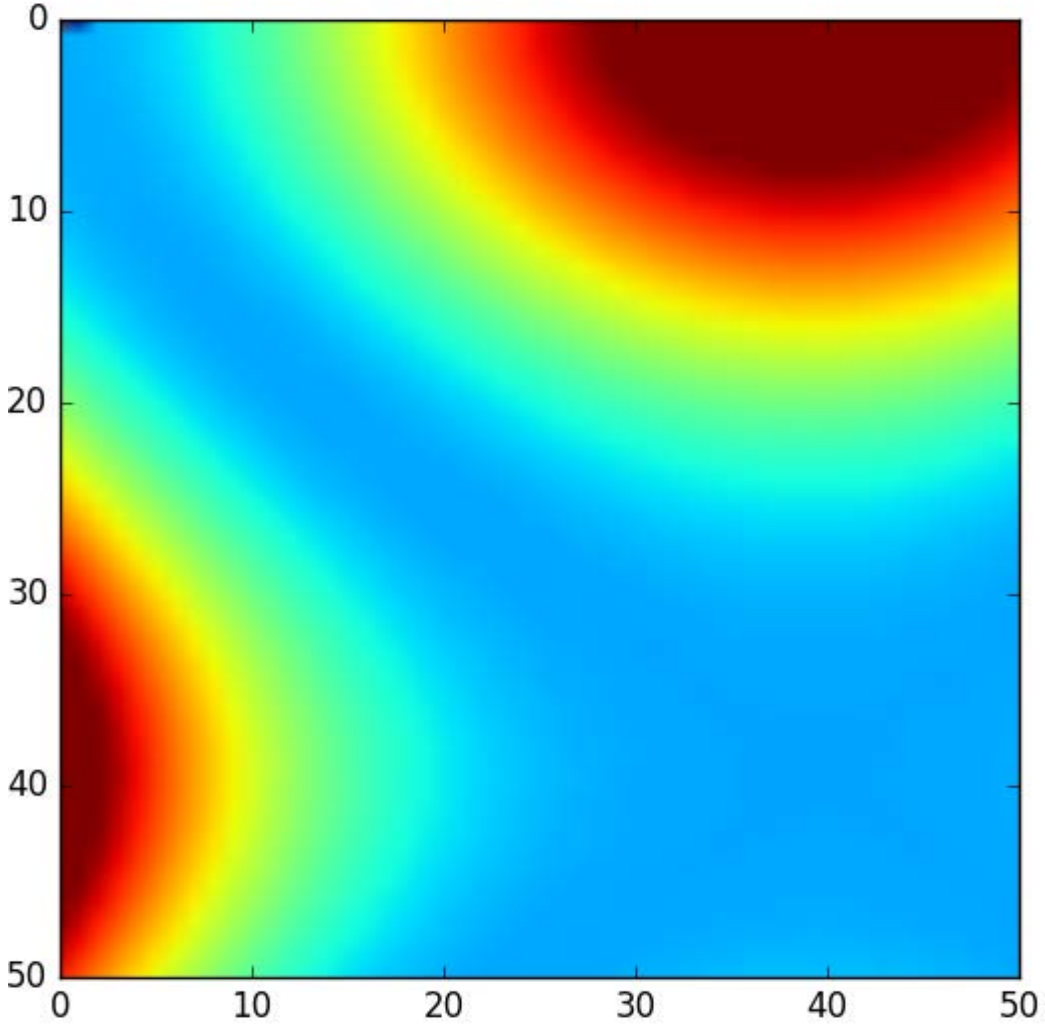


$$\text{sigm:} \quad \sigma(x) = \frac{1}{1+\exp(-x)}$$

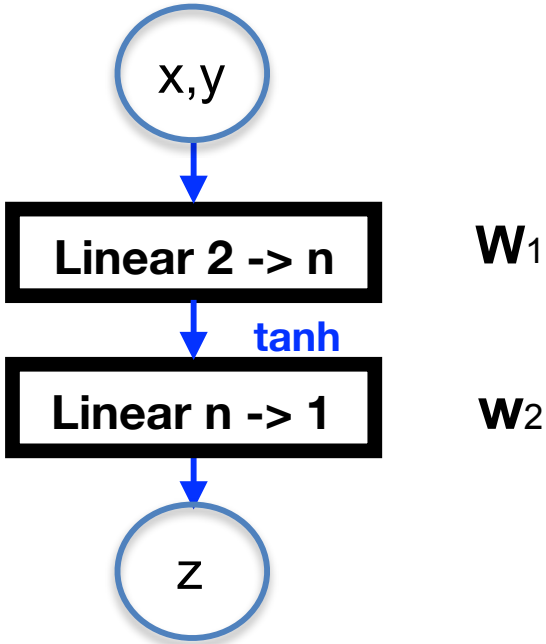
$$\text{tanh:} \quad \sigma(x) = \frac{\exp(x)-\exp(-x)}{\exp(x)+\exp(-x)}$$

- Each node defines a hyperplane.
- The resulting function is continuously differentiable.

INTERPOLATING A SURFACE



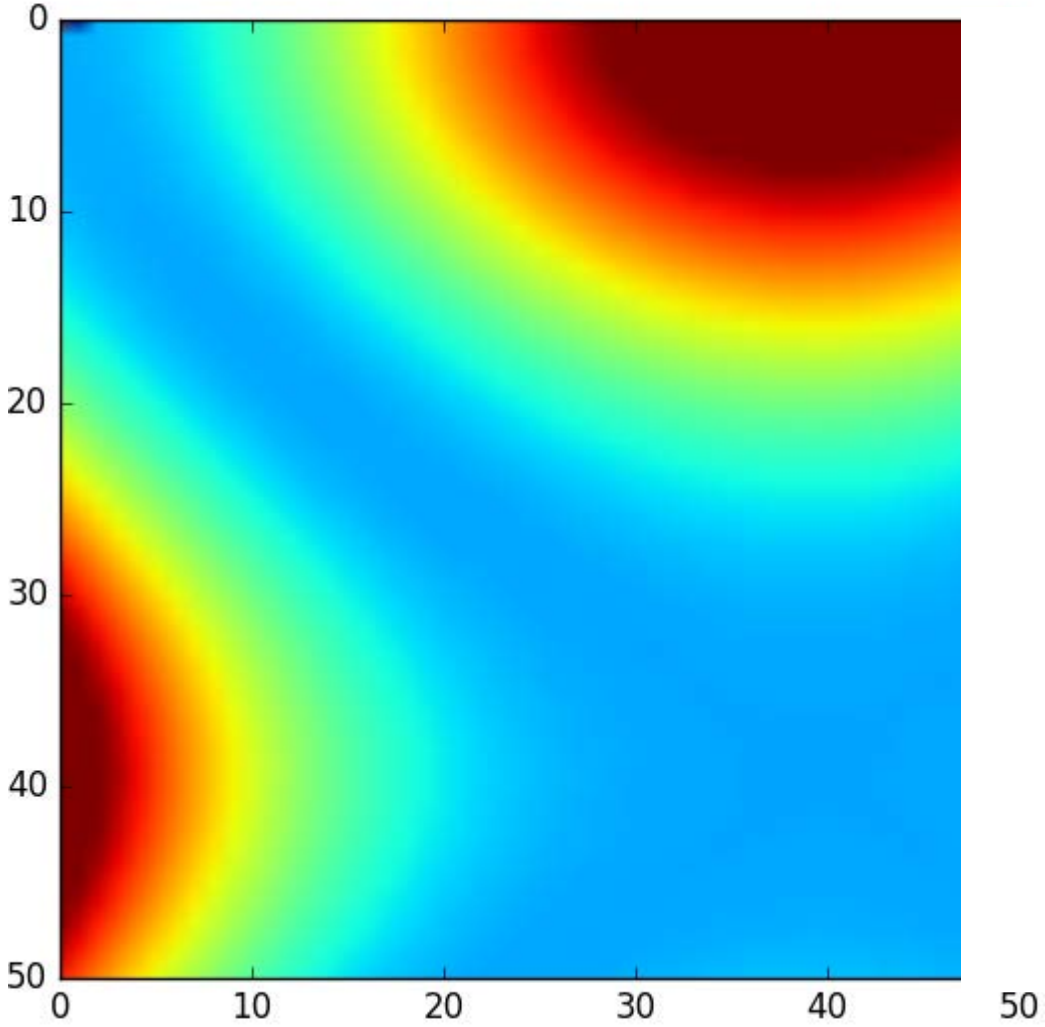
$z = 100 * (y - x^2)^2 + (1 - x)^2$



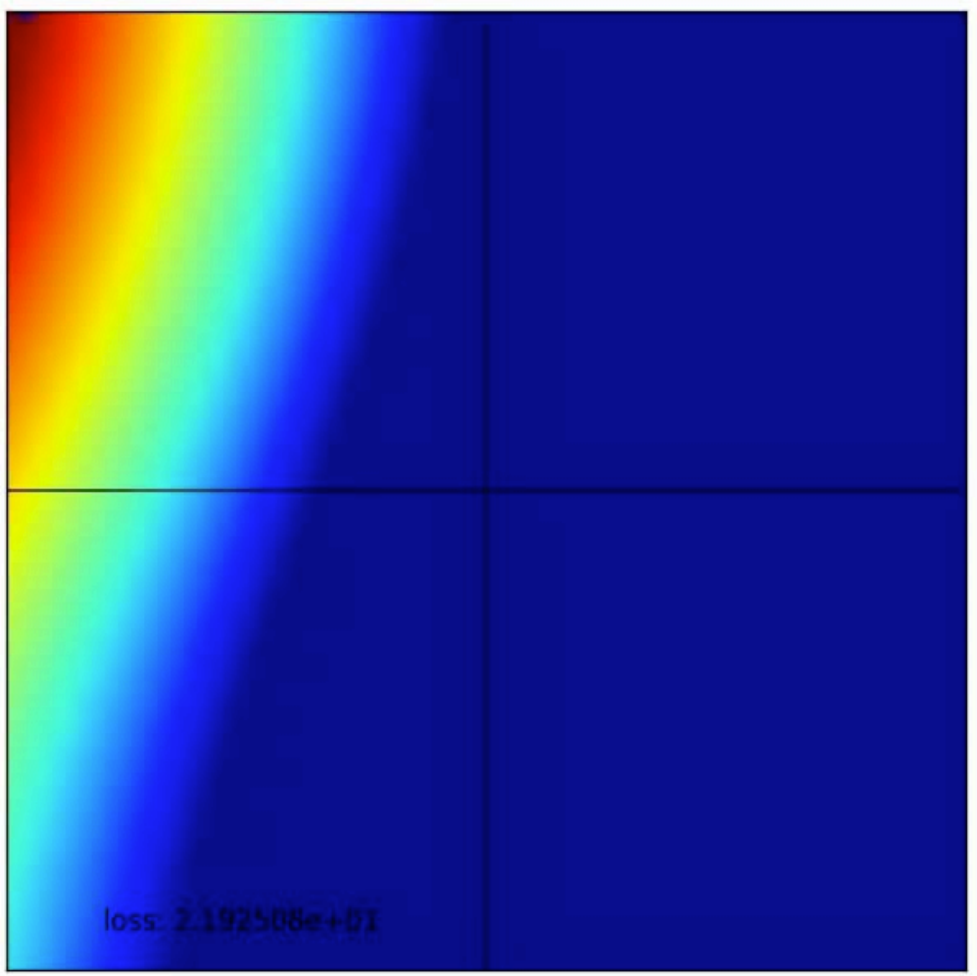
$$
\begin{aligned}
z &= f(x, y) \\
&= \mathbf{w}_2 \sigma(\mathbf{W}_1 \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \end{bmatrix}) + b_z
\end{aligned}
$$

Minimize $\sum_i (z_i - f(x_i, y_i))^2$,
with respect to $\mathbf{W}_1, \mathbf{w}_2, b_x, b_y, b_z$.

INTERPOLATING A SURFACE

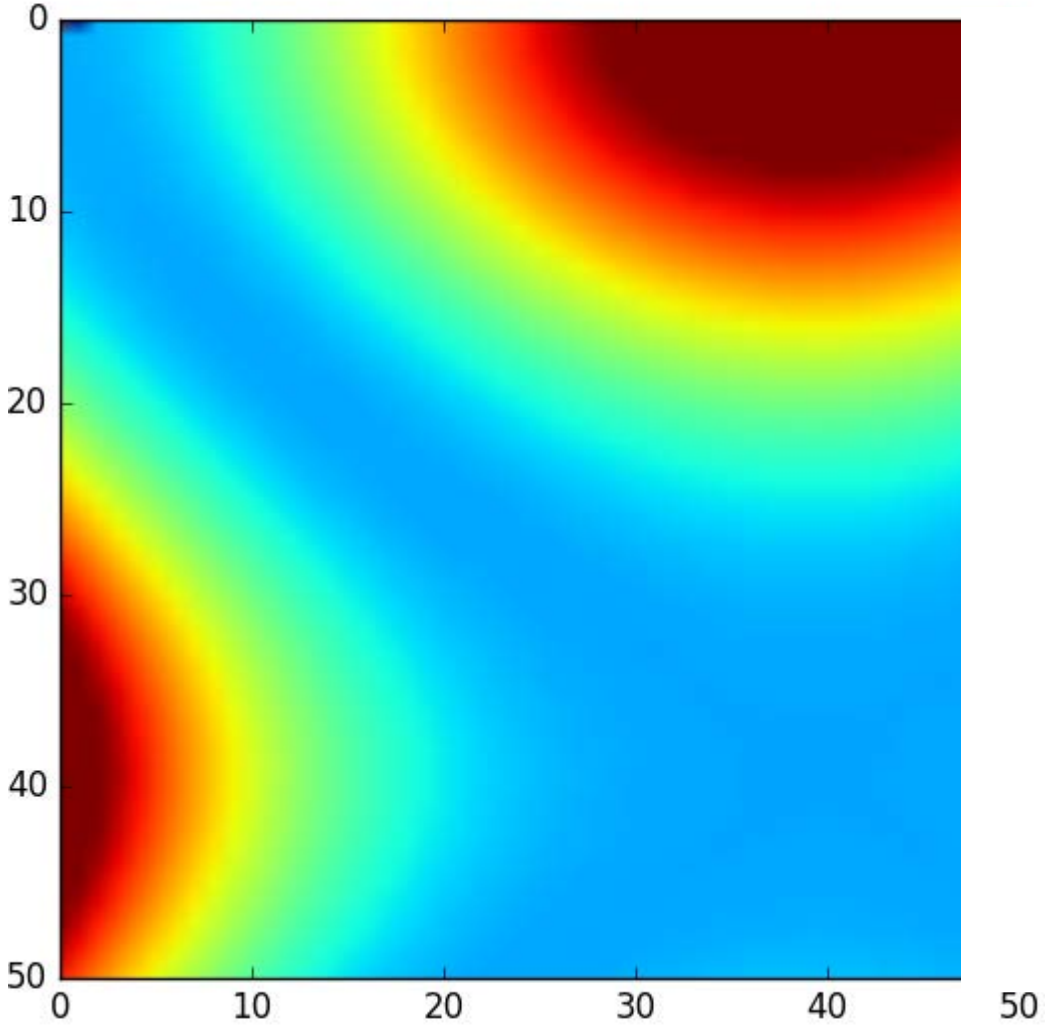


$$z = 100 * (y - x^2)^2 + (1 - x)^2$$

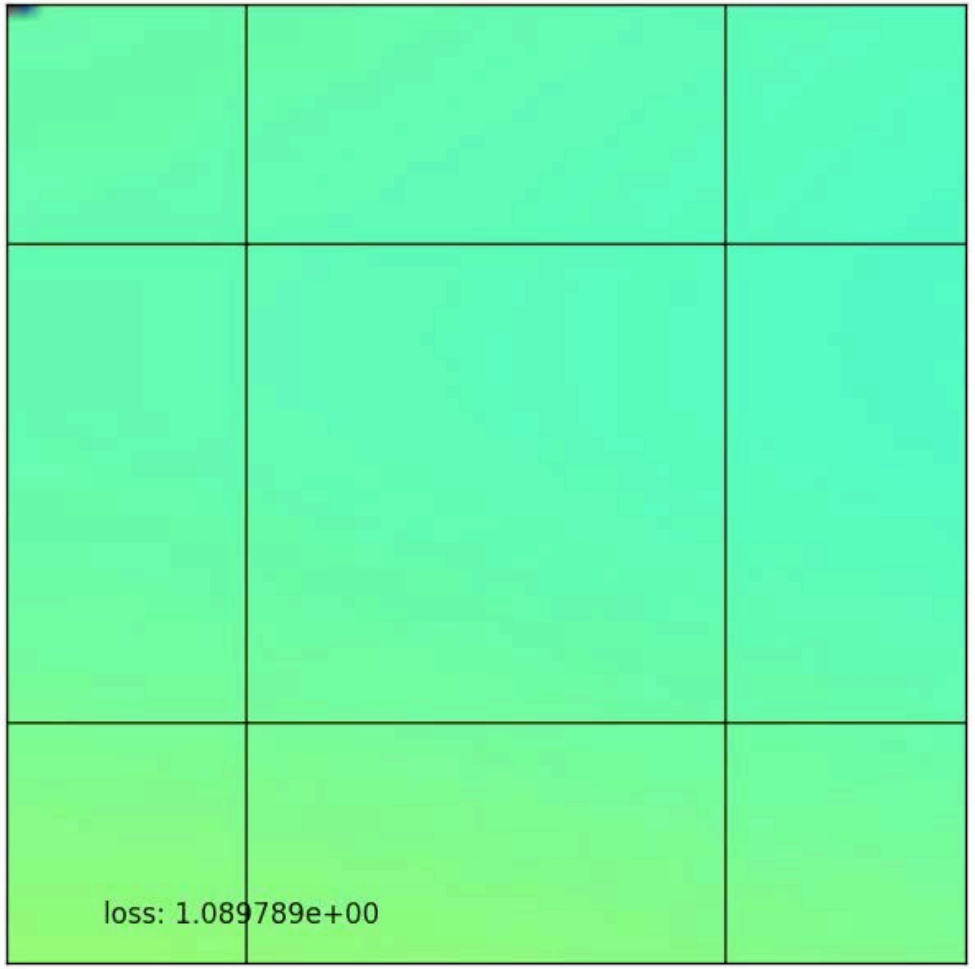


3-node hidden layer

INTERPOLATING A SURFACE

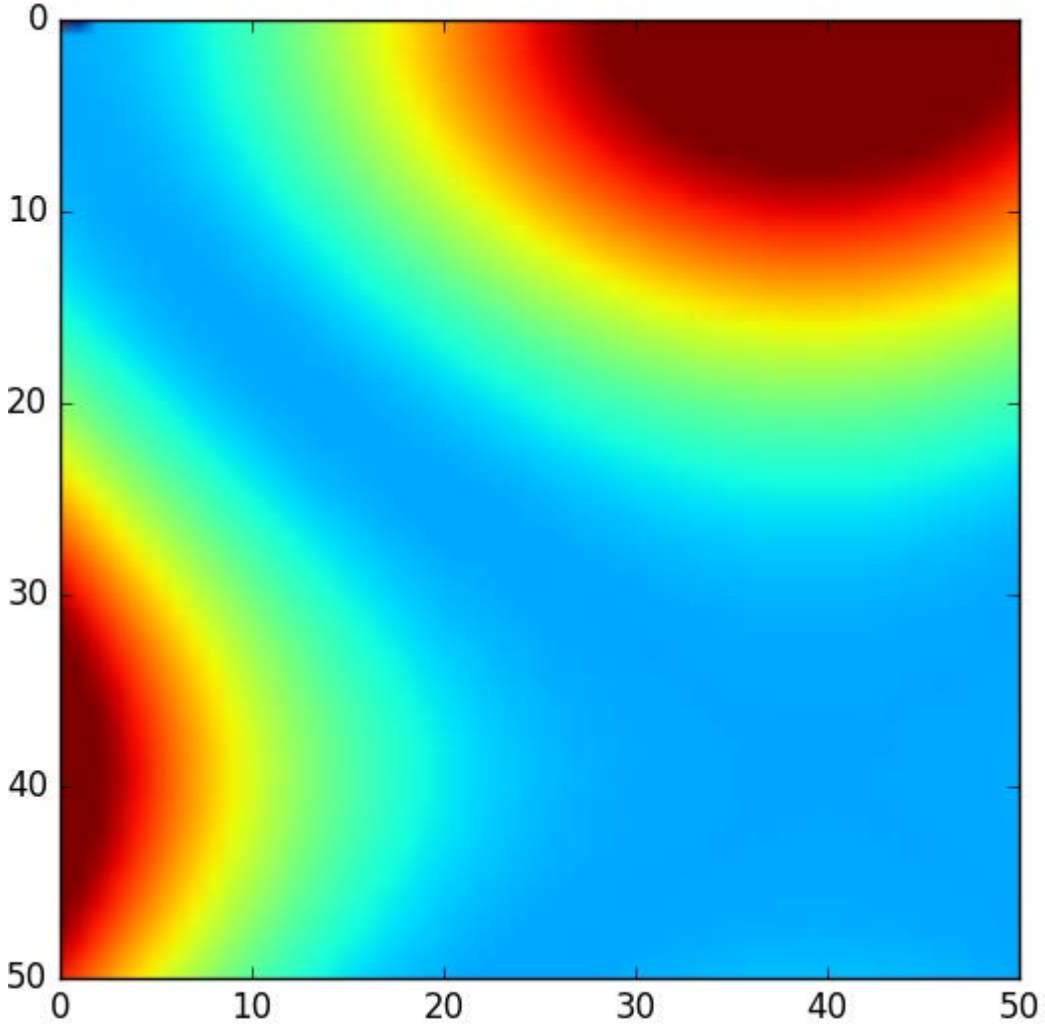


$$z = 100 * (y - x^2)^2 + (1 - x)^2$$

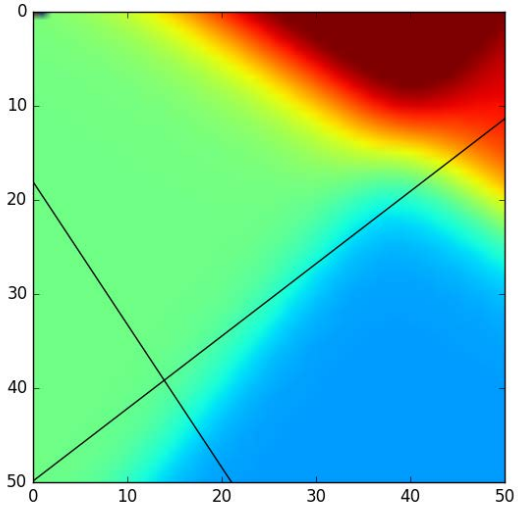


4-node hidden layer

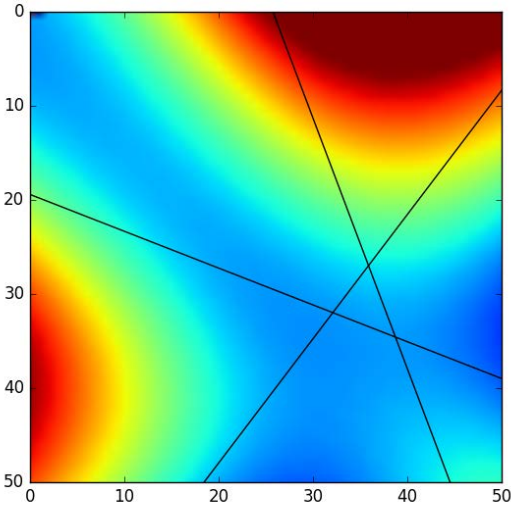
ADDING MORE NODES



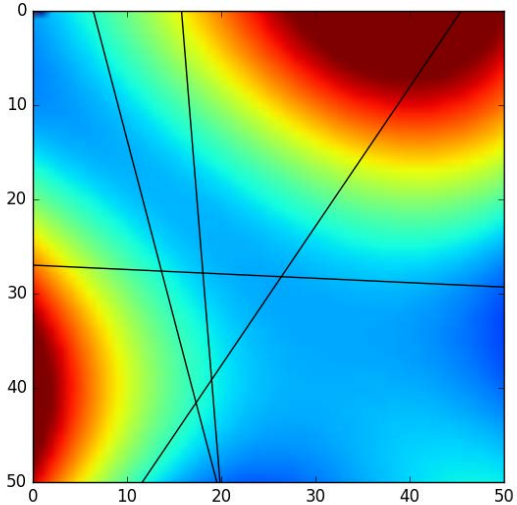
$$z = 100 * (y - x^2)^2 + (1 - x)^2$$



2 nodes -> loss 3.02e-01

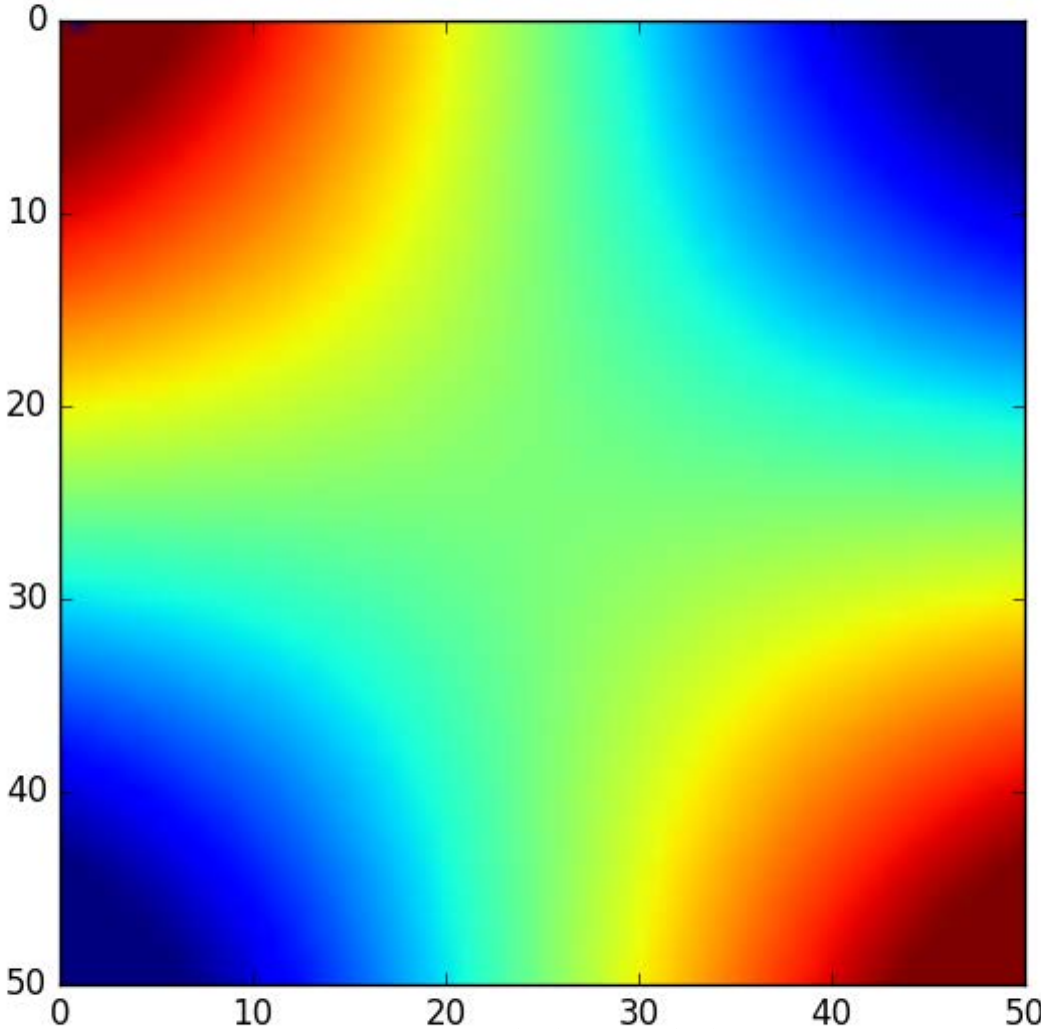


3 nodes -> loss 2.08e-02

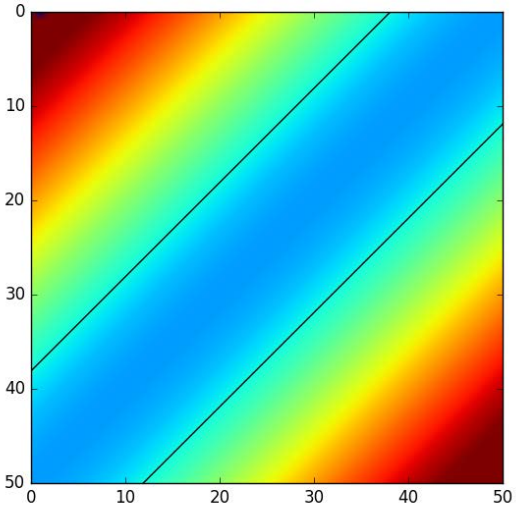


4 nodes -> loss 8.27e-03

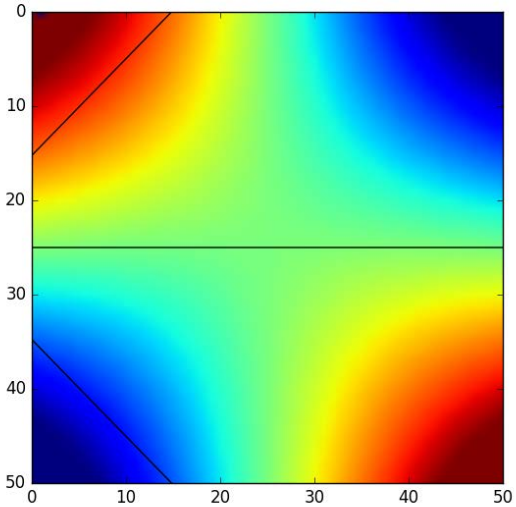
ADDING MORE NODES



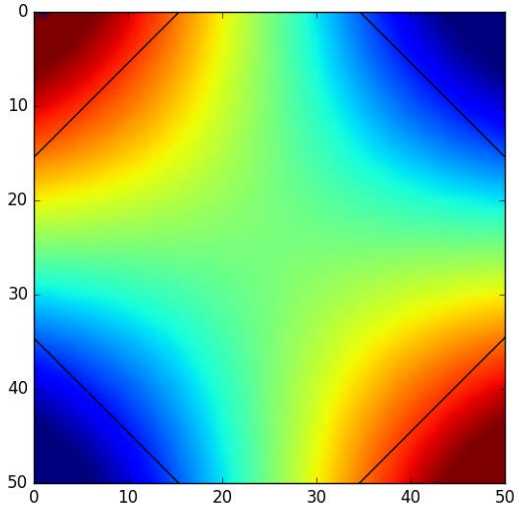
$z = sin(x)sin(y)$



2 nodes -> loss 2.61e-01

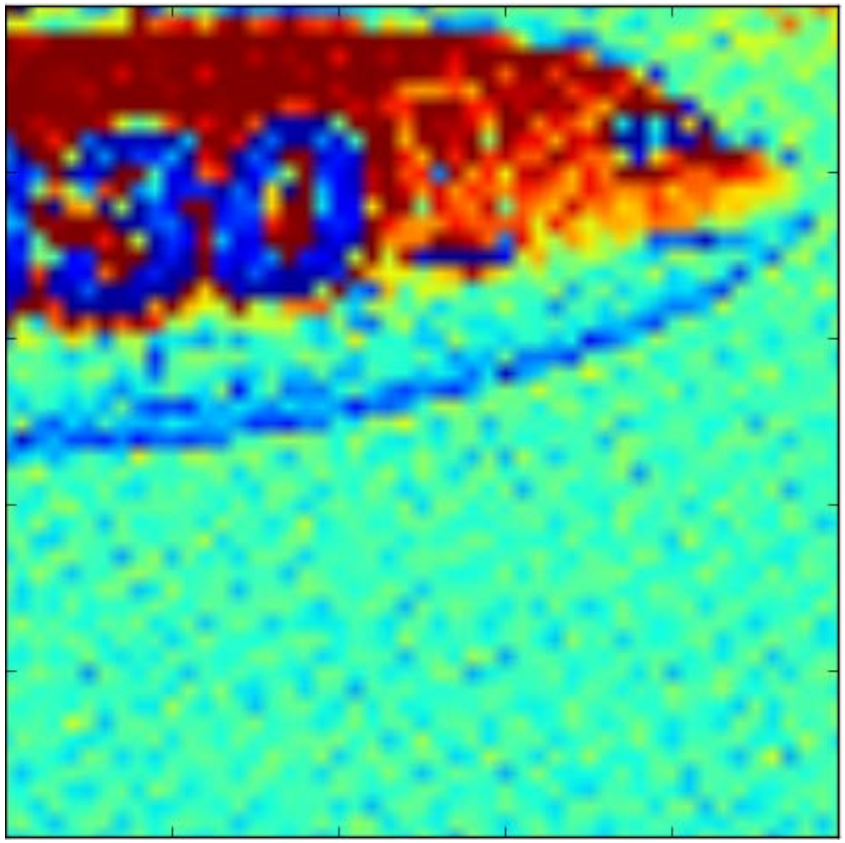


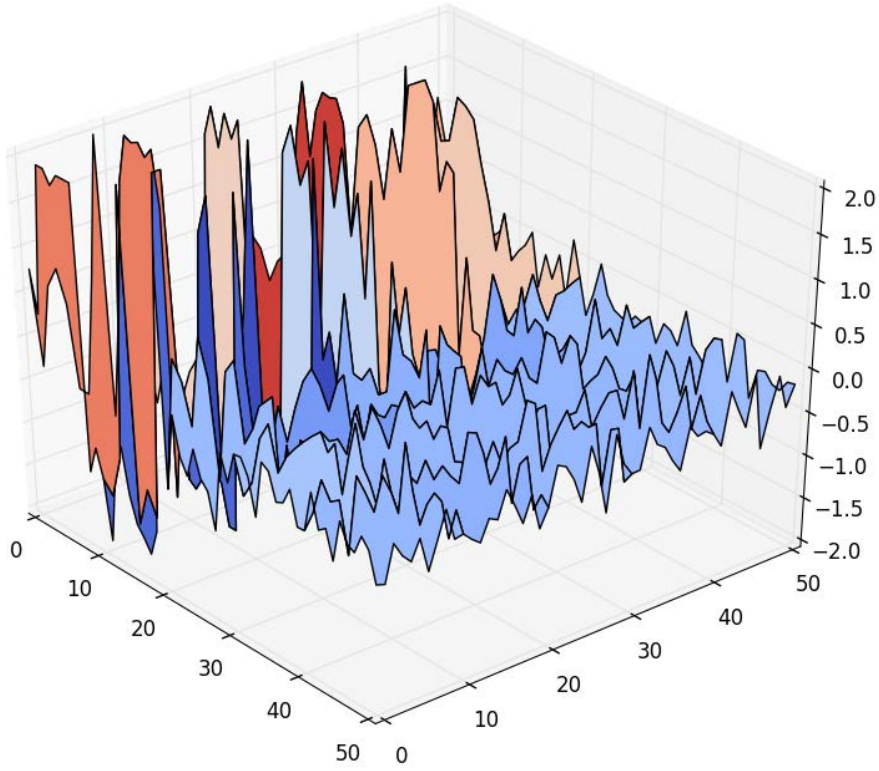
3 nodes -> loss 2.51e-04



4 nodes -> loss 3.07e-07

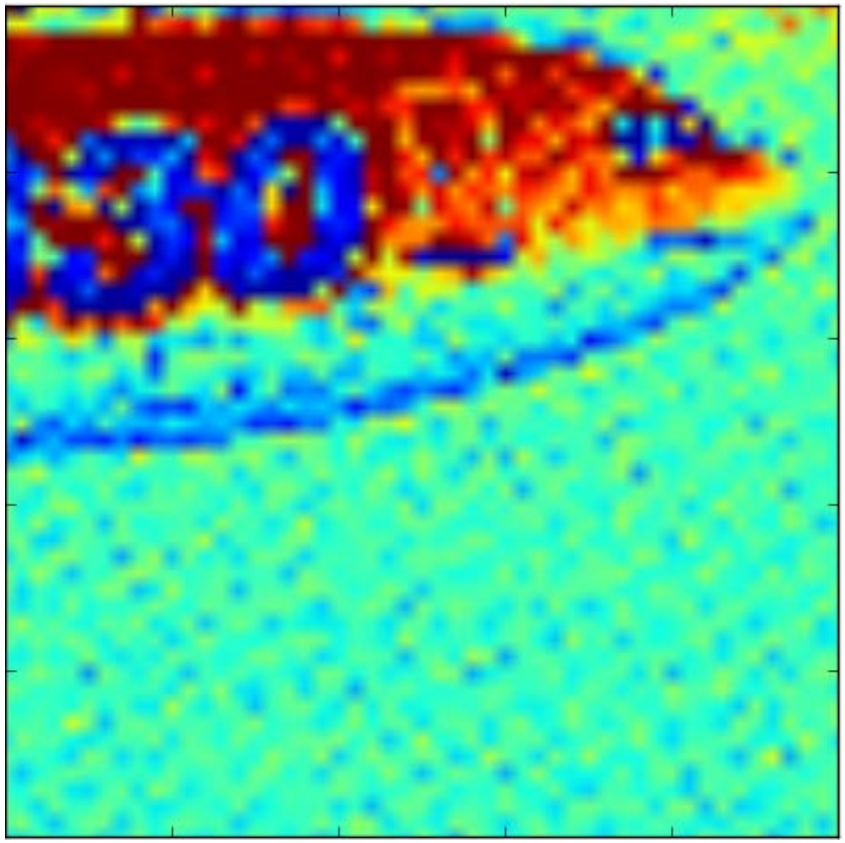
MORE COMPLEX SURFACE



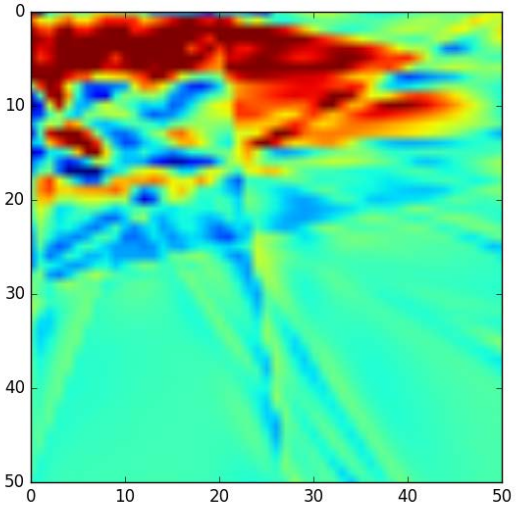


$$I = f(x, y)$$

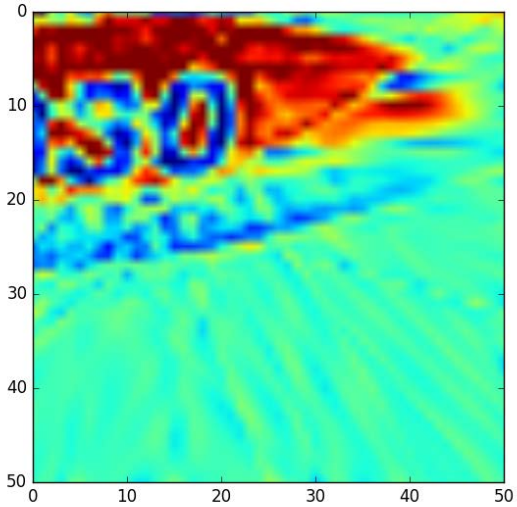
ADDING MORE NODES



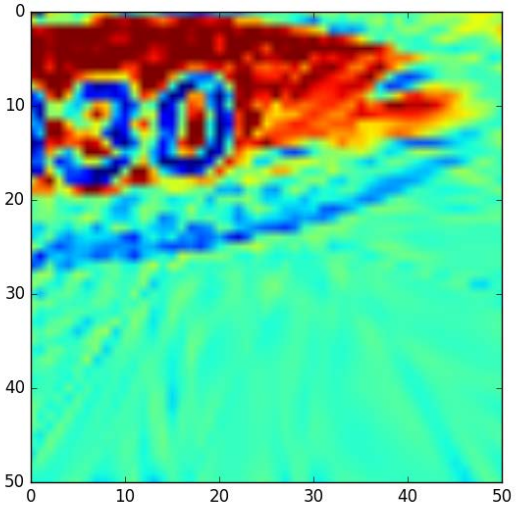
$$I = f(x, y)$$



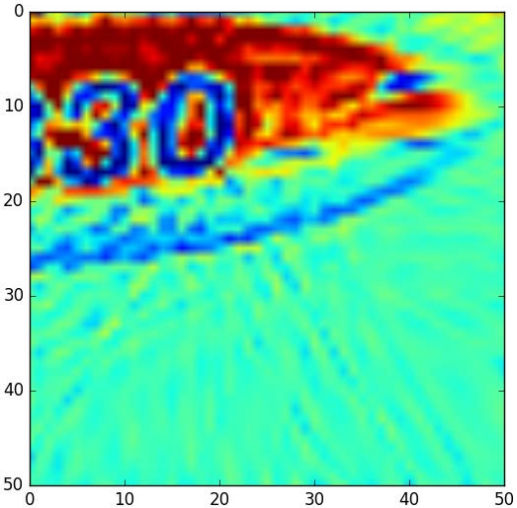
50 nodes -> loss 3.65e-01



100 nodes -> loss 2.50e-01



125 nodes -> loss 2.40e-01



300 nodes -> loss 1.92e-01

UNIVERSAL APPROXIMATION THEOREM

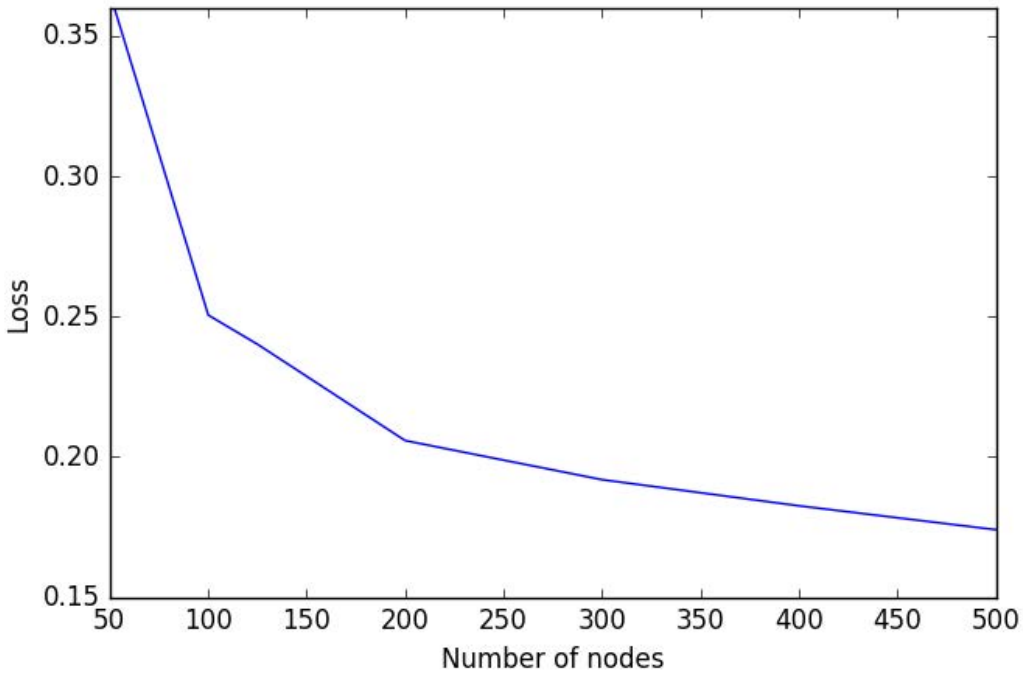
A feedforward network with a linear output layer and at least one hidden layer with any 'squashing' activation function (e.g. logistic sigmoid) can approximate any Borel measurable function (from one finite-dimensional space to another) with any desired nonzero error.

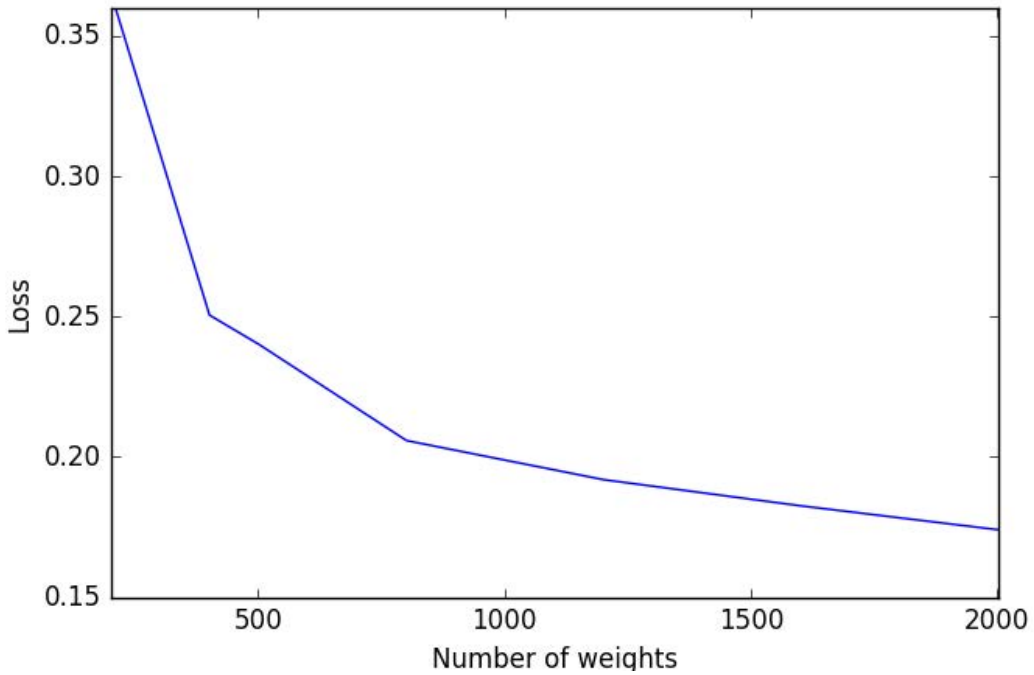
Any continuous function on a closed and bounded set of R^n is Borel-measurable.

—> In theory, any reasonable function can be approximated by a two-layer network as long as it is continuous.

[Hornik et al, 1989; Cybenko, 1989]

IN PRACTICE

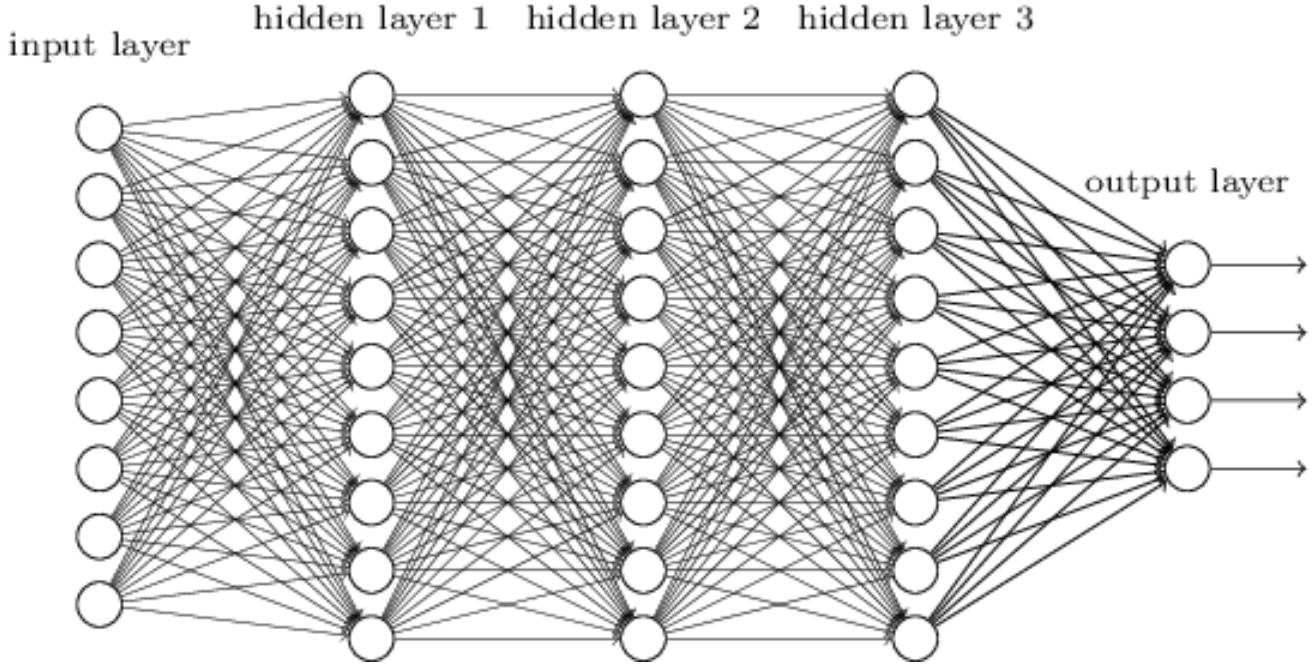




- It may take an exponentially large number of parameters for a good approximation.
- The optimization problem becomes increasingly difficult.

—> The one hidden layer perceptron may not converge to the best solution!

DEEP LEARNING



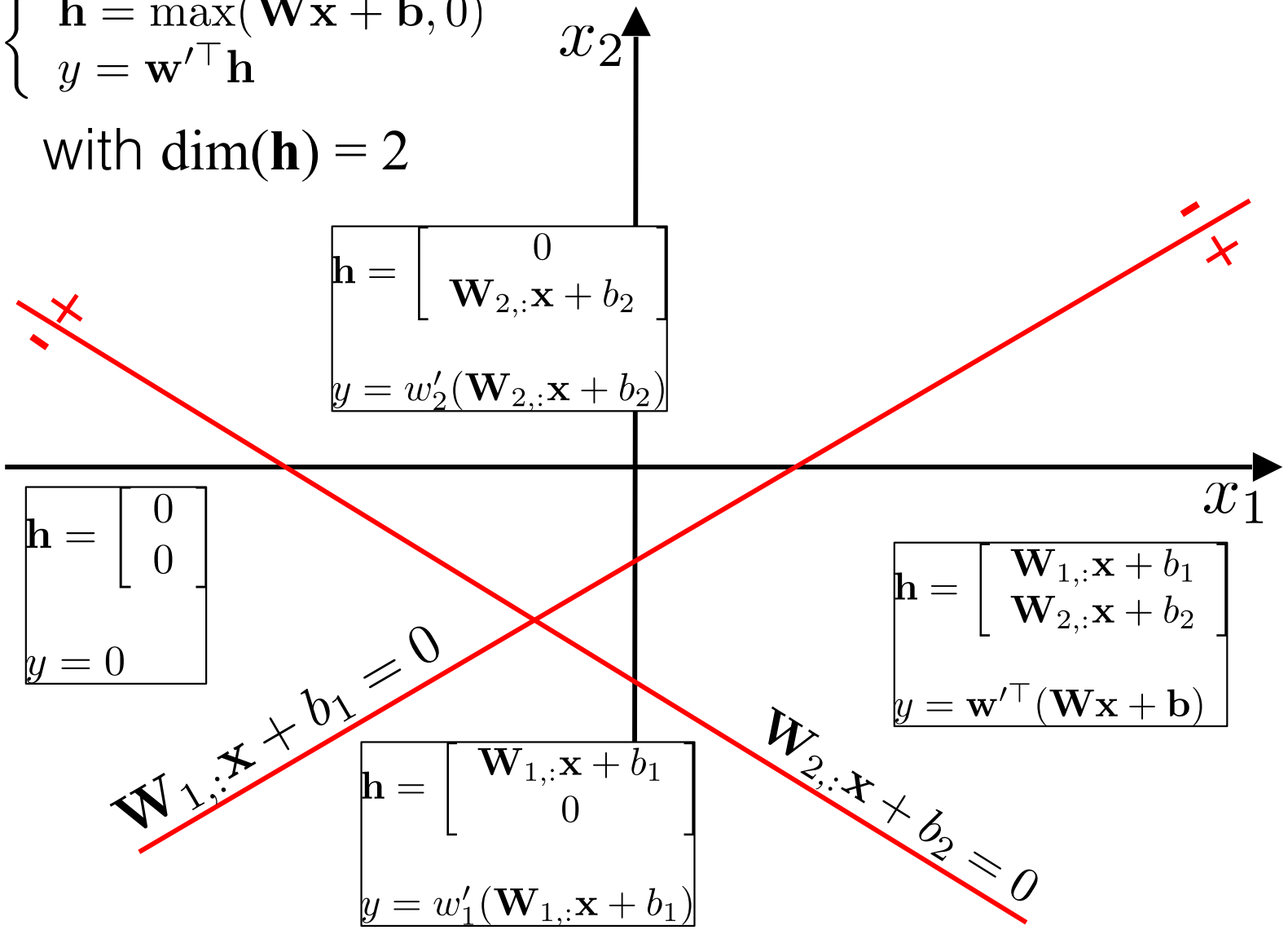
- The descriptive power of the net increases with the number of layers.
- In the case of a 1D signal, it is roughly proportional to $\prod_n W_n$ where W_n represents the width of a layer.

Telgarsky, JMLR'16

ONE LAYER: TWO HYPERPLANES

$$\begin{cases} \mathbf{h} = \max(\mathbf{W}\mathbf{x} + \mathbf{b}, 0) \\ y = \mathbf{w}'^{\top}\mathbf{h} \end{cases}$$

with $\dim(\mathbf{h}) = 2$



TWO LAYERS:
TWO HYPERPLANES PER LAYER

$$\begin{cases} \mathbf{h} = \max(\mathbf{W}\mathbf{x} + \mathbf{b}, 0) \\ \mathbf{h}' = \max(\mathbf{W}'\mathbf{h} + \mathbf{b}', 0) \\ y = \mathbf{w}''^{\top}\mathbf{h}' \end{cases}$$

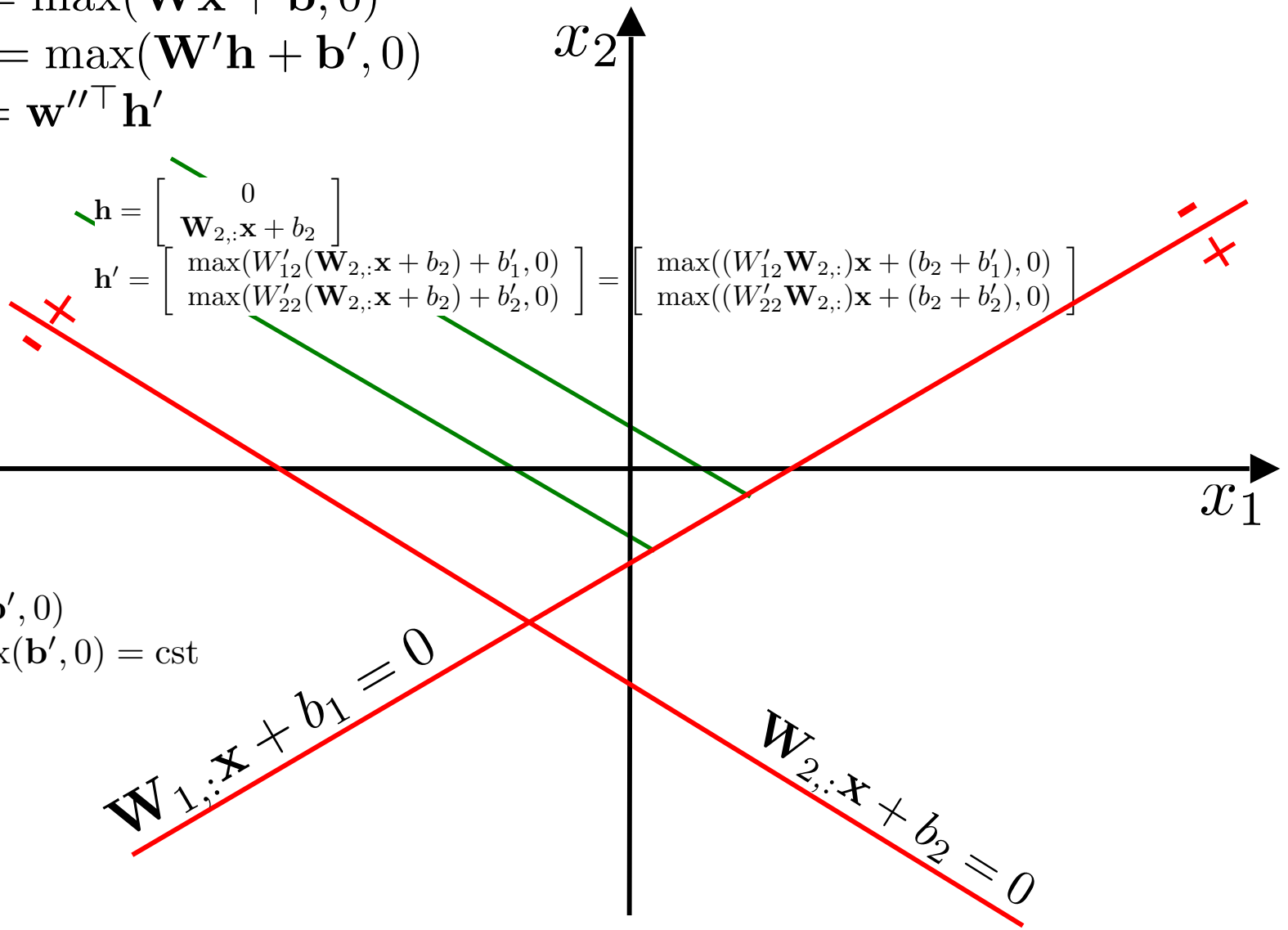
$$\mathbf{h} = \begin{bmatrix} 0 \\ \mathbf{W}_{2,:}\mathbf{x} + b_2 \end{bmatrix}$$

$$\mathbf{h}' = \begin{bmatrix} \max(W'_{12}(\mathbf{W}_{2,:}\mathbf{x} + b_2) + b'_1, 0) \\ \max(W'_{22}(\mathbf{W}_{2,:}\mathbf{x} + b_2) + b'_2, 0) \end{bmatrix} = \begin{bmatrix} \max((W'_{12}\mathbf{W}_{2,:})\mathbf{x} + (b_2 + b'_1), 0) \\ \max((W'_{22}\mathbf{W}_{2,:})\mathbf{x} + (b_2 + b'_2), 0) \end{bmatrix}$$

$$\mathbf{h} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

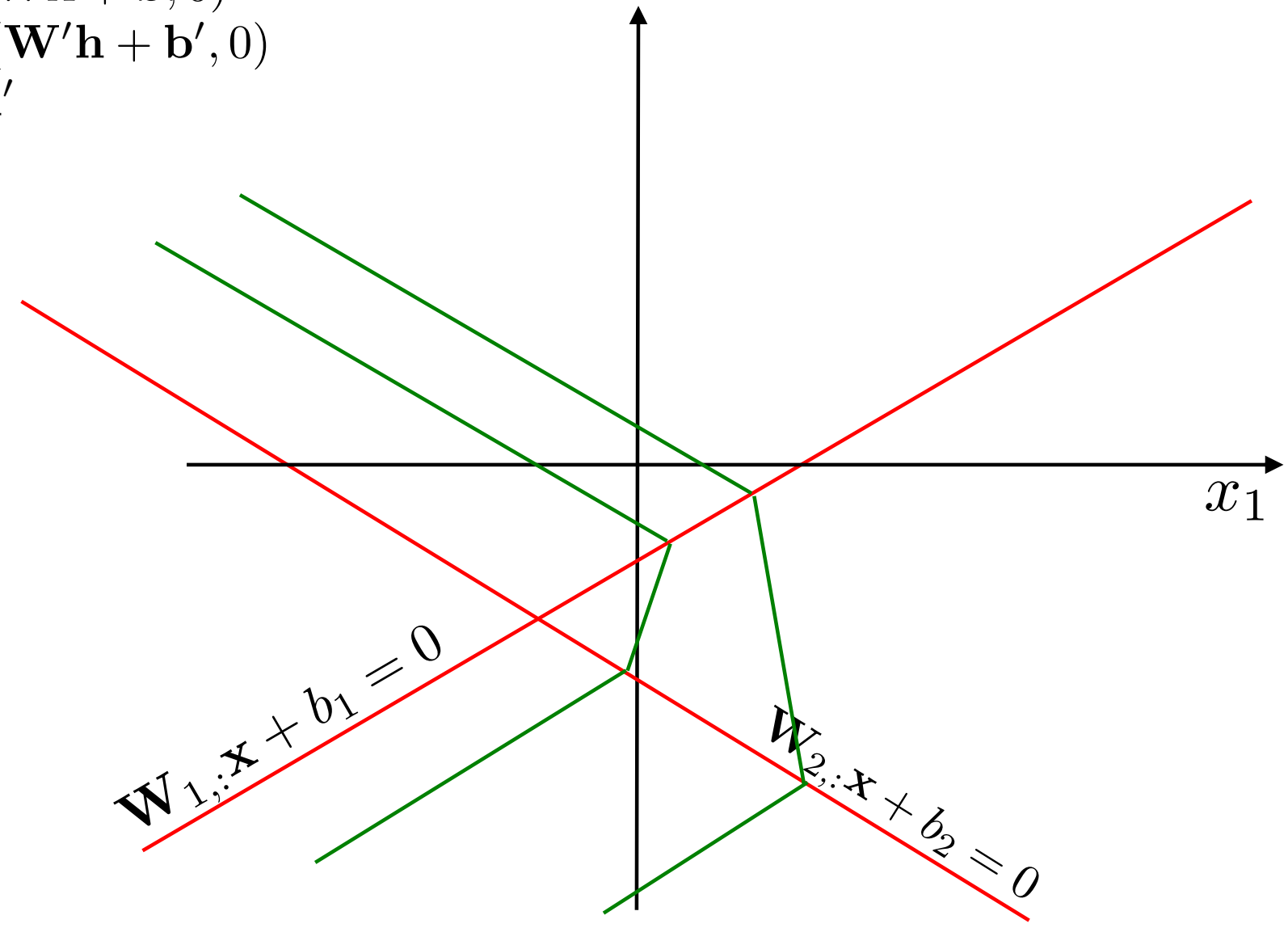
$$\mathbf{h}' = \max(\mathbf{b}', 0)$$

$$y = \mathbf{w}'' \max(\mathbf{b}', 0) = \text{cst}$$

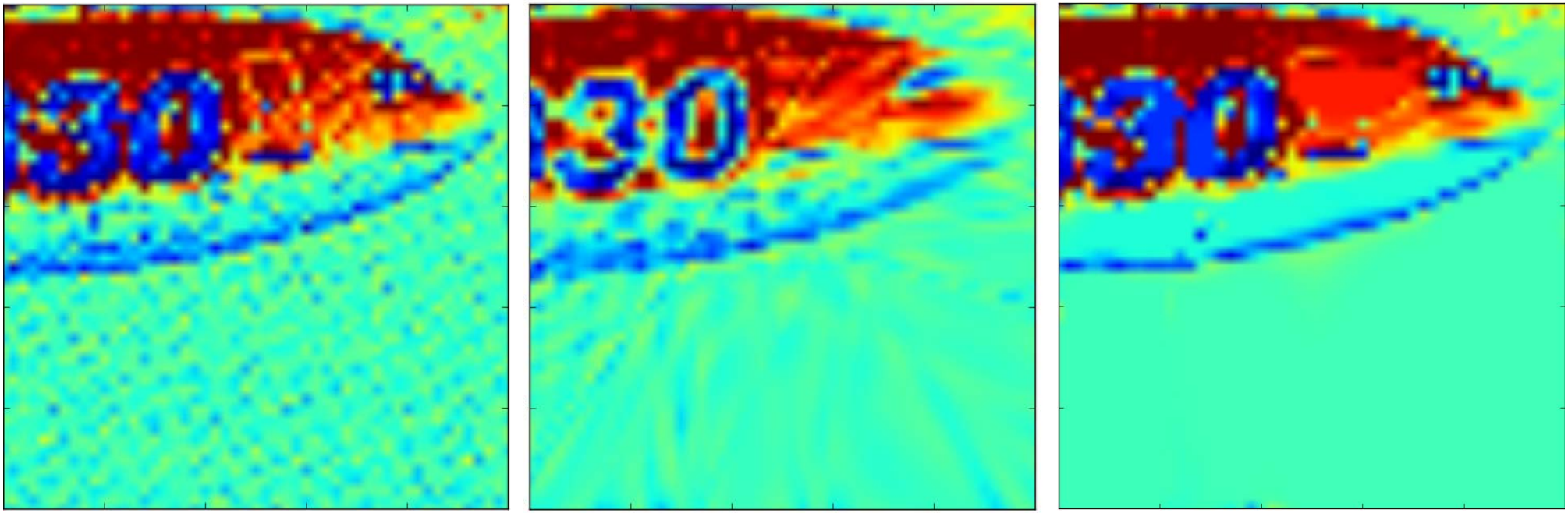


TWO LAYERS:
TWO HYPERPLANES PER LAYER

$$\begin{cases} \mathbf{h} = \max(\mathbf{W}\mathbf{x} + \mathbf{b}, 0) \\ \mathbf{h}' = \max(\mathbf{W}'\mathbf{h} + \mathbf{b}', 0) \\ y = \mathbf{w}''^{\top}\mathbf{h}' \end{cases}$$



ADDING A SECOND LAYER

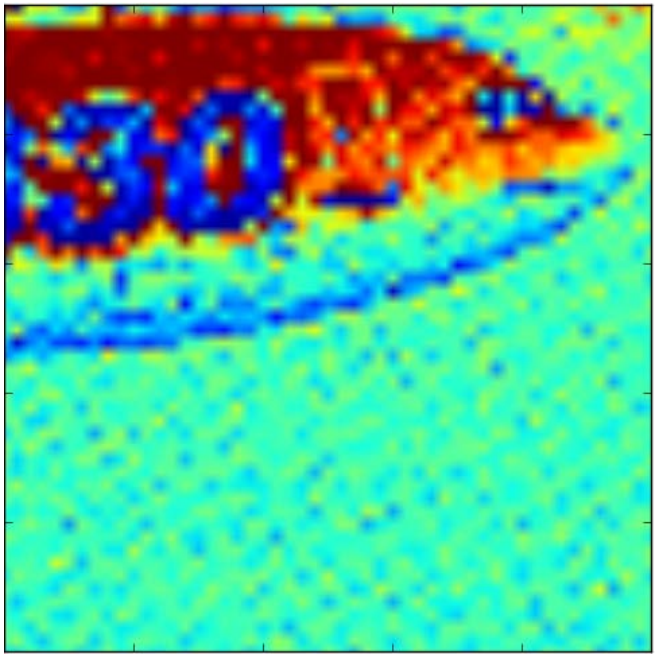


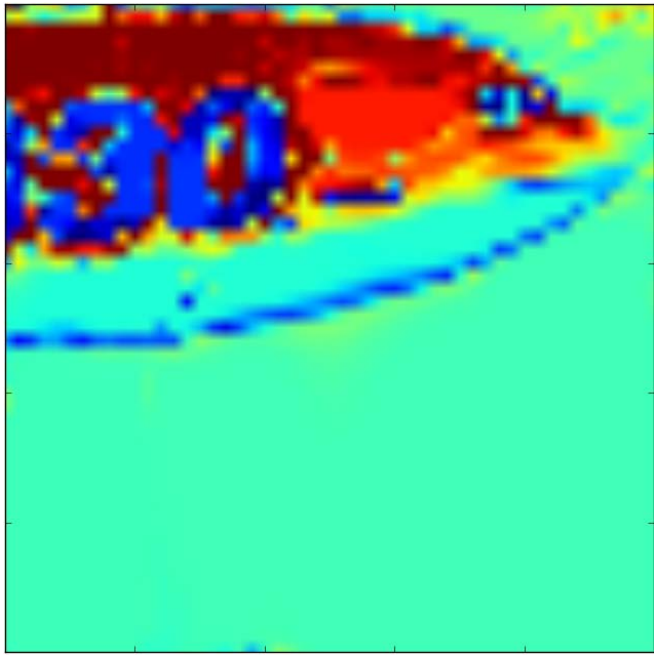
$$I = f(x, y)$$

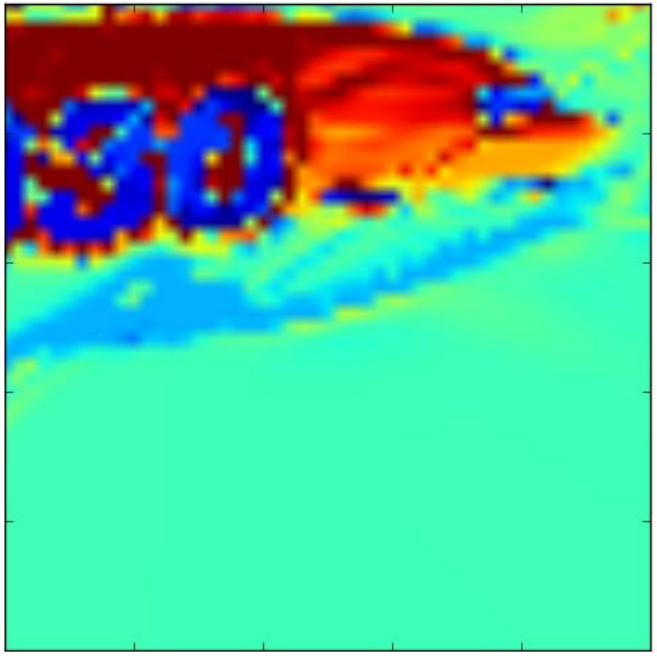
1 Layer: 125 nodes -> loss 2.40e-01 2 Layers: 20 nodes -> loss 8.31e-02

501 weights in both cases

ADDING A THIRD LAYER







$$I = f(x, y)$$

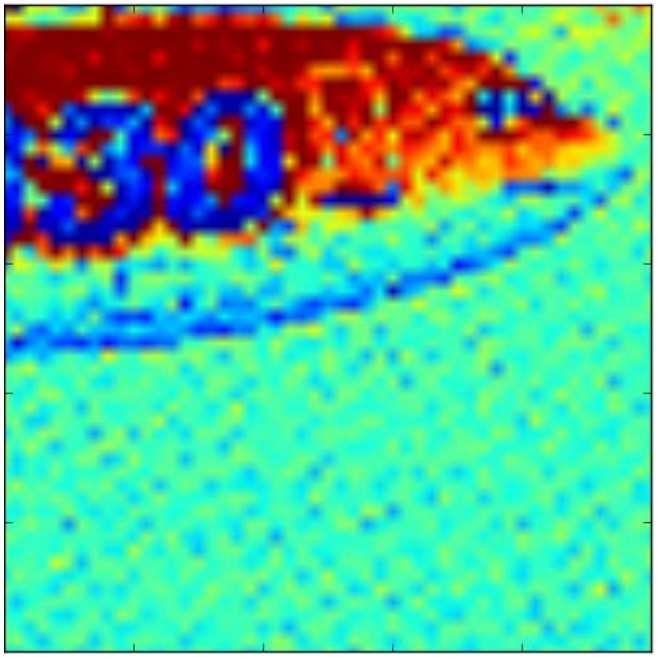
2 Layers: 20 nodes -> loss 8.31e-02

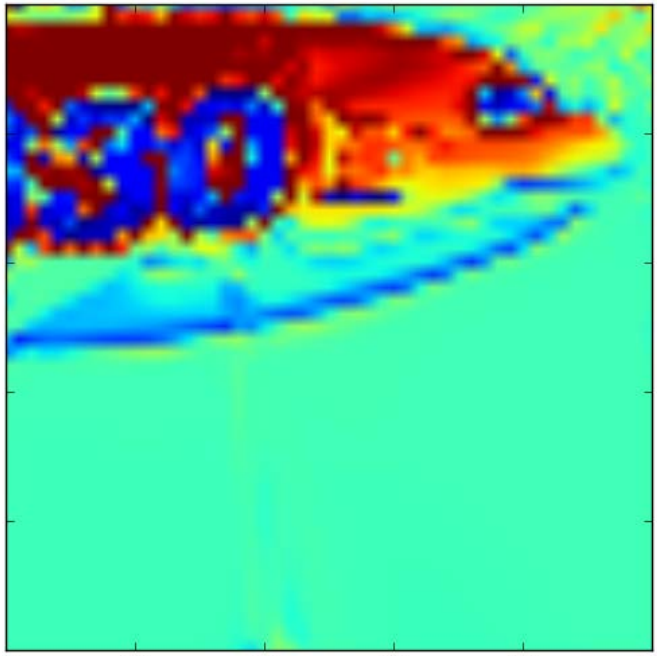
501 weights

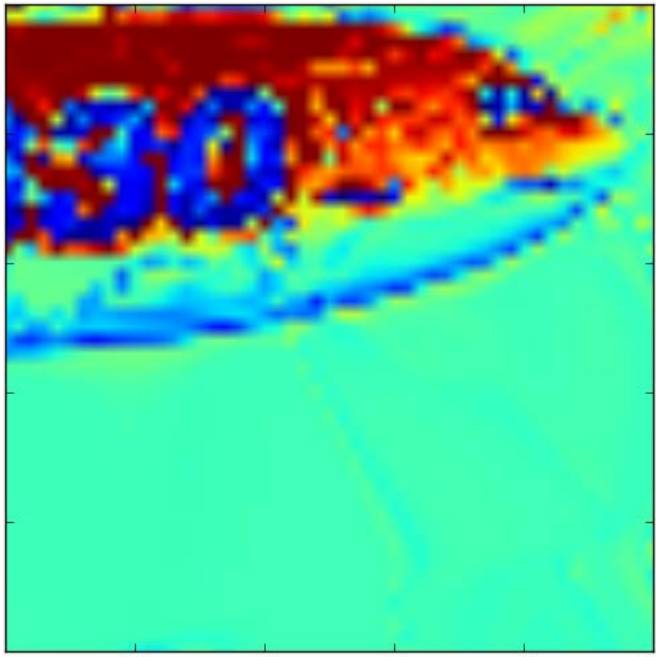
3 Layers: 14 nodes -> loss 7.55e-02

477 weights

ADDING A THIRD LAYER







$$I = f(x, y)$$

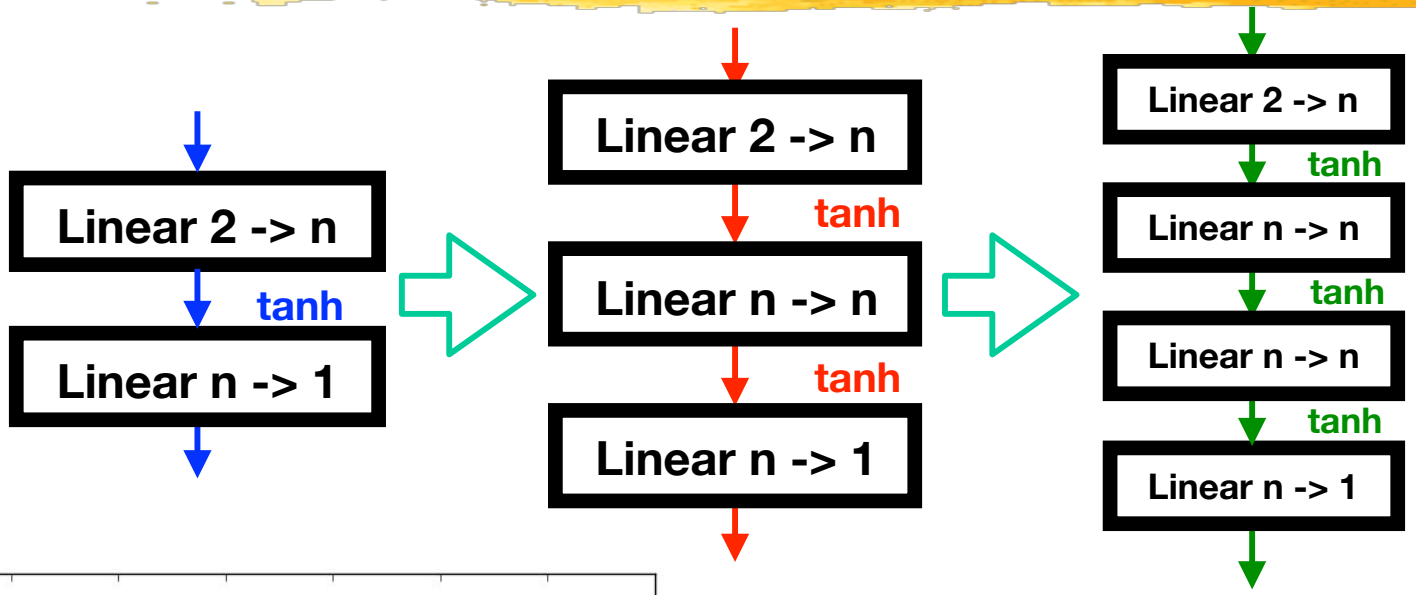
3 Layers: 15 nodes -> loss 5.93e-02

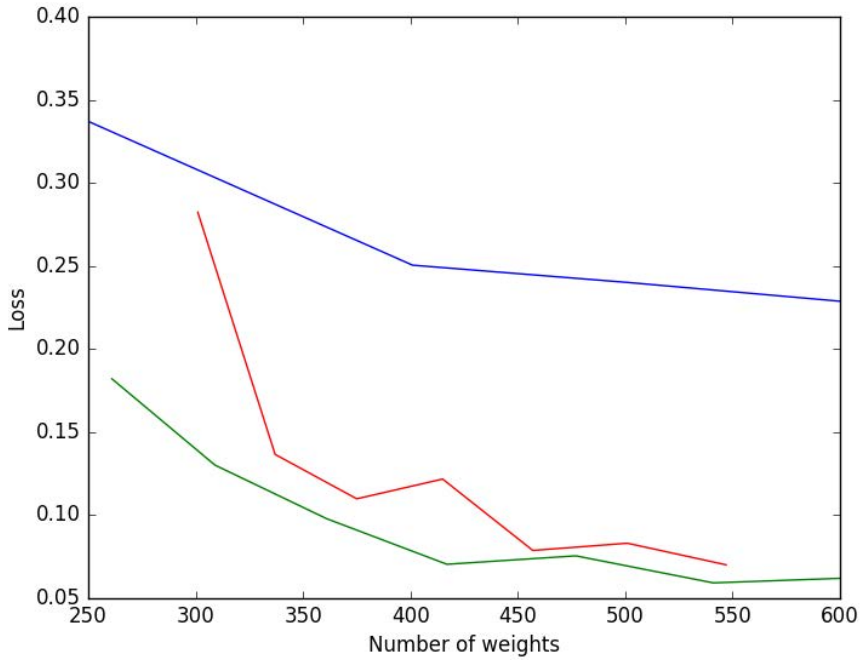
541 weights

3 Layers: 19 nodes -> loss 4.38e-02

837 weights

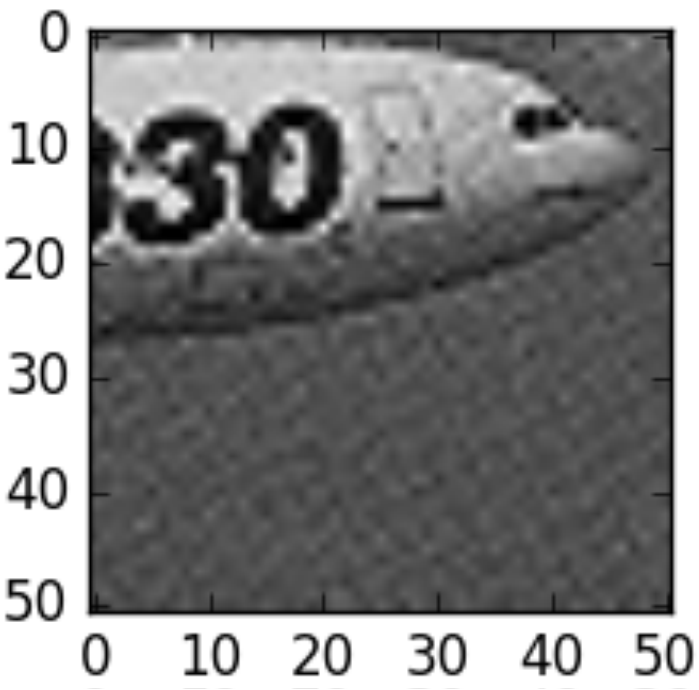
MULTILAYER PERCEPTRONS



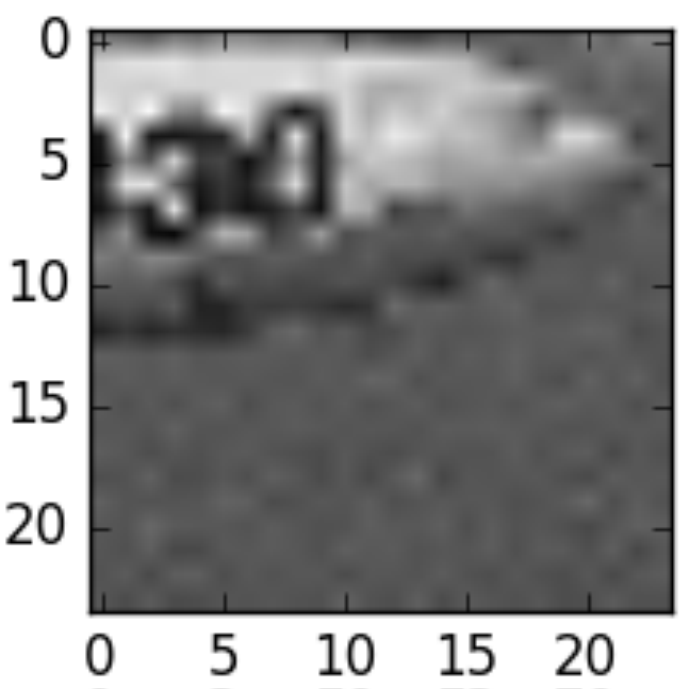


- Adding layers often yields better convergence properties.
- In current practice, deeper is usually better.

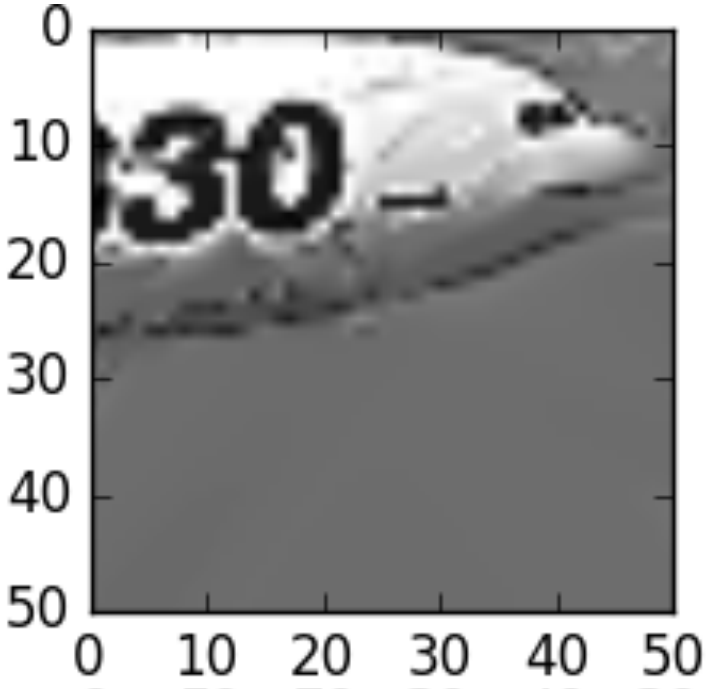
SIMPLER WAY TO INTERPOLATE



Original 51x51 image:
2601 gray level values.



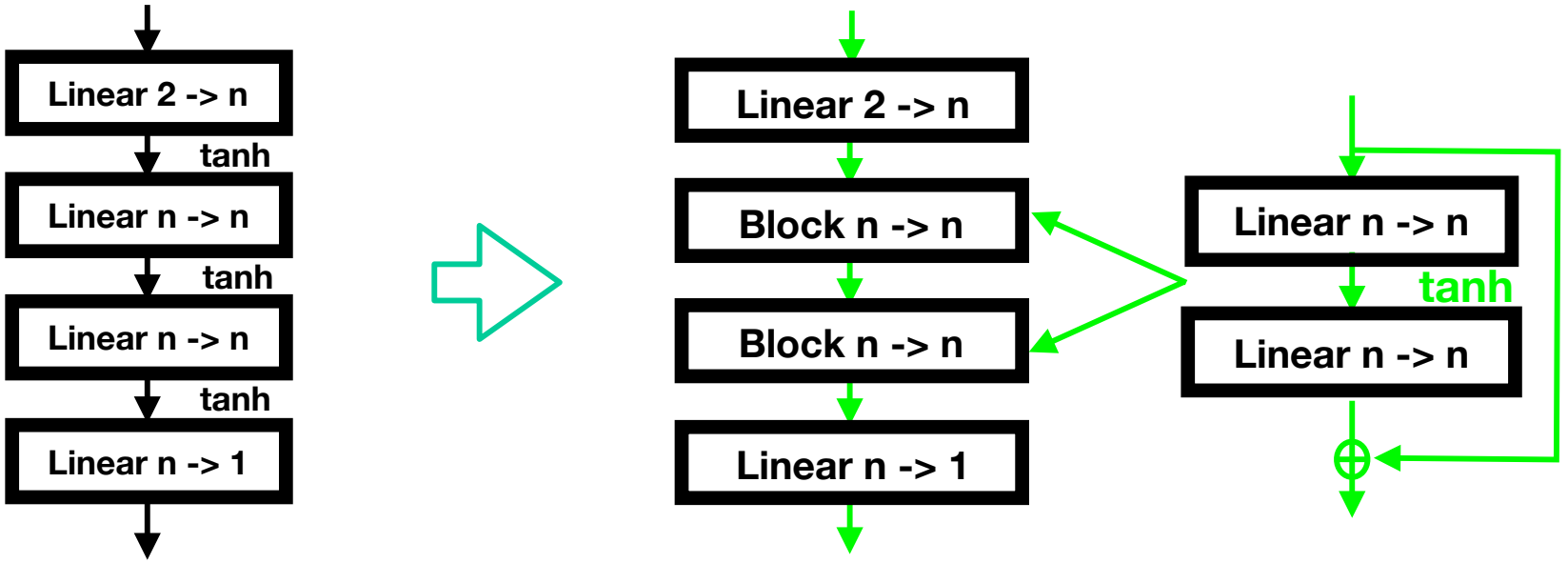
Scaled 24x24 image:
576 gray level values.



MLP 10/20/10 Interpolation:
471 weights.

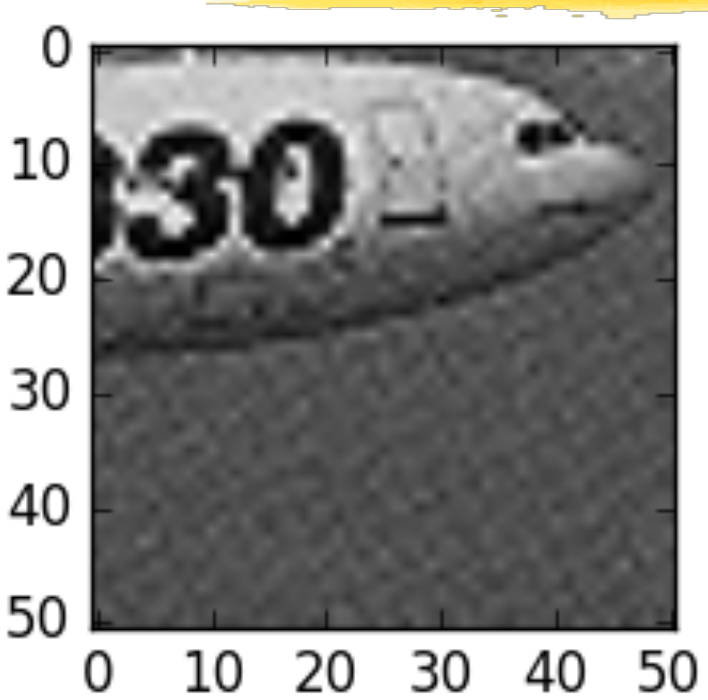
Simpler but not necessarily better!

MLP TO RESNET

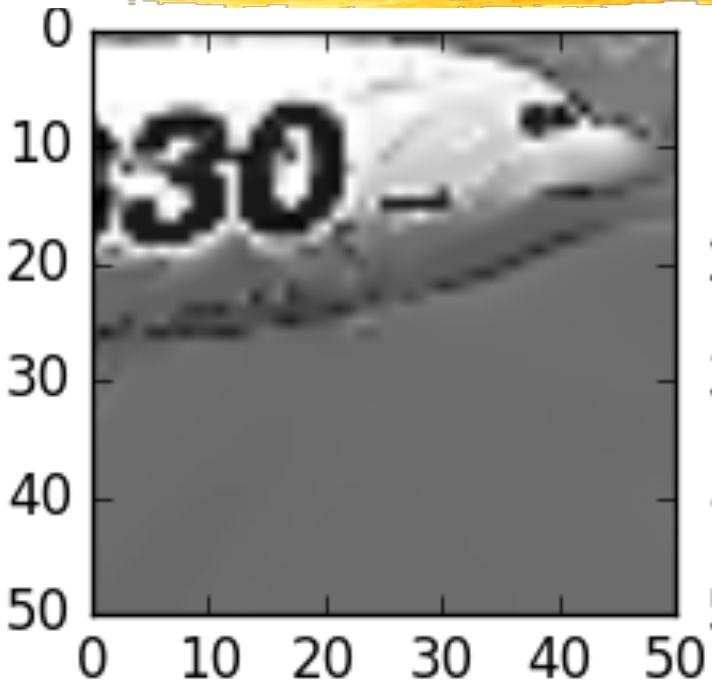


Further improvements in the convergence properties have been obtained by adding a bypass, which allows the final layers to only compute residuals.

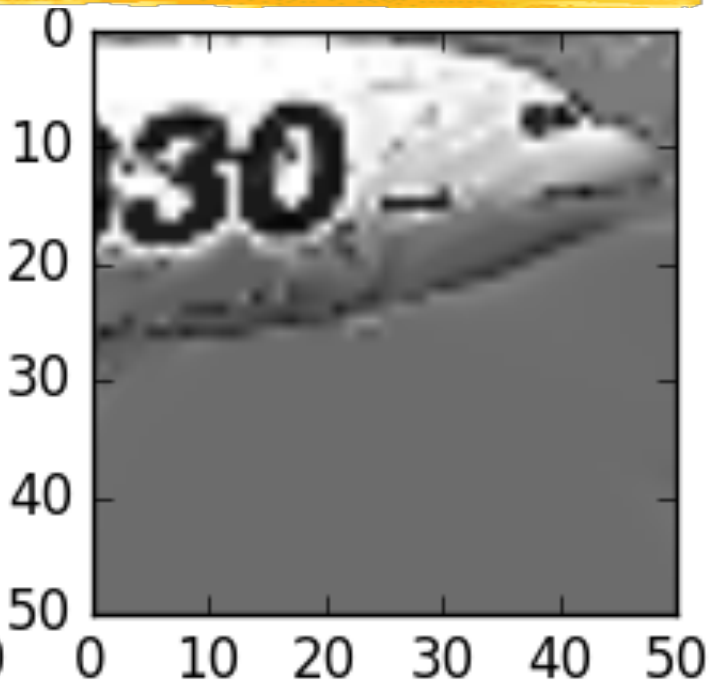
IMPROVING THE NETWORK



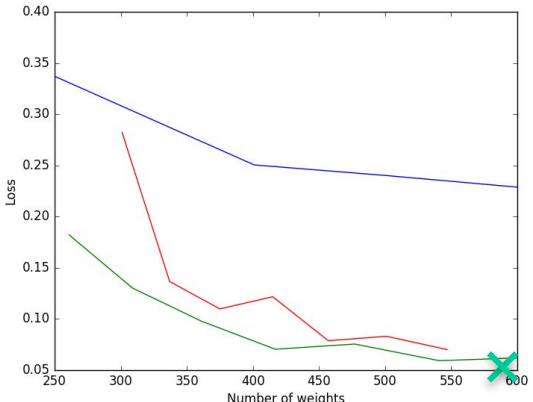
Original 51x51 image:
2601 gray level values.

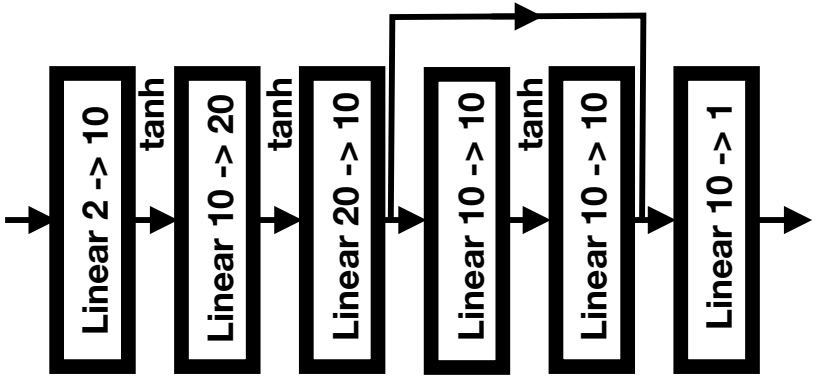


MLP 10/20/10 Interpolation:
471 weights, loss 6.43e-02.

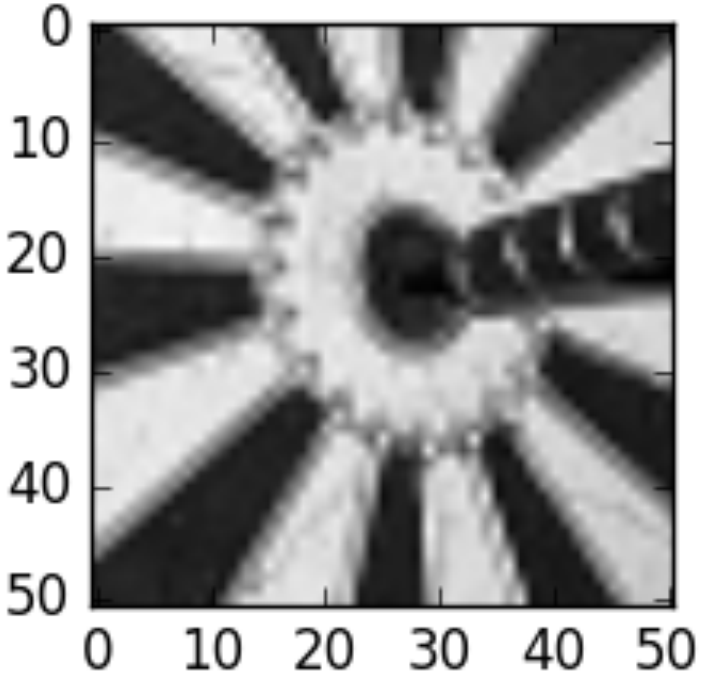


MLP 10/20/10/10 Interpolation:
581 weights, loss 5.30e-2.

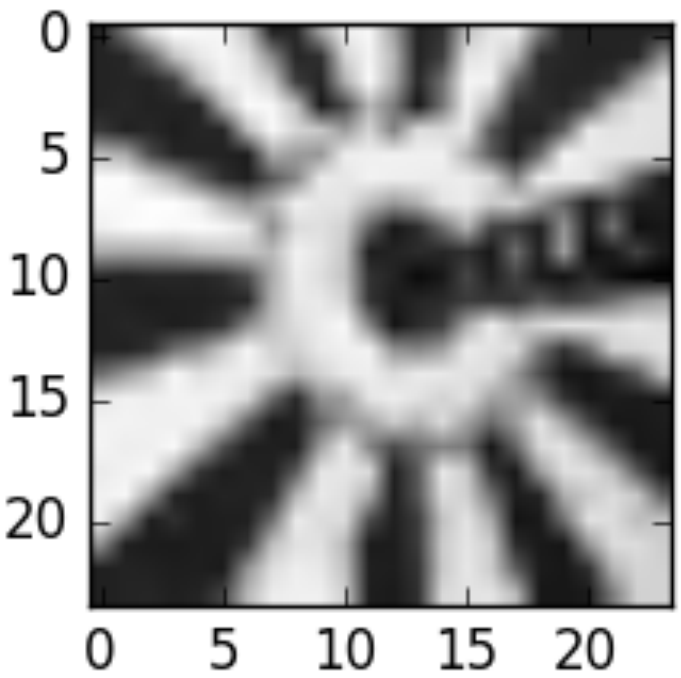




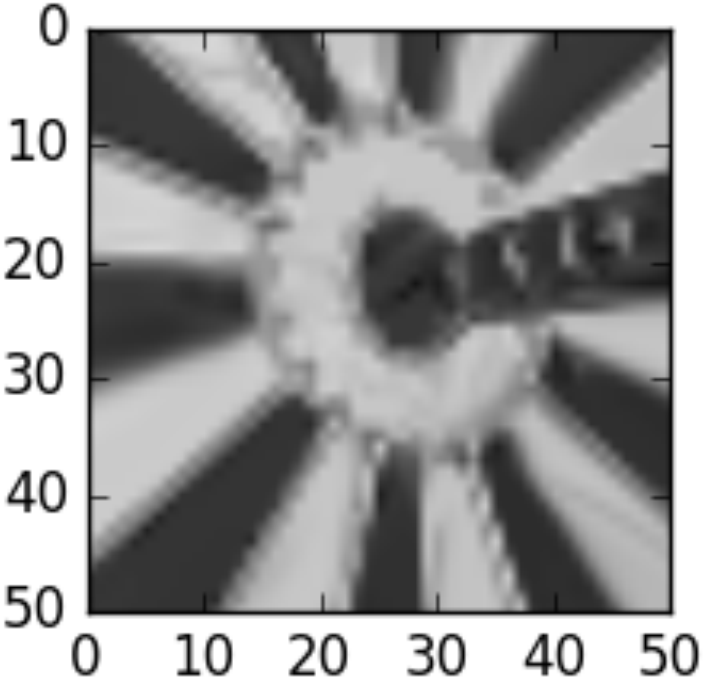
IMPROVING THE NETWORK



Original 51x51 image:
2601 gray level values.



MLP 10/20/10 Interpolation:
471 weights, loss 1.95e-02.

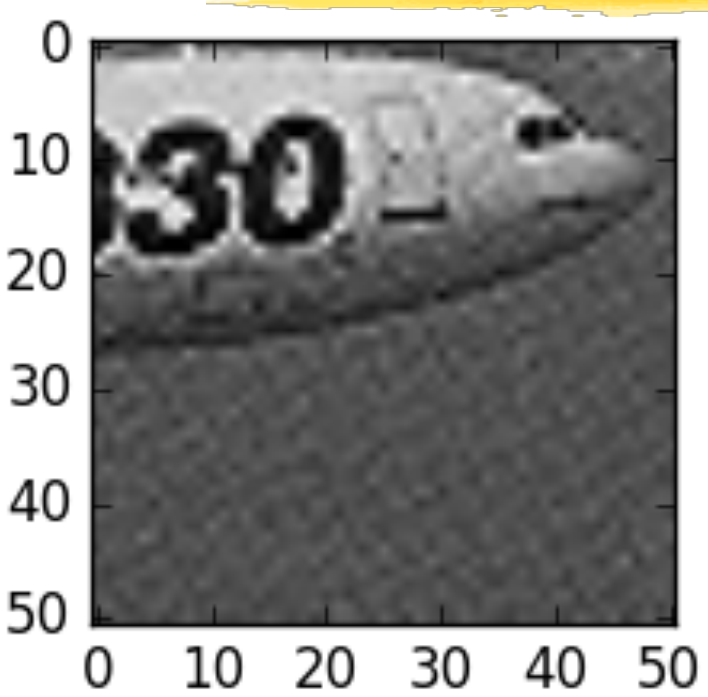


MLP 10/20/10/10 Interpolation:
581 weights, loss 1.36e-2.

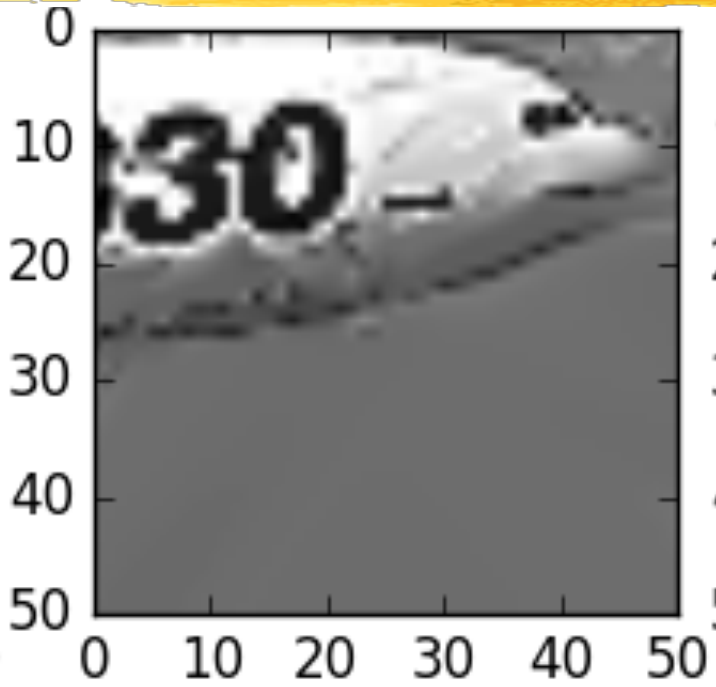
- Relatively small improvement in this case.
- The problem is probably too small.

—> Networks can behave very differently for small and large problems!

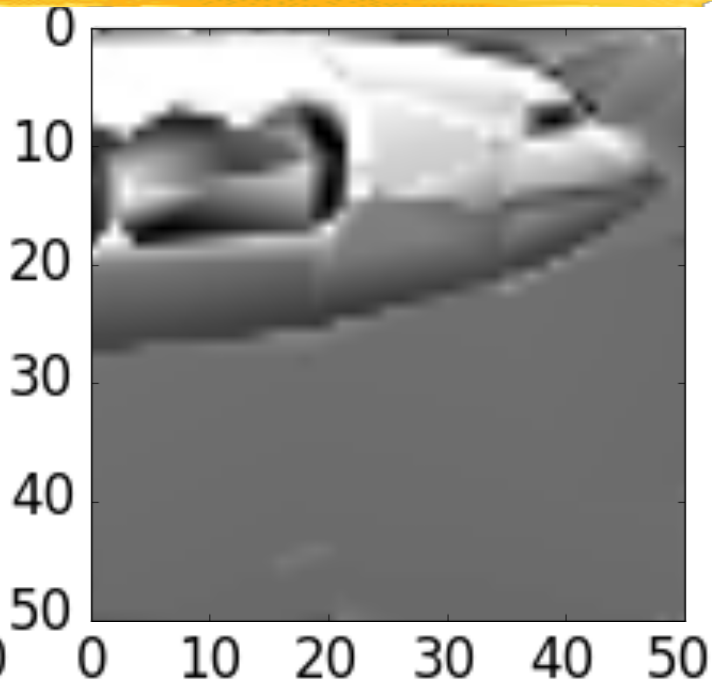
TANH vs ReLU



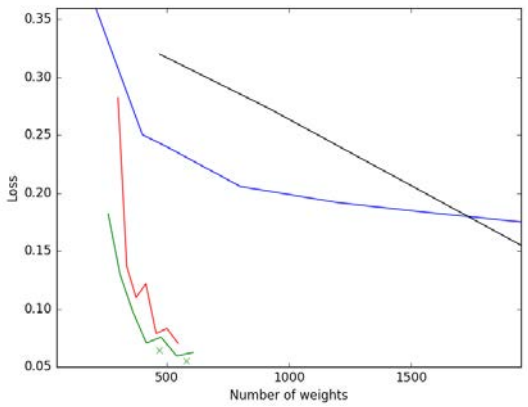
Original 51x51 image:
2601 gray level values.



MLP 10/20/10 Interpolation:
Tanh, loss 6.43e-02.

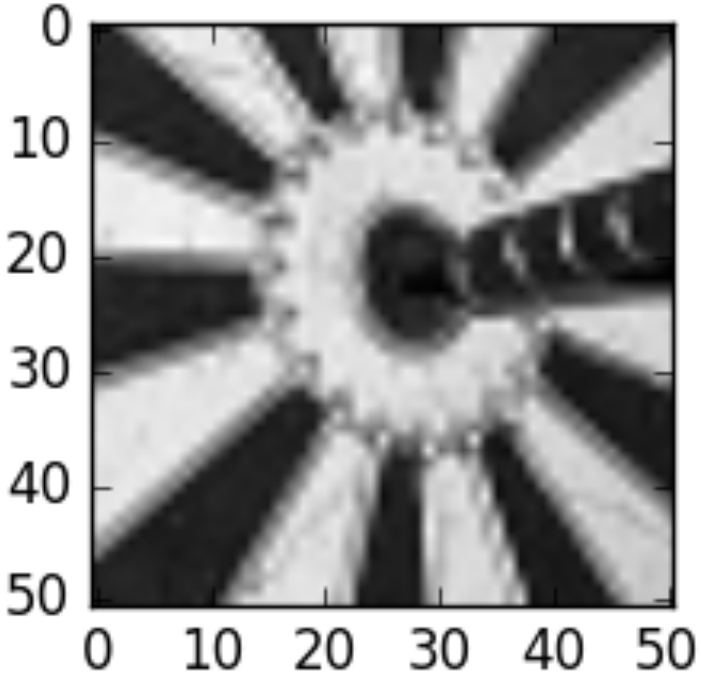


MLP 10/20/10 Interpolation:
ReLU, loss 3.07e-1.

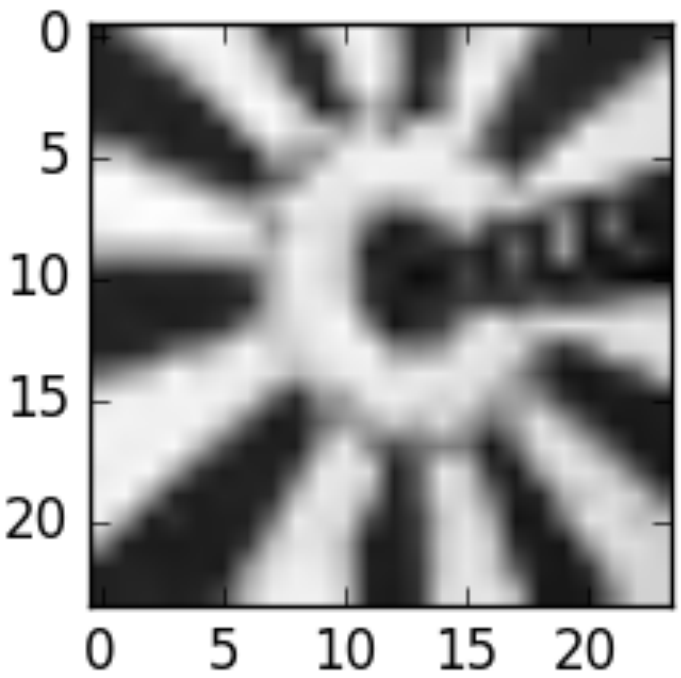


Tanh, 1 layers
Tanh, 2 layers
Tanh, 3 layers
ReLU, 3 layers
x Tanh, 4 layers

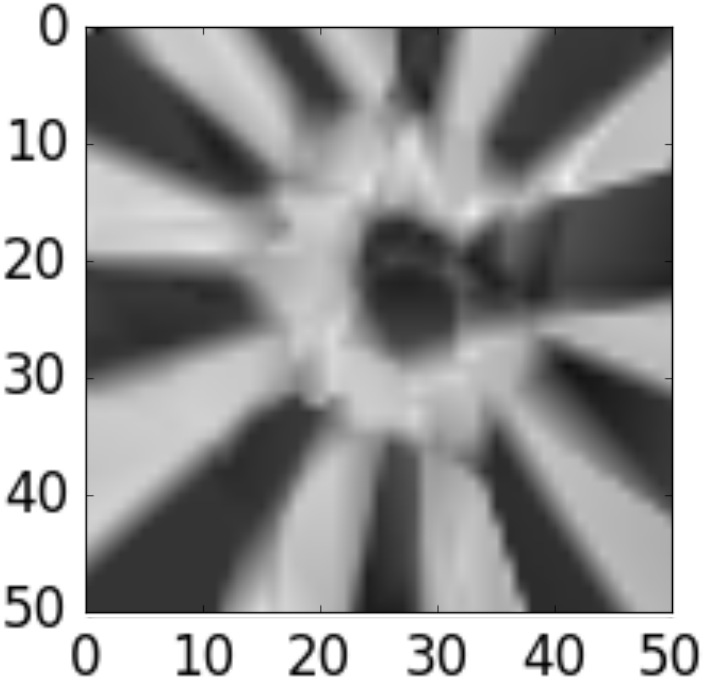
TANH vs ReLU



Original 51x51 image:
2601 gray level values.



MLP 10/20/10 Interpolation:
tanh, loss 1.95e-02.



MLP 10/20/10 Interpolation:
relu, loss 7.21e-2.

- Tanh works better than ReLU in this case.
- ReLU is widely credited with eliminating the vanishing gradient problem in large networks.

—> There is no substitute for experimentation!

MULTI LAYER PERCEPTRONS

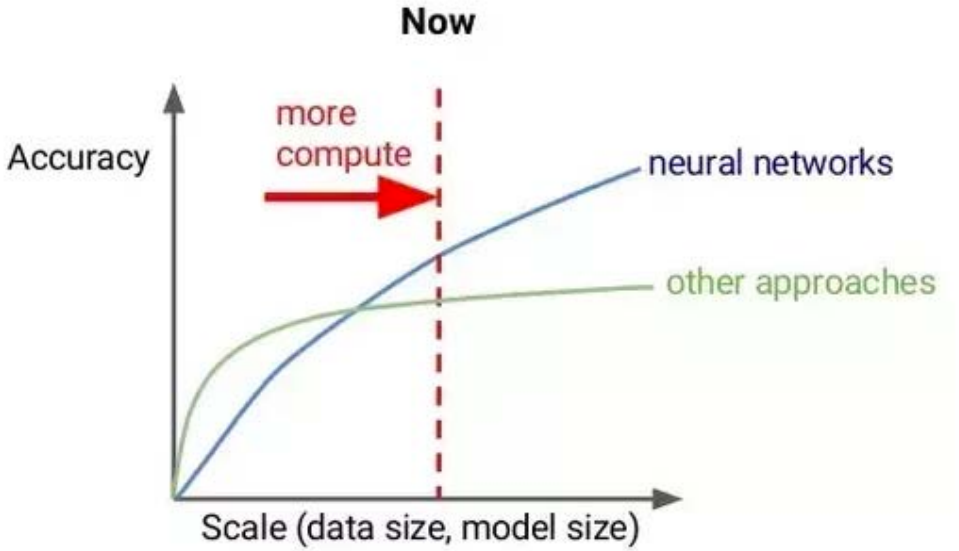
The function learned by a DNN using either the ReLU or Tanh operators is:

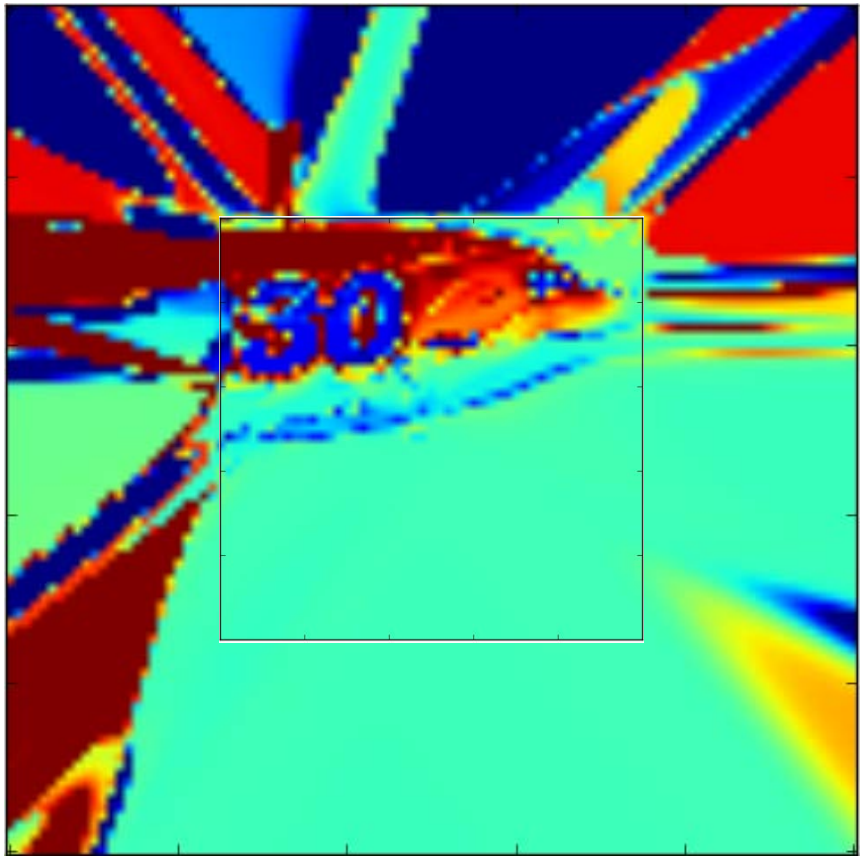
- piecewise affine or smooth;
- continuous because it is a composition of continuous functions.

Each region created by a layer is split into smaller regions:

- Their boundaries are correlated in a complex way.
- Their descriptive power is larger than shallow networks for the same number of parameters.

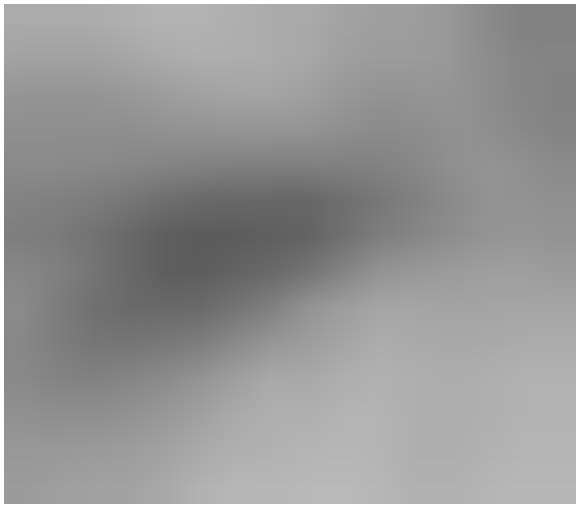
STRENGTHS AND LIMITATIONS





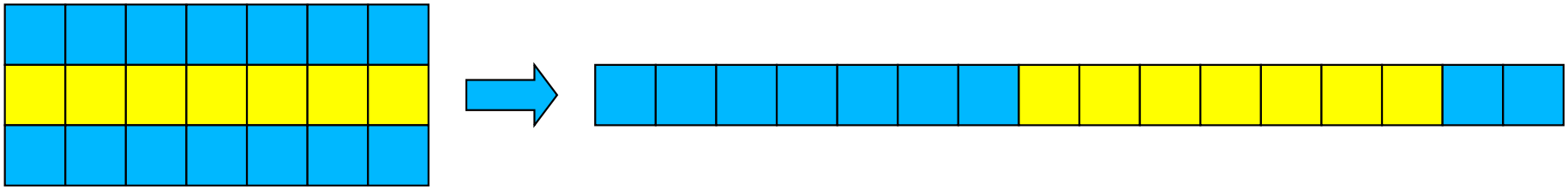
- Powerful regressors but require many parameters, and therefore large training databases.
- Excellent at interpolation but less good at extrapolation. The training data must cover all cases of interest.

DIGITAL IMAGES



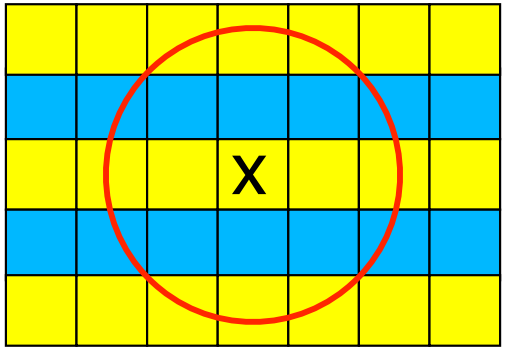
136 134 161 159 163 168 171 173 173 171 166 159 157 155
152 145 136 130 151 149 151 154 158 161 163 163 159 151
145 149 149 145 140 133 145 143 145 145 145 146 148 148
148 143 141 145 145 145 141 136 136 135 135 136 135 133
131 131 129 129 133 136 140 142 142 138 130 128 126 120
115 111 108 106 106 110 120 130 137 142 144 141 129 123
117 109 098 094 094 094 100 110 125 136 141 147 147 145
136 124 116 105 096 096 100 107 116 131 141 147 150 152
152 152 137 124 113 108 105 108 117 129 139 150 157 159
159 157 157 159 135 121 120 120 121 127 136 147 158 163
165 165 163 163 163 166 136 131 135 138 140 145 154 163
166 168 170 168 166 168 170 173 145 143 147 148 152 159
168 173 173 175 173 171 170 173 177 178 151 151 153 156
161 170 176 177 177 179 176 174 174 176 177 179 155 157
161 162 168 176 180 180 180 182 180 175 175 178 180 180

- A digital image is simply a 2D array of numbers.



- A MxN image can be represented as an MN vector, in which case neighborhood relationships are lost.
- By contrast, treating it as a 2D array preserves neighborhood relationships.

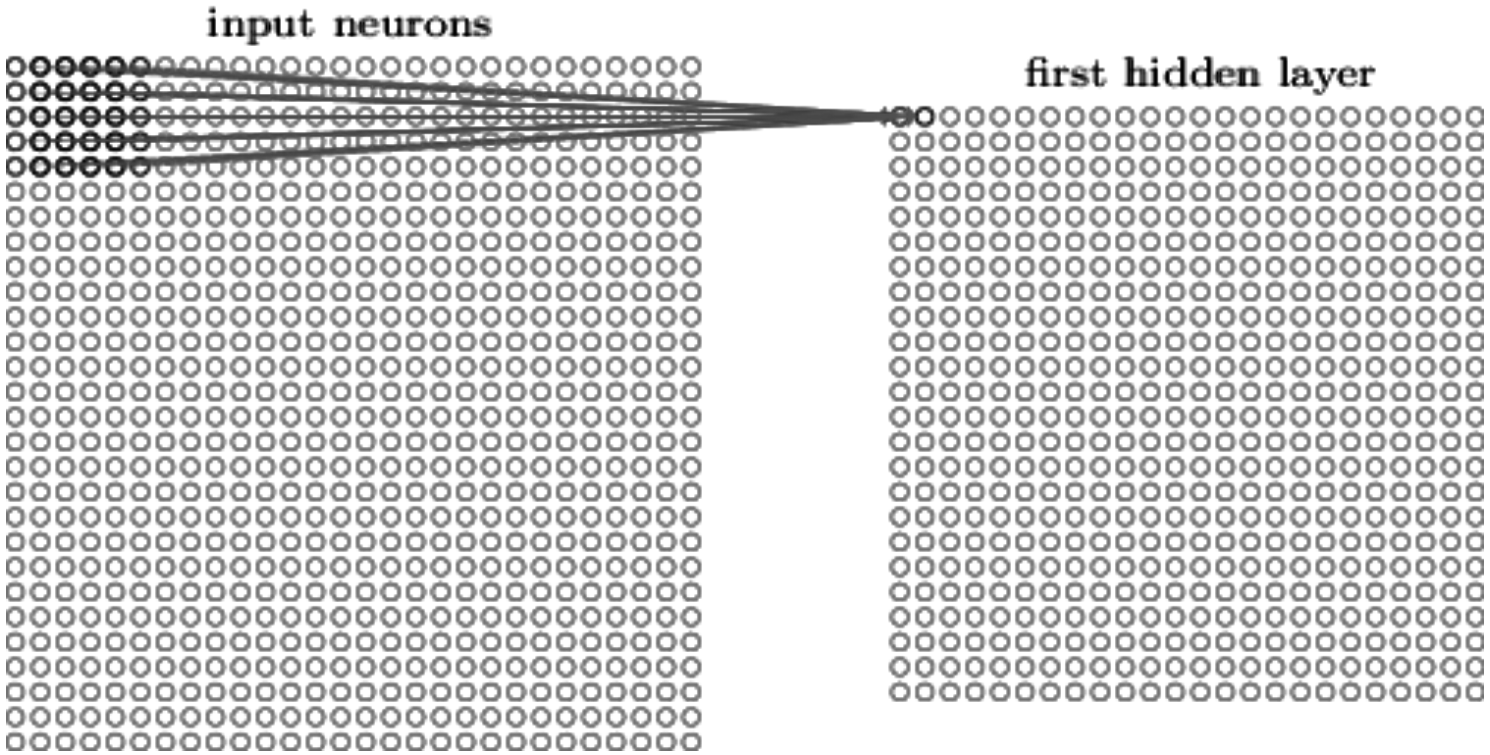
IMAGE SPECIFICITIES



- In a typical image, the values of neighboring pixels tend to be more highly correlated than those of distant ones.
- An image filter should be translation invariant.

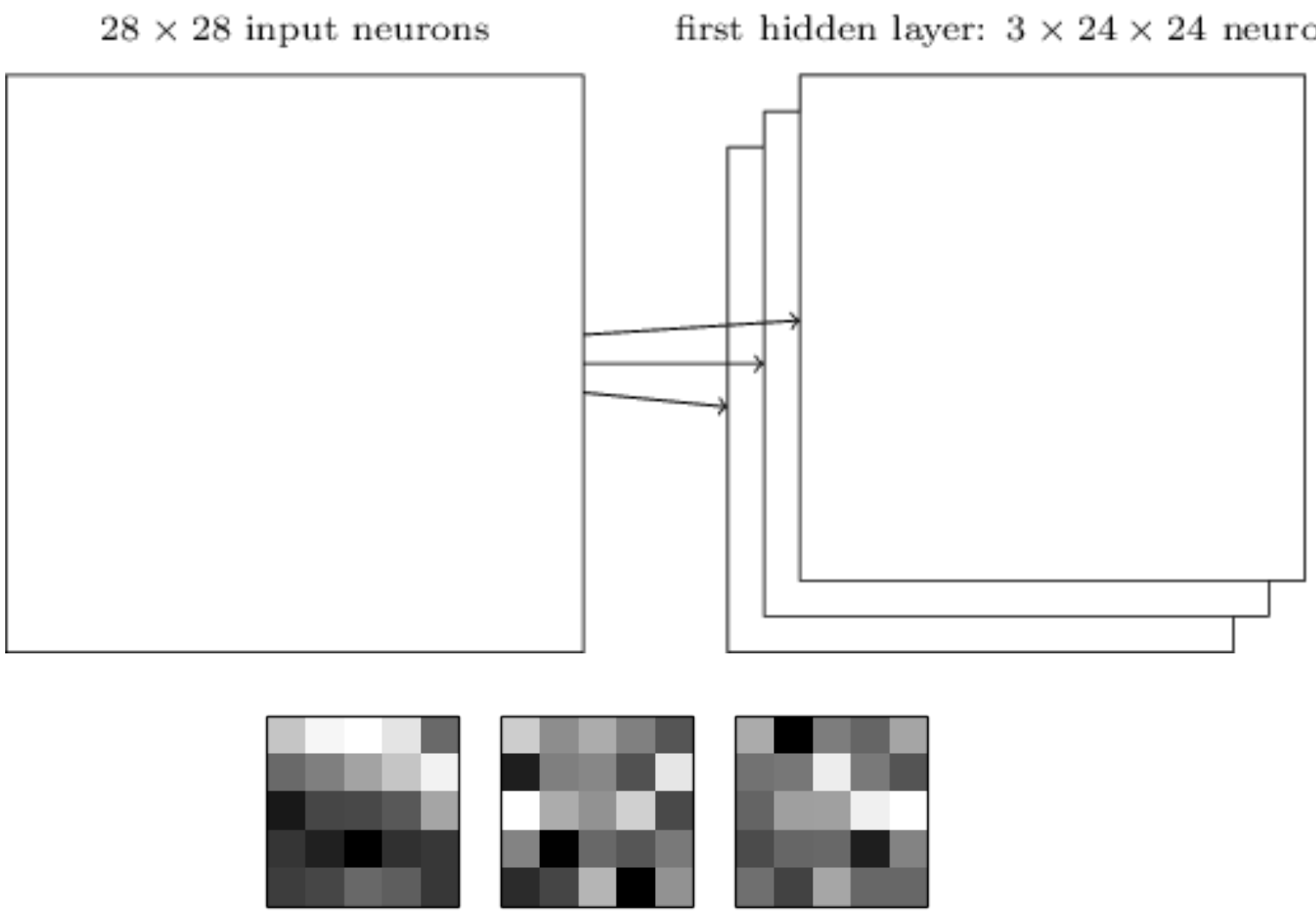
—> These two properties can be exploited to drastically reduce the number of weights required by CNNs using so-called convolutional layers.

CONVOLUTIONAL LAYER

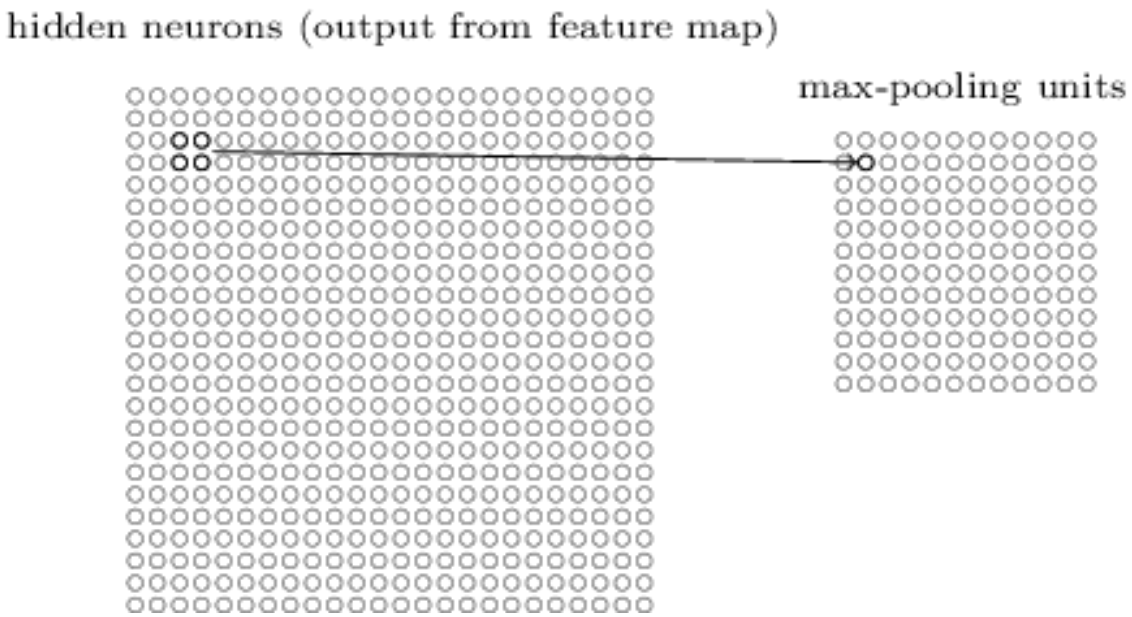


$$\sigma\left(b + \sum_{x=0}^{n_x}\sum_{y=0}^{n_y} w_{i,j}a_{i+x,j+y}\right)$$

FEATURE MAPS

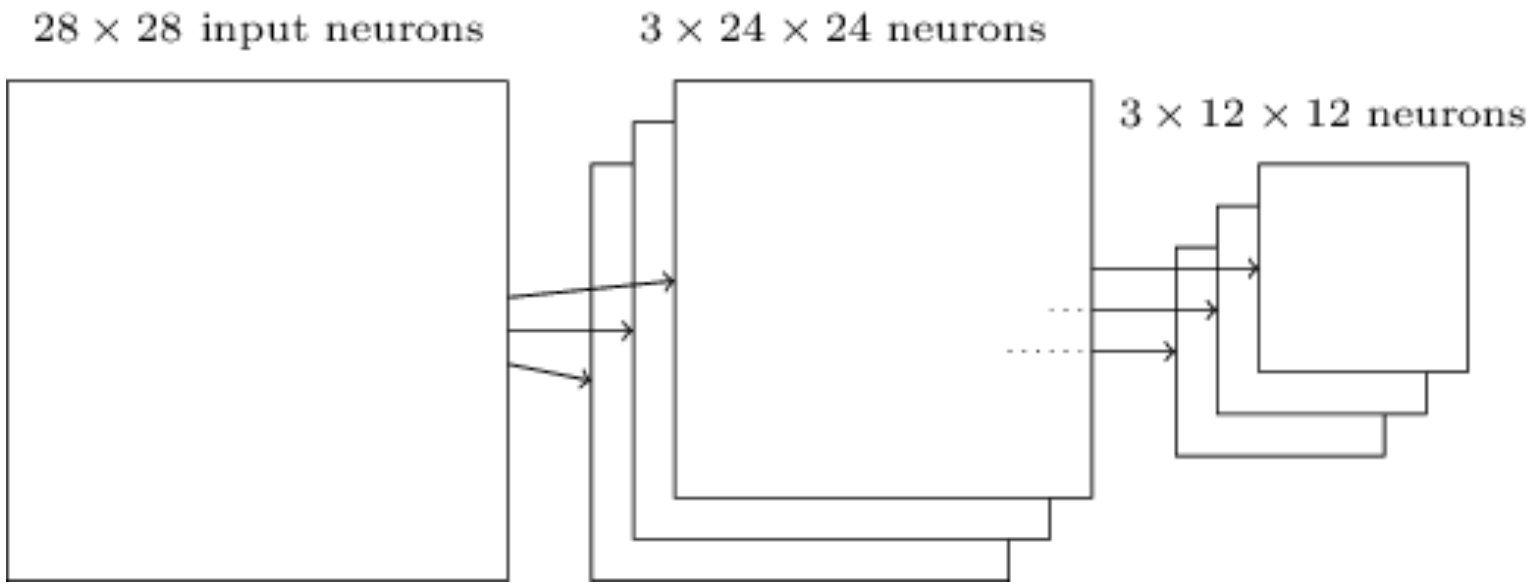


POOLING LAYER



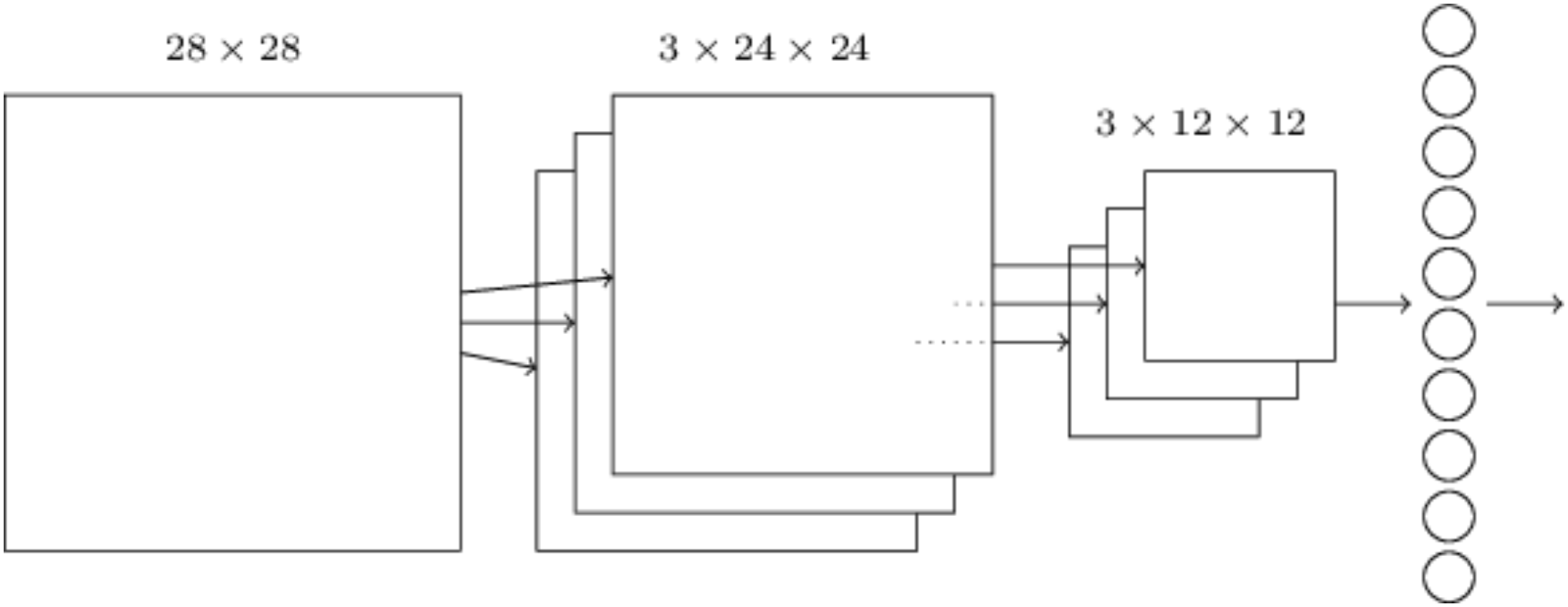
- Reduce the number of inputs by replacing all activations in a neighborhood by a single one.
- Can be thought as asking if a particular feature is present in that neighborhood while ignoring the exact location.

ADDING THE POOLING LAYERS



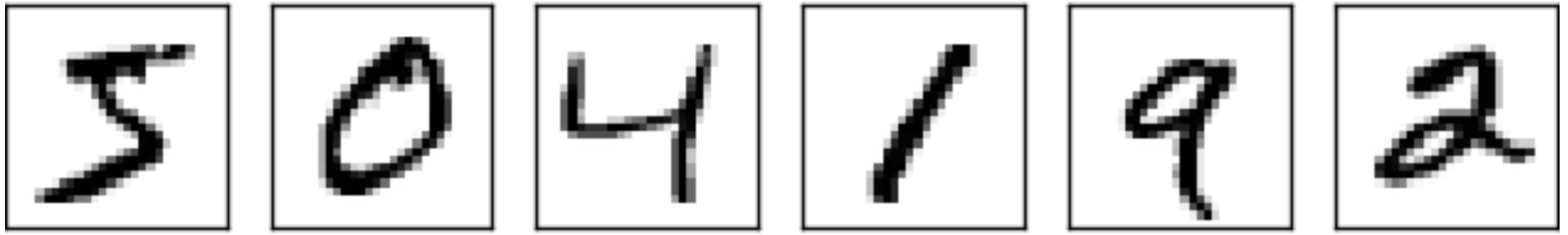
The output size is reduced by the pooling layers.

ADDING A FULLY CONNECTED LAYER



- Each neutron in the final fully connected layer is connected to all neurons in the preceding one.
- Deep architecture with many parameters to learn but still far fewer than an equivalent multilayer perceptron.

MNIST

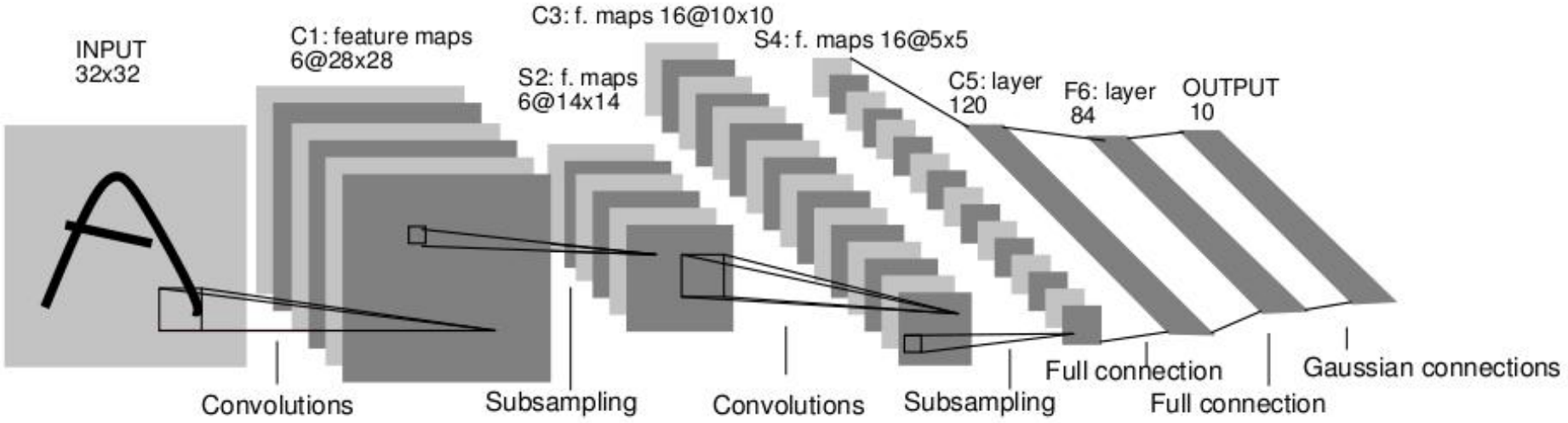


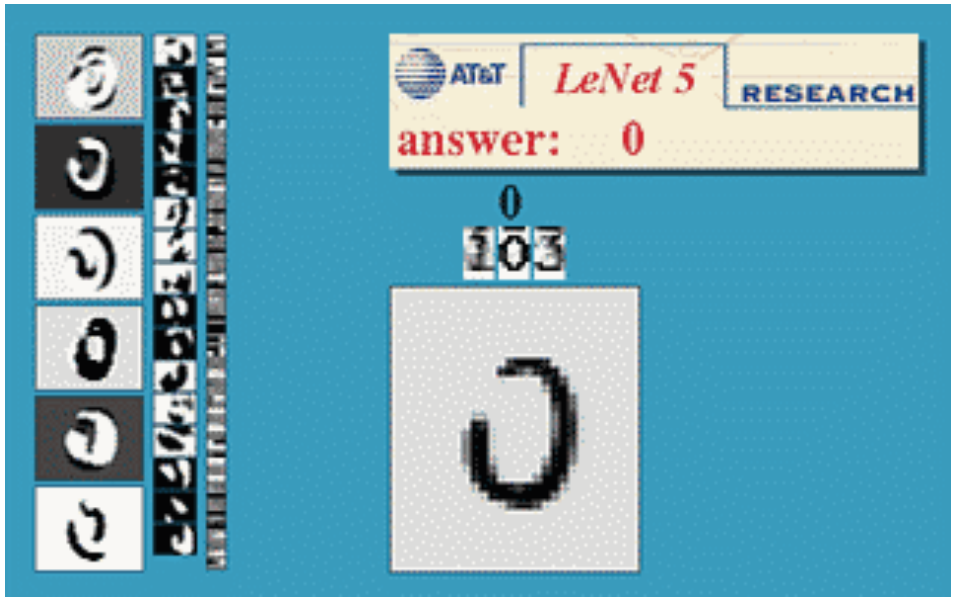
- The network takes as input 28x28 images represented as 784D vectors.

- The output is a 10D vector giving the probability of the image representing any of the 10 digits.

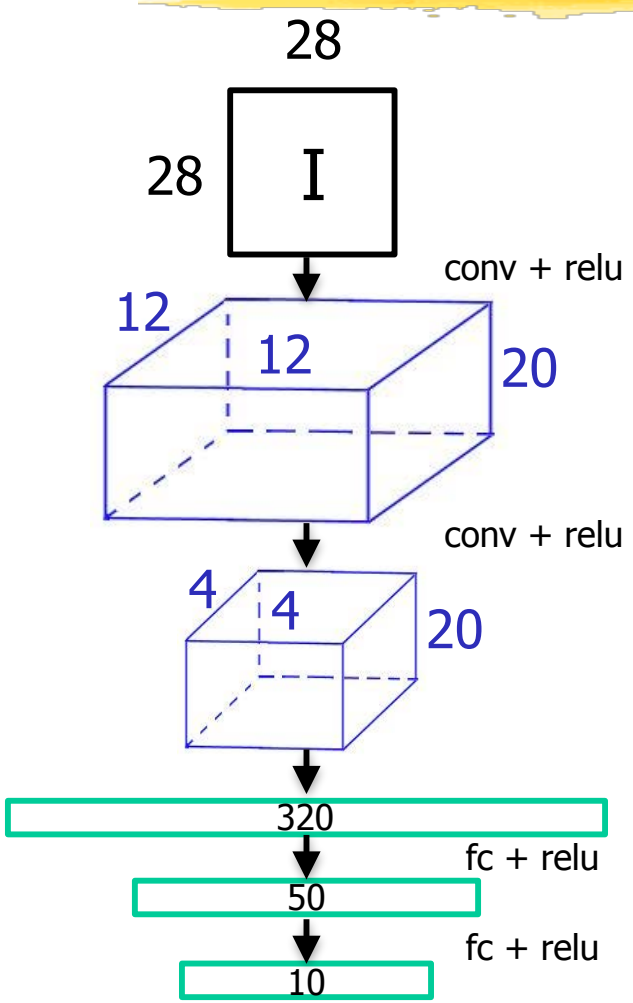
- There are 50’000 training pairs of images and the corresponding label, 10’000 validation pairs, and 5’000 testing pairs.

LeNet (1989-1999)





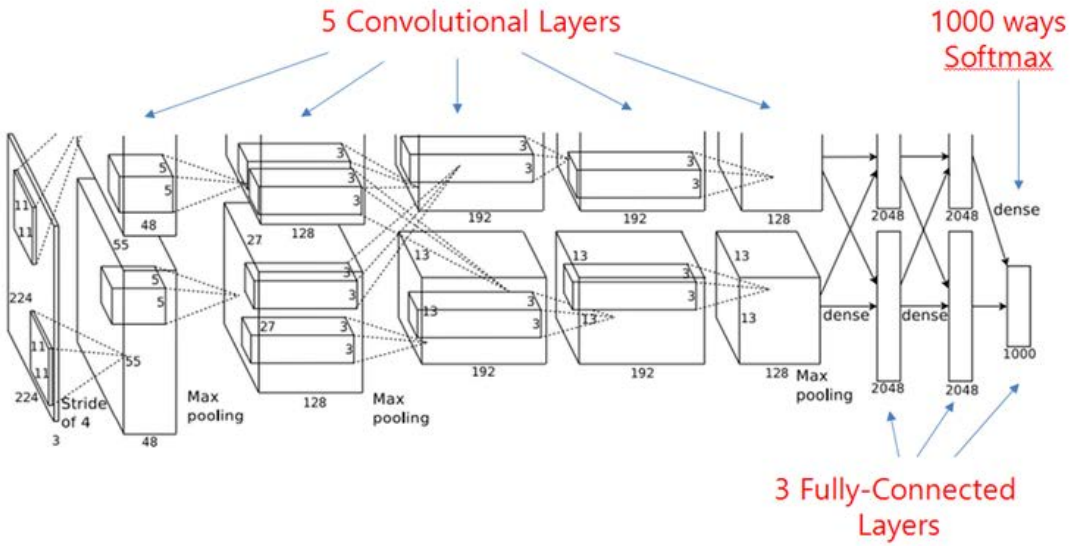
IS MAX POOL REQUIRED?



Accuracy	Train	Test
Conv 5x5, stride 1 Max pool 2x3	99.58	98.77
Conv 5x5, stride 2	99.42	98.31
Conv 5x5, stride 1 Conv 3x3, stride 2	99.38	98.57

Springenberg et al., ICLR'15

AlexNet (2012)



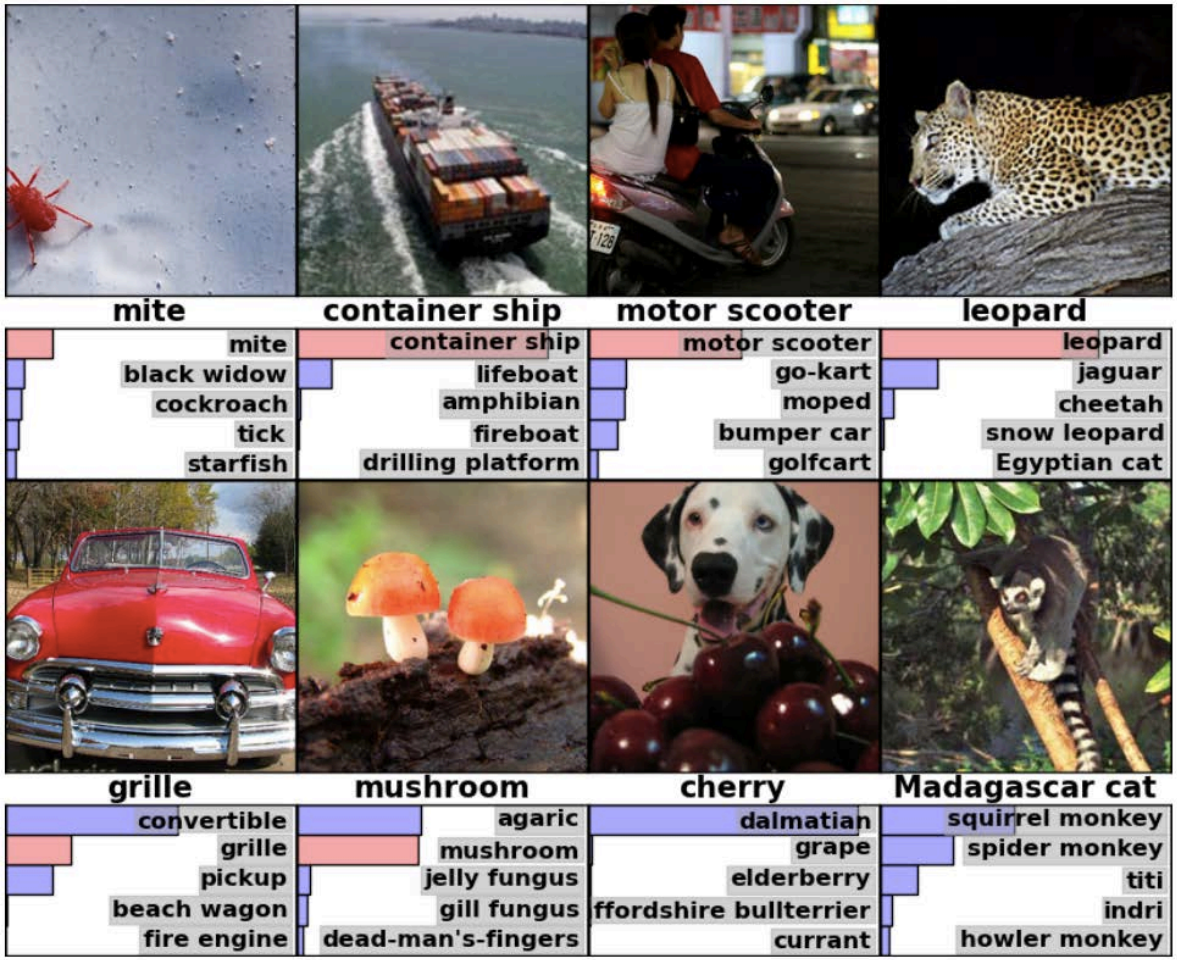
Task: Image classification

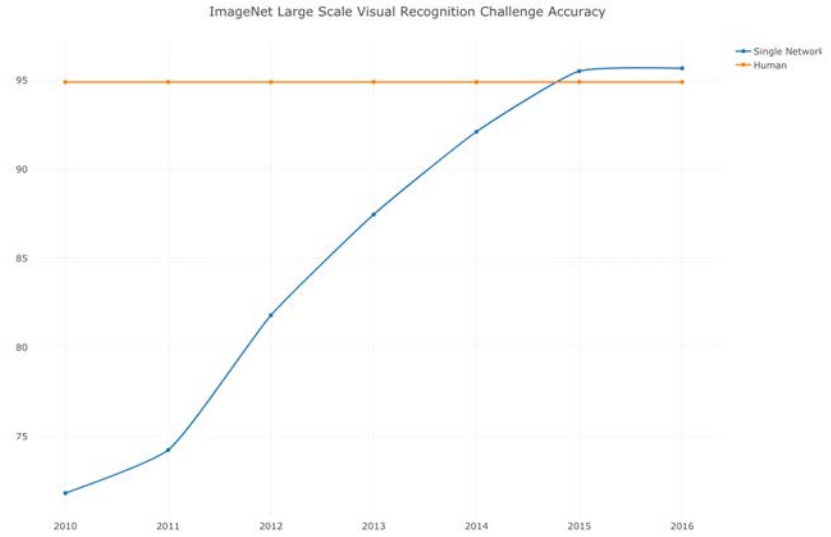
Training images: Large Scale Visual Recognition Challenge 2010

Training time: 2 weeks on 2 GPUs

Major Breakthrough: Training large networks has now been shown to be practical!!

AlexNet RESULTS

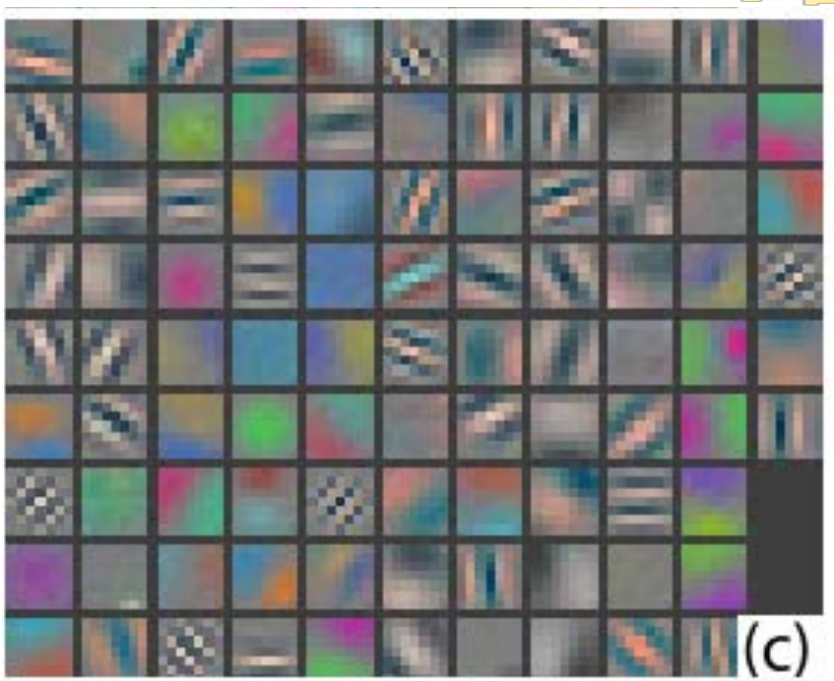




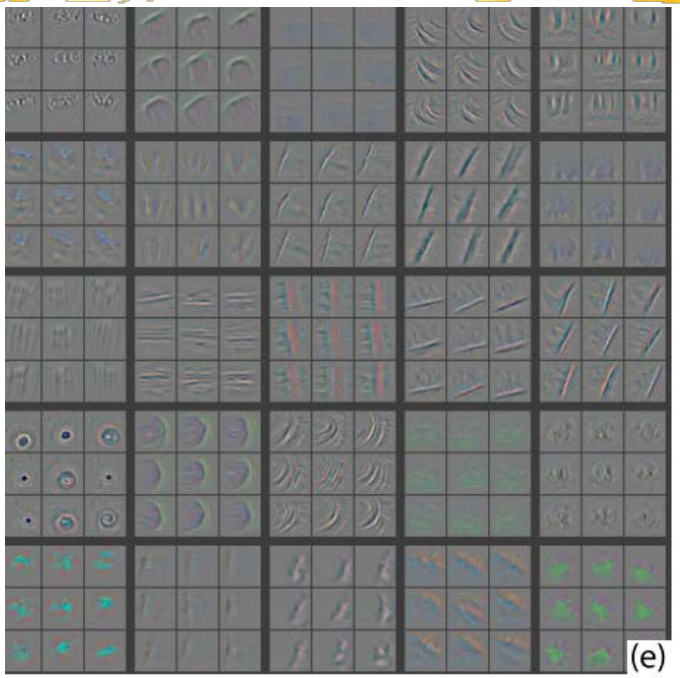
- At the 2012 ImageNet Large Scale Visual Recognition Challenge, AlexNet achieved a top-5 error of 15.3%, more than 10.8% lower than the runner up.
- Since 2015, networks outperform humans on this task.

Krizhevsky, NIPS'12

FEATURE MAPS



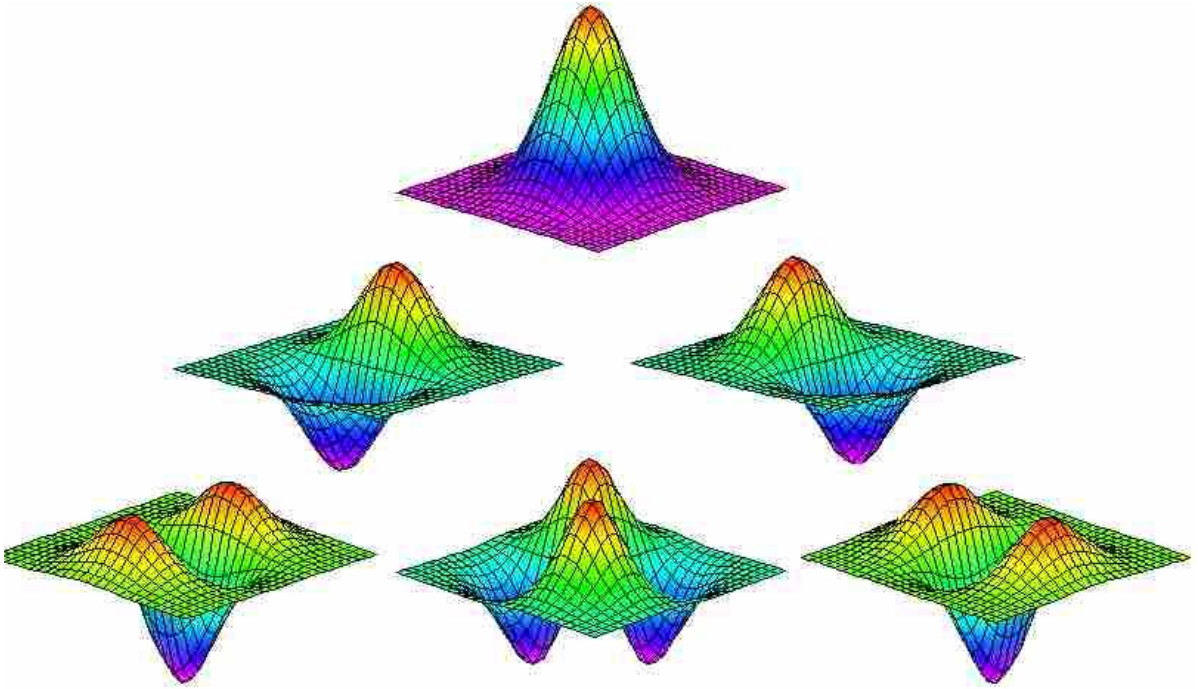
First convolutional layer

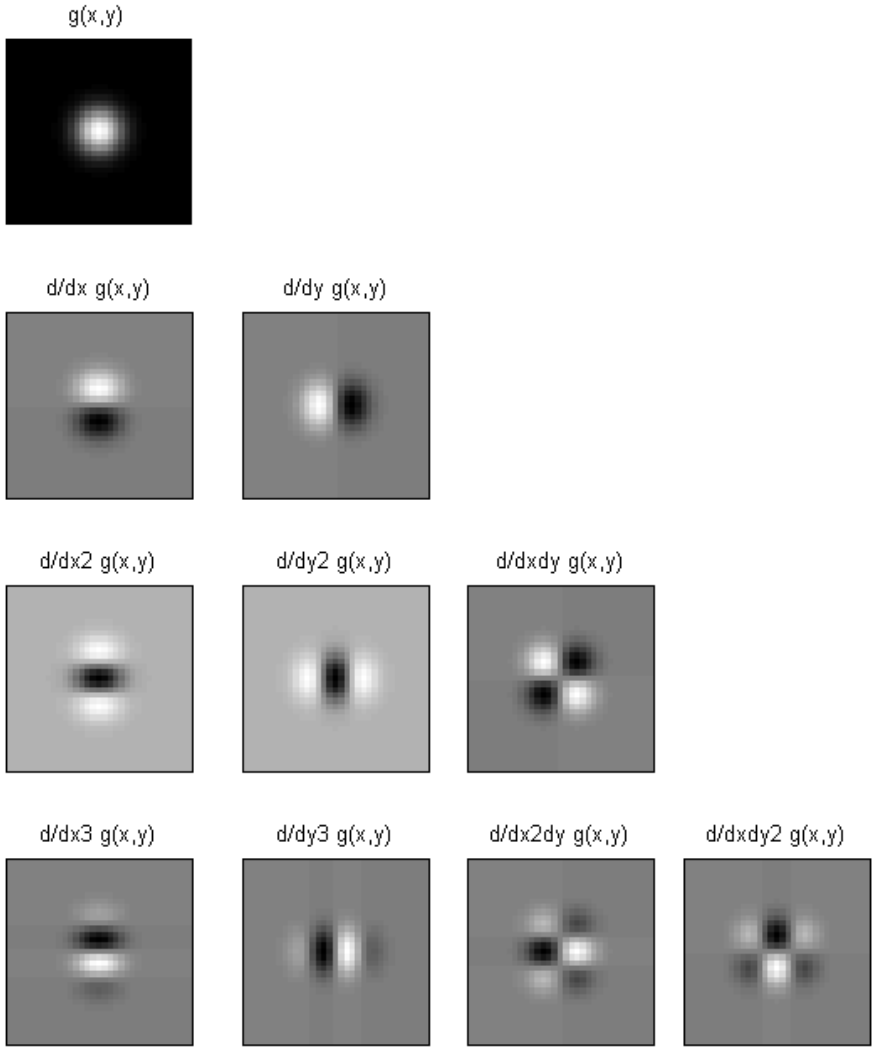


Second convolutional layer

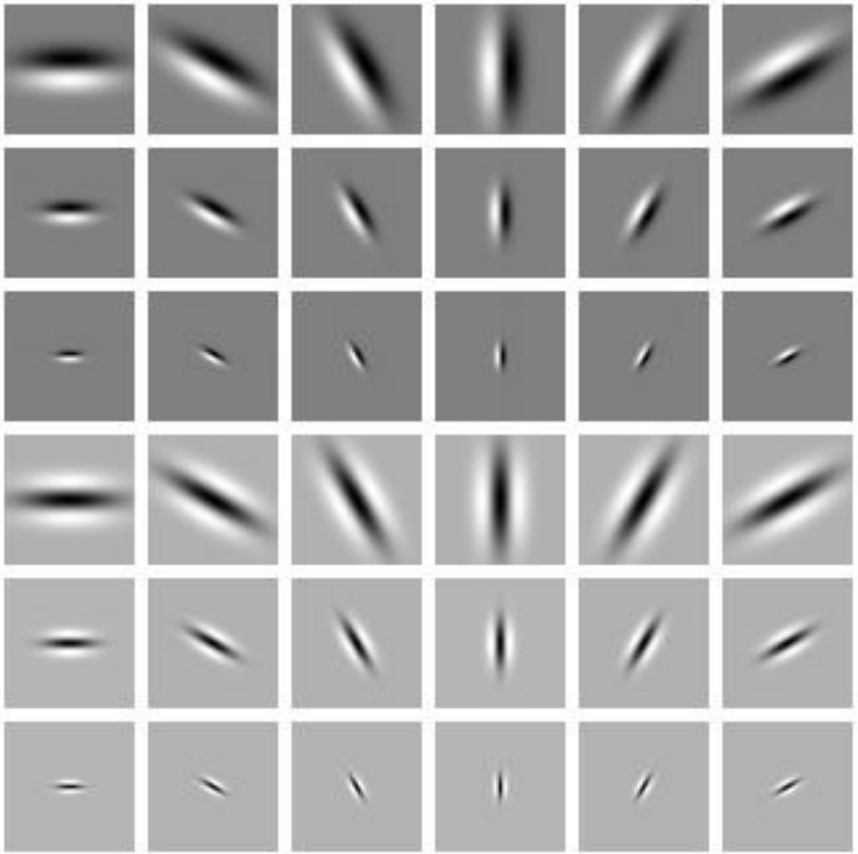
- Some of the convolutional masks seem very similar to oriented Gaussian or Gabor filters!
- Much ongoing work to better understand this.

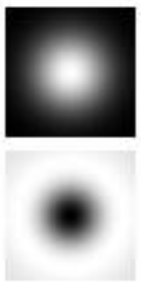
HIGHER ORDER DERIVATIVES



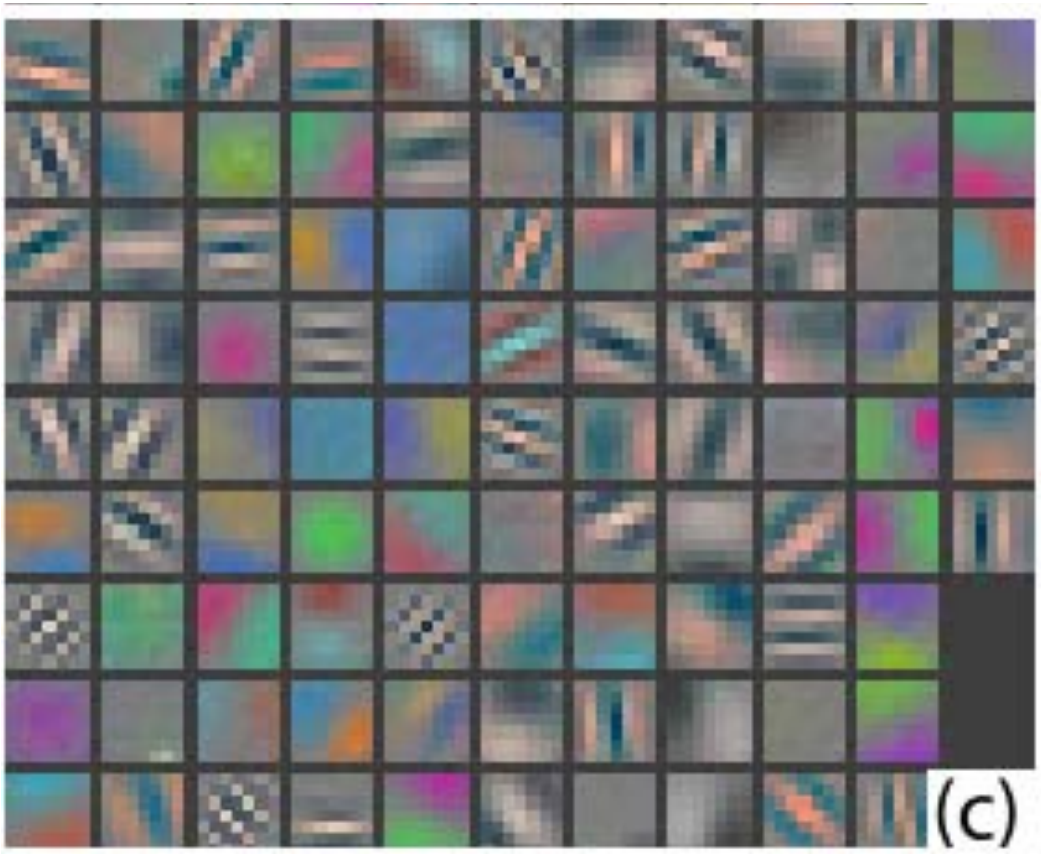


FILTER BANKS





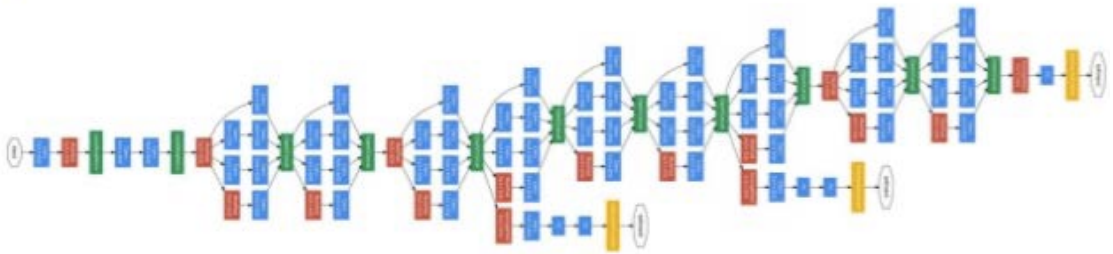
Hand-Designed



Learned

SIZE AND DEPTH MATTER

VGG19, 3 weeks of training.



"hibiscus"

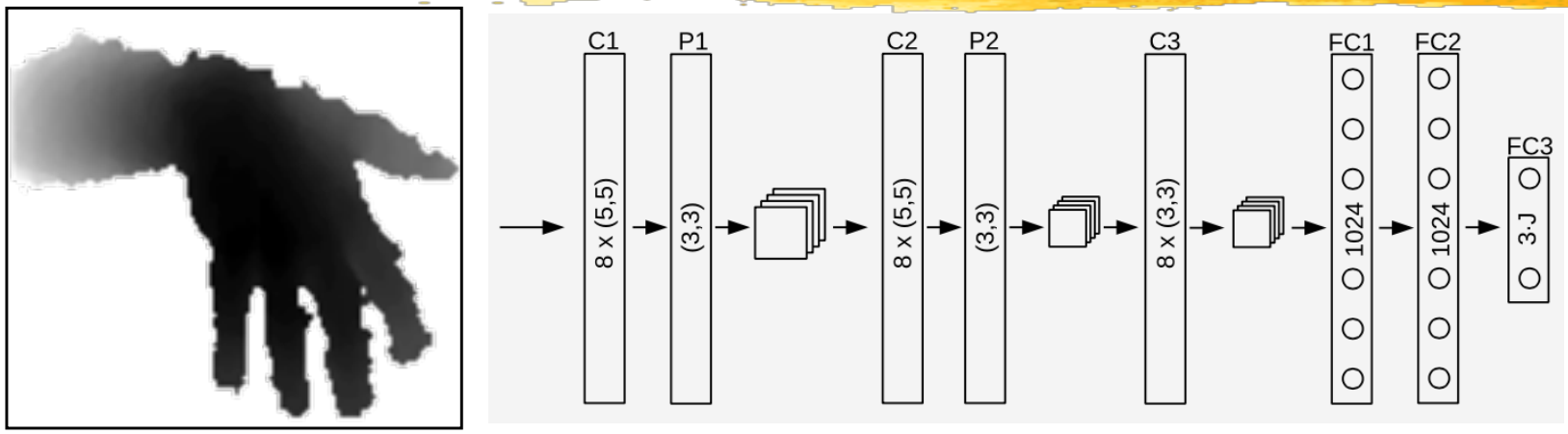
"dahlia"

GoogleLeNet.

"It was demonstrated that the representation depth is beneficial for the classification accuracy, and that state-of-the-art performance on the ImageNet challenge dataset can be achieved using a conventional ConvNet architecture."

Simonyan & Zisserman, ICLR'15

HAND POSE ESTIMATION (2015)

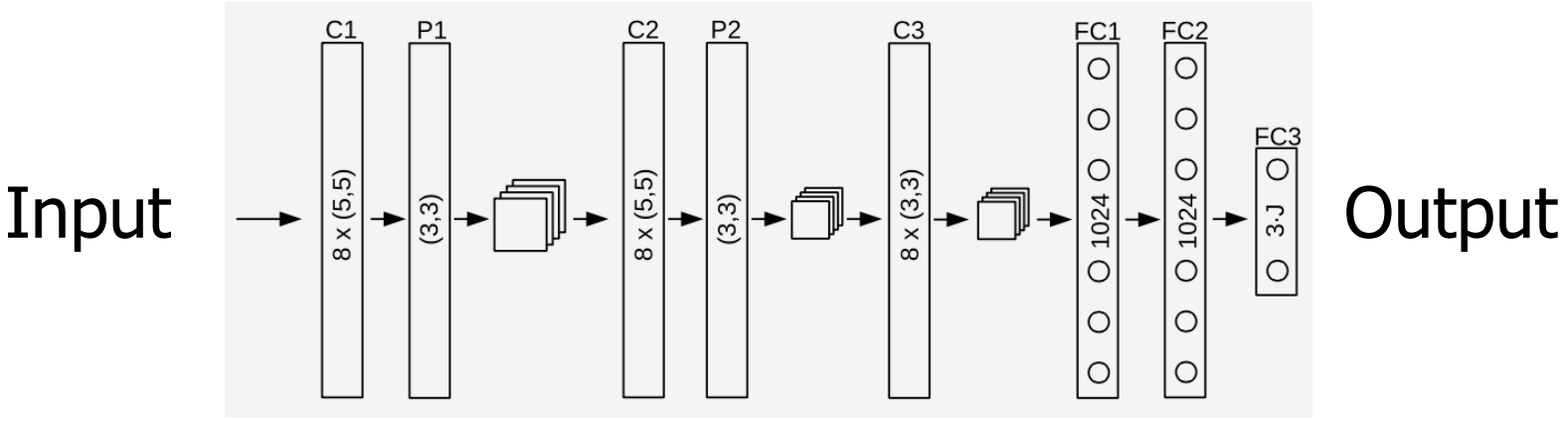


Input: Depth image.

Output: 3D pose vector.

Oberweger et al. , ICCV'15

REGRESSION



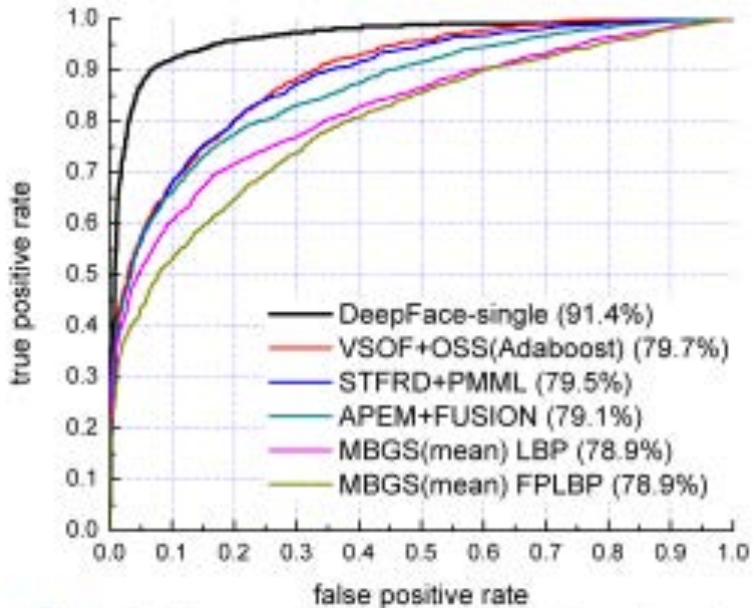
Network parameters are found by minimizing an objective function of the form

$$\min_{\mathbf{W}_l, \mathbf{B}_l} \sum_i ||\mathbf{F}(\mathbf{x}_i, \mathbf{W}_1, \ldots, \mathbf{W}_L, \mathbf{b}_1, \ldots, \mathbf{b}_L) - \mathbf{y}_i||^2$$

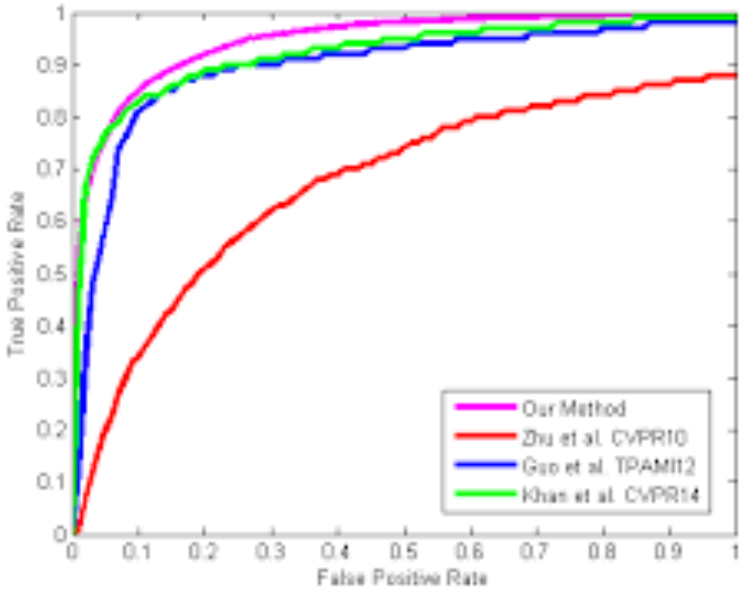
using

- stochastic gradient descent on mini-batches,
- dropout,
- hard example mining,
- ……

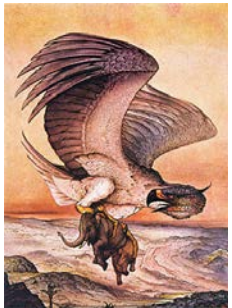
ROC HUNTING



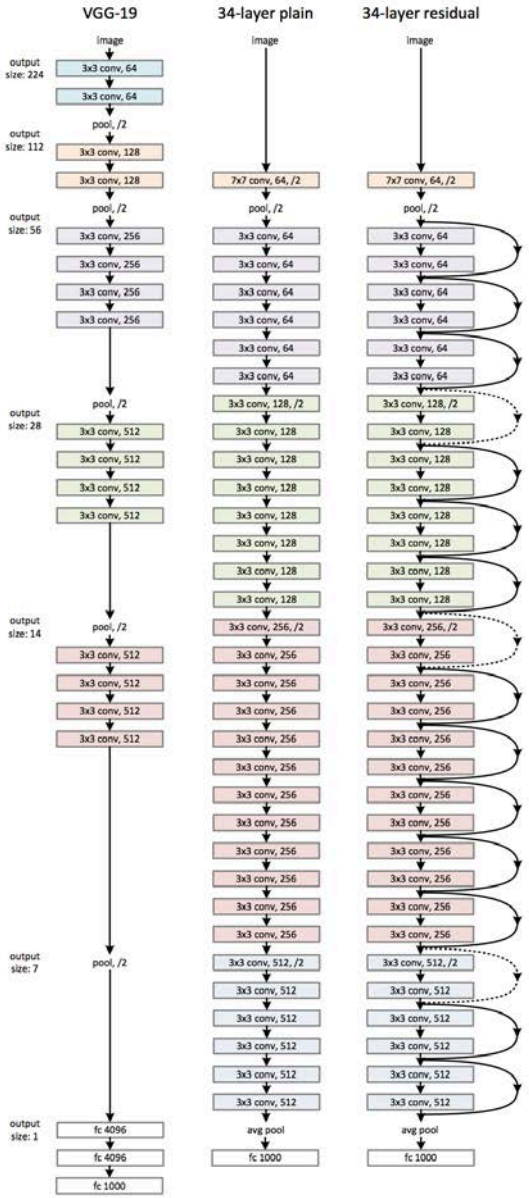
DeepFace
Taigman et al. 2014

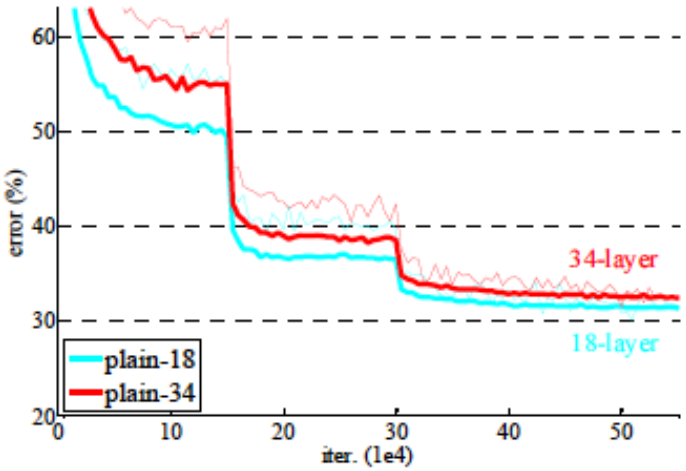


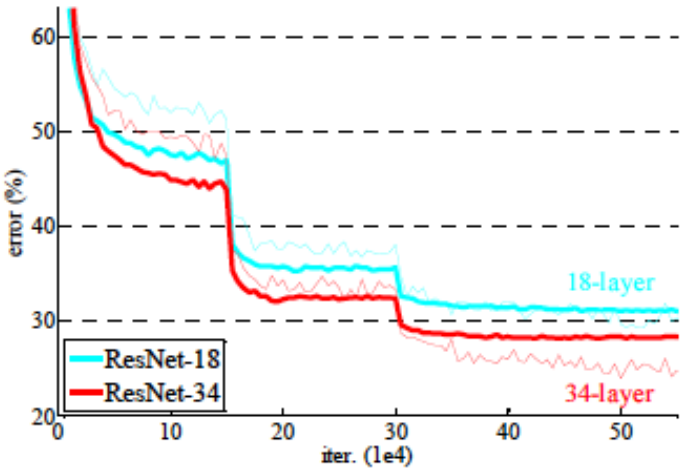
Deep Edge Detection
Shen et al. 2015



DEEPER AND DEEPER

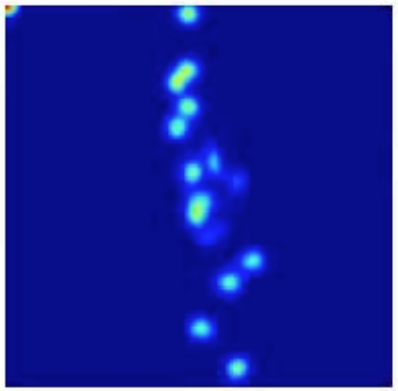


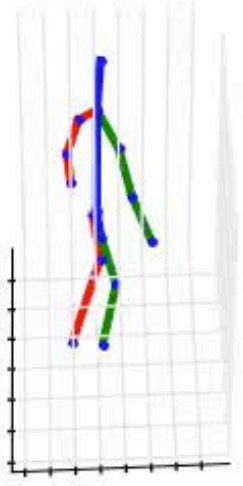


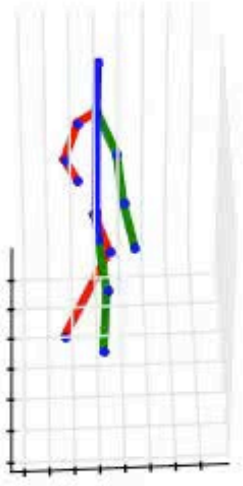


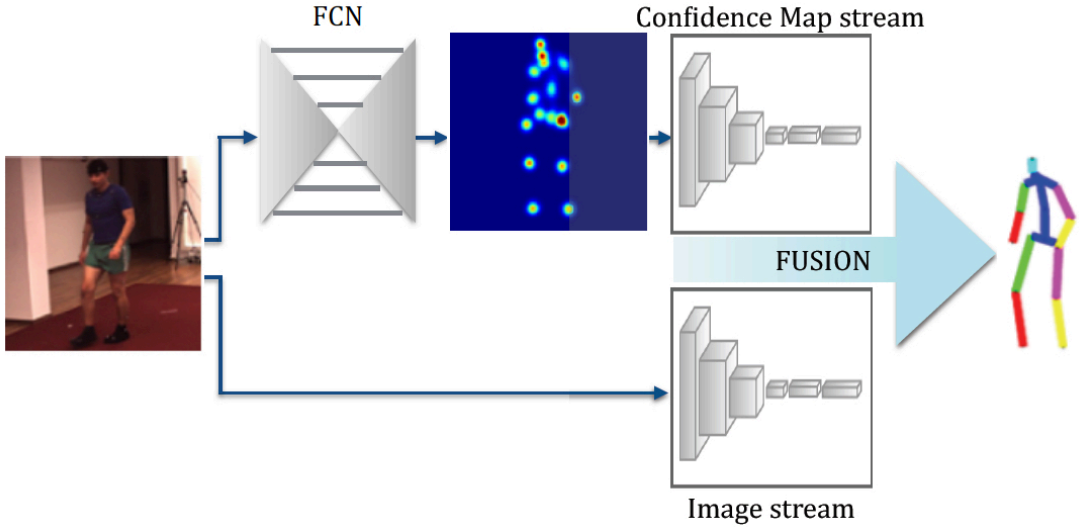
He et al. , CVPR'16

MONOCULAR POSE ESTIMATION



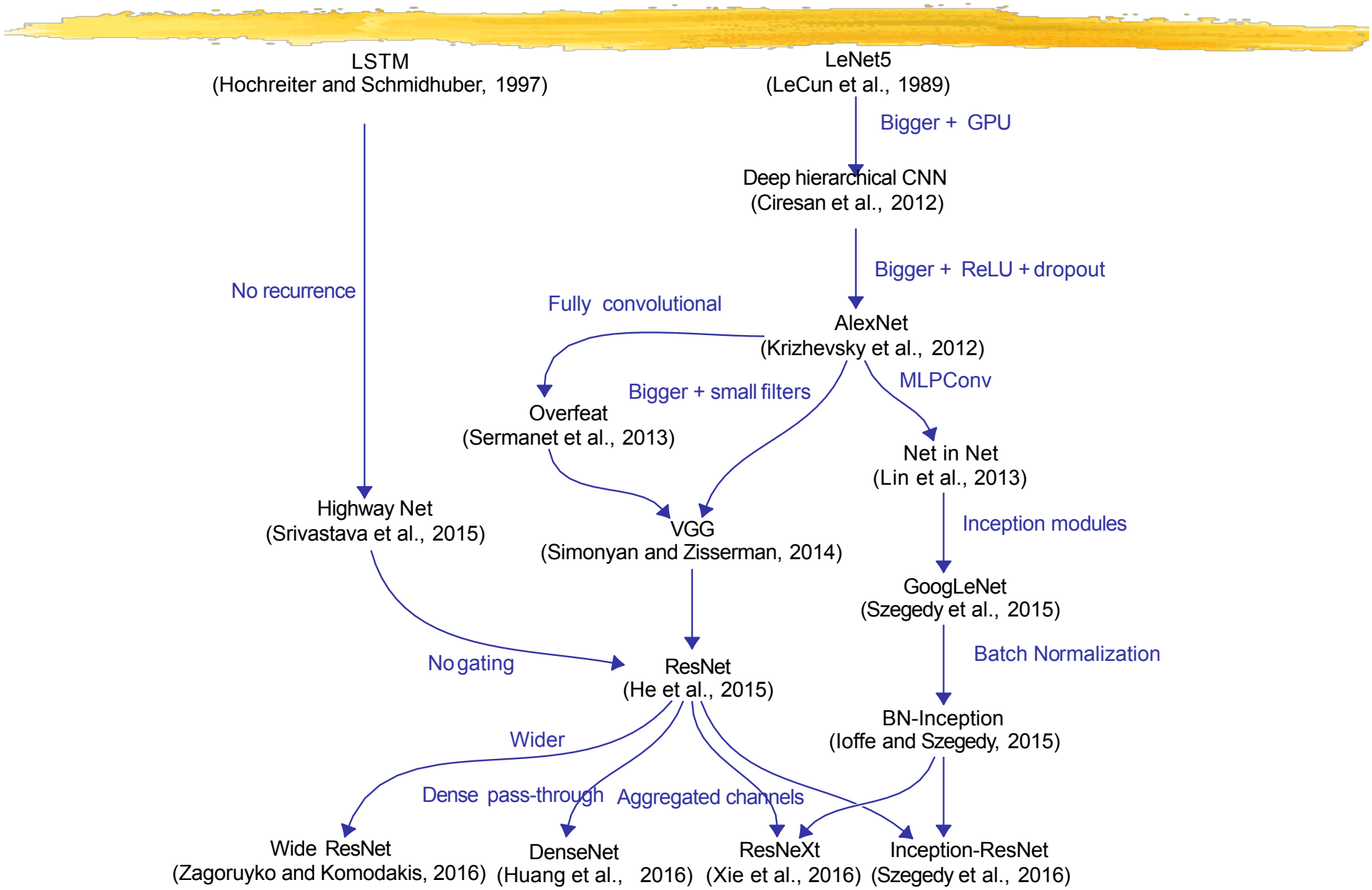






Tekin et al. , ICCV'17

IMAGE CLASSIFICATION TAXONOMY



ALPHA GO

- Uses Deep Nets to find the most promising locations to focus on.
- Performs Tree based search when possible.
- Relies on reinforcement learning and other ML techniques to train.

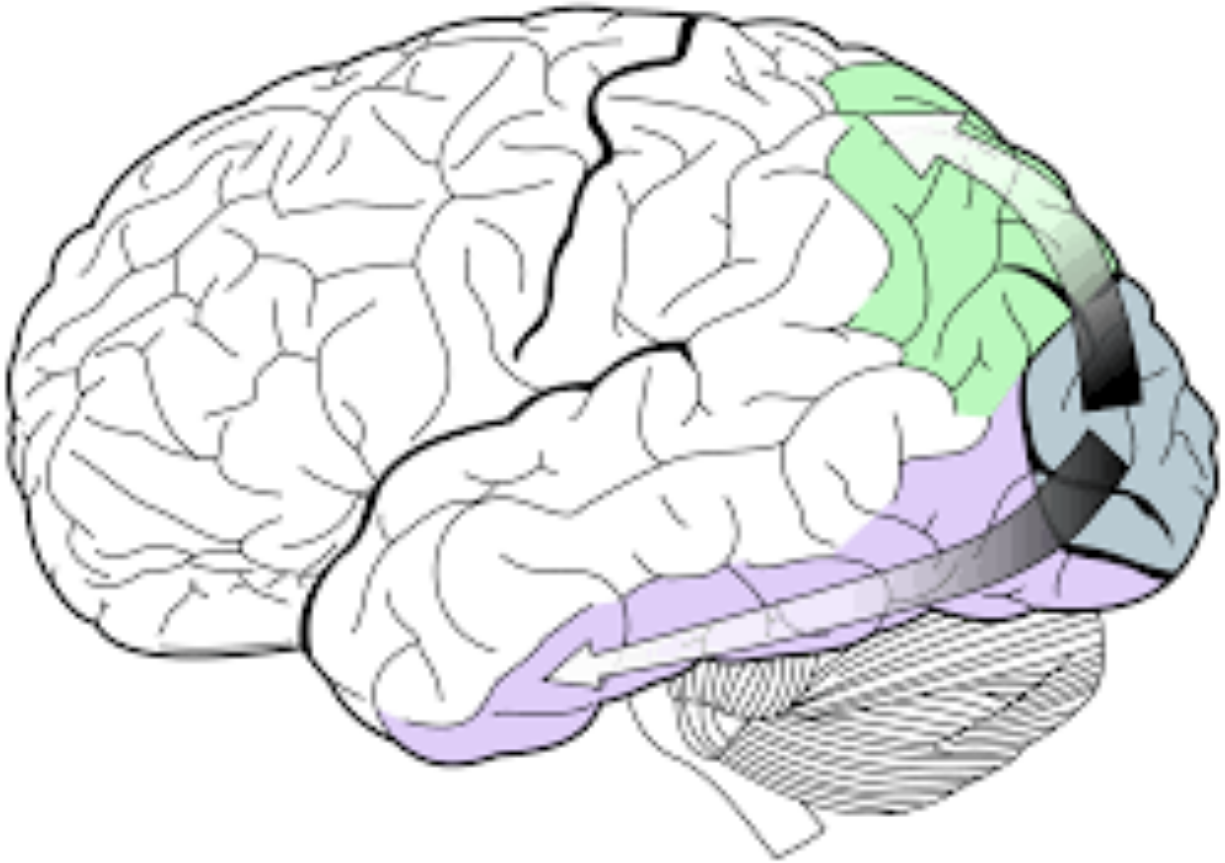
—> Beat the world champion in 2017.

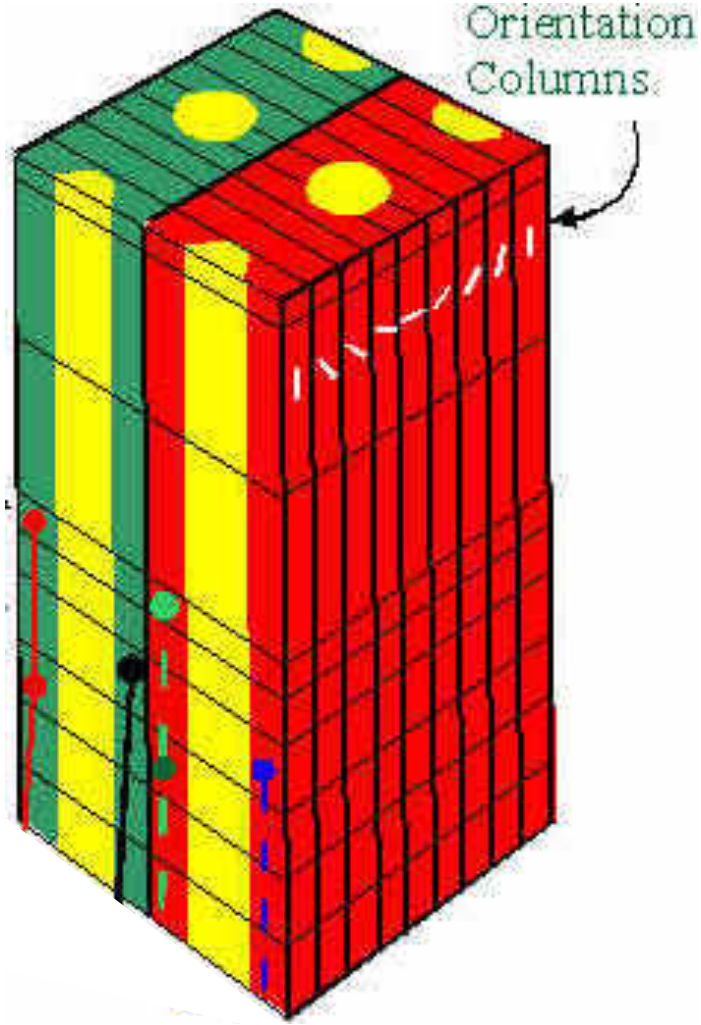
ANOTHER POINT OF VIEW

To summarize roughly the evolution of convnets for image classification:

- Standard ones are extensions of LeNet5.
- Everybody loves ReLU.
- Newer ones have 100s of channels and 10s of layers.
- They can (should?) be fully convolutional.
- Pass-through connections allow deeper "residual" nets.
- Bottleneck local structures reduce the number of parameters.
- Aggregated pathways reduce the number of parameters.

VISUAL CORTEX





AlphaGo

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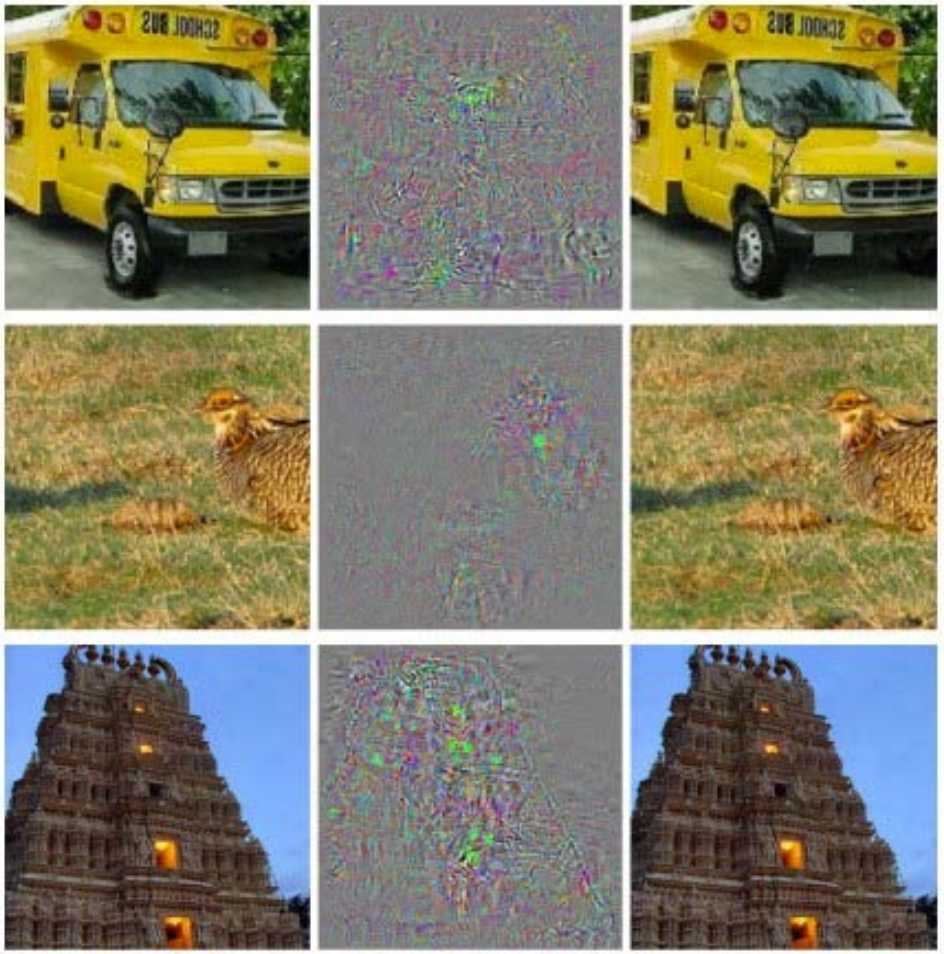
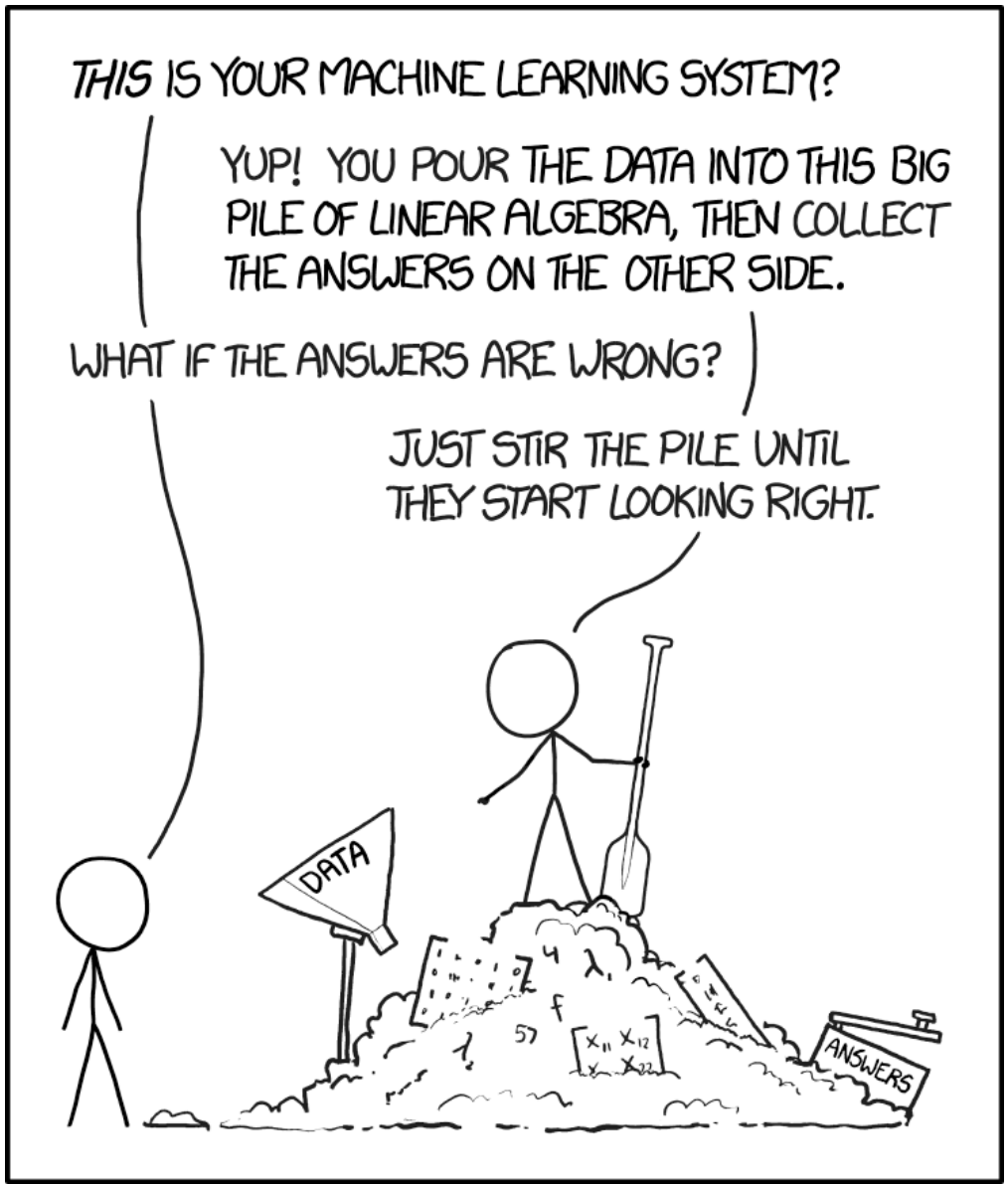
ADVERSARIAL IMAGES



Szegedy et al. 2013

XKCD'S VIEW ON THE MATTER



https://xkcd.com/

IN SHORT

- Deep Belief Networks in general and Convolutional Neural Nets in particular outperform conventional Computer Vision algorithms on many benchmarks.
- It is not fully understood why and unexpected failure cases have been demonstrated.
- They require a lot of manual tuning to perform well and performance is hard to predict.

—> Many questions are still open and there is much work left to do.