COM-303 - Signal Processing for Communications
"Mock" Midterm Exam

- This is a no-grade, take-home midterm exam: try to work on the problems as if taking a real exam, i.e.: work uninterruptedly for 2 hours, do not use the internet and use only handwritten notes.
- The solution will be discussed in class after the spring break.
- There are 6 problems with different scores for a total of 100 points.

Exercise 1. (5 points)
Consider a length-N signal $\mathbf{x} = [x[0]\, x[1] \ldots x[N-1]]^T$ and its DFT $\mathbf{X} = [X[0]\, X[1] \ldots X[N-1]]^T$.
Consider now the length-$2N$ vector

$$\mathbf{y} = [x[0] \quad -x[0] \quad x[1] \quad -x[1] \quad x[2] \quad -x[2] \quad \ldots \quad x[N-1] \quad -x[N-1]]^T$$

and express its $2N$-point DFT in terms of the N original DFT coefficients $X[k]$.

Exercise 2. (10 points)
Consider the infinte-length discrete-time signal

$$s[n] = \cos(\frac{\pi}{2}n) + \cos(\frac{5\pi}{8}n) \qquad n \in \mathbb{Z}$$

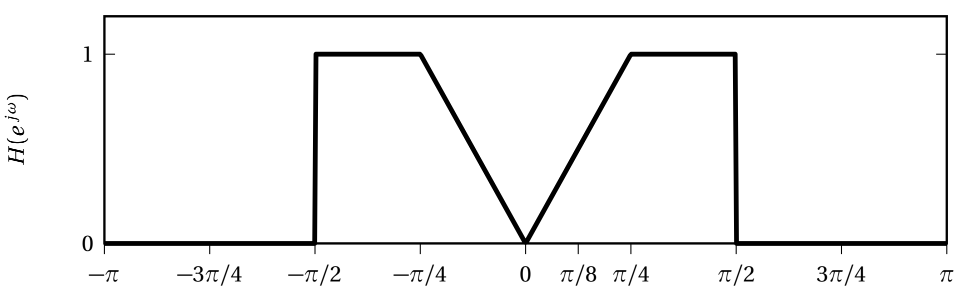
Consider now the finite-length signal

$$x[n] = s[n], \quad n = 0, 1, \ldots N-1$$

Determine the minimum value for N so that the DFT $X[k]$ satisfies the following requirements:

- $X[k]$ has only four nonzero values
- the nonzero values are non-contiguous (i.e. there should be one or more zero values for $X[k]$ between the nonzero values); this corresponds to being able to resolve the frequencies of the sinusoids of the original signal.

Exercise 3. (25 points)
Determine the impulse response $h[n]$ of an ideal filter whose real-valued frequency response $H(e^{j\omega})$ is shown in the following figure:



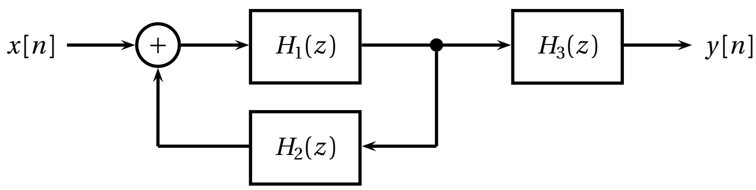
Exercise 4. (10 points)
Consider a filter with frequency response $H(e^{j\omega}) = \cos 2\omega + 3\cos 5\omega$.

(a) write out the impulse response $h[n]$

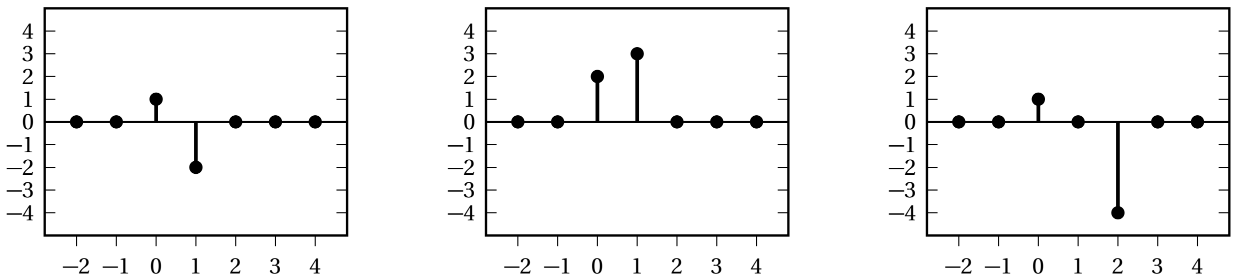
(b) what is the delay introduced by the filter?

(c) write the frequency response of a causal implementation of the filter.

Exercise 5. (30 points)
Consider the following system



where the finite-length impulse responses of the three filters $H_1(z)$, $H_2(z)$, $H_3(z)$ are as in the following figures:



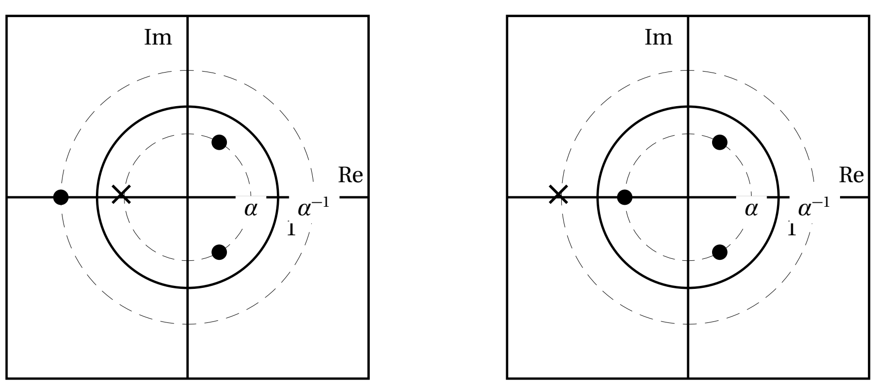
$H_1(z)$ $H_2(z)$ $H_3(z)$

(a) compute the global transfer function of the system

(b) sketch its pole-zero plot

(c) determine if the system stable

Exercise 6. (20 points)
The figures below show the pole-zero plots of two filters, $H_1(z)$ and $H_2(z)$. The poles and the zeros lie on circles of radius α and $1/\alpha$. Sketch as accurately as you can the magnitude responses of both filters, highlighting the differences if any.



$H_1(z)$ $H_2(z)$