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Reduction and linearization of MMC models for interaction and system stability studies

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Outline

- Stability studies of electric power systems
- Introduction to techniques for small-signal stability studies
  - Linearization
  - Use of eigenvalues for assessment of stability and dynamic characteristics
  - Participation factors
  - Eigenvalue parametric sensitivity
- Review of MMC operating characteristics
  - Basics of MMC modelling and control
  - Challenges of small-signal modelling of MMC systems
- Proposed state-space model of three-phase MMC HVDC terminal
  - Required assumptions of control structure and implementation of modulation
  - Main simplified equations and overview of resulting model
  - Model verification by time-domain simulations
  - Eigenvalue analysis of MMC HVDC terminal
- Summary

Stability studies of electric power systems

- General approaches for power system stability analysis
  - Time domain simulations
    - Can include all nonlinearities
    - Applicable for different levels of simplification
    - Suitable for verification of specific transients or operating conditions
    - Requires trial-and-error-based approaches
      - Controller tuning or stability improvement usually based on experience
      - No general tool for system stability analysis is directly applicable

  - Small-signal methods
    - **State-space analysis** – discussed in the following
    - Impedance-based analysis – to be presented by M. Amin

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State-space modelling of dynamic systems

- General expression of system model on state space form

$$\frac{d\mathbf{x}}{dt} = f\left(\mathbf{x}(t),\mathbf{u}(t)\right)$$

$$\mathbf{y} = g\left(\mathbf{x}(t),\mathbf{u}(t)\right)$$

$$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$$

$$\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \dots & u_m \end{bmatrix}^T$$

$$\mathbf{y} = \begin{bmatrix} y_1 & y_2 & \dots & y_k \end{bmatrix}^T$$

- Typical example

$$\underbrace{\frac{di_L}{dt}}_{\dot{x}_1} = \underbrace{\frac{1}{L}\left(-r_L \underbrace{i_L}_{x_1} + \underbrace{v_{converter}}_{f\left(x_{2\dots n},u_{2\dots n}\right)} - \underbrace{v_{source}}_{u_1}\right)}_{f(\mathbf{x},\mathbf{u})}$$

$$\underbrace{p_{out}}_{y_1} = \underbrace{\underbrace{i_L}_{x_1} \cdot \underbrace{v_{source}}_{u_1}}_{g(\mathbf{x},\mathbf{u})}$$

Equilibrium of dynamic systems

- Requirement for state-space stability analysis
  - The system must have a defined equilibrium point for a given input
  - All states must settle to a constant value in steady state

$$f\left(\mathbf{x}_0,\mathbf{u}_0\right)=0 \longrightarrow \dot{\mathbf{x}}=0$$

    - All state variables have zero derivatives in the equilibrium point

- General methods for stability analysis of dynamic systems cannot be applied if the equilibrium for any of the state variables is a time-dependent trajectory
- Stability analysis cannot be directly applied to electrical systems with sinusoidal voltages and currents
  - Traditional approach for power system analysis: quasi-stationary phasor models
  - Dynamic analysis of balanced three-phase electrical systems: **modelling in synchronously rotating dq reference frame**

Small signal modelling

- Most physical systems have some degree of nonlinearity
  - Still, stability and response to small-signal perturbations can often be studied by applying linear methods
    - Dynamics around a steady-sate operating point can be expressed by:

$$\mathbf{x} = \mathbf{x}_0 + \Delta\mathbf{x} \qquad \mathbf{u} = \mathbf{u}_0 + \Delta\mathbf{u}$$

    - Stability can be studied by analysing the small-signal dynamics

- Linearization by first order Taylor expansion:

$$\frac{d\Delta x_1}{dt} = \frac{df(\mathbf{x},\mathbf{u})}{dx_1}\Delta x_1 + \frac{df(\mathbf{x},\mathbf{u})}{dx_2}\Delta x_2 + \ldots + \frac{df(\mathbf{x},\mathbf{u})}{du_1}\Delta u_1 + \frac{df(\mathbf{x},\mathbf{u})}{du_2}\Delta u_2 + \ldots$$

- A linearized small-signal system can be expressed on the general form:

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\cdot\Delta\mathbf{x} + \mathbf{B}\cdot\Delta\mathbf{u} \qquad \Delta\mathbf{y} = \mathbf{C}\cdot\Delta\mathbf{x} + \mathbf{D}\cdot\Delta\mathbf{u}$$

Eigenvalues of a dynamic system

- The eigenvalues of a dynamic system are defined as the solution to the characteristic equation

$$\det\left(\mathbf{A} - \lambda\mathbf{I}\right) = 0$$

- A system with n states will give n eigenvalues as solution
  - Eigenvalues are either single real poles or pairs of complex conjugate poles

$$\lambda_i = \alpha + j\omega$$

- The small-signal stability of the system can be determined from the eigenvalues
  - The system is asymptotically stable if all eigenvalues have negative real values
  - When at least one eigenvalues has a positive real part, the system is unstable
  - If at least one eigenvalues has zero real part, the system is marginally stable and it is not possible to determine the stability of a linearized system from small-signal analysis

Definition of eigenvectors

- The eigenvalues of a system satisfy the following equation:

$$\mathbf{A}\mathbf{\Phi}_i = \lambda_i \mathbf{\Phi}_i$$

  - Where the column vector Φ_i is the right eigenvector associated with the eigenvalue λ_i
  - The corresponding left eigenvector is defined as a row vector by:

$$\mathbf{\Psi}_i \mathbf{A} = \lambda_i \mathbf{\Psi}_i$$

  - Any multiple of an eigenvector is also a solution to the equations above
  - Usually eigenvectors are scaled so that:

$$\mathbf{\Psi}_i \mathbf{\Phi}_i = 1$$

Definition of modal matrices

- The right and left eigenmatrices and the diagonal eigenvalue matrix are defined by:

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 & \dots & \mathbf{\Phi}_n \end{bmatrix}$$

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{\Psi}_1 & \mathbf{\Psi}_2 & \dots & \mathbf{\Psi}_n \end{bmatrix}^T$$

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & \lambda_n \end{bmatrix}$$

- Thus the following identities can be defined:

$$\mathbf{A\Phi} = \mathbf{\Phi\Lambda} \longrightarrow \mathbf{\Phi}^{-1}\mathbf{A\Phi} = \mathbf{\Lambda}$$

$$\mathbf{\Psi\Phi} = \mathbf{I} \longrightarrow \mathbf{\Psi A\Phi} = \mathbf{\Lambda}$$

Dynamic response expressed by eigenvalues

- A transformed, decoupled, system on diagonal form can be defined by:

$$\dot{\mathbf{z}} = \mathbf{\Psi A \Phi} \cdot \mathbf{z} = \mathbf{\Lambda} \cdot \mathbf{z}$$

- Due to the diagonal form, the time response of the mode corresponding to each eigenvalue can be written directly in the time domain as:

$$z_i(t) = z_i(0) e^{\lambda_i t}$$

- Transforming the system back into a representation based on the original states:

$$\Delta \mathbf{x}(t) = \mathbf{\Phi} \cdot \mathbf{z}(t)$$

- Substituting previous matrix definitions result in:

$$\Delta \mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{\Phi}_i \underbrace{\mathbf{\Psi}_i \Delta \mathbf{x}(0)}_{z_i(0)=c_i} \cdot e^{\lambda_i \cdot t}$$

Interpretation of eigenvalues

- Time response of each state

$$\Delta x_i(t) = \Phi_{i1} c_1 \cdot e^{\lambda_1 \cdot t} + \Phi_{i2} c_2 \cdot e^{\lambda_2 \cdot t} + \ldots + \Phi_{in} c_n \cdot e^{\lambda_n \cdot t}$$

$$\lambda_i = \alpha + j\omega$$

- Single real pole results in:

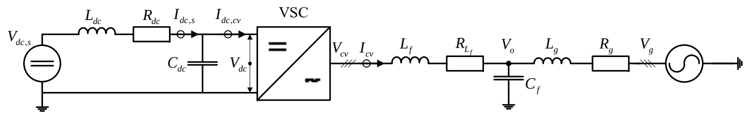
$$\lambda_i = \alpha \longrightarrow f(t) = ke^{\alpha t}$$

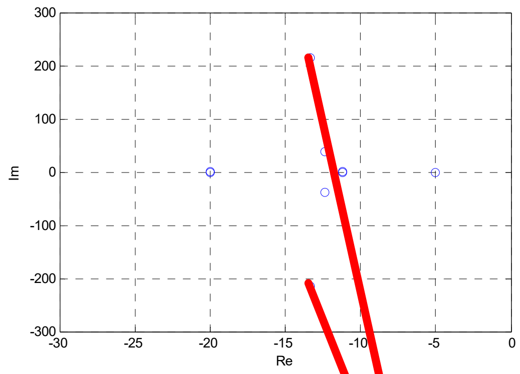
- Complex conjugate poles result in:

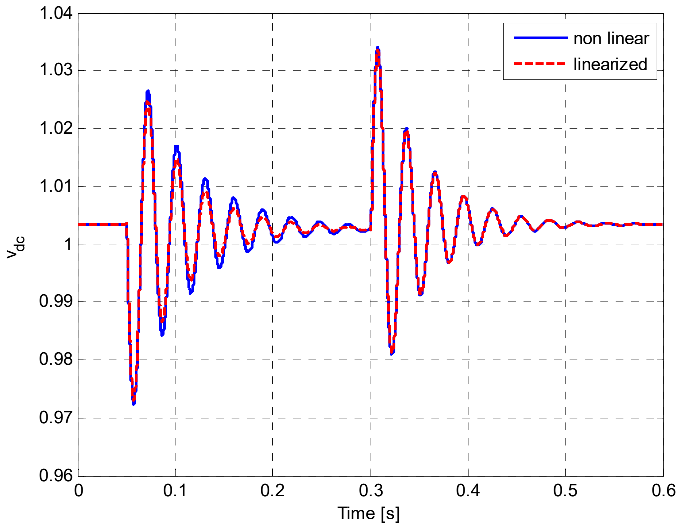
$$\lambda_i = \alpha + j\omega \longrightarrow f(t) = ke^{\alpha t} \sin(\omega t + \varphi)$$

Example of eigenvalue analysis

- From paper published at PEMC 2014
  - Results presented at Workshop in February 2014
- Voltage source converter with inductor in dc-link
  - State space model with 17 states
  - Nonlinear system but the small-signal dynamics are accurately captured by a linearized model
  - Poorly damped oscillations in dc voltage and currents
    - Corresponding complex conjugate pair of eigenvalues







$\lambda_1 = -500$	$\lambda_9 = -475$
$\lambda_2 = -50$	$\boldsymbol{\lambda_{10,11} = -13.3 \pm 214}$
$\lambda_{3,4} = -4436 \pm 21$	$\lambda_{12,13} = -12.4 \pm 37{,}6$
$\lambda_{5,6} = -274 \pm 3048$	$\lambda_{14,15} = -20 \pm 0.80$
$\lambda_{7,8} = -307 \pm 2397$	$\lambda_{16,17} = -11.2 \pm 0.10$

Participation factors

- The participation factor matrix is defined as:

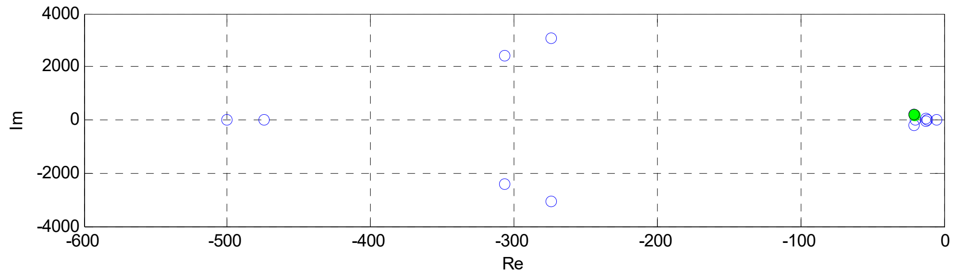
$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n1} & \cdots & p_{nn} \end{bmatrix} = \begin{bmatrix} \phi_{11}\Psi_{11} & \phi_{12}\Psi_{21} & \cdots & \phi_{1n}\Psi_{n1} \\ \phi_{21}\Psi_{12} & \phi_{22}\Psi_{22} & \cdots & \phi_{2n}\Psi_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{n1}\Psi_{1n} & \phi_{n2}\Psi_{2n} & \cdots & \phi_{nn}\Psi_{nn} \end{bmatrix}$$

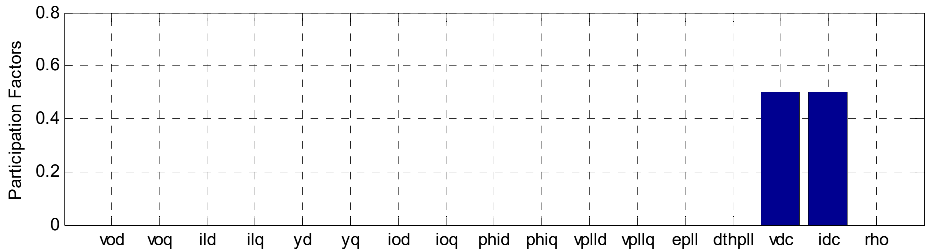
- Participation factor: $p_{ki} = \Phi_{ki}\Psi_{ik}$
  - A measure of the relative participation of state *k* in the mode *i* and vice versa

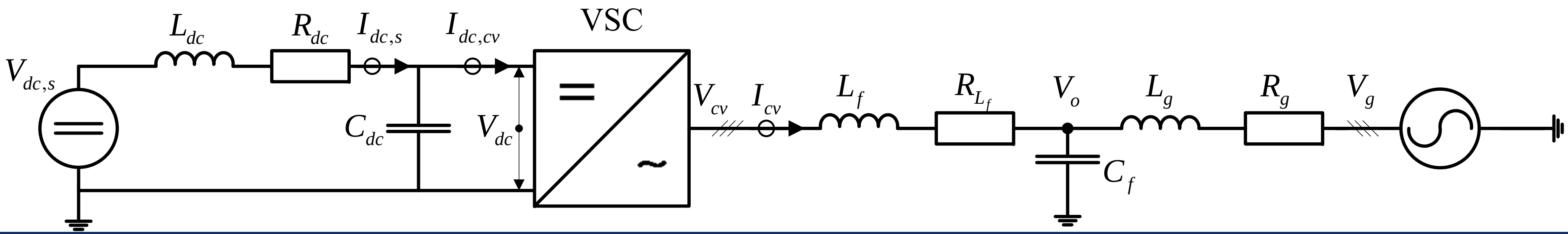
- Can be used to identify what states are mainly contributing to a poorly damped mode
  - Will indicate what states it can be suitable to act on with damping controllers
- Suitable approach for identifying properties of large complex systems

Example of participation factor analysis

- The same system as used for eigenvalue example
- Participation factor analysis for the poorly damped mode
- Only two states participating in the mode
  - Capacitor voltage v_{dc}
  - DC inductor current i_{dc}
- Damping of the oscillations can be implemented by using either of the two state variables as feedback signal







Eigenvalue parametric sensitivity

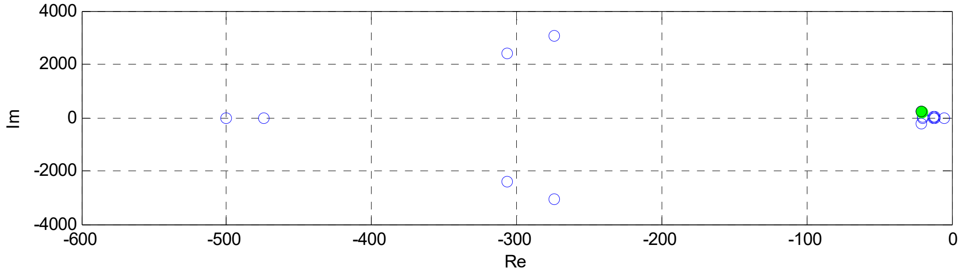
- The parametric sensitivity expresses the derivative of an eigenvalue with respect to a specific parameter

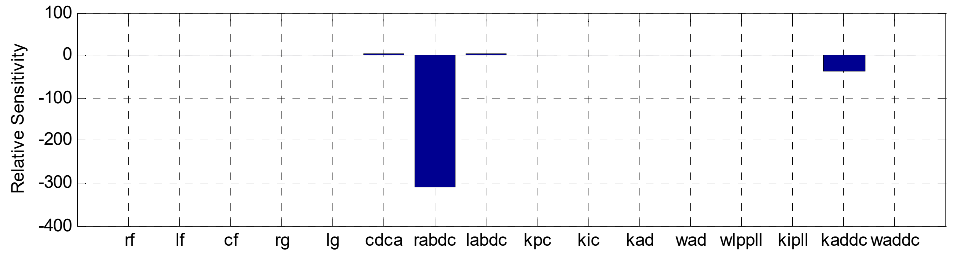
$$\alpha_{i,k} = \frac{d\lambda_i}{d\rho_k} = \frac{\boldsymbol{\Phi}_i^T \dfrac{\partial \mathbf{A}}{\partial \rho_k} \boldsymbol{\Psi}_i}{\boldsymbol{\Phi}_i^T \boldsymbol{\Psi}_i}$$

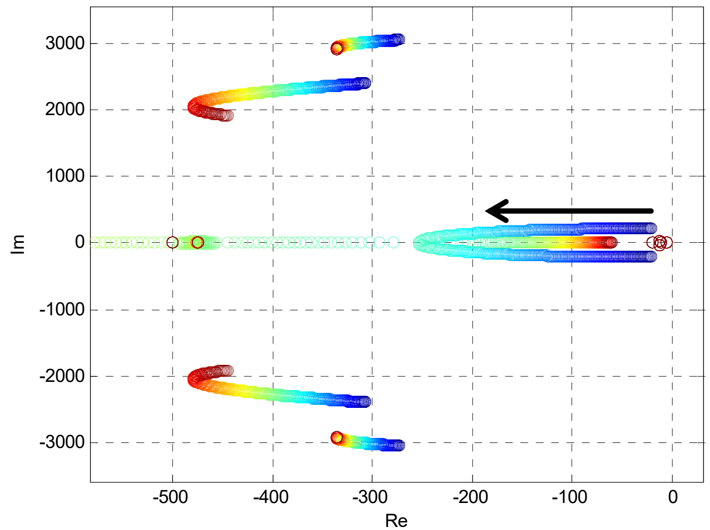
- Considering *k* changeable parameter, the eigenvalue parametric sensitivities defines an n by k matrix with complex elements
  - The real part of the elements express how much an eigenvalue moves along the real axis for a change of the corresponding parameter
  - The imaginary part of the elements express how much the eigenvalue moves in the imaginary axis for a change of the corresponding parameter
- Suitable tool for identifying what parameters can be most effectively used to improve the dynamic response of a system

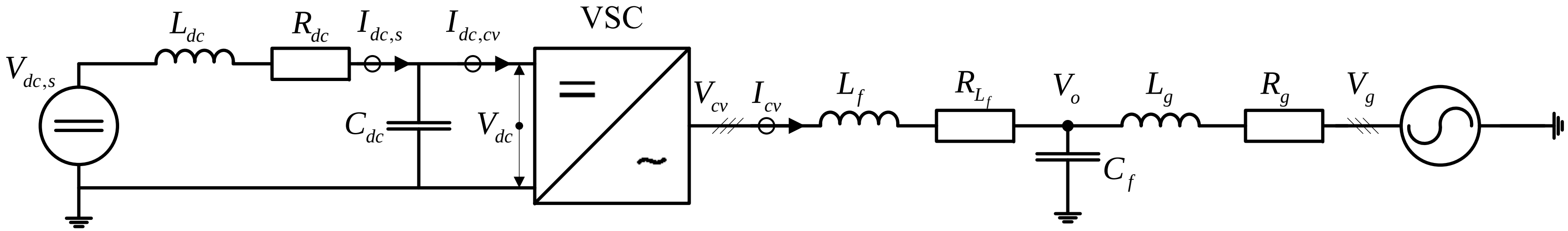
Example of parametric sensitivity analysis

- The same example as in previous slides
- Parametric sensitivity analysis for the identified poorly damped mode
  - Gain of damping term is the most suitable way to damp the oscillation
  - The sensitivity to the damping term gives the tangent to the pole trajectory









Applications of eigenvalue analysis for power systems

- Suitable for analyzing small-signal stability and dynamics in large and complex systems
  - Impacts of parameter variations on dynamic characteristics can be easily studied
  - Oscillation modes and stability problems that are difficult to interpret and prevent by experience-based and manual approaches can be identified and analyzed
  - Tools for linear system analysis can be applied to prevent instability and improve dynamic performance
    - Participation factor analysis can be used to identify the states involved in particular oscillation modes
      - Can be used to design damping controllers
    - Parameter sensitivity analysis can be used to identify most suitable controller gains to modify for improving dynamic response
      - Optimization methods can be applied to ensure small-signal stability and improve the dynamic response of a system
- Long application history for rotor angle stability analysis in large power systems
  - Not many systematic application studies for power electronic systems