Find ODEs for and in the case of foetal development (note , so !)

Predefined reserve dynamics state that change in reserve density is linear in :

Through the chain rule for differentiation we have the relationship between and :

We obtain by filling in our previous expression for :

For growth rate , we have

Gathering terms with

Multiplying all with and isolating

Inserting this in our expression for

Combining these in one fraction

Rewriting in terms of (which is properly defined even when ):

For the increase in structural length we have

Rewriting in terms of (which is properly defined even when ):

Maturity dynamics:

Challenge: we know . What is ? The initial state is

## Von Bertalanffy growth

This occurs at constant reserve density

This has solution

With being the structural length at which growth ceases:

with heating length

And being the von Bertalanffy growth rate

In this case, equals

## V1 morph growth

Reserve turnover and heating are now proportional to structural length

For constant reserve density , this has solution

With

In this case, equals

Then, maturity dynamics can be written as

With prefactor

This has solution

With integration constant

## Foetal growth

For the foetus in the uterus, the reserve density is assumed to be infinite, i.e., . Then, we obtain for the specific growth rate of structural volume (note for the foetus )

The increase in structural length then equals

This implies structural length increases linearly, whereas structural volume increases cubically:

Checked against p 64 (book)

Under these conditions, the catabolic power equals

Expressed in time instead of :

The total energy spent on production of the foetus, , is the combination of the total energy directed to the foetus during development, , and the reserve of the foetus at time of birth, .

The time integral of is easily obtained as is a cubic function of time:

Thus

Thus, we obtain

Rearranging

The [“Comments on Dynamic Energy Budget theory” document](https://www.bio.vu.nl/thb/research/bib/Kooy2010_c.pdf) lists in section 2.6.2 (“Foetal costs”, p 42) the expression

Inserting and

Inserting ,

Inserting

This is identical to our earlier expression for if .

## Ultimate reproduction rate

To compute the ultimate reproduction rate, we first compute the ultimate catabolic flux

The ultimate reproduction rate then becomes