

Algorithms

Notes for Professionals

Chapter 6: Check if a tree is BST or not

Section 6.1: Algorithm to check if a given binary tree is BST or not

A binary tree is BST if it satisfies any one of the following condition:

1. It is empty
2. It has no subtrees
3. For every node x in the tree all the keys (if any) in the left sub tree must be less than x and all the keys (if any) in the right sub tree must be greater than x .

So a straightforward recursive algorithm would be:

```
isBST(root):
    if root == NULL:
        return true
    // Check values in left subtree
    if root->left != NULL:
        min_key_in_left = find_min_key(root->left)
        if max_key_in_left > root->key:
            return false
    // Check values in right subtree
    if root->right != NULL:
        min_key_in_right = find_min_key(root->right)
        if min_key_in_right < root->key:
            return false
    return isBST(root->left) && isBST(root->right)
```

The above recursive algorithm is correct but inefficient, because it traverses each node multiple times.

Another approach to minimize the multiple visits of each node is to remember the keys in the subtree we are visiting. Let the minimum possible value of any key in the subtree be K_{min} . When we start from the root of the tree, the range of values in the tree is (K_{min}, K_{max}) . We will use this idea to develop a more efficient algorithm.

```
isBST(root, min, max):
    if root == NULL:
        return true
    // Is the current node key out of range?
    if root->key < min || root->key > max:
        return false
    // Check if left and right subtree is BST
    return isBST(root->left, min, root->key) && isBST(root->right, root->key, max)
```

It will be initially called as:

```
isBST(my_tree, root, KEY_MIN, KEY_MAX)
```

Another approach will be to do inorder traversal of the Binary tree. If the inorder traversal produces a sorted sequence of keys then the given tree is a BST. To check if the inorder sequence is sorted remember the value of

Chapter 15: Applications of Dynamic Programming

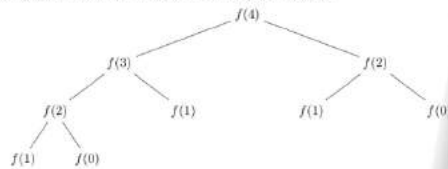
The basic idea behind dynamic programming is breaking a complex problem down to several small and simple problems that are repeated. If you can identify a simple subproblem that is repeatedly calculated, odds are there is a dynamic programming approach to the problem.

As this topic is titled Applications of Dynamic Programming, it will focus more on applications rather than the process of creating dynamic programming algorithms.

Section 15.1: Fibonacci Numbers

Fibonacci Numbers are a prime subject for dynamic programming as the traditional recursive approach makes a lot of repeated calculations. In these examples I will be using the base case of $F(0) = F(1) = 1$.

Here is an example recursive tree for $Fibonacci(4)$, note the repeated computations:



Non-Dynamic Programming $O(2^n)$ Runtime Complexity, $O(n)$ Stack complexity

```
def fibonacci(n):
    if n <= 1:
        return n
    return fibonacci(n-1) + fibonacci(n-2)
```

This is the most intuitive way to write the problem. At most the stack space will be $O(n)$ as you descend the first recursive branch making calls to $fibonacci(n-1)$ until you hit the base case $n = 2$.

The $O(2^n)$ runtime complexity proof that can be seen here: [Computational complexity of Fibonacci Sequence](#). The main point to note is that the runtime is exponential, which means the runtime for this will double for every subsequent term, $fibonacci(15)$ will take twice as long as $fibonacci(14)$.

Memorized $O(n)$ Runtime Complexity, $O(n)$ Space complexity, $O(n)$ Stack complexity

```
memo = {}
def fibonacci(n):
    if n <= 1:
        return n
    if n in memo:
        return memo[n]
    memo[n] = fibonacci(n-1) + fibonacci(n-2)
    return memo[n]
```

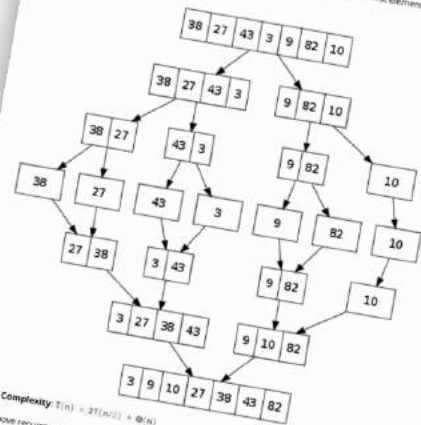
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Chapter 30: Merge Sort

Section 30.1: Merge Sort Basics

Merge Sort is a divide-and-conquer algorithm. It divides the input list of length n in half successively until there are n lists of size 1. Then, pairs of lists are merged together with the smaller first element among the pair of lists being added in each step. Through successive merging and through comparison of first elements, the sorted list is built.

An example:



Time Complexity: $T(n) = 2T(n/2) + O(n)$

The above recurrence can be solved either using Recurrence Tree method or Master method. It falls in case II of Master Method and solution of the recurrence is $O(n \log n)$. Time complexity of Merge Sort is $O(n \log n)$ in all 3 cases (best, average and bad) as merge sort always divides the array in two halves and take linear time to merge two halves.

Auxiliary Space: $O(n)$

Algorithmic Paradigm: Divide and Conquer

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About

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Chapter 1: Getting started with algorithms

Section 1.1: A sample algorithmic problem

An algorithmic problem is specified by describing the complete set of *instances* it must work on and of its output after running on one of these instances. This distinction, between a problem and an instance of a problem, is fundamental. The algorithmic *problem* known as *sorting* is defined as follows: [Skiena:2008:ADM:1410219]

- Problem: Sorting
- Input: A sequence of n keys, a_1, a_2, \dots, a_n .
- Output: The reordering of the input sequence such that $a'_1 \leq a'_2 \leq \dots \leq a'_{n-1} \leq a'_n$

An *instance* of sorting might be an array of strings, such as { Haskell, Emacs } or a sequence of numbers such as { 154, 245, 1337 }.

Section 1.2: Getting Started with Simple Fizz Buzz Algorithm in Swift

For those of you that are new to programming in Swift and those of you coming from different programming bases, such as Python or Java, this article should be quite helpful. In this post, we will discuss a simple solution for implementing swift algorithms.

Fizz Buzz

You may have seen Fizz Buzz written as Fizz Buzz, FizzBuzz, or Fizz-Buzz; they're all referring to the same thing. That "thing" is the main topic of discussion today. First, what is FizzBuzz?

This is a common question that comes up in job interviews.

Imagine a series of a number from 1 to 10.

```
1 2 3 4 5 6 7 8 9 10
```

Fizz and Buzz refer to any number that's a multiple of 3 and 5 respectively. In other words, if a number is divisible by 3, it is substituted with fizz; if a number is divisible by 5, it is substituted with buzz. If a number is simultaneously a multiple of 3 AND 5, the number is replaced with "fizz buzz." In essence, it emulates the famous children game "fizz buzz".

To work on this problem, open up Xcode to create a new playground and initialize an array like below:

```
// for example
let number = [1,2,3,4,5]
// here 3 is fizz and 5 is buzz
```

To find all the fizz and buzz, we must iterate through the array and check which numbers are fizz and which are buzz. To do this, create a for loop to iterate through the array we have initialised:

```
for num in number {
    // Body and calculation goes here
}
```

After this, we can simply use the "if else" condition and module operator in swift ie - % to locate the fizz and buzz


```

for num in number {
    if num % 3 == 0 {
        print("\(num) fizz")
    } else {
        print(num)
    }
}

```

Great! You can go to the debug console in Xcode playground to see the output. You will find that the "fizzes" have been sorted out in your array.

For the Buzz part, we will use the same technique. Let's give it a try before scrolling through the article — you can check your results against this article once you've finished doing this.

```

for num in number {
    if num % 3 == 0 {
        print("\(num) fizz")
    } else if num % 5 == 0 {
        print("\(num) buzz")
    } else {
        print(num)
    }
}

```

Check the output!

It's rather straight forward — you divided the number by 3, fizz and divided the number by 5, buzz. Now, increase the numbers in the array

```

let number = [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15]

```

We increased the range of numbers from 1-10 to 1-15 in order to demonstrate the concept of a "fizz buzz." Since 15 is a multiple of both 3 and 5, the number should be replaced with "fizz buzz." Try for yourself and check the answer!

Here is the solution:

```

for num in number {
    if num % 3 == 0 && num % 5 == 0 {
        print("\(num) fizz buzz")
    } else if num % 3 == 0 {
        print("\(num) fizz")
    } else if num % 5 == 0 {
        print("\(num) buzz")
    } else {
        print(num)
    }
}

```

Wait...it's not over though! The whole purpose of the algorithm is to customize the runtime correctly. Imagine if the range increases from 1-15 to 1-100. The compiler will check each number to determine whether it is divisible by 3 or 5. It would then run through the numbers again to check if the numbers are divisible by 3 and 5. The code would essentially have to run through each number in the array twice — it would have to run the numbers by 3 first and then run it by 5. To speed up the process, we can simply tell our code to divide the numbers by 15 directly.

Here is the final code:

```

for num in number {

```



```
if num % 15 == 0 {  
    print("\(num) fizz buzz")  
} else if num % 3 == 0 {  
    print("\(num) fizz")  
} else if num % 5 == 0 {  
    print("\(num) buzz")  
} else {  
    print(num)  
}  
}
```

As Simple as that, you can use any language of your choice and get started

Enjoy Coding

Chapter 2: Algorithm Complexity

Section 2.1: Big-Theta notation

Unlike Big-O notation, which represents only upper bound of the running time for some algorithm, Big-Theta is a tight bound; both upper and lower bound. Tight bound is more precise, but also more difficult to compute.

The Big-Theta notation is symmetric: $f(x) = \Theta(g(x)) \Leftrightarrow g(x) = \Theta(f(x))$

An intuitive way to grasp it is that $f(x) = \Theta(g(x))$ means that the graphs of $f(x)$ and $g(x)$ grow in the same rate, or that the graphs 'behave' similarly for big enough values of x .

The full mathematical expression of the Big-Theta notation is as follows:

$\Theta(f(x)) = \{g: N_0 \rightarrow R \text{ and } c_1, c_2, n_0 > 0, \text{ where } c_1 < \text{abs}(g(n) / f(n)), \text{ for every } n > n_0 \text{ and abs is the absolute value} \}$

An example

If the algorithm for the input n takes $42n^2 + 25n + 4$ operations to finish, we say that is $O(n^2)$, but is also $O(n^3)$ and $O(n^{100})$. However, it is $\Theta(n^2)$ and it is not $\Theta(n^3)$, $\Theta(n^4)$ etc. Algorithm that is $\Theta(f(n))$ is also $O(f(n))$, but not vice versa!

Formal mathematical definition

$\Theta(g(x))$ is a set of functions.

$\Theta(g(x)) = \{f(x) \text{ such that there exist positive constants } c_1, c_2, N \text{ such that } 0 \leq c_1 \cdot g(x) \leq f(x) \leq c_2 \cdot g(x) \text{ for all } x > N\}$

Because $\Theta(g(x))$ is a set, we could write $f(x) \in \Theta(g(x))$ to indicate that $f(x)$ is a member of $\Theta(g(x))$. Instead, we will usually write $f(x) = \Theta(g(x))$ to express the same notion - that's the common way.

Whenever $\Theta(g(x))$ appears in a formula, we interpret it as standing for some anonymous function that we do not care to name. For example the equation $T(n) = T(n/2) + \Theta(n)$, means $T(n) = T(n/2) + f(n)$ where $f(n)$ is a function in the set $\Theta(n)$.

Let f and g be two functions defined on some subset of the real numbers. We write $f(x) = \Theta(g(x))$ as $x \rightarrow \text{infinity}$ if and only if there are positive constants K and L and a real number x_0 such that holds:

$K|g(x)| \leq f(x) \leq L|g(x)|$ for all $x \geq x_0$.

The definition is equal to:

$f(x) = O(g(x))$ and $f(x) = \Omega(g(x))$

A method that uses limits

if $\lim_{x \rightarrow \text{infinity}} f(x)/g(x) = c \in (0, \infty)$ i.e. the limit exists and it's positive, then $f(x) = \Theta(g(x))$

Common Complexity Classes

Name	Notation	n = 10	n = 100
Constant	$\Theta(1)$	1	1
Logarithmic	$\Theta(\log(n))$	3	7
Linear	$\Theta(n)$	10	100