Appendix To:

Visualization, Identification, and Estimation in the Linear Panel Event-Study Design

Simon Freyaldenhoven

Federal Reserve Bank of Philadelphia

Christian Hansen

University of Chicago

Jorge Pérez Pérez

Banco de México

Jesse M. Shapiro

*Brown University and NBER**

This version: May 7, 2021

## List of Tables

List of Figures

*Emails: simon.freyaldenhoven@phil.frb.org, chansen1@chicagobooth.edu, jorgepp@banxico.org.mx, jesse_shapiro_1@brown.edu.

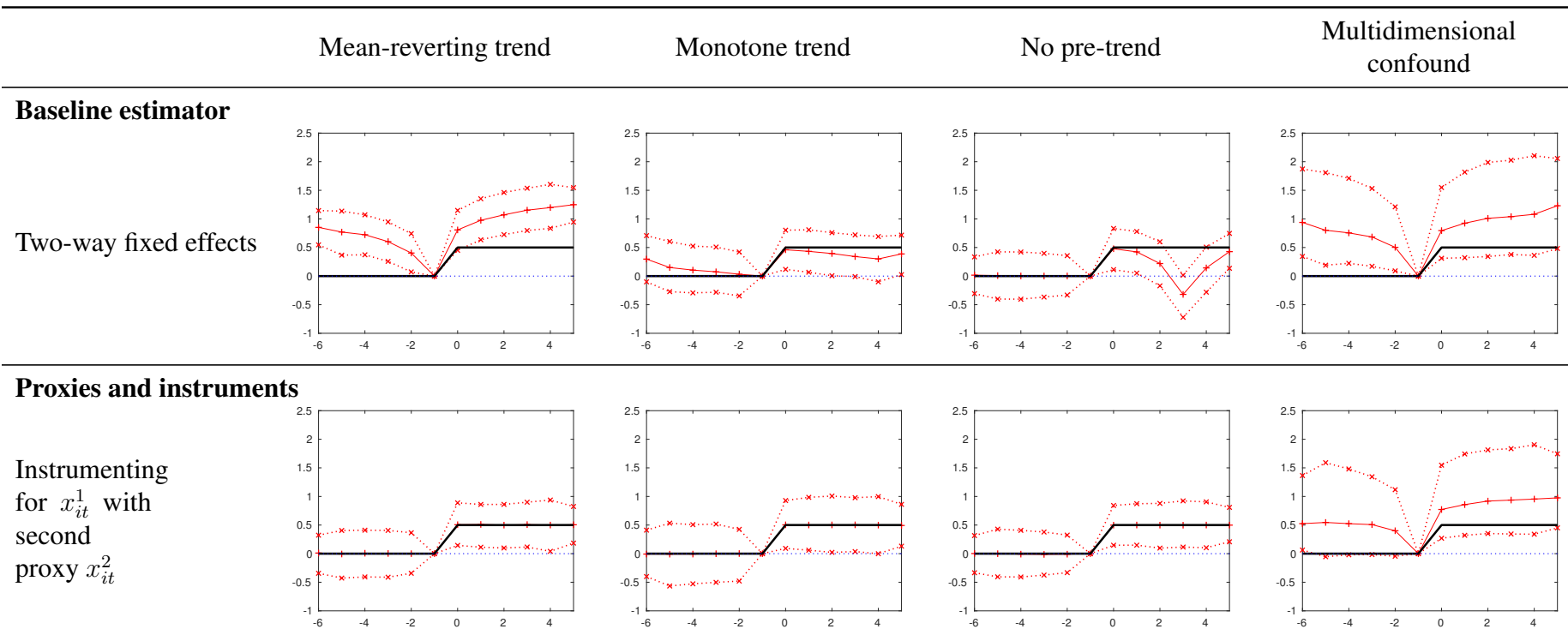|	Mean-reverting trend	Monotone trend	No pre-trend	Multidimensional confound
Observed				
Outcome	$y_{it} = 0.5\left(\sqrt{0.5}\alpha_i + \sqrt{0.5}\gamma_t\right) + 0.5z_{it} + \sqrt{0.5}C_{it} + 0.5\varepsilon_{it}$			
Policy	$z_{i1} = 0;\ \Delta z_{it} = \mathbf{1}\left(\left\{z_{i,t-1} = 0 \text{ and } C_{i,t+P} + 0.5\zeta^z_{i,t+P} < -2\right\}\right)$ for $t \geq 2$, and			
	$P = -1$	$P = 6$	$P = 3$	$P = -1$
Unobserved				
Fixed effects	$\gamma_t = 0.5\gamma_{t-1} + \sqrt{0.75}\zeta^{\gamma}_t$			
Confound C_{it}	$C_{it} = \sqrt{1-\sigma_\eta}\left[\sqrt{1-\sigma_a-\sigma_d}\lambda_i' F_t + \sqrt{\sigma_a}a_i + \sqrt{\sigma_d}d_t\right] + \sqrt{\sigma_\eta}\eta_{it}$, with			
	$d_t = 0.5d_{t-1} + \sqrt{0.75}\zeta^d_t$;			
	$\sigma_a = \sigma_d = \sigma_\eta = 0.5$			$\sigma_\eta = 0, \sigma_a = \sigma_d = 0.1,$
	$\eta_{it} = \rho_\eta \eta_{i,t-1} + \sqrt{1-\rho_\eta^2}\zeta^\eta_{it}$; and			$\lambda_{ji} = \frac{1}{\sqrt{2}}\left(1 + \zeta^\lambda_{ji}\right),\ F_{jt} = \rho_{F_j}F_{j,t-1} + \sqrt{1-\rho_{F_j}^2}\zeta^F_{jt}$
	$\rho_\eta = 0.6$	$\rho_\eta = 0.95$	$\rho_\eta = 0.4$	$\rho_{F_1} = 0.9, \rho_{F_2} = 0.4$
If available				
Proxies	$x_{it} = 0.5\left(\sqrt{0.5}h_i + \sqrt{0.5}g_t\right) + \sqrt{0.5}\left[\sqrt{1-\theta}C_{it} + \sqrt{\theta}\lambda_{1i}F_{1t}\right] + 0.5u_{it}$, with			
	$g_t = 0.5g_{t-1} + \sqrt{0.75}\zeta^g_t$; $u_{it} = 0.5u_{i,t-1} + \sqrt{0.75}\zeta^u_{it}$			
	$\theta = 0$			$\theta = 1$

Appendix Table 1: Detailed description of the data-generating processes underlying the simulations presented in Section 4. We generate data for the time periods $t = -4, -3, \ldots, T + 5$ to avoid missing observations for leads and lags of the policy and to allow for a policy adoption rule that is consistent with a forward/backward-looking decision-maker on either end of the sample. All random variables not otherwise defined, including initial conditions for autoregressive processes, are distributed as i.i.d. standard normal. In the "Monotone trend" DGP, we set $z_{it} = z_{i,t-1}$ for $t \geq T$. In the "No pre-trend" DGP, we set $z_{it} = z_{i,t-1}$ for $t \geq T + 3$.

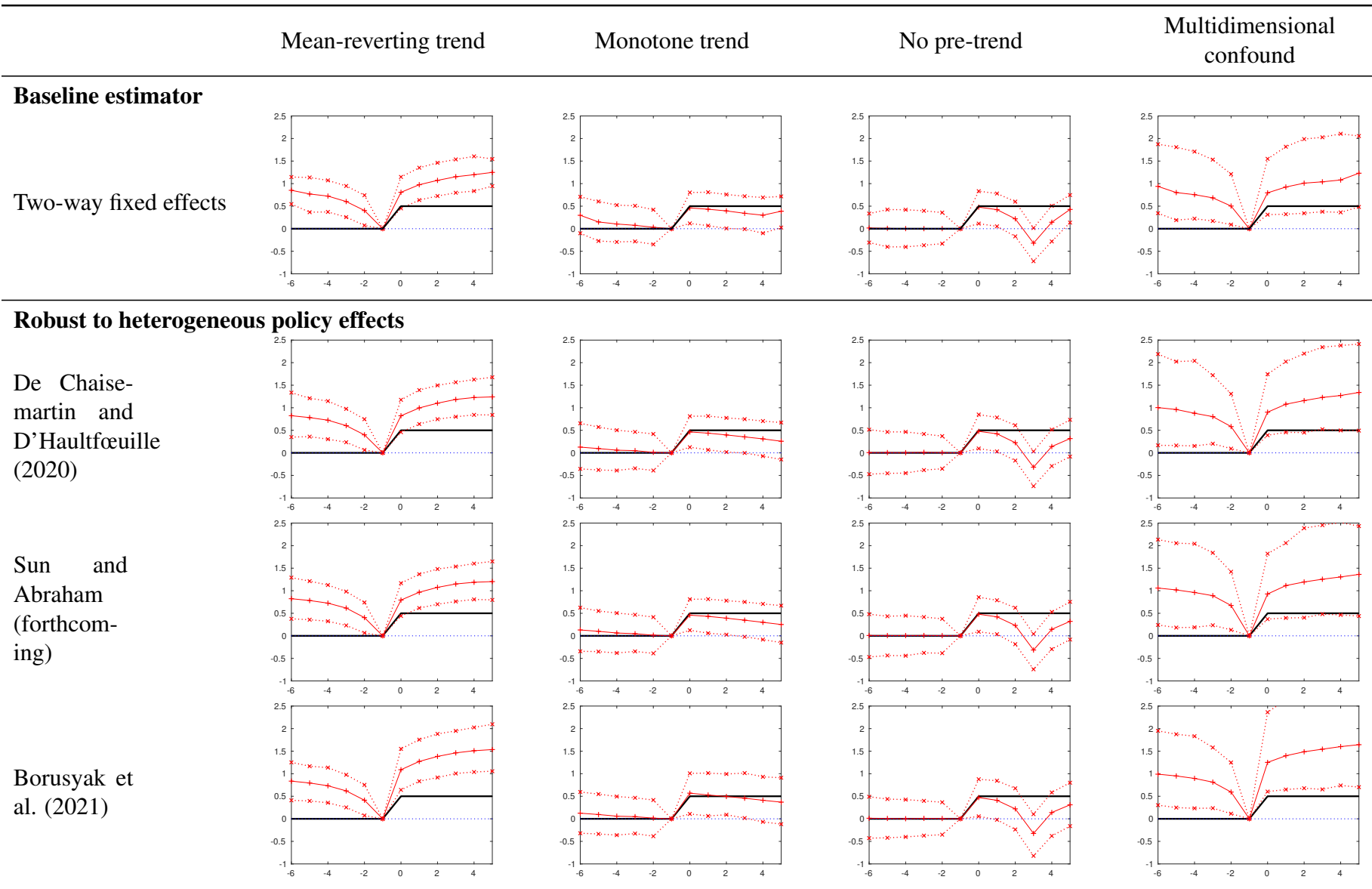|	Mean-reverting trend	Monotone trend	No pre-trend	Multidimensional confound
Baseline estimator				
Two-way fixed effects				
Conditional on observables				
Controlling for x_{it}				
Unit-specific linear trend				
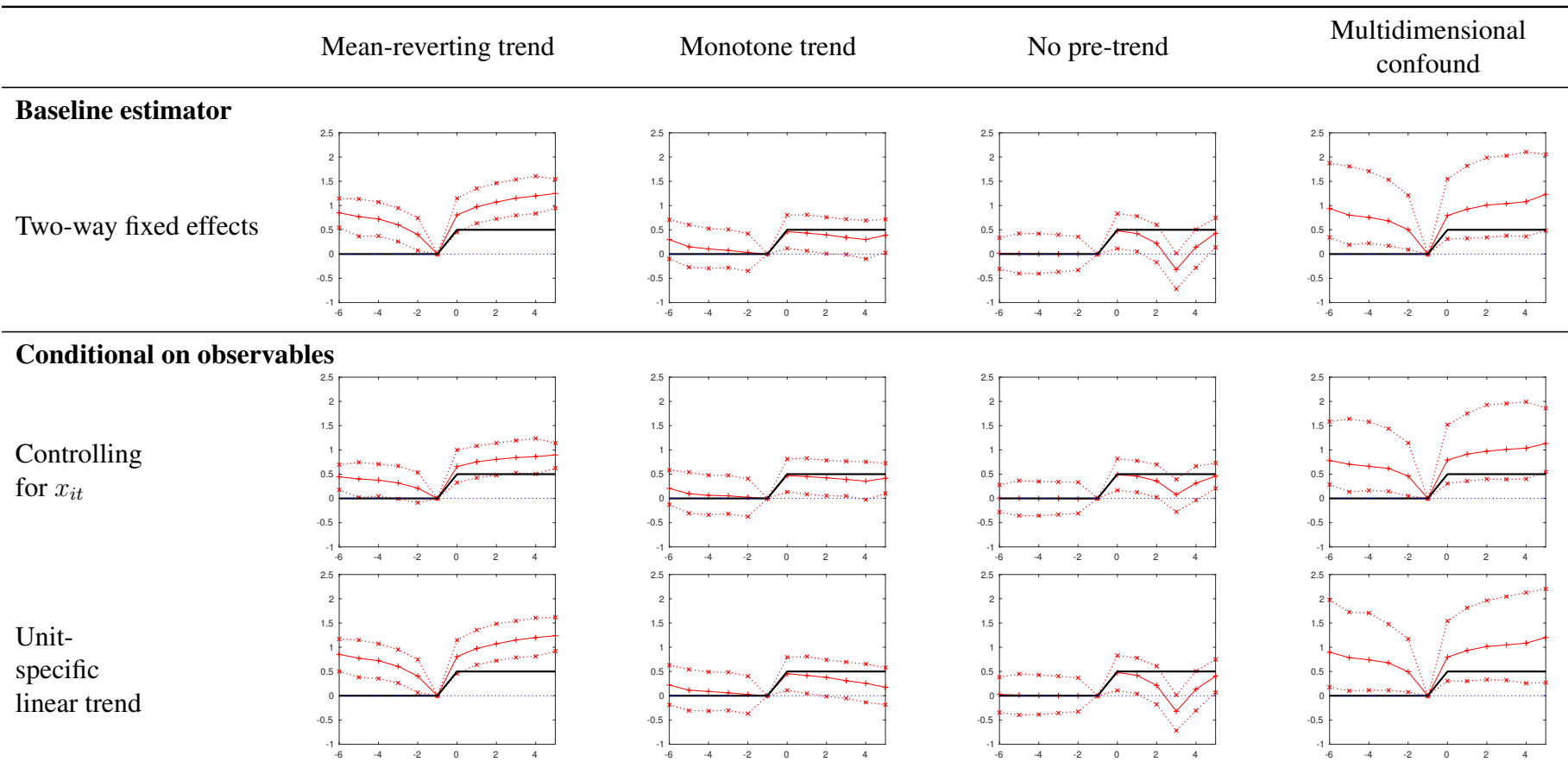
Appendix Figure 1: Additional "conditional on observables" estimators across DGPs. The true treatment effect is depicted in solid black. For each value of k indicated on the x-axis, red series correspond to the 2.5th (dotted, marked by x's), 50th (solid, marked by +'s), and 97.5th (dotted, marked by x's) percentiles across 1000 simulations for each $\hat{\delta}_k$. The first row ("Two-way fixed effects") implements a two-way fixed effects estimator. The second row ("Controlling for x_{it}") includes the proxy x_{it} directly in the controls q_{it}. The third row ("Unit-specific linear trend") allows for unit-specific linear time trends.
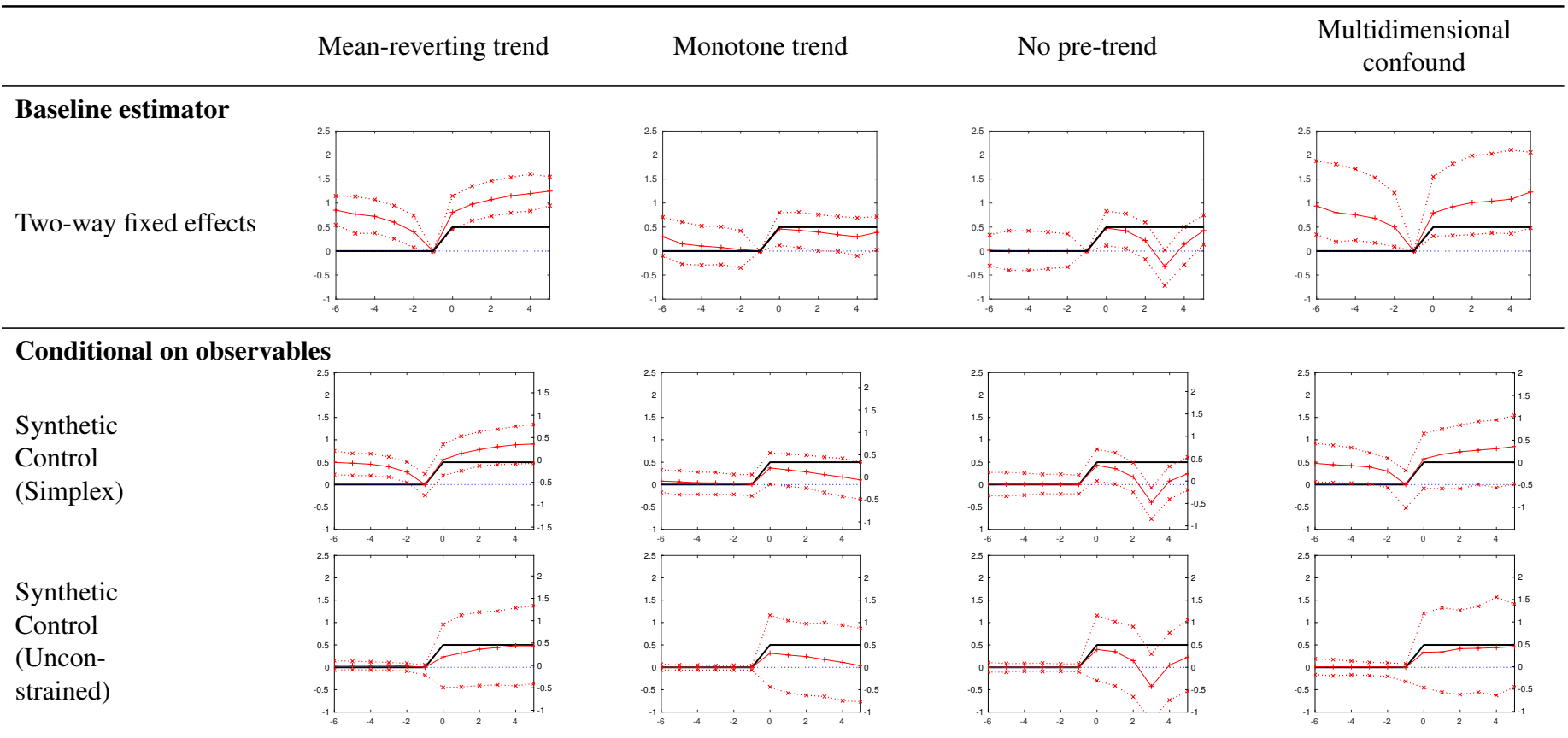
Appendix Figure 2: Synthetic control estimators across DGPs. The true treatment effect is depicted in solid black. For each value of k indicated on the x-axis, red series correspond to the 2.5th (dotted, marked by x's), 50th (solid, marked by +'s), and 97.5th (dotted, marked by x's) percentiles across 1000 simulations for each $\hat{\delta}_k$. The right axis depicts the estimated treatment effect relative to the synthetic control. On the left axis, estimates are normalized such that the median treatment effect is zero the period before the event. The donor pool for unit i in cohort $T^*(i)$ consists of all units j with $T^*(j) > T^*(i) + 5$. The first row ("Two-way fixed effects") implements a two-way fixed effects estimator. The second row ("Synthetic control (Simplex)") estimates $\hat{\delta}_k$ using the `synth` package (Abadie et al. 2020), restricting the weights on the donor units to the simplex. The third row ("Synthetic control (Unconstrained)") estimates $\hat{\delta}_k$ using the `augsynth` package (Ben-Michael et al. 2020), allowing donor weights outside the simplex but imposing an ℓ_2-penalty of 0.001.

	Mean-reverting trend	Monotone trend	No pre-trend	Multidimensional confound

Baseline estimator

Two-way fixed effects

Proxies and instruments

Instrumenting for x_{it}^1 with second proxy x_{it}^2

Appendix Figure 3: Additional "proxies and instruments" estimator across DGPs. The true treatment effect is depicted in solid black. For each value of k indicated on the x-axis, red series correspond to the 2.5th (dotted, marked by x's), 50th (solid, marked by +'s), and 97.5th (dotted, marked by x's) percentiles across 1000 simulations for each $\hat{\delta}_k$. The first row ("Two-way fixed effects") implements a two-way fixed effects estimator. The second row ("Instrumenting for x_{it}^1 with second proxy x_{it}^2") considers a measurement-error correction, assuming the availability of two proxies for the confound, by instrumenting for one proxy, x_{it}^1, with the other, x_{it}^2.

	Mean-reverting trend	Monotone trend	No pre-trend	Multidimensional confound
Baseline estimator				
Two-way fixed effects				
Robust to heterogeneous policy effects				
De Chaise-martin and D'Haultfœuille (2020)				
Sun and Abraham (forthcoming)				
Borusyak et al. (2021)				

Appendix Figure 4: Additional estimators designed to allow for heterogeneous effects of the policy. The true treatment effect is depicted in solid black. For each value of k indicated on the x-axis, red series correspond to the 2.5th (dotted, marked by x's), 50th (solid, marked by +'s), and 97.5th (dotted, marked by x's) percentiles across 1000 simulations for each $\hat{\delta}_k$. The first row ("Two-way fixed effects") implements a two-way fixed effects estimator. The second row ("De Chaisemartin and D'Haultfœuille (2020)") uses the `did_multiplegt` package (de Chaisemartin et al. 2019). The third row ("Sun and Abraham (forthcoming)") uses the interaction-weighted estimator described in Sun and Abraham (forthcoming) Section 4.1. The fourth row ("Borusyak et al. (2021)") uses the `did_imputation` package (Borusyak et al. 2021).

References

Alberto Abadie, Alexis Diamond, and Jens Hainmueller. Synth: Stata module to implement synthetic control methods for comparative case studies, 2020. URL `https://ideas.repec.org/c/boc/bocode/s457334.html`.

Eli Ben-Michael, Avi Feller, and Jesse Rothstein. The augmented synthetic control method. 2020. URL `https://github.com/ebenmichael/augsynth`.

Kirill Borusyak, Xavier Jaravel, and Jann Spiess. DID_IMPUTATION package for Revisiting Event Study Designs: Robust and Efficient Estimation, 2021. URL `https://www.dropbox.com/sh/r7wy4wjpwxy5uyy/AADxVrOHP653damnqpy_OhN1a?dl=0`.

Clément de Chaisemartin, Xavier D'Haultfoeuille, and Yannick Guyonvarch. DID_MULTIPLEGT: Stata module to estimate sharp Difference-in-Difference designs with multiple groups and periods, 2019. URL `https://ideas.repec.org/c/boc/bocode/s458643.html`.