Visualization, Identification, and Estimation in the Linear Panel Event-Study Design

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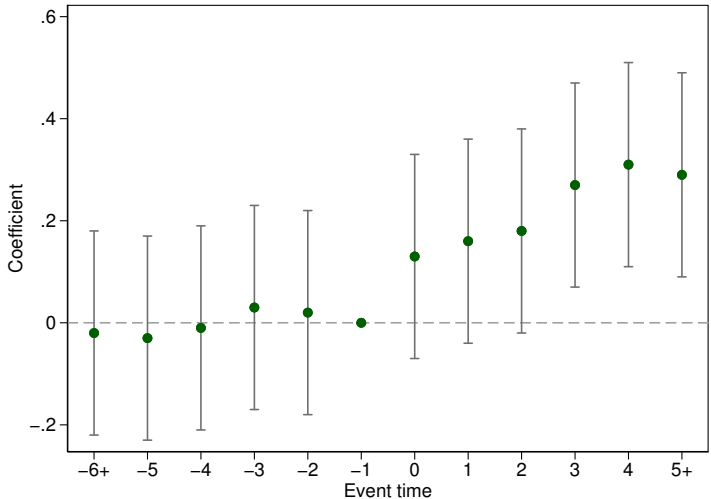
Estimation and Event-Study Plots

Linear panel model

$$y_{it} = \alpha_i + \gamma_t + q_{it}'\psi + \sum_{m=-G}^{M} \beta_m z_{i,t-m} + C_{it} + \varepsilon_{it} \qquad \text{(linear panel model)}$$

- Unit fixed effects α_i and time fixed effects γ_t
- Observed controls q_{it}
- Unobserved confound C_{it} potentially related to policy z_{it}
- Unobserved error ε_{it} unrelated to policy z_{it}
- Parameters of interest $\{\beta_m\}_{m=-G}^{M}$
  - No *ceteris paribus* effect of policy more than G periods in the past or M periods in the future

Typical event-study plot



Building the plot

$$y_{it} = \alpha_i + \gamma_t + q'_{it}\psi + \sum_{m=-G}^{M} \beta_m z_{i,t-m} + C_{it} + \varepsilon_{it} \qquad \text{(linear panel model)}$$

For the event-study plot we want to:

- Show cumulative effects of the policy $\rightarrow$ replace z_{it} with Δz_{it}
- Show pre-G and post-M dynamics $\rightarrow$ add L_G extra leads and L_M extra lags

Estimating equation

$$y_{it} = \sum_{k=-G-L_G}^{M+L_M-1} \delta_k \Delta z_{i,t-k} + \delta_{M+L_M} z_{i,t-M-L_M} + \delta_{-G-L_G-1}(-z_{i,t+G+L_G}) + \alpha_i + \gamma_t + q_{it}'\psi + C_{it} + \varepsilon_{it}$$

(estimating equation)

- Will refer to index k as *event time*
- Will refer to vector δ as *event time path* of outcome

Interpretation under staggered adoption

$$\ldots \sum_{k=-G-L_G}^{M+L_M-1} \delta_k \Delta z_{i,t-k} + \delta_{M+L_M} z_{i,t-M-L_M} + \delta_{-G-L_G-1}(-z_{i,t+G+L_G}) \ldots$$

(key part of estimating equation)

Say that for each unit i, z_{it} starts at 0 and switches to 1 at time $t^*(i)$. Then:

$$\begin{aligned} \Delta z_{i,t-k} &= \mathbf{1}\{t^*(i) = t-k\} \\ z_{i,t-M-L_M} &= \mathbf{1}\{t^*(i) \leq t-M-L_M\} \\ 1 - z_{i,t+G+L_G} &= \mathbf{1}\{t^*(i) > t+G+L_G\} \end{aligned}$$

Interpretation as cumulative effects of policy

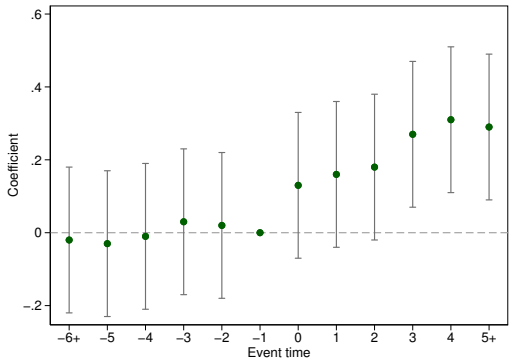
$$\ldots \sum_{k=-G-L_G}^{M+L_M-1} \delta_k \Delta z_{i,t-k} + \delta_{M+L_M} z_{i,t-M-L_M} + \delta_{-G-L_G-1}(-z_{i,t+G+L_G}) \ldots$$

(key part of estimating equation)

Under the linear panel model, and for general z_{it},

$$\delta_k = \begin{cases} 0 \quad \text{for} \quad k < -G \\ \sum_{m=-G}^{k} \beta_m \quad \text{for} -G \leq k \leq M \\ \sum_{m=-G}^{M} \beta_m \quad \text{for} \quad k > M. \end{cases}$$

Definition of plot



Points on plot correspond to $\{(k, \hat{\delta}_k)\}_{k=-G-L_G-1}^{k=M+L_M}$.