Appendix To:
Visualization, Identification, and Estimation in the Linear Panel Event-Study Design

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List of Tables

List of Figures

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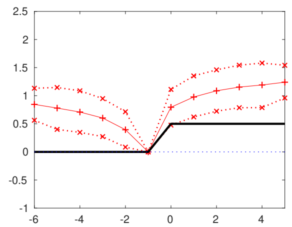
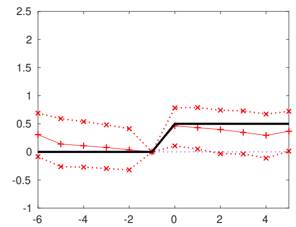
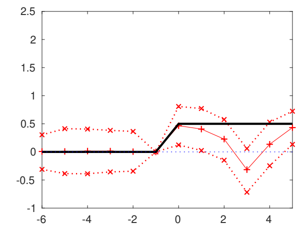
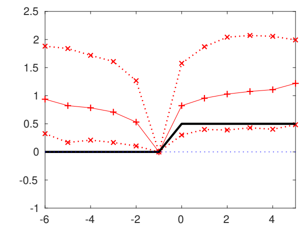
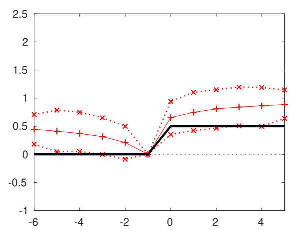
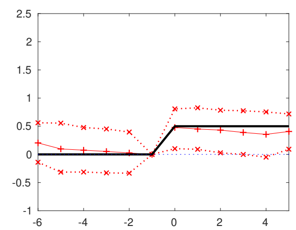
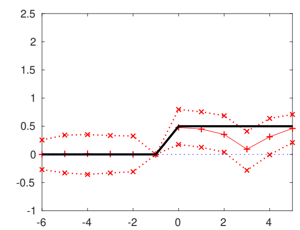
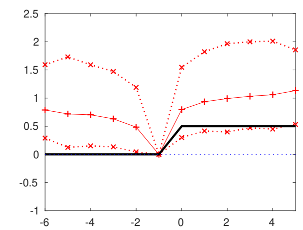
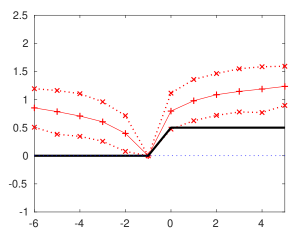
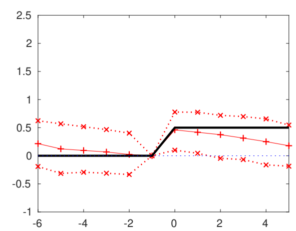
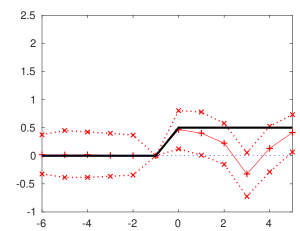
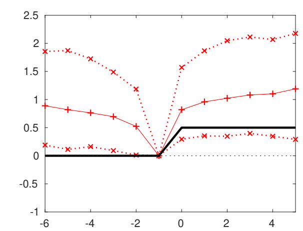
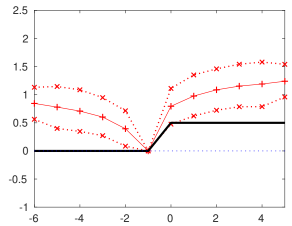
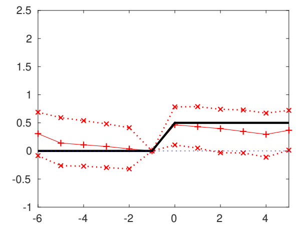
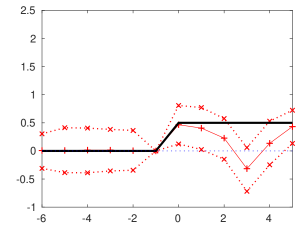
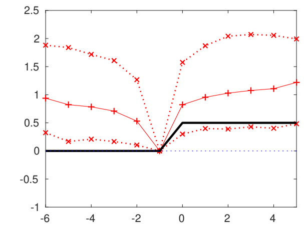
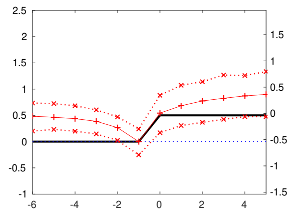
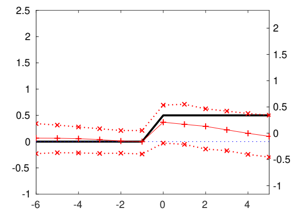
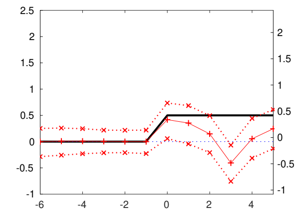
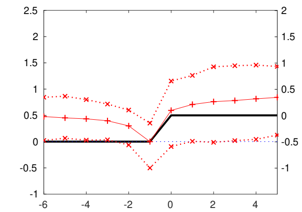
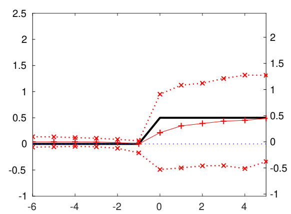
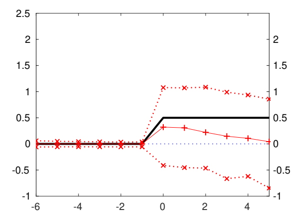
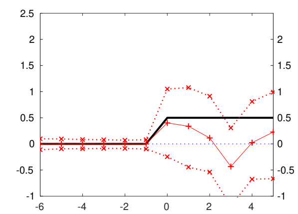
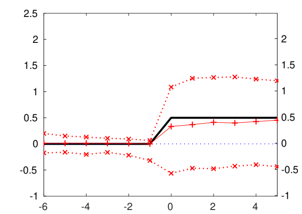
<table>
<tr><th></th><th>Mean-reverting trend</th><th>Monotone trend</th><th>No pre-trend</th><th>Multidimensional confound</th></tr>
<tr><td colspan="5">Observed</td></tr>
<tr><td>Outcome</td><td colspan="4">$y_{it} = 0.5\left(\sqrt{0.5}\alpha_i + \sqrt{0.5}\gamma_t\right) + 0.5 z_{it} + \sqrt{0.5}C_{it} + 0.5\varepsilon_{it}$</td></tr>
<tr><td rowspan="2">Policy</td><td colspan="4">$z_{i1} = 0$; $\Delta z_{it} = \mathbf{1}\left(\left\{z_{i,t-1} = 0 \text{ and } C_{i,t+P} + 0.5\zeta^z_{i,t+P} < -2\right\}\right)$ for $t \geq 2$, and</td></tr>
<tr><td>$P = -1$</td><td>$P = 6$</td><td>$P = 3$</td><td>$P = -1$</td></tr>
<tr><td colspan="5">Unobserved</td></tr>
<tr><td>Fixed effects</td><td colspan="4">$\gamma_t = 0.5\gamma_{t-1} + \sqrt{0.75}\zeta^\gamma_t$</td></tr>
<tr><td rowspan="4">Confound C_{it}</td><td colspan="4">$C_{it} = \sqrt{1-\sigma^2_\eta}\left[\sqrt{1-\sigma^2_a-\sigma^2_d}\lambda'_i F_t + \sqrt{\sigma^2_a}a_i + \sqrt{\sigma^2_d}d_t\right] + \sqrt{\sigma^2_\eta}\eta_{it}$, with
$d_t = 0.5 d_{t-1} + \sqrt{0.75}\zeta^d_t$;</td></tr>
<tr><td colspan="3">$\sigma^2_a = \sigma^2_d = \sigma^2_\eta = 0.5$, $F_t = 0$</td><td>$\sigma^2_\eta = 0, \sigma^2_a = \sigma^2_d = 0.1, \dim(F_t) = 2$</td></tr>
<tr><td colspan="3">$\eta_{it} = \rho_\eta \eta_{i,t-1} + \sqrt{1-\rho^2_\eta}\zeta^\eta_{it}$; and</td><td>$\lambda_{ji} = \frac{1}{\sqrt{2}}\left(1+\zeta^\lambda_{ji}\right)$, $F_{jt} = \rho_{F_j}F_{j,t-1} + \sqrt{1-\rho^2_{F_j}}\zeta^F_{jt}$</td></tr>
<tr><td>$\rho_\eta = 0.6$</td><td>$\rho_\eta = 0.95$</td><td>$\rho_\eta = 0.4$</td><td>$\rho_{F_1} = 0.9, \rho_{F_2} = 0.4$</td></tr>
<tr><td colspan="5">If available</td></tr>
<tr><td rowspan="2">Proxies</td><td colspan="4">$x_{it} = 0.5\left(\sqrt{0.5}\alpha^x_i + \sqrt{0.5}\gamma^x_t\right) + \sqrt{0.5}\left[\sqrt{1-\theta}\left(0.5[a_i + d_t] + \sqrt{0.5}\eta_{it}]\right) + \sqrt{\theta}\lambda_{1i}F_{1t}\right] + 0.5u_{it}$, with
$\gamma^x_t = 0.5\gamma^x_{t-1} + \sqrt{0.75}\zeta^{\gamma^x}_t$; $u_{it} = 0.5u_{i,t-1} + \sqrt{0.75}\zeta^u_{it}$</td></tr>
<tr><td colspan="3">$\theta = 0$</td><td>$\theta = 1$</td></tr>
</table>

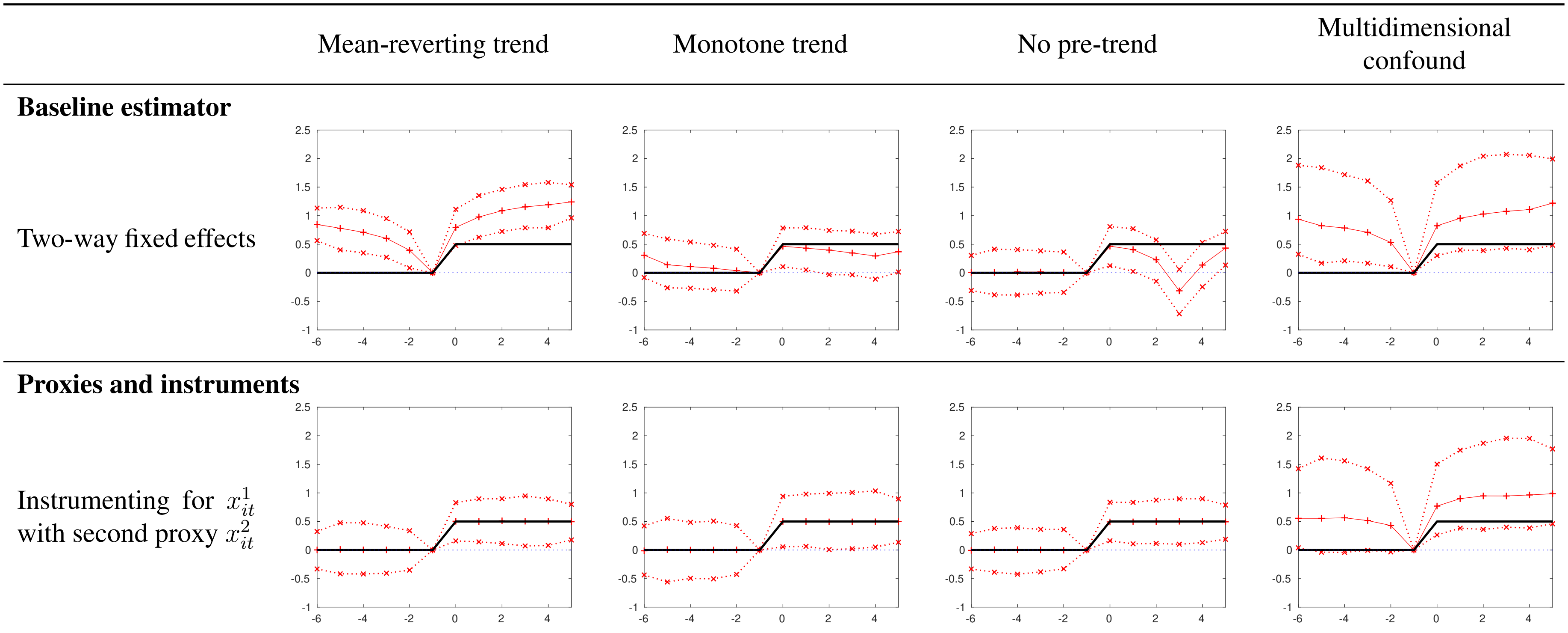
Appendix Table 1: Detailed description of the data-generating processes underlying the simulations presented in Section 4. We generate data for the time periods $t = -4, -3, \ldots, T+5$ to avoid missing observations for leads and lags of the policy and to allow for a policy adoption rule that is consistent with a forward/backward-looking decision-maker on either end of the sample. All random variables not otherwise defined, including initial conditions for autoregressive processes, are distributed as i.i.d. standard normal. We set $z_{it} = z_{i,t-1}$ if $t + P > T + 5$.

	Mean-reverting trend	Monotone trend	No pre-trend	Multidimensional confound
Baseline estimator				
Two-way fixed effects				
Conditional on observables				
Controlling for x_{it}				
Unit-specific linear trend				

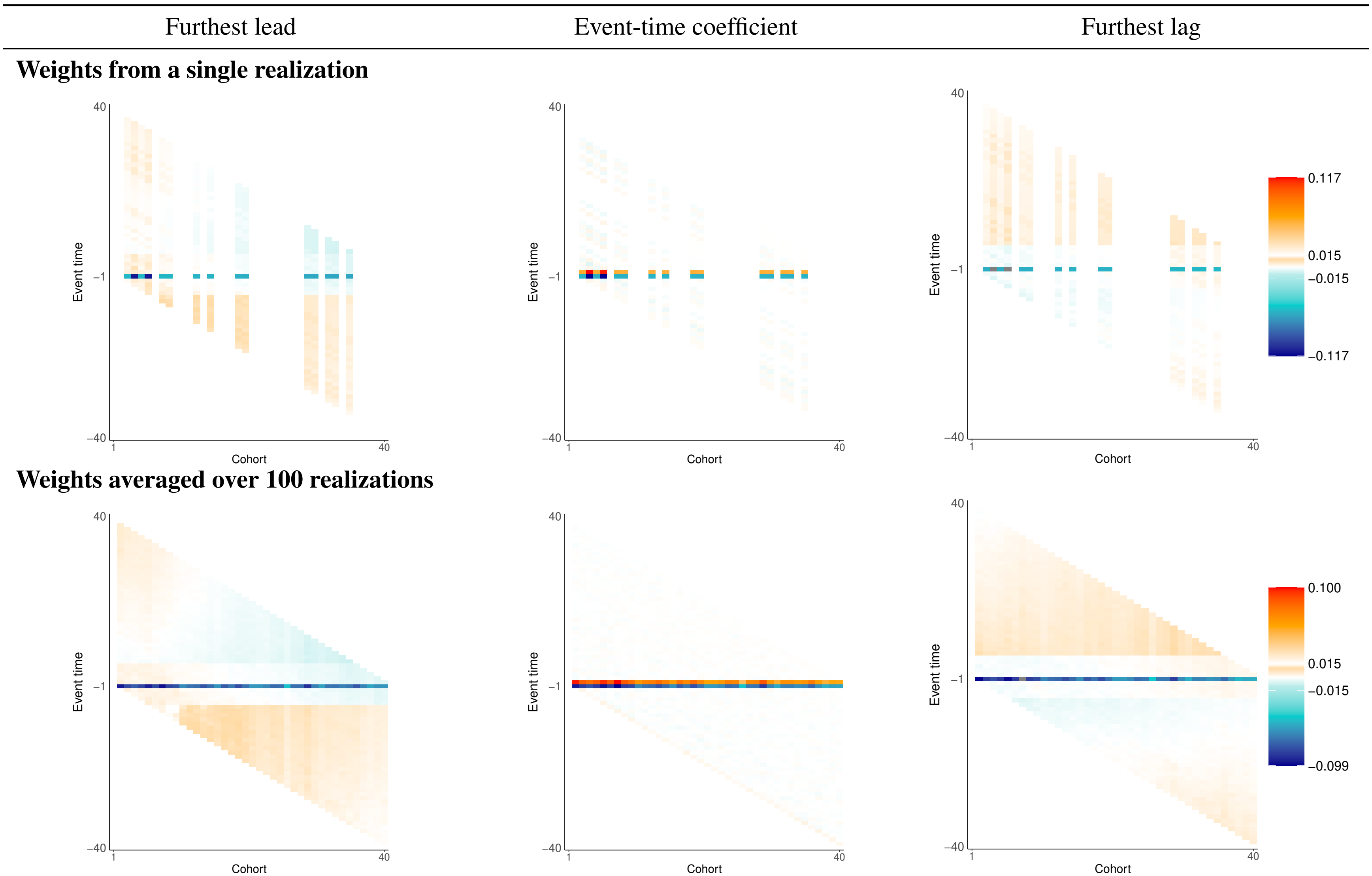
Appendix Figure 1: Additional "conditional on observables" estimators across DGPs. The true treatment effect is depicted in solid black. For each value of k indicated on the x-axis, the series correspond to the 2.5th (dotted, marked by x's), 50th (solid, marked by +'s), and 97.5th (dotted, marked by x's) percentiles across 1000 simulations for each $\hat{\hat{\delta}}_k$. The first row ("Two-way fixed effects") implements a two-way fixed effects estimator. The second row ("Controlling for x_{it}") includes the proxy x_{it} directly in the controls q_{it}. The third row ("Unit-specific linear trend") allows for unit-specific linear time trends.

	Mean-reverting trend	Monotone trend	No pre-trend	Multidimensional confound
Baseline estimator				
Two-way fixed effects				
Conditional on observables				
Synthetic control (simplex)				
Synthetic control (un-constrained)				

Appendix Figure 2: Synthetic control estimators across DGPs. The true treatment effect is depicted in solid black. For each value of k indicated on the x-axis, the series correspond to the 2.5th (dotted, marked by x's), 50th (solid, marked by +'s), and 97.5th (dotted, marked by x's) percentiles across 1000 simulations for each $\hat{\delta}_k$. The right axis depicts the estimated treatment effect relative to the synthetic control. On the left axis, estimates are normalized such that the median treatment effect is zero the period before the event. The donor pool for unit i in cohort $T^*(i)$ consists of all units j with $T^*(j) > T^*(i) + 5$. The first row ("Two-way fixed effects") implements a two-way fixed effects estimator. The second row ("Synthetic control (Simplex)") estimates $\hat{\delta}_k$ using the `synth` package (Abadie et al. 2020), restricting the weights on the donor units to the simplex. The third row ("Synthetic control (Unconstrained)") estimates $\hat{\delta}_k$ using the `augsynth` package (Ben-Michael et al. 2020), allowing donor weights outside the simplex but imposing an ℓ_2-penalty of 0.001.



Appendix Figure 3: Additional "proxies and instruments" estimator across DGPs. The true treatment effect is depicted in solid black. For each value of k indicated on the x-axis, the series correspond to the 2.5th (dotted, marked by x's), 50th (solid, marked by +'s), and 97.5th (dotted, marked by x's) percentiles across 1000 simulations for each $\hat{\delta}_k$. The first row ("Two-way fixed effects") implements a two-way fixed effects estimator. The second row ("Instrumenting for x_{it}^1 with second proxy x_{it}^2") considers a measurement-error correction, assuming the availability of two proxies for the confound, by instrumenting for one proxy, x_{it}^1, with the other, x_{it}^2.



Appendix Figure 4: Graphical illustration of weights underlying coefficients. The figure shows the estimated weights underlying the coefficients δ_{-6} ("Furthest lead"), δ_0 ("Event-time coefficient"), and δ_5 ("Furthest lag") from the model in (2) with $M + L_M = 5$ and $G + L_G = 5$. In each plot, the coloring of each cell denotes the estimated weight that corresponds to the given cohort and event time. The first row ("Weights from a single realization") shows the estimated weights from a single realization from the "Mean-reverting trend" DGP. The second row ("Weights averaged over 100 realizations") shows the average estimated weights across 100 realizations of the "Mean-reverting trend" DGP. The weights are defined following Proposition 1 of Sun and Abraham (forthcoming) and estimated following equation (13) of Sun and Abraham (forthcoming) using the package `eventstudyweights` (Sun 2021).

References

Alberto Abadie, Alexis Diamond, and Jens Hainmueller. SYNTH: Stata module to implement synthetic control methods for comparative case studies, 2020. URL `https://ideas.repec.org/c/boc/bocode/s457334.html`.

Eli Ben-Michael, Avi Feller, and Jesse Rothstein. AUGSYNTH: Augmented synthetic control method, 2020. URL `https://github.com/ebenmichael/augsynth`.

Liyang Sun. EVENTSTUDYWEIGHTS: Stata module to estimate the implied weights on the cohort-specific average treatment effects on the treated (CATTs) (event study specifications), 2021. URL `https://ideas.repec.org/c/boc/bocode/s458833.html`.

Liyang Sun and Sarah Abraham. Estimating dynamic treatment effects in event studies with heterogeneous treatment effects. *Journal of Econometrics*, forthcoming.