MATH 601. HW 1
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(1.4) Let $\mathbb{C} \sqcup \mathbb{C} = \{(1, z)|z \in C\} \cup \{(z, 1)|z \in C\}$. Consider the map $\pi_3 : \mathbb{C} \sqcup \mathbb{C} \to \mathrm{Set}(\mathbb{CP}^1)$ where $(z, 1)$ maps to the line spanned by $(z, 1)$, $\mathrm{span}(z, 1)$ and likewise $(1, z) \mapsto \mathrm{span}(1, z)$.

Notice that this map is surjective as for any line $\ell = \mathrm{span}(a, b)$ we can rescale the vector (a, b) to be either $(a/b, 1)$ or $(1, 0)$ which then maps to ℓ with π_3. Finally we can look at the quotient space of $\mathbb{C} \sqcup \mathbb{C}$ where we identify inputs that share the same output, which is the equivalence relation $(z, 1)\tilde{}(1, 1/z)$.

(2.5) As shown in Exercise 2.4 we have that S^2 is homeomorphic to $\mathbb{CP}^1$. Likewise by definition of the quotient topology we have a surjective continuous function $\pi_2 : S^3 \to \mathbb{CP}^1 \cong S^2$. Let $\ell \in \mathbb{CP}^1 \cong S^2$ and consider an element of the preimage $p \in \pi_2^{-1}(\ell) \subseteq S^4 \subseteq \mathbb{C}^2$. We also know from 1.3 that any point $e^{i\theta}p \mapsto \ell$. And so the entire preimage $\pi_2^{-1}(\ell) \cong S^1$ by the map $e^{i\theta} \mapsto e^{i\theta}p$.