# Reading Group on Lie Algebras and Lie Groups

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## Main Resources

(WilErd) Introduction to Lie Algebras, Erdmann and Wildon

(Hump) Introduction to Lie Algebras and Repn Thy, Humphreys

## Topics

**General Topics.**

(1) The Basics, with many motivating examples
- Chapters 1, 2, 4 of WilErd, and Chpters 1,2,3 of Hump.
- Semisimple lie algebras (WilErd Def 4.6 Ch 4, Ch 3 Hump)
- Engel's theorem (WilErd Ch 6, Hump Ch 3)
- Lie Theorem (WilErd Ch 6, Hump Ch 4)

(2) Representations of $SL_2$ (WilErd CH 8, Hump Ch 7)
- Probably need WilErd Ch 7

(3) Killing form (WilErd Ch 9.3, Hump Ch 5)

(4) Root Space Decompositions (WilErd Ch 10, Hump Ch 8)
- Cartan subalgs (WilErd Ch 10.1) or Maximal Toral Subalgebras (Hump 8.1). These notations I guess are equivalent (WilErd Appendix C)

(5) Dynkin diagrams (WilErd Ch 11.4, 13.1, Hump Ch 11.2)
- Classification of all simple lie algebras

(6) Universal Enveloping Algebra and Highest weight modules (WilErd 15.2, Hump Ch 17)

**Special Topics.** Topics primarily found from What are applications of Lie groups/algebras in mathematics? And Why study Lie algebras?

- Lie Groups and Applications to "Geometry": "groups of symmetries of geometric objects"
  - Groups of Lie Type (WilErd Ch 15.3)
  - Some Notes
  - Helgason's notes
  - "Lie groups provide a way to express the concept of a continuous family of symmetries for geometric objects"

  - Some references and Books: John Lee's "Introduction to Smooth Manifolds", Spivak's "comprehensive introduction to differential geometry" and Sharpe's Differential Geometry Text.
  - Chern-Weil theory
- Applications to Harmonic Analysis
  - Peter-Weyl theorem
  - Automorphic Forms and number theory
  - Zauner's Conjecture
- Applications to Differential Equations
  - "Lie algebras arise as the infinitesimal symmetries of differential equations"
  - A reference book: Olver, Peter J., Applications of Lie groups to differential equations
- Computer Vision

## An Outline (10-12 weeks)