Complex Geometry. HW 5

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(9.2) Consider two meromorphic function from $\mathbb{CP}^1$ to $\mathbb{C}$, the function $F(X:Y)=\frac{\prod^n(a_iX-b_iY)}{\prod^n(c_iX-d_iY)}$ and $G(X:Y)=\frac{\prod^n(e_iX-f_iY)}{\prod^n(g_iX-g_iY)}$. Notice that their product $F(X:Y)G(X:Y)=\frac{\prod^n(a_iX-b_iY)}{\prod^n(c_iX-d_iY)}\frac{\prod^n(e_iX-f_iY)}{\prod^n(g_iX-g_iY)}$ added the zeros of FG are precisely the zeros from F and G separately. Likewise the poles of FG are precisely the poles from F and G separately. This means the the divisor is $div(FG)=div(F)+div(G)$

(9.4) Let $s:\mathbb{CP}^1\to L$ be a section for the line bundle $\pi:L\to\mathbb{CP}^1$. Let the $supp(div(s))$ be the points with non-zero coefficient in the divisor, meaning this are precisely the zeros and poles of s. This gives us a natural bijection, that trivializes the line bundle for all fibers other then the one in the support.

Consider the map $T_s:\pi^{-1}(\mathbb{CP}^1-supp(div(s)))\to(\mathbb{CP}^1-supp(div(s)))\times\mathbb{C}$ that maps $p\in\pi^{-1}(\mathbb{CP}^1-supp(div(s)))$ and maps it to the point $(\pi(p),\frac{p}{s(\pi(p))})$ where because both p and $s(\pi(p))$ live in the same fiber, $\pi^{-1}(\pi(p))$ we can interpret $\frac{p}{s(\pi(p))}$ as an element of $\mathbb{C}$, where we relate p with a value in $\mathbb{C}$ with the corresponding non-canonical linear isomorphism, and $s(\pi(p))$ with the same linear isomorphism. It is then the case that $\frac{p}{s(\pi(p))}$ is invariant under the choice on non-canonical isomorphism and is defined for all points p not in the fibers of the support of the divisor. This is invertible precisely because we divide by a non-zero complex number $s(\pi(p))$