

READING GROUP ON LIE ALGEBRAS AND LIE GROUPS
(NEVER DID THIS, INSTEAD I TOOK MATH 160)
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MAIN RESOURCES

(WilErd) Introduction to Lie Algebras, Erdmann and Wildon

(Hump) Introduction to Lie Algebras and Repn Thy, Humphreys

(Hall) Lie Groups, Lie Algebras and Representations, Hall

TOPICS

General Topics.

(1) The Basics of Lie Groups, with many motivating examples

- Matrix Lie Groups Theory (Hall Chap 1: 1.3 1.4 1.5) **Covered in MATH 601**
- Symplectic Groups (Hall ch 1.2.4) The Heisenberg Group (Hall 1.2.6)
- The Matrix Exponential (Hall Ch 2)
- The Lie Algebra of a Matrix Lie Group (Hall Ch 3.3) **Covered in MATH 601**

(2) The Basics of Lie Algebras, with many motivating examples

- Chapters 1, 2, 4 of WilErd, and Chpters 1,2,3 of Hump. **Covered in MATH 601**
- Semisimple lie algebras (WilErd Def 4.6 Ch 4, Ch 3 Hump)
- Engel's theorem (WilErd Ch 6, Hump Ch 3)
- Lie Theorem (WilErd Ch 6, Hump Ch 4)

(3) Representations of \mathfrak{sl}_2 (WilErd CH 8, Hump Ch 7) **Covered in MATH 601**

- Probably need WilErd Ch 7

(4) Killing form (WilErd Ch 9.3, Hump Ch 5)

(5) Root Space Decompositions (WilErd Ch 10, Hump Ch 8)

- Cartan subalgs (WilErd Ch 10.1) or Maximal Toral Subalgebras (Hump 8.1). These notations I guess are equivalent (WilErd Appendix C)

(6) Dynkin diagrams (WilErd Ch 11.4, 13.1, Hump Ch 11.2)

- Classification of all simple lie algebras

(7) Universal Enveloping Algebra and Highest weight modules (WilErd 15.2, Hump Ch 17)

Special Topics. Topics primarily found from What are applications of Lie groups/algebras in mathematics? And Why study Lie algebras?

- Lie Groups and Applications to “Geometry”: “groups of symmetries of geometric objects”
 - Groups of Lie Type (WilErd Ch 15.3)
 - Some Notes
 - Helgason’s notes
 - “Lie groups provide a way to express the concept of a continuous family of symmetries for geometric objects”
 - Some references and Books: John Lee’s “Introduction to Smooth Manifolds”, Spivak’s “comprehensive introduction to differential geometry” and Sharpe’s Differential Geometry Text.
 - Chern-Weil theory
- Applications to Harmonic Analysis
 - Peter-Weyl theorem
 - Automorphic Forms and number theory
 - Zauner’s Conjecture
- Applications to Differential Equations
 - “Lie algebras arise as the infinitesimal symmetries of differential equations”
 - A reference book: Olver, Peter J., Applications of Lie groups to differential equations
- Computer Vision

AN OUTLINE (10-12 WEEKS)