Master's Thesis Proposal

Memoization of Incremental Computation for Generic Data Types

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Todo list

1 Introduction

- What is the problem? Illustrate with an example.

Example Haskell Data Type

Calculate a result over the Data Type

Cache the incremental computation over the data type

1.1 Research Questions

(A) A library needs to be implemented which contains the generic `merkle`, `cataMerkle` and `cataMerkleWithMap` functions

(B) Then using that library, what parameters can be tweaked to have the best ratio of performance and memory usage?

(C) What type of equivalence is needed to reuse the incremental computation?

(D) What type of data structures are the best for storing the incremental computation?

(E) Could be used to perform static analysis in a more performant manner.

2 Background

2.1 An Efficient Algorithm for Type-Safe Structural Diffing

The paper *An Efficient Algorithm for Type-Safe Structural Diffing* by Victor Cacciari Miraldo and Wouter Swierstra presents an efficient datatype-generic algorithm called *hdiff* to compute the difference between two values of any algebraic datatype. In particular, the algorithm readily works over the abstract syntax tree (AST) of a programming language[8].

To make the *hdiff* algorithm work, an implementation of which common subtree needs to be defined. The `wcs` function is a function that when given two trees and a subtree, returns the position of the subtree inside the trees if both contain the subtree. Otherwise, the function returns nothing. An example of a naive implementation would be:

```
wcs :: Tree -> Tree -> Tree -> Maybe Int
wcs s d x = elemIndex x (subtrees s ∩ subtrees d)
```

Here the function `subtrees` enumerates all the subtrees of a given tree. Then `elemIndex` returns the index when the subtree is found, otherwise it returns nothing.

The paper identifies two inefficiencies using this naive implementation. (A) Furthermore, enumerating all subtrees is exponential; (B) checking trees for equality is linear in the size of the tree.

To improve the first inefficiency of the naive `wcs` implementation is to use cryptographic hash functions to compare the equality of the trees. To check the trees for equality in constant time the trees are decorated with a hash at every node in the tree. Then, using the precomputed hash and the root node of the given tree, the hash of a subtree is calculated in constant time.

The second inefficiency of the naive `wcs` implementation is improved by using a `Trie`[2] datastructure. Given that a `Hash` is just a `[Char]`, this makes the `Trie` datastructure the preferred choice to store the enumerated subtrees. And because the `Hash` has a constant size the `Trie` lookups are efficient and runs in amortized constant time.

2.2 Sums of Products for Mutually Recursive Datatypes

The paper *Sums of Products for Mutually Recursive Datatypes* written by Victor Cacciari Miraldo and Alejandro Serrano[7] presents a new approach to generic programming using recursive positions to handle mutually recursive families and the *sum-of-products* structure. This work (`generics-msrop`) is later used by the paper *An Efficient Algorithm for Type-Safe Structural Diffing* by Victor Cacciari Miraldo and Wouter Swierstra[8] to define the generic version of their diffing algorithm. Compared to existing generic programming libraries, `generics-mrsop` has *deep explicit recursion, sums of products* and supports *mutually recursive datatypes*.

Explicit recursion   There are two ways to represent values. One contains the information on what properties of a datatype are recursive. The other does not contain that information. If we do not know explicitly if the property is recursive, then only one layer of the value can be formed into a generic representation. This is called *shallow* encoding. If we explicitly keep track of the recursive property, then the entire value can be transformed into a generic representation. This is called *deep* encoding. Using the *deep* encoding more datatypes can be defined generically (e.g., a generic *map* or generic Zipper datatype).

Sums of Products   The `generic-sop` library uses a list of lists of types. The outer list represents the sum and the inner list represents the product. The sum represents the choice between two constructors; the product represents a combination of two constructors. An example of a `Code` representation of a `BinTree` is

```
data BinTree a = Leaf a
               | Node (BinTree a) (BinTree a)


Code_BinTree(Bin a) = `[`[a], `[Bin a, Bin a]]
```

Here the ` sign in the code promotes the definition to the type-level instead of a run-time value. The use of *Sums of Products* makes it considerably easier to represent generic datatypes.

Mutually recursive datatypes   Most of the generic programming libraries are restricted to only allowing recursion on the same datatype, which is the one being defined. Mutually recursive datatypes are recursively defined in each other's terms, meaning that most generic programming libraries do not support mutually recursive datatypes. This limits the ability to generically represent the syntax of many programming languages. Thus `generic-sop` introduces recursive positions on a type level, which can be used to define mutually recursive datatypes.

2.3   Concise, Type-Safe, and Efficient Structural Diffing

The paper *Concise, Type-Safe, and Efficient Structural Diffing* written by Erdweg, Sebastian and Szabó, Tamás and Pacak, André presents a structural diffing algorithm called *truediff* [4]. *truediff* ensures that the patches produces are concise and type safe, and with a performance by an order of magnitude higher than Gumtree[5] and the *hdiff* [8] algorithm.

To compute the difference between a source tree and a target tree, *truediff* operates in four steps: (1) prepare subtree equivalence relations; (2) find reusable candidates; (3) select reusable candidates; (4) and compute the edit script.

The equivalence relations used in step 1, exist out of two equivalence relations, both encoded through cryptographic hashes. The first equivalence relation is used to identify reusable candidates. The second equivalence relation is used to identify preferred reusable candidates. The paper found that using structural equivalence to identify candidates and literal equivalence to select preferred candidates yields very concise edit scripts.

3   Selective memoization

The paper *Selective Memoization* by Umut A. Acar, Guy E. Blelloch and Robert Harper[1] presents a framework for applying memoization selectively. Also, it describes what type of key issues there are with implementing memoization efficiently: (A) equality; (B) precise dependences and (C) space management.

For equality, the cost of having an equality test can negate the advantage of using memoization. In the paper, there are a few approaches proposed to alleviate this problem. The first is based on the equality test not having to be exact. So, for expensive equality tests, it could determine to skip the test or use a less expensive equality test.

The second approach suggested is to ensure that there is only one copy of every value, known as a "hash consing". If there is only one copy, equality can then be implemented by comparing locations. The problem with hash consing is it demands a large amount of memory and has trouble working with the garbage collection. An alternative proposed by Pugh and Teitelbaum[10], is lazy structure sharing.

Maybe add an explanation of what lazy structure sharing is

For precise dependences

Add precise dependences

Add space management

4   Preliminary Results

The prototype algorithm is written using simpler self-defined generic datatypes with a fixpoint, which are defined in Appendix A and B. An example of how the generic datatypes can be used is:

```
data Tree a = Leaf a
            | Node (Tree a) a (Tree a)


type TreeG a = Fix (TreeF a)
type TreeF a = K a                      -- Leaf
            :+: ((I :*: K a) :*: I) -- Node
```

Using the generic datatypes a `merkle` function can be defined, where at every recursive step of the datatype a `Hash` is stored. To merkelize a datatype, the datatype has to have the `Merkelize` constraint. The `Merkelize` type class is a class containing a single function `merkleIn` which converts the once unpacked `Fix` datatype into a unpacked `Fix` which contains a Hash at every recursive step[1].

```
merkle :: Merkelize f => Fix f -> Fix (f :*: K Hash)
merkle = In . merkleIn . unFix

class (Functor f) => Merkelize f where
  merkleIn :: (Merkelize g)
           => f (Fix g) -> (f :*: K Hash) (Fix (g :*: K Hash))
```

The generic datatypes can also use a `cata` function. The `cata` or catamorphism is a generalization of the concept of a fold, which means it deconstructs a data structure into its underlying functor[3].

```
cata :: Functor f => (f a -> a) -> Fix f -> a
cata alg t = alg (fmap (cata alg) (unFix t))
```

The `cata` function can then be used to, for example, calculate the sum of all the values of the nodes and the leaves of the tree.

```
cataSum :: TreeG Int -> Int
cataSum = cata (\case
  Inl (K x)                    -> x
  Inr (Pair (Pair (I l, K x), I r)) -> l + x + r)
```

---

[1]The implementation of the generic datatypes for the `Merkelize` type class can be found in Appendix C.

To keep track of the incremental computation of the summation of the tree, a HashMap[6] is used. The calculation of the incremental step is inserted into the HashMap and a pair of the HashMap and the result is returned. The implementation for the `TreeG` datatype is:

```
cataMerkleTree :: TreeG Int -> (Map Hash Int, Int)
cataMerkleTree t = cata sumTree merkleTree
  where
    merkleTree :: Fix (TreeF a :*: K Hash)
    merkleTree = merkle t

    sumTree :: (TreeG Int :*: K Hash) Int -> Int
    sumTree (Pair (px, K h)) = case px of
      -- Leaf
      Inl (K x)
        -> (M.insert h x M.empty, x)
      -- Node
      Inr (Pair (Pair (I (xl, ml), K x), I (xr, mr)))
        -> let n = x + xl + xr
           in (M.insert h n (ml <> mr), n)
```

Then using the previously generated HashMap, we can then calculate the result reusing the previously incremental computations:

```
cataMerkleTreeWithMap :: Map Hash Int -> TreeG Int -> (Int, Map Hash Int)
cataMerkleTreeWithMap m (In (Pair (x, K h))) =
  case lookup h m of
    Just n   -> (n, m)
    Nothing -> case x of
      Inl (K x) -> (x, insert h x empty)
      Inr (Pair (Pair (I l, K x), I r)) -> (x', m')
        where
          (xl, ml) = cataMerkleTreeWithMap m l
          (xr, mr) = cataMerkleTreeWithMap ml r
          x' = x + xl + xr
          m' = insert h x' mr
```

Using the previously defined `cata` functions we can determine the performance of the functions by using the `criterion`[9] package. For a benchmark in `criterion`, first, the environment is set up. Then the bench function is executed multiple times within a certain timeframe. The result of the multiple executions is used to calculate the mean and standard deviation of the

time executed.

The results of the `cataSum`, `cataMerkleTree` and `cataMerkleTreeWithMap` is seen in the graph.
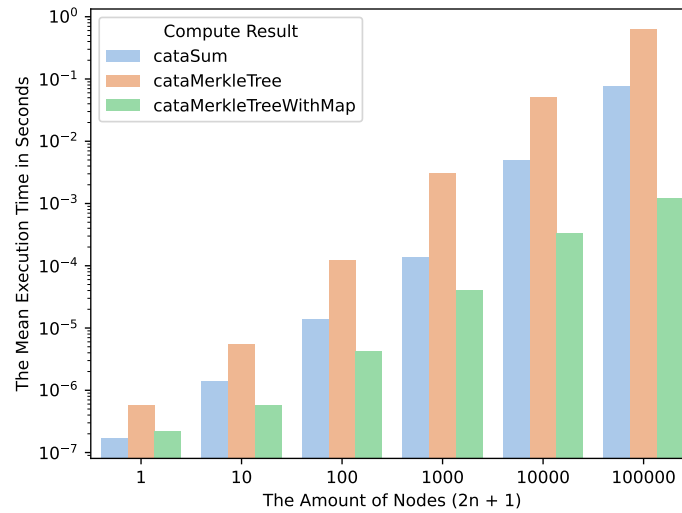

Figure 1: Compute the result

4.1   Future challenges

The problem with this implementation is that it only works for the `TreeG` datatype. The goal would be to create a generic function, where only the `cataSum` would be defined and the result would automatically contain the intermediate results. A generic definition could look something like this:

```
cataMerkle :: (f a -> a) -> Fix (f :*: K Hash) -> State (Map Hash a) a
```

Using the `cataMerkle` function would lead to only needing to implement the `cataSum` function and the intermediate results are then automatically stored.

5   Timetable and Planning

- What will I do with the remainder of my thesis?

- Give an approximate estimation/timetable for what you will do and when you will be done.

5.1   Exploratory topics

During the first part of the Thesis project, multiple topics are thought of that need further research/implementation in the second part of the Thesis project.

To implement the generic `merkle`, `cata` and `cataWithMap` functions, the `generics-msrop` library described in Section 2.2 is a good candidate to use for implementing these functions. This is because it supports mutually recursive datatypes, meaning that a large group of datatypes are supported.

Describe the parameter tweaking using the selective memoization paper

In the paper *Concise, Type-Safe, and Efficient Structural Diffing* in Section 2.3 they describe using two equivalence relations instead of one. Using two equivalence relations could lead to more opportunities for reusing computed results. However, further research is needed on how feasible it is using two equivalence relations.

For the data structures used for storing the incremental computation, the easiest to use would be using a `HashMap`. But, the paper *An Efficient Algorithm for Type-Safe Structural Diffing* described in Section 2.1 suggest using a different data structure, the `Trie` data structure. Further research could be done in comparing the performance and memory usage of both data structures.

5.2   Schedule

Week	Date	Category	Work
Week 1	28 feb - 04 mar	Implementation	Generic CataMerkle
Week 2	07 mar - 11 mar	Implementation	Generic CataMerkle
Week 3	14 mar - 18 mar	Implementation	Using Generic MSROP library
Week 4	21 mar - 25 mar	Implementation	Using Generic MSROP library
Week 5	28 mar - 01 apr	Experiments	Creating benchmarks
Week 6	04 apr - 08 apr	Experiments	Creating benchmarks
Week 7	11 apr - 15 apr	Experiments	Using real-world data for benchmarks
Week 8	18 apr - 22 apr	Experiments	Using real-world data for benchmarks
Week 9	25 apr - 29 apr	Analysis	Test using different parameters
Week 10	02 may - 06 may	Analysis	Test using different parameters
Week 11 - 10	08 mar - 15 apr	Writing	Experiments
Week 12 - 13	18 apr - 06 may	Writing	Discussion
Week 13	09 may - 13 may	Writing	Conclusion
Week 14 - 17	16 may - 3 jun	Feedback	Process final feedback
Week 18 - 19	6 jun - 17 jun	Time Left	Vacation / Overdue work
Week 20	20 jun - 24 jun	Finalize	Finalizing the Thesis
Week 21	27 jun - 1 jul	Vacation	-
Week 27	08 aug - 12 aug	Submission	Presentation & Thesis hand-in
Week 28	15 aug	Finish	End Date Research Project

Table 1: Planning per week

6   Appendix

A   Definition Generic Datatypes

```
data I r       = I r
data K a r     = K a
data (:+:) f g r = Inl (f r) | Inr (g r)
data (:*:) f g r = Pair (f r, g r)
```

B   Definition Fixpoint

```
data Fix f = In { unFix :: f (Fix f) }


instance Eq (f (Fix f)) => Eq (Fix f) where
    f == g = unFix f == unFix g


instance Show (f (Fix f)) => Show (Fix f) where
    show = show . unFix
```

C   Implementation Merkelize

```
instance (Show a) => Merkelize (K a) where
  merkleIn (K x) = Pair (K x, K h)
    where
      h = hashConcat [hash "K", hash x]


instance Merkelize I where
  merkleIn (I x) = Pair (I prevX, K h)
    where
      prevX@(In (Pair (_, K ph))) = merkle x
      h = hashConcat [hash "I", ph]


instance (Merkelize f, Merkelize g) => Merkelize (f :+: g) where
  merkleIn (Inl x) = Pair (Inl prevX, K h)
    where
      (Pair (prevX, K ph)) = merkleIn x
      h = hashConcat [hash "Inl", ph]
  merkleIn (Inr x) = Pair (Inr prevX, K h)
```

```
    where
      (Pair (prevX, K ph)) = merkleIn x
      h = hashConcat [hash "Inr", ph]


instance (Merkelize f, Merkelize g) => Merkelize (f :*: g) where
  merkleIn (Pair (x, y)) = Pair (Pair (prevX, prevY), K h)
      where
        (Pair (prevX, K phx)) = merkleIn x
        (Pair (prevY, K phy)) = merkleIn y
        h = hashConcat [hash "Pair", phx, phy]
```

D   Results of computing the sum of a Tree

Amount	Action	Mean	Stddev
1	Generate (Result, Map)	5.662e-07	1.195e-08
1	Generate (Result, Map) with Map	2.208e-07	5.237e-09
1	Generate Result	1.713e-07	1.721e-09
10	Generate (Result, Map)	5.456e-06	6.462e-08
10	Generate (Result, Map) with Map	5.744e-07	8.788e-09
10	Generate Result	1.401e-06	1.132e-08
100	Generate (Result, Map)	1.205e-04	2.379e-06
100	Generate (Result, Map) with Map	4.165e-06	6.188e-08
100	Generate Result	1.358e-05	1.826e-07
1000	Generate (Result, Map)	3.024e-03	9.485e-05
1000	Generate (Result, Map) with Map	3.955e-05	6.024e-07
1000	Generate Result	1.387e-04	1.708e-06
10000	Generate (Result, Map)	5.018e-02	2.108e-03
10000	Generate (Result, Map) with Map	3.280e-04	5.800e-06
10000	Generate Result	4.994e-03	1.123e-04
100000	Generate (Result, Map)	6.253e-01	2.174e-02
100000	Generate (Result, Map) with Map	1.228e-03	1.836e-05
100000	Generate Result	7.592e-02	1.505e-03

Table 2: Compute the result

References

[1] Umut A Acar, Guy E Blelloch, and Robert Harper. "Selective memoization". In: *ACM SIGPLAN Notices* 38.1 (2003), pp. 14–25.

[2] Peter Brass. *Advanced data structures*. Vol. 193. Cambridge University Press Cambridge, 2008, pp. 336–356.

[3] *Catamorphisms*. URL: https://wiki.haskell.org/Catamorphisms (visited on Jan. 26, 2022).

[4] Sebastian Erdweg, Tamás Szabó, and André Pacak. "Concise, type-safe, and efficient structural diffing". In: *Proceedings of the 42nd ACM SIGPLAN International Conference on Programming Language Design and Implementation*. 2021, pp. 406–419.

[5] Jean-Rémy Falleri et al. "Fine-grained and accurate source code differencing". In: *Proceedings of the 29th ACM/IEEE international conference on Automated software engineering*. 2014, pp. 313–324.

[6] *Map type*. URL: https://hackage.haskell.org/package/containers-0.4.0.0/docs/Data-Map.html (visited on Feb. 8, 2022).

[7] Victor Cacciari Miraldo and Alejandro Serrano. "Sums of products for mutually recursive datatypes: the appropriationist's view on generic programming". In: *Proceedings of the 3rd ACM SIGPLAN International Workshop on Type-Driven Development*. 2018, pp. 65–77.

[8] Victor Cacciari Miraldo and Wouter Swierstra. "An efficient algorithm for type-safe structural diffing". In: *Proceedings of the ACM on Programming Languages* 3.ICFP (2019), pp. 1–29.

[9] Bryan O'Sullivan. URL: http://www.serpentine.com/criterion/ (visited on Feb. 8, 2022).

[10] William Pugh and Tim Teitelbaum. "Incremental computation via function caching". In: *Proceedings of the 16th ACM SIGPLAN-SIGACT symposium on Principles of programming languages*. 1989, pp. 315–328.