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Generic Incremental Computation for Regular Datatypes

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Title Explanation – Incremental Computation

Generic Incremental Computation for Regular Datatypes

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Incremental computation is an approach to improve performance by only recomputing result for changed input

Title Explanation – Example Incremental Computation

```haskell
fib :: Int -> Int
fib 0 = 0
fib 1 = 1
fib n = fib (n - 1) + fib (n - 2)
```

Call Hierarchy

```
                  fib(4)
               /          \
         fib(3)            fib(2)
          /   \             /   \
    fib(2)    fib(1) fib(1)    fib(0)
    /   \
fib(1)  fib(0)
```

Memoization

stores the result of a computation and returns the cached result when the same input occurs again.

Title Explanation – Example Incremental Computation

New Call Hierarchy

```
            fib(4)
         /          \
     fib(3)          fib(2)
      /   \
  fib(2)   fib(1)
   /   \
fib(1)  fib(0)
```

Function Call	Result
fib(2)	1
fib(3)	2
fib(4)	3

Cached Results

Title Explanation – Generic

Generic Incremental Computation for Regular Datatypes

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Generic refers to *datatype-generic programming*, which is a form of abstraction that allows defining functions that can operate on a large class of datatypes.

Title Explanation – Generic Example

```haskell
data List a = Nil | Cons a (List a) -- Haskell Notation [] | x : []

length :: List a -> Int
length Nil          = 0
length (Cons _ t) = 1 + length t

data Tree a = Leaf | Node (Tree a) a (Tree a)

length :: Tree a -> Int
length Leaf _         = 1
length (Node l _ r) = 1 + length l + length r
```

Title Explanation – Generic Example

```haskell
gLength :: (Generic f) => f a -> Int
gLength = ...
```

A *single* `length` function can be written, that can operate on lists, trees, and many other datatypes

```haskell
> gLength (Cons 1 (Cons 2 (Cons 3 Nil))) -- List Int
  3
```

```haskell
> gLength (Node Leaf 1 Leaf) -- Tree Int
  3
```

Title Explanation – Regular Datatypes

Generic Incremental Computation for Regular Datatypes

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Regular datatypes are recursive datatypes, which can only recurse into themselves, such as lists, binary trees, etc.

Regular Datatypes

```
data List a = Nil | Cons a (List a)

data Tree a = Leaf | Node a (Tree a) (Tree a)
```

Not Regular Datatypes

```
data Tree a = Empty | Node a (Forest a)

data Forest a = Nil | Cons (Tree a) (Forest a)
```

Title Explanation – Summary

Generic Incremental Computation for Regular Datatypes

Goal – What does this solve?

Goal: implement an incremental algorithm which performs better than the non-incremental algorithm for large regular datatypes

Goal – Example Problem

```haskell
data Tree a = Leaf a | Node (Tree a) a (Tree a)

sumTree :: Tree Int -> Int
sumTree (Leaf x)       = x
sumTree (Node l x r) = x + sumTree l + sumTree r

exampleTree = Node (Node (Leaf 1) 3 (Leaf 2)) 5 (Node (Leaf 1) 3 (Leaf 2))
```

Visual representation

```
                5
              /   \
sumTree(    3       3     ) = 17
           / \     / \
          1   2   1   2
```

Goal – Example Problem

Memoized version of the `sumTree`

Example Tree

```
      5
    /   \
   3     3
  / \   / \
 1   2 1   2
```

<table>
<tr><th>Tree</th><th>Result</th></tr>
<tr><td><pre>
      5
    /   \
   3   3
  / \   / \
 1   2 1   2
</pre></td><td>17</td></tr>
<tr><td><pre>
   3
  / \
 1   2
</pre></td><td>6</td></tr>
</table>

Cached Results

Solution – Using Hash function

A *hash function* is a process of transforming input into an arbitrary fixed-size value (i.e., digest), where the same input always generates the same output

```
       5
     /   \
    3     3          ->     1da16c7c48e429b4ad6e6c88d941d2bd
   / \   / \
  1   2 1   2                            Digest
```

Solution – Storing the Digests

A *Merkle Tree* is a data structure which integrates the *digests*, which represents the internal structure, within the data structure

```
data TreeH a = LeafH Digest a
             | NodeH Digest (TreeH a) a (TreeH a)
```

```
merkle :: Tree Int -> TreeH Int
merkle l@(Leaf x)     = LeafH (hash l) x
merkle b@(Node l x r) = NodeH (hash b) l' x r'
  where
    l' = merkle l
    r' = merkle r
```

Solution – Using Digests

```
sumTreeInc m (LeafH d x) = case lookup d m of
  Just z  -> (z, m)
  Nothing -> (x, insert d x m)
```

. . .

```
SumTreeInc
```

Solution – Updating the Digests

The *Zipper* is a technique for keeping track of how the data structure is being traversed through

```haskell
data Cxt a = L (Tree a) a | R (Tree a) a

type Loc a = (Tree a, [Cxt a])

enter :: Tree a -> Loc a
enter t = (t, [])
```

Solution – Zipper - Left

```
     5                           5
   /   \                           \
  3     3      ->      3            3
 / \   / \            / \          / \
1  2   1  2          1   2        1   2
```

```
left :: Loc a -> Loc a
left (Node l x r, cs) = (l, (L r x):cs)
```

Solution – Zipper - Right

```
     5                    5
   /   \                 /
  3     3      ->       3          3
 / \   / \             / \        / \
1  2   1  2           1   2      1   2
```

```haskell
right :: Loc a -> Loc a
right (Node l x r, cs) = (r, (R l x):cs)
```

Solution – Zipper - Up

```
     5
    /
   3          3    ->     5
  / \        / \         / \
 1   2      1   2       3   3
                       / \ / \
                      1  2 1  2
```

```haskell
up :: Loc a -> Loc a
up (t, (L r x):cs) = (Node l x r, cs)
up (t, (R l x):cs) = (Node l x r, cs)
```

Generic Programming – Pattern Functors

Primitive Type Constructors

```
data U r         = U                 -- Empty constructor
data I r         = I r               -- Recursive position
data K a r       = K a               -- Constant
data (f :+: g) r = L (f r) | R (g r) -- Sums (Choice)
data (f :*: g) r = (f r) :*: (g r)   -- Products (Combine)
```

Pattern functor for the `Tree` datatype

```
data Tree a = Leaf a | Node (Tree a) a (Tree a)

type instance PF (Tree a) = K a                  -- Leaf
                        :+: (I :*: K a :*: I) -- Node
```

Generic Implementation – Hashing Primitive Type Constructors

```
class Hashable f where
  hash :: f (Merkle g) -> Digest
```

- The `f` represents the primitive type constructors (i.e., `U`, `I`, `K`, `:+:`, `:*:`)
- The `Merkle g` is the type of the recursive position (i.e., `I r`).
- `Merkle g` contains the `Digest` of its internal structure.
- The `hash` function only converts a single layer of the pattern functor.

Generic Implementation – Hashing Primitive Type Constructors

```haskell
instance Hashable U where
  hash _ = hash "U"

instance (Show a) => Hashable (K a) where
  hash (K x) = digestConcat [hash "K", hash x]
```

Generic Implementation – Hashing Primitive Type Constructors

```
instance (Hashable f, Hashable g) => Hashable (f :+: g) where
  hash (L x) = digestConcat [hash "L", hash x]
  hash (R x) = digestConcat [hash "R", hash x]

instance (Hashable f, Hashable g) => Hashable (f :*: g) where
  hash (x :*: y) = digestConcat [hash "P", hash x, hash y]
```

Generic Implementation – Hashing Primitive Type Constructors

```
class Hashable f where
  hash :: f (Merkle g) -> Digest
```

```
instance Hashable I where
  hash (I x) = digestConcat [digest "I", getDigest x]
    where
      getDigest :: Fix (f :*: K Digest) -> Digest
      getDigest (In (_ :*: K h)) = h
```

Generic Implementation – Generic Merkle Tree

```haskell
merkleG :: Hashable f => f (Merkle g) -> (f :*: K Digest) (Merkle g)
merkleG f = f :*: K (hash f)

merkle :: (Regular a, Hashable (PF a), Functor (PF a))
       => a -> Merkle (PF a)
merkle = In . merkleG . fmap merkle . from
```

Generic Implementation – Cata Merkle

```haskell
cataMerkleState :: (Functor f, Traversable f)
                => (f a -> a) -> Merkle f -> State (HashMap Digest a) a
cataMerkleState alg (In (x :*: K d))
  = do m <- get
       case lookup d m of
         Just a  -> return a
         Nothing -> do y <- mapM (cataMerkleState alg) x
                       let r = alg y
                       modify (insert d r) >> return r
```

Generic Implementation – Cata Sum

```haskell
cataSum :: Merkle (PF (Tree Int)) -> (Int, HashMap Digest Int)
cataSum = cataMerkle
  (\case
    L (K x)                     -> x
    R (I l :*: K x :*: I r) -> l + x + r
  )

> cataSum (merkle (Node (Leaf 1) 2 (Leaf 3)))
    (6, {"931090e5": 1, "7d1ef1c9": 3, "ba811ed5": 6})
```

Generic Implementation – Pattern Synonyms

Pattern synonyms add an abstraction over patterns, which allows the user to move additional logic from guards and case expressions into patterns.

```
cataSum :: Merkle (PF (Tree Int)) -> (Int, HashMap Digest Int)
cataSum = cataMerkle
  (\case
    Leaf_ x     -> x
    Node_ l x r -> l + x + r
  )
```

Experiments – Method

Three functions:

- `Cata Sum`
  - Non-incremental algorithm
- `Generic Cata Sum`
  - Incremental algorithm with an empty cache
- `Incremental Cata Sum`
  - Incremental algorithm with a filled cache

Three scenarios:

- Worst case: ...
- Average case: ...
- Best case: ...

Experiments – Results

I am fast as f*ck boy!

Conclusion – Summary

- ...