# Using scores to improve language modelling movie plot summaries

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**Abstract**

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## 1 Introduction

In recent years, several successes have been booked for applying semantic analysis on user comments of movies. In this report we use those same techniques, but apply them to plot summaries of movies. Using these corpus of text we try to determine whether the contents of these summaries and the score these movies are rated with on the popular online movie database IMDb [2] are correlated. We do this by comparing the performance on two latent Dirichlet allocation models – one with and another without using the scores.

First we describe the characteristics of the problem and take a closer look at the data set in Section 2. Next, we outline the model and Gibbs sampler in Section 3. We describe which experiments we ran in Section 4 along with their results. Finally, we provide our final remarks in Section 5.

This project is part of the Natural Language Processing course of the UvA from Fall 2014.

# 2 Problem

In this section we describe the characteristics of the problem and take a closer look at the data set that we use.

## 2.1 Data set

The texts that we use in ths model are summaries of movies. These summaries have been written by users of the popular online movie database IMDb, with the intent to outline the events that occur in the movie.

This is fundamentally different from movie reviews, as the author is not supposed to convey his or her own opinion of the movie in the summary. However, this can obviously never be fully prevented, since the author has seen the movie in question and is willing to spend time and effort to write the summary. In this light we make the following assumptions.

1. An important assumption we make is that the authors wrote the summaries voluntarily, without any compensation or external influence which might affect the writing of the author.
2. Additionally, we assume that the summary also contains the authors personal opinion on the movie, although this does not have to be explicitly mentioned.

Only if these assumptions hold, can we try to find a correlation between the summary and the score of the movie.

An example plot summary from IMDb can be found below. It is from the movie Big Fish (2003) and is written by a user who wanted to remain anonymous.

> *The story revolves around a dying father and his son, who is trying to learn more about his dad by piecing together the stories he has gathered over the years. The son winds up re-creating his father's elusive life in a series of legends and myths inspired by the few facts he knows. Through these tales, the son begins to understand his father's great feats and his great failings.*
>
> by Anonymous

# 3 Approach

## 3.1 Model

In this section we describe the extended topic based model we used. It is taken from [1].

The generative version of LDA is as follows:

1. Draw topic proportions $\theta \mid \alpha \sim \text{Dir}(\alpha)$.
2. For each word:
   (a) Draw topic assignment $z_n \mid \theta \sim \text{Mult}(\theta)$.
   (b) Draw word $w_n \mid z_n, \beta_{1:K} \sim \text{Mult}(\beta_{zn})$.

Its graphical representation can be seen in Figure 3.1. It makes use of the hyperparameters $\alpha$, which influences the topic distributions of a document, and of $\beta$, which influences the topic definitions of each topic.

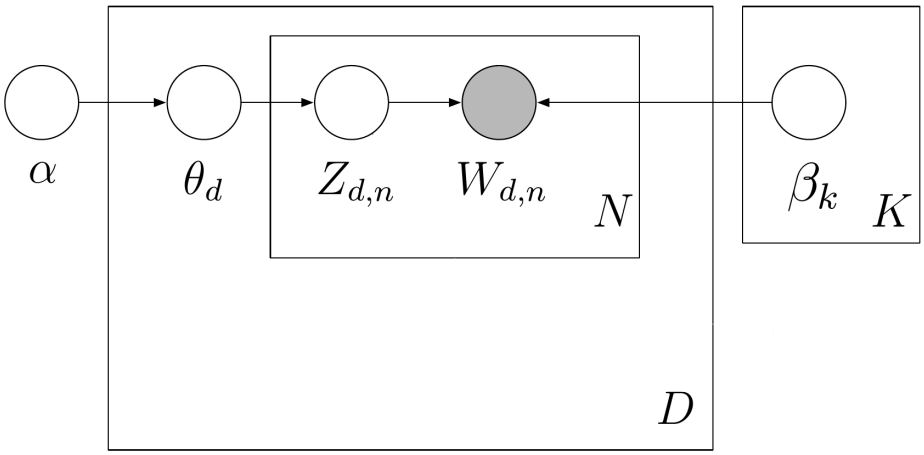


Figure 1: A graphical representation of traditional LDA model.

We however, use an extended version of LDA, which makes use of the given scores. Because of this, a third step is added to the generative process:

1. Draw topic proportions $\theta \mid \alpha \sim \text{Dir}(\alpha)$.
2. For each word:
   (a) Draw topic assignment $z_n \mid \theta \sim \text{Mult}(\theta)$.
   (b) Draw word $w_n \mid z_n, \beta_{1:K} \sim \text{Mult}(\beta_{zn})$.
3. Draw response variable $y \mid z_{1:N}, \eta, \sigma^2 \sim \mathcal{N}(\eta^\top \bar{z}, \sigma^2)$.

This version can be called supervised LDA or SLDA. Its graphical representation can be seen in Figure 3.1. Note that we use a symmetric $\alpha$ and $\beta$, i.e. $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_K)$, where $\forall i, j \leq K : \alpha_i = \alpha_j$ and similar for $\beta$.

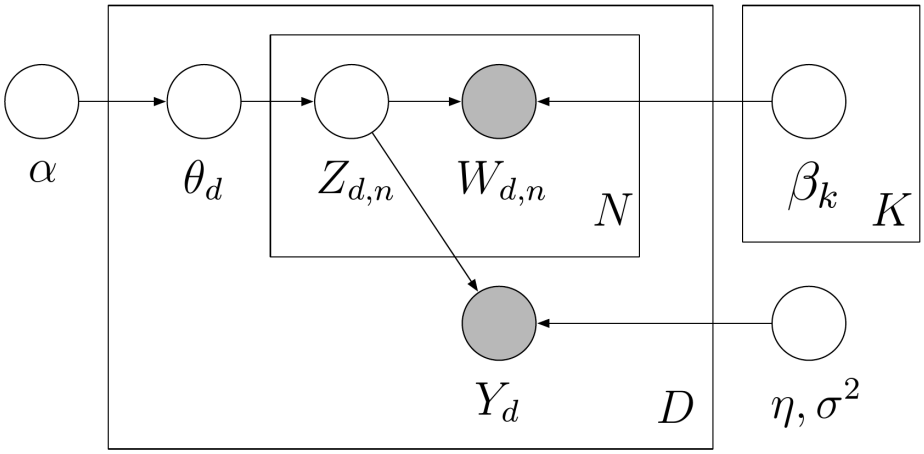


Figure 2: A graphical representation of our modified LDA model.

### 3.1.1 Collapsed Supervised Latent Dirichlet Allocation

In this section we will give the derivations for the collapsed supervised LDA model.

We start by recalling the full likelihood expression (i.e. for all variables, both latent and visible) of the model:

$$P(\Phi, \Theta, S, Z, W \mid \alpha, \beta, \eta, \sigma) = \left[\prod_{k=1}^{K} \mathrm{Dir}(\varphi_k \mid \beta)\right] \times \left[\prod_{d=1}^{D} \mathrm{Dir}(\theta_d \mid \alpha)\mathcal{N}(s_d \mid \eta^\top \bar{z}_d, \sigma) \prod_{i=1}^{N_d} \mathrm{Mult}(z_{di} \mid \theta_d)\, \mathrm{Mult}(w_{di} \mid \varphi_{z_{di}})\right] \tag{1}$$

The final collapsed SLDA will only give probabilities for $S, Z$ and $W$. To achieve this, we will first integrate out the latent variable $\Phi$. The first thing that can be noticed is that every $\varphi_k$ is independently sampled, and thus can be integrated separately:

$$P(\Theta, S, Z, W \mid \alpha, \beta, \eta, \sigma) = \int_{\varphi_0} \int_{\varphi_1} \cdots \int_{\varphi_K} P(\Phi, \Theta, S, Z, W \mid \alpha, \beta, \eta, \sigma) \tag{2}$$

At this point we need to rewrite part of Equation 1 in order to be able to continue the derivation:

$$\prod_{d=1}^{D} \prod_{i=1}^{N_d} \mathrm{Mult}(w_{di} \mid \varphi_{z_{di}}) = \prod_{d=1}^{D} \prod_{w=1}^{W} \prod_{k=1}^{K} \mathrm{Mult}(w \mid \varphi_k)^{N_{dk}}, \tag{3}$$

where $N_d$ represents the total number of words within document $d$, and $N_{dk}$

represents the number of words within document $d$ assigned to topic $k$.

$$P(\Theta, S, Z, W \mid \alpha, \beta, \eta, \sigma) = \left[\prod_{k=1}^{K} \int_{\varphi_k} \mathrm{Dir}(\varphi_k \mid \beta) \prod_{d=1}^{D} \prod_{w=1}^{W} \mathrm{Mult}(w \mid \varphi_k)^{N_{dk}}\right] \times \left[\prod_{d=1}^{D} \mathrm{Dir}(\theta_d \mid \alpha)\mathcal{N}(s_d \mid \eta^\top \bar{z}_d, \sigma) \prod_{i=1}^{N_d} \mathrm{Mult}(z_{d,i} \mid \theta_d)\right] \tag{4}$$

We can now make use of the definition of the Dirichlet-Multinomial distribution in order to solve all the integrals:

$$P(\Theta, S, Z, W \mid \alpha, \beta, \eta, \sigma) = \left[\prod_{k=1}^{K} \frac{\Gamma(W\beta)}{\Gamma(N_k + W\beta)} \prod_{w=1}^{W} \frac{\Gamma(N_{kw} + \beta)}{\Gamma(\beta)}\right] \times \left[\prod_{d=1}^{D} \mathrm{Dir}(\theta_d \mid \alpha)\mathcal{N}(s_d \mid \eta^\top \bar{z}_d, \sigma) \prod_{i=1}^{N_d} \mathrm{Mult}(z_{di} \mid \theta_d)\right] \tag{5}$$

We can proceed in an analogous fashion in order to integrate out the latent $\Theta$ parameter:

$$P(S, Z, W \mid \alpha, \beta, \eta, \sigma) = \int_{\theta_0} \int_{\theta_1} \dots \int_{\theta_D} p(\Theta, S, Z, W \mid \alpha, \beta, \eta, \sigma) \tag{6}$$

This gives us:

$$P(S, Z, W \mid \alpha, \beta, \eta, \sigma) = \left[\prod_{k=1}^{K} \frac{\Gamma(W\beta)}{\Gamma(N_k + W\beta)} \prod_{w=1}^{W} \frac{\Gamma(N_{kw} + \beta)}{\Gamma(\beta)}\right] \times \left[\prod_{d=1}^{D} \mathcal{N}(s_d \mid \eta^\top \bar{z}_d, \sigma) \frac{\Gamma(K\alpha)}{\Gamma(N_d + K\alpha)} \prod_{k=1}^{K} \frac{\Gamma(N_{dk} + \alpha)}{\Gamma(\alpha)}\right] \tag{7}$$

Note that $\Gamma(\beta)$ and $\Gamma(\alpha)$ are constant in these products, so they can be moved outside of the product.

$$P(S, Z, W \mid \alpha, \beta, \eta, \sigma) = \left[\prod_{k=1}^{K} \frac{\Gamma(W\beta)}{\Gamma(\beta)^W \Gamma(N_k + W\beta)} \prod_{w=1}^{W} \Gamma(N_{kw} + \beta)\right] \times \left[\prod_{d=1}^{D} \mathcal{N}(s_d \mid \eta^\top \bar{z}_d, \sigma) \frac{\Gamma(K\alpha)}{\Gamma(\alpha)^K \Gamma(N_d + K\alpha)} \prod_{k=1}^{K} \Gamma(N_{dk} + \alpha)\right] \tag{8}$$

### 3.1.2 Collapsed Gibbs Sampler

Using Bayes' theorem, we know that:

$$P(Z \mid S, W, \dots) = \frac{P(S, W \mid Z, \dots)P(Z \mid \dots)}{P(S, W \mid \dots)} \propto P(S, W, Z \mid \dots), \tag{9}$$

where we did not write all model hyperparameters for the sake of clarity. Thus, in order to implement a Gibbs sampler for this model, we have that:

$$P(z_{di} = k \mid Z^{\setminus i}, S, W, \alpha, \beta, \eta, \sigma) \propto P(z_{di} = k, Z^{\setminus i}, S, W, \alpha, \beta, \eta, \sigma) =$$

More formulas still need to be written here...

We can further simplify the previous expression by removing those factors that are constant across all possible values for $k$, resulting in the following final collapsed Gibbs sampler:

$$P(z_{di} = k \mid Z^{\setminus i}, S, W, \dots) \propto \left[\prod_{k'} \frac{\prod_w \Gamma(N^{\setminus i}_{k'w} + \mathbb{I}(k' = k \wedge w = w_{di}) + \beta)}{\Gamma(N^{\setminus i}_{k'} + \mathbb{I}(k' = k) + W\beta)}\right] \times$$

$$\mathcal{N}\left(s_d \;\middle|\; \eta^T \frac{N^{\setminus i}_{dk'} + \mathbb{I}(k' = k)}{N_d}, \sigma\right) \prod_{k'} \Gamma(N^{\setminus i}_{dk'} + \mathbb{I}(k' = k) + \alpha), \tag{10}$$

where we used $\frac{N_{dk}}{N_d} \equiv \bar{z_d}$.

The Gibbs sampler can also be expressed in log-space probabilities, in order to get a more numerically-stable implementation:

$$\log P(z_{di} = k \mid Z^{\setminus i}, S, W, \alpha, \beta, \eta, \sigma)$$
$$\propto \sum_{k',w} \left[\log \Gamma(N^{\setminus i}_{k'w} + \mathbb{I}(k' = k \wedge w = w_{di}) + \beta) - \log \Gamma(N^{\setminus i}_{k'} + \mathbb{I}(k' = k) + W\beta)\right]$$
$$- \frac{1}{2\sigma^2}\left(s_d - \eta^\top \frac{N^{\setminus i}_{dk'} + \mathbb{I}(k' = k)}{N_d}\right)^2 + \sum_{k'} \log \Gamma(N^{\setminus i}_{dk'} + \mathbb{I}(k' = k) + \alpha)$$

### 3.1.3 Rewriting the log-gamma function

The logarithm of the gamma function can be rewritten as follows [3]:

$$\log \Gamma(z) = -\gamma z - \log z + \sum_{j=1}^{\infty} \left[\frac{z}{j} - \log\left(1 + \frac{z}{j}\right)\right], \tag{11}$$

where $\gamma$ is the Euler-Mascheroni constant. We apply this to $\sum_k \sum_w \log \Gamma(N_{kw} + \beta)$, which then becomes:

$$= \sum_{k,w} -\gamma(N_{kw} + \beta) - \log(N_{kw} + \beta) + \sum_{j=1}^{\infty} \frac{N_{kw} + \beta}{j} - \log\left(1 + \frac{N_{kw} + \beta}{j}\right)$$
$$= -\gamma(N + KW\beta) - \sum_{k,w} \log(N_{kw} + \beta) - \sum_{j=1}^{\infty} \frac{N_{kw} + \beta}{j} - \log\left(\frac{N_{kw} + \beta + j}{j}\right)$$

Note that the term $-\gamma(N + KW\beta)$ serves as a normalisation constant for this dataset. Since we do not need the exact probabilities but only the proportional probabilities during the algorithms execution, we can discard those terms.

$$
\begin{aligned}
&\Rightarrow -\sum_{k,w} \log(N_{kw} + \beta) - \sum_{j=1}^{\infty} \frac{N_{kw} + \beta}{j} - \log\left(\frac{N_{kw} + \beta + j}{j}\right) \\
&= -\sum_{k,w} \log(N_{kw} + \beta) - \sum_{j=1}^{\infty} \frac{N_{kw} + \beta}{j} - \log(N_{kw} + \beta + j) + \log(j) \\
&= -\sum_{k,w} \log(N_{kw} + \beta) - \sum_{j=1}^{\infty} \left(\frac{N_{kw} + \beta}{j} + \log(j)\right) + \sum_{j=1}^{\infty} \log(N_{kw} + \beta + j) \\
&= \sum_{j=1}^{\infty} \left(\frac{N + KW\beta}{j} + KW \log(j)\right) - \sum_{k,w} \log(N_{kw} + \beta) + \sum_{j=1}^{\infty} \log(N_{kw} + \beta + j) \\
&= \sum_{j=1}^{\infty} \left(\frac{N + KW\beta}{j} + KW \log(j)\right) - \sum_{k,w} \sum_{j=0}^{\infty} \log(N_{kw} + \beta + j)
\end{aligned}
$$

Again, the term $\sum_{j=1}^{\infty} \frac{N+KW\beta}{j} + KW \log(j)$ is a constant for this dataset, so we can discard it. This results in the following proportionality:

$$\sum_{k,w} \log \Gamma(N_{kw} + \beta) \propto -\sum_{k,w} \sum_{j=0}^{\infty} \log(N_{kw} + \beta + j) \tag{12}$$

Using similar steps, we can also simplify

$$\sum_{k} \log \Gamma(N_{k} + W\beta) \propto -\sum_{k} \sum_{j=0}^{\infty} \log(N_{k} + W\beta + j) \tag{13}$$

$$\sum_{k,d} \log \Gamma(N_{dk} + \alpha) \propto -\sum_{k,d} \sum_{j=0}^{\infty} \log(N_{dk} + \alpha + j) \tag{14}$$

$$\sum_{d} \log \Gamma(N_{d} + K\alpha) \propto -\sum_{d} \sum_{j=0}^{\infty} \log(N_{d} + K\alpha + j) \tag{15}$$

Putting these together gives us the following proportionality:

$$
\begin{aligned}
&\log P(z_{di} = k \mid Z^{\setminus i}, S, W, \alpha, \beta, \eta, \sigma) \\
&\propto \sum_{j=0}^{\infty} \sum_{k'} \log(N_{k'}^{\setminus i} + \mathbb{I}(k' = k) + W\beta + j) - \log(N_{dk'}^{\setminus i} + \mathbb{I}(k' = k) + \alpha + j) \\
&- \sum_{j=0}^{\infty} \sum_{k',w} \log(N_{k'w}^{\setminus i} + \mathbb{I}(k' = k \wedge w = w_{di}) + \beta + j) - \frac{1}{2\sigma^2} \left(s_d - \eta^\top \frac{N_{dk'}^{\setminus i} + \mathbb{I}(k' = k)}{N_d}\right)^2
\end{aligned}
$$

In practice, however, we can not calculate the infinite terms of the above sums, and thus we would stop the approximation at some arbitrary iteration $J$:

$$\log P(z_{di} = k \mid Z^{\setminus i}, S, W, \alpha, \beta, \eta, \sigma)$$

$$\propto -\frac{1}{2\sigma^2}\left(s_d - \eta^\top \frac{N_{dk'}^{\setminus i} + \mathbb{I}(k' = k)}{N_d}\right)^2 + \sum_{k'}\sum_{j=0}^{J} \log(N_{k'}^{\setminus i} + \mathbb{I}(k' = k) + W\beta + j)$$

$$- \log(N_{dk'}^{\setminus i} + \mathbb{I}(k' = k) + \alpha + j) + \sum_{w} \log(N_{k'w}^{\setminus i} + \mathbb{I}(k' = k \wedge w = w_{di}) + \beta + j)$$

### 3.1.4 Estimating response parameters

The maximum a posteriori (MAP) estimate for the hyperparameter $\eta$ can be inferred from the gradient of the complete model likelihood:

$$\begin{aligned}\nabla_{\eta_k} \log P(S, Z, W \mid \alpha, \beta, \eta, \sigma) &= \sum_d \frac{1}{\sigma^2}\frac{N_{dk}}{N_d}\left(s_d - \eta^T \frac{N_{d\cdot}}{N_d}\right) \\ &= \sum_d \frac{s_d \frac{N_{dk}}{N_d}}{\sigma^2} - \sum_d \frac{\frac{N_{dk}}{N_d}\left(\eta^T \frac{N_{d\cdot}}{N_d}\right)}{\sigma^2}\end{aligned} \tag{16}$$

For the MAP estimate, the gradient should be zero. This allows us to rewrite the above formula into:

$$\begin{aligned}\sum_d s_d \frac{N_{dk}}{N_d} &= \sum_d \frac{N_{dk}}{N_d}\left(\sum_{k'} \eta_{k'} \frac{N_{dk'}}{N_d}\right) \\ \sum_d s_d \frac{N_{dk}}{N_d} &= \sum_d \frac{N_{dk}}{N_d}\left(\eta_k \frac{N_{dk}}{N_d} + \sum_{k' \neq k} \eta_{k'} \frac{N_{dk'}}{N_d}\right) \\ \sum_d s_d \frac{N_{dk}}{N_d} &= \eta_k \sum_d \left(\frac{N_{dk}}{N_d}\right)^2 + \sum_d \left(\frac{N_{dk}}{N_d} \sum_{k' \neq k} \eta_{k'} \frac{N_{dk'}}{N_d}\right)\end{aligned}$$

Further rewriting finally gives us the MAP estimate for $\eta$:

$$\begin{aligned}\eta_k \sum_d \left(\frac{N_{dk}}{N_d}\right)^2 &= \sum_d \left(s_d \frac{N_{dk}}{N_d} - \frac{N_{dk}}{N_d} \sum_{k' \neq k} \eta_{k'} \frac{N_{dk'}}{N_d}\right) \\ \eta_k \sum_d \left(\frac{N_{dk}}{N_d}\right)^2 &= \sum_d \frac{N_{dk}}{N_d}\left(s_d - \sum_{k' \neq k} \eta_{k'} \frac{N_{dk'}}{N_d}\right) \\ \eta_k &= \frac{\sum_d \frac{N_{dk}}{N_d}\left(s_d - \sum_{k' \neq k} \eta_{k'} \frac{N_{dk'}}{N_d}\right)}{\sum_d \left(\frac{N_{dk}}{N_d}\right)^2}\end{aligned} \tag{17}$$

From our experiments, we noticed that trying to apply the formula as described in Equation 17 as an update rule for $\eta$ does not converge. Instead, the following

update can be used:

$$\eta_k^{new} \leftarrow (1-\gamma)\eta_k^{old} + \gamma \frac{\sum_d \frac{N_{dk}}{N_d}\left(s_d - \sum_{k' \neq k} \eta_{k'} \frac{N_{dk'}}{N_d}\right)}{\sum_d \left(\frac{N_{dk}}{N_d}\right)^2 + \varepsilon}, \tag{18}$$

where we used $1 \gg \gamma > 0$ in order for the previous series to converge and $1 \gg \varepsilon > 0$ as a smoothing constant.

# 4 Experiments

For our experiments we chose to use two different performance measures: perplexity and inverse accuracy.

## 4.1 Perplexity

The perplexity measure is commonly used within LDA literature, and we include it for reference purposes. It is defined as follows:

$$\text{Perplexity} \equiv \exp\left\{-\frac{1}{N}\sum_{i=1}^{N} \log P(x_i)\right\}, \tag{19}$$

where $P(x_i)$ represents the probability the model gives for the $i$-th item within the test set. The lower the perplexity, the less "surprised" the model is of seeing its input, which indicates a better model.

This measure has a shortcoming, though. It is enough for the model to give a probability of zero to a word to drive this performance measure to infinity. This does not mean, however, that the model is infinitely bad, since the rest of the items might still have large probabilities. To overcome this problem, we also measure the performance in terms of the average inverse accuracy of the model.

## 4.2 Inverse accuracy

The average inverse accuracy of the model is defined as follows:

$$\text{Accuracy}^{-1} \equiv \frac{1}{\frac{1}{N}\sum_{i=1}^{N} P(x_i)} \tag{20}$$

This measure should be interpreted as the number of words the model incorrectly predicts for each correctly predicted word. The lower this magnitude, the better the model.

## 4.3 Results

We created a first experiment in order to check the validity of our approximation for the Gibbs sampler of our model. We selected different values for $J$ and

compared the predictive performance for each case. We ran three different MCMC chains and then averaged the results. Within each execution, 100 movies were randomly selected as the training set, and 20 as the testing set. We used 5 samples, taken every 4 steps of the Gibbs sampler, with the first 20 iterations discarded as the burn-in period. Our findings are summarized in Table 1:

| J | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Perplexity | 4100 | 4125 | 4082 | 4251 | 3754 | 3881 |
| Accuracy$^{-1}$ | 858 | 871 | 823 | 863 | 877 | 815 |

Table 1: Experiment results for varying values of $J$ with 100/20 movies as training/test set.

We can observe that the predictive performance tends to improve with larger values of $J$. This is to be expected, since the higher $J$ is, the better the approximation of the log-gamma function becomes. This improvement is not, however, very consistent even when averaging across 3 chains, and thus we theorize that the extra computational time spent on making $J$ higher is better used if $J$ is kept as low as possible and more movies and/or iterations are used instead. We then conclude this approximation is interesting, at least from a computational perspective.

We devised a second experiment in order to find out good values for the number of topics to use in our models. Again we did an equivalent set up to the one for the previous experiment, but this time changing the number of topics and whether or not the Gibbs sampler uses the movie score information.

Perplexity measure:

| LDA topics | | 25 | 50 | 100 |
|---|---|---|---|---|
| Using scores? | No | 4128 | 5333 | 6954 |
| | Yes | **4005** | 5503 | 7082 |

Inverse accuracy measure:

| LDA topics | | 25 | 50 | 100 |
|---|---|---|---|---|
| Using scores? | No | 845 | 1318 | 2049 |
| | Yes | **805** | 1372 | 2067 |

Figure 3: Experiment results for varying values of $K$, with $J = 1$ and 100/20 movies as training/test set.

We can see how all models start overfitting with, at least, more than 25 topics. This is most likely due to the fact that we only used 100 movie plot summaries for training, which is a rather small dataset. We also observe that, for 25 topics, not only the best predictive performance is achieved, but this is further improved by using the movie scores. This improvement is very small, but we could systematically observe it in all the tests we run for this work.

# 5 Discussion

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### References


[1] David M. Blei, Jon D. McAuliffe; *Supervised Topic Models*; Neural Information Processing Systems 21 (2007)

[2] IMDb.com, Inc.; *IMDb, the world's most popular and authorative source for movie, TV and celebrity content*; `www.imdb.com`

[3] Geogre Boros, Victor H. Moll, *Irresistible Integrals: symbolics, analysis and experiments in the evaluation of integrals*; Cambridge University Press (2004)