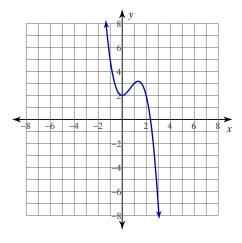
## Intervals of Increase and Decrease

For each problem, find the x-coordinates of all critical points, find all discontinuities, and find the open intervals where the function is increasing and decreasing.

1) 
$$y = -x^3 + 2x^2 + 2$$



2) 
$$y = x^3 - 11x^2 + 39x - 47$$

3) 
$$y = -x^4 + 3x^2 - 3$$

4) 
$$y = \frac{x^2}{4x + 4}$$

$$5) \ \ y = \frac{3x^2 - 3}{x^3}$$

6) 
$$y = (2x - 8)^{\frac{2}{3}}$$

7) 
$$y = -\frac{1}{5}(x-4)^{\frac{5}{3}} - 2(x-4)^{\frac{2}{3}} - 1$$

## **Critical thinking question:**

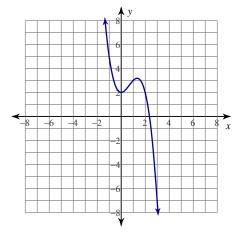
8) If functions f and g are increasing on an interval, show that f + g is increasing on the same interval.

9) Give an example where functions f and g are increasing on the interval  $(-\infty,\infty)$ , but where f-g is decreasing.

## Intervals of Increase and Decrease

For each problem, find the x-coordinates of all critical points, find all discontinuities, and find the open intervals where the function is increasing and decreasing.

1) 
$$y = -x^3 + 2x^2 + 2$$



Critical points at:  $x = 0, \frac{4}{3}$  No discontinuities exist.

Increasing:  $\left(0, \frac{4}{3}\right)$  Decreasing:  $\left(-\infty, 0\right), \left(\frac{4}{3}, \infty\right)$ 

2) 
$$y = x^3 - 11x^2 + 39x - 47$$

Critical points at: x = 3,  $\frac{13}{3}$  No discontinuities exist.

Increasing:  $(-\infty, 3), \left(\frac{13}{3}, \infty\right)$  Decreasing:  $\left(3, \frac{13}{3}\right)$ 

3) 
$$y = -x^4 + 3x^2 - 3$$

Critical points at:  $x = -\frac{\sqrt{6}}{2}$ , 0,  $\frac{\sqrt{6}}{2}$  No discontinuities exist.

Increasing:  $\left(-\infty, -\frac{\sqrt{6}}{2}\right), \left(0, \frac{\sqrt{6}}{2}\right)$  Decreasing:  $\left(-\frac{\sqrt{6}}{2}, 0\right), \left(\frac{\sqrt{6}}{2}, \infty\right)$ 

4) 
$$y = \frac{x^2}{4x + 4}$$

Critical points at: x = -2, 0 Discontinuity at: x = -1 Increasing:  $(-\infty, -2)$ ,  $(0, \infty)$  Decreasing: (-2, -1), (-1, 0)

$$5) \ \ y = \frac{3x^2 - 3}{x^3}$$

Critical points at: 
$$x = -\sqrt{3}$$
,  $\sqrt{3}$  Discontinuity at:  $x = 0$  Increasing:  $(-\sqrt{3}, 0)$ ,  $(0, \sqrt{3})$  Decreasing:  $(-\infty, -\sqrt{3})$ ,  $(\sqrt{3}, \infty)$ 

6) 
$$y = (2x - 8)^{\frac{2}{3}}$$

Critical point at: 
$$x = 4$$
 No discontinuities exist.  
Increasing:  $(4, \infty)$  Decreasing:  $(-\infty, 4)$ 

7) 
$$y = -\frac{1}{5}(x-4)^{\frac{5}{3}} - 2(x-4)^{\frac{2}{3}} - 1$$

Critical points at: x = 0, 4 No discontinuities exist. Increasing: (0, 4) Decreasing:  $(-\infty, 0)$ ,  $(4, \infty)$ 

## **Critical thinking question:**

8) If functions f and g are increasing on an interval, show that f + g is increasing on the same interval.

We know that if 
$$x_1 < x_2$$
, then  $f(x_1) < f(x_2)$  and  $g(x_1) < g(x_2)$ . Therefore,  $f(x_1) + g(x_1) < f(x_2) + g(x_2)$ .

9) Give an example where functions f and g are increasing on the interval  $(-\infty,\infty)$ , but where f-g is decreasing.

Many answers. Ex: f = x and g = 2x