Aspects of superradiant scattering off Kerr black holes

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Outline

- What is superradiance ?
- Kerr black hole and the Penrose process
- Newman-Penrose formalism and the Teukolsky equation
- Analytical approximated solutions
- Numerical methods
- Scattering of planes waves
- Conclusions and future work

Default

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Example

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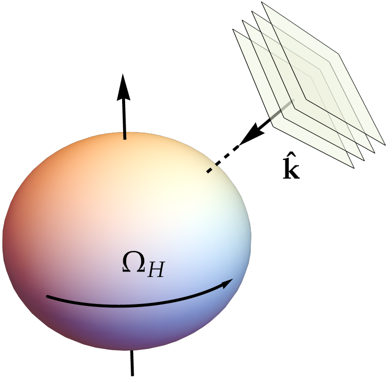


Boyer-Lindquist coordinates

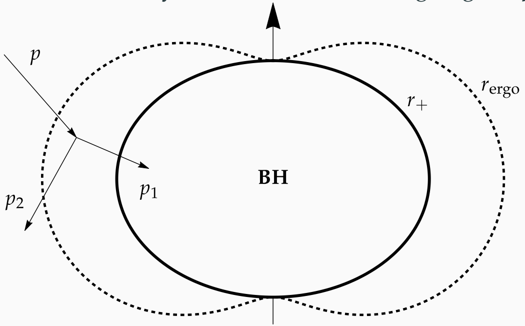
$$g = \left(1 - \frac{2Mr}{\rho^2}\right) \mathrm{d}t^2 + 2a \sin^2\theta \frac{(r^2 + a^2 - \Delta)}{\rho^2} \mathrm{d}t \mathrm{d}\varphi$$
$$- \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}{\rho^2} \sin^2\theta \mathrm{d}\varphi^2 - \frac{\rho^2}{\Delta} \mathrm{d}r^2 - \rho^2 \mathrm{d}\theta^2$$

$$[\, \Delta = r^2 - 2Mr + a^2 \,, \;\; \rho^2 = r^2 + a^2 \cos^2\theta \,]$$

- BH angular momentum $J = aM \quad [a = 0 \;\Rightarrow \text{Schwarzschild}]$
- Killing vectors $\boldsymbol{k} = \partial_t$ (stationary) and $\boldsymbol{m} = \partial_\varphi$ (axisymmetric)
- Horizons at $g^{rr} = 0 \;\Rightarrow\; \Delta = 0 \;\Rightarrow\; r = r_\pm \equiv M \pm \sqrt{M^2 - a^2}$
  Cosmic censorship conjecture $\;\Rightarrow\; |a| \leq M$
- Infinite redshift boundary $g_{tt} = 0 \;\Rightarrow$ Ergoregion $(g_{tt} < 0)$

$$r_+ < r < r_{\text{ergo}}(\theta) \equiv M + \sqrt{M^2 - a^2 \cos^2\theta}$$

- Particle decays into two inside the ergoregion: $\boldsymbol{p} = \boldsymbol{p_1} + \boldsymbol{p_2}$



Killing horizon

Hypersurface $r = r_+$ with normal null vector

$$\boldsymbol{\xi} = \boldsymbol{k} + \Omega_H \boldsymbol{m}$$

Event horizon angular momentum:

$$\Omega_H = \frac{a}{2Mr_+}$$

- We may have $E_1 = \boldsymbol{k} \cdot \boldsymbol{p_1} < 0 \;\Rightarrow E_2 = E + |E_1| > E$
- Local energy condition $\boldsymbol{\xi} \cdot \boldsymbol{p_1} > 0$ at $r = r_+ \;\Rightarrow E_1 - \Omega_H L_1 > 0$

$$\psi \sim e^{-i\omega t + im\varphi} \;\Rightarrow\; \frac{\delta J}{\delta M} = \frac{\hbar m}{\hbar \omega} \;\Rightarrow\; \delta M < 0 \;\wedge\; \boxed{\omega(\omega - \Omega_H m) < 0}$$

Energy extraction !

Newman-Penrose formalism

- Electromagnectic perturbations ($A^\mu \ll 1$)

$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} = 8\pi T_{\mu\nu} = 0 + \mathcal{O}(A^2) \;\Rightarrow \text{fixed background (Kerr)}$$

- Tetrad frame of complex null vectors $(e_1, e_2, e_3, e_4) = (\mathfrak{l}, \mathfrak{n}, \mathfrak{m}, \bar{\mathfrak{m}} \equiv \mathfrak{m}^*)$ with $\mathfrak{l} \cdot \mathfrak{m} = \mathfrak{n} \cdot \mathfrak{m} = 0\,,\;\; \mathfrak{l} \cdot \mathfrak{n} = 1\,,\;\; \mathfrak{m} \cdot \bar{\mathfrak{m}} = -1$

$$\underbrace{\mathbf{E}\,,\,\mathbf{B}}_{\text{6 real components}} \longrightarrow \boxed{\begin{aligned} \phi_0 &= F_{\mu\nu} \mathfrak{l}^\mu \mathfrak{m}^\nu \\ \phi_1 &= \tfrac{1}{2} F_{\mu\nu} (\mathfrak{l}^\mu \mathfrak{n}^\nu - \mathfrak{m}^\mu \bar{\mathfrak{m}}^\nu) \\ \phi_2 &= F_{\mu\nu} \bar{\mathfrak{m}}^\mu \mathfrak{n}^\nu \end{aligned}}$$

- Equations written using tetrad derivatives $(\mathbb{D}, \mathbb{\Delta}, \mathbb{\delta}, \bar{\mathbb{\delta}})$ and spin connection $\gamma_{cab} = (e_c)^\mu (e_b)^\nu \nabla_\nu (e_a)_\mu \;\Rightarrow\; \lambda, \pi, \tau, \varrho, \varepsilon, \sigma, \kappa, \gamma, \mu, \nu, \alpha, \beta$

$$\begin{aligned} \nabla_\mu F^{\mu\nu} &= 0 \\ \nabla_{[\mu} F_{\nu\rho]} &= 0 \end{aligned} \longrightarrow \begin{cases} \mathbb{D}\phi_2 - \bar{\mathbb{\delta}}\phi_1 = -\lambda\phi_0 + 2\pi\phi_1 + (\varrho - 2\varepsilon)\phi_2 \\ \mathbb{\Delta}\phi_1 - \mathbb{\delta}\phi_2 = \nu\phi_0 - 2\mu\phi_1 + (2\beta - \tau)\phi_2 \\ \mathbb{D}\phi_1 - \bar{\mathbb{\delta}}\phi_0 = (\pi - 2\alpha)\phi_0 + 2\varrho\phi_1 - \kappa\phi_2 \\ \mathbb{\Delta}\phi_0 - \mathbb{\delta}\phi_1 = (2\gamma - \mu)\phi_0 - 2\tau\phi_1 + \sigma\phi_2 \end{cases}$$

Newman-Penrose formalism

Kinnersley tetrad

$$\mathfrak{l} = \frac{1}{\Delta}\left(r^2 + a^2,\, \Delta,\, 0,\, a\right)$$
$$\mathfrak{n} = \frac{1}{2\rho^2}\left(r^2 + a^2,\, -\Delta,\, 0,\, a\right)$$
$$\mathfrak{m} = \frac{1}{\sqrt{2}\bar{\rho}}\left(i\,a\sin\theta,\, 0,\, 1,\, i\csc\theta\right)$$

$$\left[\,\bar{\rho} = r + i\,a\cos\theta\,\right]$$

- $\mathfrak{l}$, $\mathfrak{n}$ are the doubly degenerate principal null of directions of the Weyl tensor

$$\mathfrak{l} \sim (\boldsymbol{e}_t + \boldsymbol{e}_r) \qquad \text{(ingoing)}$$
$$\mathfrak{n} \sim \tfrac{1}{2}(\boldsymbol{e}_t - \boldsymbol{e}_r) \quad \text{(outgoing)}$$

- $\mathfrak{l}$, $\mathfrak{n}$ are geodesic ($\kappa = \lambda = 0$) and shear-free ($\nu = \sigma = 0$)
  $\Rightarrow$ equations decouple

- Reformulation of the fields: $\Phi_n = (\sqrt{2}\bar{\rho})^n\,\phi_n \quad (n = 0, 1, 2)$
- Spacetime symmetry $\Rightarrow \Phi_n \sim e^{-i\omega t + im\phi} \Rightarrow \partial_t \to -i\omega\,,\ \partial_\varphi \to im$
- Rewrite operators: $\mathbb{D} = \mathcal{D}_0\,,\ \mathbb{A} = -\frac{\Delta}{2\rho^2}\mathcal{D}_0^\dagger$

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\emph{emphasize} text, \alert{accent} parts
or show \textbf{bold} results.
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The theme provides sensible defaults to *emphasize* text, accent parts or show **bold** results.

Coordinate frame

- Vector (coordinate) basis

$$\boldsymbol{e}_\mu \equiv \frac{\partial}{\partial x^\mu} \quad (\mu = t, r, \theta, \varphi)$$

- Spacetime metric

$$g_{\mu\nu}$$

- Tensorial components

$$A_\mu \ , \ F_{\alpha\beta}$$

- Covariant derivative

$$A_{\mu;\nu} = A_{\mu,\nu} - \Gamma^\rho_{\mu\nu} A_\rho$$

- Levi-Civita connection

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma} (g_{\sigma\mu,\nu} + g_{\sigma\mu,\nu} - g_{\mu\nu,\sigma})$$

Tetrad frame

- Tetrad (non-coordinate) basis

$$\boldsymbol{e}_a \equiv (e_a)^\mu \frac{\partial}{\partial x^\mu} \quad (a = 1, 2, 3, 4)$$

- Raising/lowering tetrad indices

$$\eta_{ab} = \boldsymbol{e}_a \cdot \boldsymbol{e}_b = (e_a)^\mu (e_b)^\nu g_{\mu\nu}$$

- Tetrad (NP) scalars

$$A_a = (e_a)^\mu A_\mu \ , \ F_{ab} = (e_a)^\alpha (e_b)^\beta F_{\alpha\beta}$$

- Intrinsic derivative

$$A_{a|b} \equiv (e_a)^\mu (e_b)^\nu A_{\mu;\nu} = A_{a,b} - \gamma_{cab} A^c$$

- Spin connection

$$\gamma_{cab} = (e_c)^\mu (e_a)_{\mu;\nu} A_{\mu;\nu} = (e_b)^\nu$$

Kinnersley tetrad

$$\mathfrak{l} = \frac{1}{\Delta}\left(r^2 + a^2,\, \Delta,\, 0,\, a\right)$$
$$\mathfrak{n} = \frac{1}{2\rho^2}\left(r^2 + a^2,\, -\Delta,\, 0,\, a\right)$$
$$\mathfrak{m} = \frac{1}{\sqrt{2}\bar{\rho}}\left(ia\sin\theta,\, 0,\, 1,\, i\csc\theta\right)$$

$$\begin{aligned} {}_{\pm1}Z_{\ell m} \simeq &-4\bar{\omega}(\bar{\omega} - m\bar{\Omega}_H)\,(2-\tau)(2\bar{\omega}\tau)^{2\ell} \\ &\times \left[\frac{(\ell-1)!(\ell+1)!}{(2\ell)!(2\ell+1)!}\right]^2 \prod_{n=1}^{\ell}\left(n^2 + \frac{4\varpi^2}{\tau^2}\right) \end{aligned}$$

Typography

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or show \textbf{bold} results.
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becomes

The theme provides sensible defaults to *emphasize* text, accent parts or show **bold** results.

Font feature test

- Regular
- *Italic*
- SmallCaps
- **Bold**
- ***Bold Italic***
- **Bold SmallCaps**
- `Monospace`
- *`Monospace Italic`*
- **`Monospace Bold`**
- ***`Monospace Bold Italic`***

Lists

Items

- Milk
- Eggs
- Potatos

Enumerations

1. First,
2. Second and
3. Last.

Descriptions

PowerPoint Meeh.

Beamer Yeeeha.

Animation

- This is important

Animation

- This is important
- Now this

Animation

- This is important
- Now this
- And now this

Animation

- This is really important
- Now this
- And now this

Three different block environments are pre-defined and may be styled with an optional background color.

Default

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Alert

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Example

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Block content.

Alert

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Example

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Math

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

Quotes

Veni, Vidi, Vici

Frame footer

Metropolis defines a custom beamer template to add a text to the footer. It can be set via

```
\setbeamertemplate{frame footer}{My custom footer}
```

Conclusion

Summary

Get the source of this theme and the demo presentation from

`github.com/matze/mtheme`

Questions?

Line plots

BAR CHARTS

Backup slides

Sometimes, it is useful to add slides at the end of your presentation to refer to during audience questions.

The best way to do this is to include the `appendixnumberbeamer` package in your preamble and call `\appendix` before your backup slides.

Metropolis will automatically turn off slide numbering and progress bars for slides in the appendix [1].

References

[1] J. G. Rosa, *Superradiance in the sky*, Phys. Rev. **D95**, 064017 (2017), arXiv:1612.01826 [gr-qc] .