Superradiant scattering at Kerr black holes

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UNIVERSIDADE DO PORTO

MASTER'S THESIS

Superradiant scattering at Kerr black holes

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Abstract

Faculdade de Ciências da Universidade do Porto
Departamento de Física e Astronomia

Master of Science

Superradiant scattering at Kerr black holes

by José SÁ

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Resumo

Faculdade de Ciências da Universidade do Porto
Departamento de Física e Astronomia

Mestre de Ciência

Superradiant scattering at Kerr black holes

por José SÁ

Tradução em português do "Abstract" escrito em inglês mais a cima. A página é centrada vertical e horizontalmente, podendo espandir para o espaço superior da página em branco ...

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Abbreviations

QM	Quantum Mechanics
BH	Black Hole
GR	General Relativity
GW	Gravitational Wave
LIGO	Laser Interferometric Gravitational Wave Observatory
SWSH	Spin-Weighted Spheroidal Harmonic

Chapter 1

Superradiance

1.1 Introduction

The first appearance of the concept of *superradiance* was in 1954, when Dicke [1] showed that a gas could be excited by a pulse into "superradiant states" from thermal equilibrium and then emit coherent radiation.

Zel'dovich [2, 3] showed that a absorbing surface rotating with an angular velocity Ω could scatter incident wave with frequency ω which satisfies

$$\omega - m\Omega < 0 \tag{1.1}$$

where m is the usual azimuthal number of the monochromatic plane wave relative to the rotation axis. Condition (1.1) was to become one of the most important results of (rotational) superradiance as it presents itself in multiple examples in the literature, *i.e.* Vavilov-Cherekov effect and anomalous Doppler effect.

Actually, radiation amplification can be traced to birth of Quantum Mechanics, in the beginnings of the 20th century. First studies of the Dirac equation by Klein [4] revealed the possibility of electrons propagating in a region with a sufficiently large potential barrier without the expected dampening from non-relativistic QM tunnel effect. Due to some confusion, this result was wrongly interpreted by some authors as fermionic superradiance, as if the reflected current by the barrier could be greater than the incident current. The problem was named *Klein paradox* by Sauter [5] and this misleading result was due to a incorrect calculation of the group velocities of the reflected and transmitted waves.

Today, it is known that fermionic currents cannot be amplified for this particular problem [6], result that was correctly obtained by Klein in is original paper. On the contrary, superradiant scattering can indeed occur for bosonic fields.

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Spin-weighted spherical harmonics

SWSHs play an important role BH physics and was first introduced by Teukolsky when considering non-scalar wave perturbations on a Kerr background, obtaining a separable master equation in four dimensions. After the usual change of coordinates, the polar differential equation goes as

$$\frac{1}{S}\frac{\mathrm{d}}{\mathrm{d}x}\left((1-x^2)\frac{\mathrm{d}S}{\mathrm{d}x}\right)+(cx)^2-2csx-\frac{(m+sx)^2}{1-x^2}+s=-\lambda \tag{A.1}$$

with $x=\cos\theta$, where λ is the eigenvalue for a given SWSH solution. Periodic boundary conditions on the azimuthal wave function constrains m to the integers.

A.1 Connection with spheroidal harmonics

By setting $s=0$ (scalar) and $c=0$ (spherical), then it's clear that (A.1) appears as a generalization of the spherical harmonics equation. In this last case, the solution are given by the associated Legendre polynomials, $P_\ell^m(x)$, for which the eigenvalue is $\ell(\ell+1)$, restricted to the condition of $|m|\leq\ell$. The closed form for spherical harmonics, after normalization, is

$${}_0Y_\ell^m(x)=(-1)^m\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\,P_\ell^m(x) \tag{A.2}$$

where P_ℓ^m are the associated Legendre polynomials which can be obtained using the famous Rodrigues' formula.

A.2 Spin raising/lowering differential operators

A.3 Generalized addition of angular momentum formula

A.4 Some useful harmonics

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