Testing fundamental physics with astrophysical and cosmological observations

José Sá[a] **, João G. Rosa**[a]

[a] *Departamento de Física, Universidade de Aveiro, Campus de Santiago, 3810-183 Aveiro, Portugal*

E-mail: jose.a.sa@ua.pt, joao.rosa@ua.pt

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Some text transitioning from MSc thesis to research work ...

2 Superradiant scattering

The general scattering theory of perturbations off rotating black holes (BH) has long been known in General Relativity (GR) [1]. We will take particular focus on neutral BHs since any intrinsic charge would increase electromagnetic (EM) forces on opposite-charged plasma The Kerr BH is a vacuum solution for the Einstein's field equations, which generalized the well-known Schwarzschild spherical geometry.

In this work we use the Boyer-Linquist coordinates [2], where the metric takes the form

$$\begin{aligned} ds^2 = \left(1 - \frac{2Mr}{\rho^2}\right) \mathrm{d}t^2 - 2a \sin^2\theta \frac{(r^2 + a^2 - \Delta)}{\rho^2} \mathrm{d}t\mathrm{d}\varphi \\ - \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2\theta}{\rho^2} \sin^2\theta \mathrm{d}\varphi^2 - \frac{\rho^2}{\Delta} \mathrm{d}r^2 - \rho^2 \mathrm{d}\theta^2 \end{aligned} \tag{2.1}$$

where $\Delta = r^2 - 2Mr + a^2$, $\bar{\rho} = r + ia\cos\theta$ and $\rho^2 \equiv |\bar{\rho}|^2$.

It was Newman and Penrose that develop the necessary formalism of spinor calculus for the study of perturbations [3]. The NP formalism focus on choosing a non-local tetrad $(\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}})$ of complex null vectors and projecting the relevant tensors in this basis. For example, for electromagnetic waves we use the Faraday to define the relevant NP scalars

$$\phi_0 = F_{\mu\nu} l^\mu m^\nu \ , \qquad \phi_2 = F_{\mu\nu} \bar{m}^\mu n^\nu \ , \tag{2.2}$$

while for gravitational perturbations we use the Weyl tensor

$$\psi_0 = -C_{\mu\nu\sigma\rho} l^\mu m^\nu l^\sigma m^\rho \ , \qquad \psi_4 = -C_{\mu\nu\sigma\rho} n^\mu \bar{m}^\nu n^\sigma \bar{m}^\rho \ . \tag{2.3}$$

Choosing a suitable tetrad [4], Teukolsky showed that is possible to obtain a separable wave equation for all types of massless perturbations (scalar, electromagnetic, gravitational) that are characterized by a *spin-weight* parameter s [5–8]. Due to the underlying symmetries of the geometry we can perform a mode decomposition of the form

$$\Upsilon_s = \int \mathrm{d}\omega \sum_{\ell,m} e^{-i\omega t + im\varphi} \, {}_sS_{\ell m}(\theta) \, {}_sR_{\ell m}(r) \ , \tag{2.4}$$

where Υ_0 obeys the Klein-Gordon wave equation in curved spacetime, $g^{\mu\nu}\nabla_\mu\partial_\nu\Upsilon_0 = 0$. The other bosonic perturbations are characterized by two polarizations mixed differently in the corresponding complex NP quantities $\Upsilon_{+1} = \phi_0$ and $\Upsilon_{-1} = 2(\bar{\rho}^*)^2\phi_2$ for electromagnetic waves and $\Upsilon_{+2} = \psi_0$ and $\Upsilon_{-2} = 4(\bar{\rho}^*)^4\psi_4$ for gravitational waves.

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