Insira o título da dissertação, projeto ou relatório de estágio, letra Arial Bold, tamanho ajustado a caixa de texto 12x12 cm, justificado à esquerda

Nome do autor

Nome do curso

Departamento

2017

Orientador

Nome do Orientador, Categoria, Faculdade

Coorientador

Nome do Coorientador, Categoria, Faculdade

LOGO 3

LOGO 2

U.PORTO

FACULDADE DE CIÊNCIAS
UNIVERSIDADE DO PORTO

Todas as correções determinadas pelo júri, e só essas, foram efetuadas.

O Presidente do Júri,

Porto, ______/______/________

MSc

UNIVERSIDADE DO PORTO

MASTER'S THESIS

Superradiance

Author:
José SÁ

Supervisor:
João ROSA

Co-supervisor:
Orfeu BERTOLAMI

A thesis submitted in fulfilment of the requirements for the degree of Master of Science

at the

Faculdade de Ciências da Universidade do Porto
Departamento de Física e Astronomia

July 2017

Acknowledgements

The acknowledgements and the people to thank go here, don't forget to include your project advisors…

UNIVERSIDADE DO PORTO

Abstract

Faculdade de Ciências da Universidade do Porto

Departamento de Física e Astronomia

Master of Science

Superradiance

by José SÁ

The Thesis Abstract is written here (and usually kept to just this page). The page is kept centered vertically so can expand into the blank space above the title too...

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

UNIVERSIDADE DO PORTO

Resumo

Faculdade de Ciências da Universidade do Porto

Departamento de Física e Astronomia

Mestre de Ciência

Nome da tese em português

por José SÁ

Tradução em português do "Abstract" escrito em inglês mais a cima. A página é centrada vertical e horizontalmente, podendo espandir para o espaço superior da página em branco . . .

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Contents

List of Figures

Abbreviations

QM	**Q**uantum **M**echanics
BH	**B**lack **H**ole
GR	**G**eneral **R**elativity
SWSH	**S**pin-**W**eighted **S**pheroidal **H**armonic

Notation and Conventions

Units

Text random and a new citation

Tensors and GR related

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Chapter 1

Superradiance

1.1 Introduction

The first appearance of the concept of *supperradiance* was in 1954, when Dicke [1] showed that a gas could be excited by a pulse into "superradiant states" from thermal equilibrium and then emit coherent radiation.

Actually, radiation amplification can be traced to birth of Quantum Mechanics, in the beginnings of the 20th century. First studies of the Dirac equation by Klein [2] revealed the possibility of electrons propagating in a region with a sufficiently large potential barrier without the expected dampening from non-relativistic QM tunnel effect. Due to some confusion, this result was wrongly interpreted by some authors as fermionic superradiance, as if the reflected current by the barrier could be greater than the incident current. The problem was named *Klein paradox* by Sauter [3] and this misleading result was due to a incorrect calculation of the group velocities of the reflected and transmitted waves. Today, it is known that fermionic currents cannot be amplified for this particular problem [4, 5], which was correctly calculated in Klein's original paper. On the contrary, superradiant scattering could indeed occur for bosonic fields.

Further calculations from Zel'dovich [6, 7] showed that a absorbing surface rotating with an angular velocity Ω could scatter incident wave with frequency ω which satisfies

$$\omega - m\Omega < 0\,, \tag{1.1}$$

where m is the usual azimuthal number of the monochromatic plane wave relative to the rotation axis. Condition (**??**) was to become one of the most important results of (rotational) superradiance as it presents itself in multiple examples in the literature, *i.e.* Vavilov-Cherekov effect and anomalous Doppler effect.

1.2 Klein paradox

1.2.1 Fermions

Following chronologically, Klein used Dirac equation to study electrons in a step potential potential [2]. This equation states

$$(\gamma^{\mu}\partial_{\mu} - m)\Psi = 0\,, \tag{1.2}$$

where m is the electron mass and $\{\gamma_{\mu}, \gamma_{\nu}\} = 2\eta_{\mu\nu}$. The problem can be greatly simplified by considering in the pseudo-Euclidean space $\mathbb{E}^{1,1}$, for which a valid representation of the gamma matrices is

$$\gamma^0 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\,, \quad \gamma^1 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}\,. \tag{1.3}$$

Furthrmore will be used the ansatz $\Psi = e^{-i\omega t}\psi$

1.2.2 Bosons

Klein and Gordon develop their equation in which describes scalar fields

1.3 Black hole superradiance

Chapter 2

Mathematical formalism

2.1 General Relativity

2.2 Kinnersley tetrad

2.3 Newman-Penrose formalism

Appendix A

Spin-weighted spheroidal harmonics

SWSHs play an important role BH physics and was first introduced by Teukolsky when considering non-scalar wave perturbations on a Kerr background, obtaining a separable master equation in four dimensions. After the usual change of coordinates, the polar differential equation goes as

$$\frac{1}{S}\frac{\mathrm{d}}{\mathrm{d}x}\left((1-x^2)\frac{\mathrm{d}S}{\mathrm{d}x}\right)+(cx)^2-2csx-\frac{(m+sx)^2}{1-x^2}+s=-\lambda \tag{A.1}$$

with $x=\cos\theta$, where λ is the eigenvalue for a given SWSH solution. Periodic boundary conditions on the azimuthal wave function constrains m to the integers.

A.1 Connection with spheroidal and spherical harmonics

By setting $s=0$ (scalar) and $c=0$ (spherical), then it's clear that (A.1) appears as a generalization of the spherical harmonics equation. In this last case, the solution are given by the associated Legendre polynomials, $P_\ell^m(x)$, for which the eigenvalue is $\ell(\ell+1)$, restricted to the condition of $|m|\leq\ell$. The closed form for spherical harmonics, after normalization, is

$${}_0Y_\ell^m(x)=(-1)^m\sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}}\,P_\ell^m(x) \tag{A.2}$$

where P_ℓ^m are the associated Legendre polynomials which can be obtained using the famous Rodrigues' formula.

Bibliography

[1] R. H. Dicke, *Coherence in Spontaneous Radiation Processes*, Phys. Rev. **93**, 99 (1954).

[2] O. Klein, *Die Reflexion von Elektronen an einem Potentialsprung nach der relativistischen Dynamik von Dirac*, Z. Phys. **53**, 157 (1929).

[3] F. Sauter, *Uber das Verhalten eines Elektrons im homogenen elektrischen Feld nach der relativistischen Theorie Diracs*, Z. Phys. **69**, 742 (1931).

[4] C. A. Manogue, *The Klein paradox and superradiance*, Ann. Phys. **181**, 261 (1988).

[5] R. Brito, V. Cardoso and P. Pani, *Superradiance*, Lecture Notes in Physics, Vol. 906 (Springer International Publishing, 2015) arXiv:1501.06570 .

[6] Y. B. Zel'dovich, *Generation of Waves by a Rotating Body*, Zh. Eksp. Teor. Fiz. Pis'ma Red. **14**, 270 (1971), [JETP Lett. **14**, 180 (1971)].

[7] Y. B. Zel'dovich, *Amplification of Cylindrical Electromagnetic Waves Reflected from a Rotating Body*, Zh. Eksp. Teor. Fiz. **35**, 2076 (1972), [Sov. Phys. JETP **35**, 1085 (1972)].