Optical lattice potential calculation and simulation

Guanghui Su

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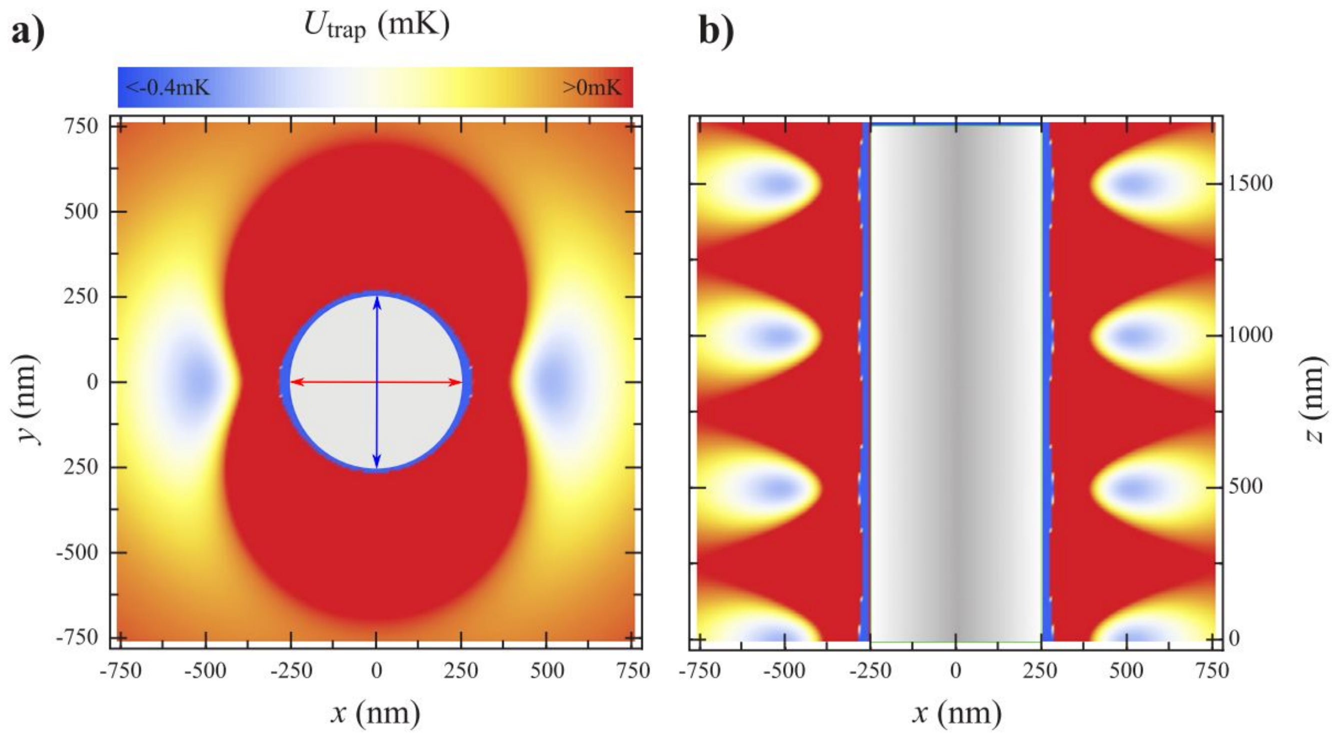
- Optical lattice potential

1. Cross-section potential
2. 3D distribution



Lorentz model

$$\mathbf{d}(t) = -e\mathbf{x}(t) = \alpha(\omega)\mathbf{E}(t)$$

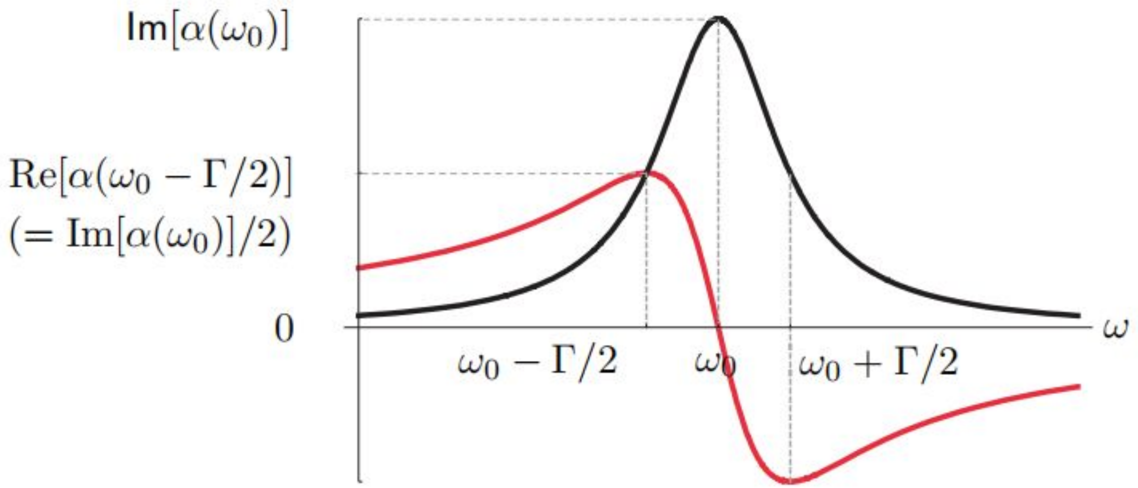
$$U = -\frac{1}{2}\langle \mathbf{d}(t)\cdot\mathbf{E}(t)]\rangle = -\frac{1}{4}|E_0|^2\mathrm{Re}[\alpha(\omega)].$$

$$\ddot{\mathbf{x}}(t) + \Gamma_\omega\dot{\mathbf{x}}(t) + \omega_0^2\mathbf{x}(t) = -\frac{e}{m_e}\mathbf{E}(t),$$

$$\Gamma_\omega = \frac{e^2\omega^2}{6\pi\varepsilon_0 m_e c^3},$$

$$\alpha(\omega) = \frac{e^2}{m_e}\frac{1}{\omega_0^2 - \omega^2 - i\Gamma_\omega\omega},$$

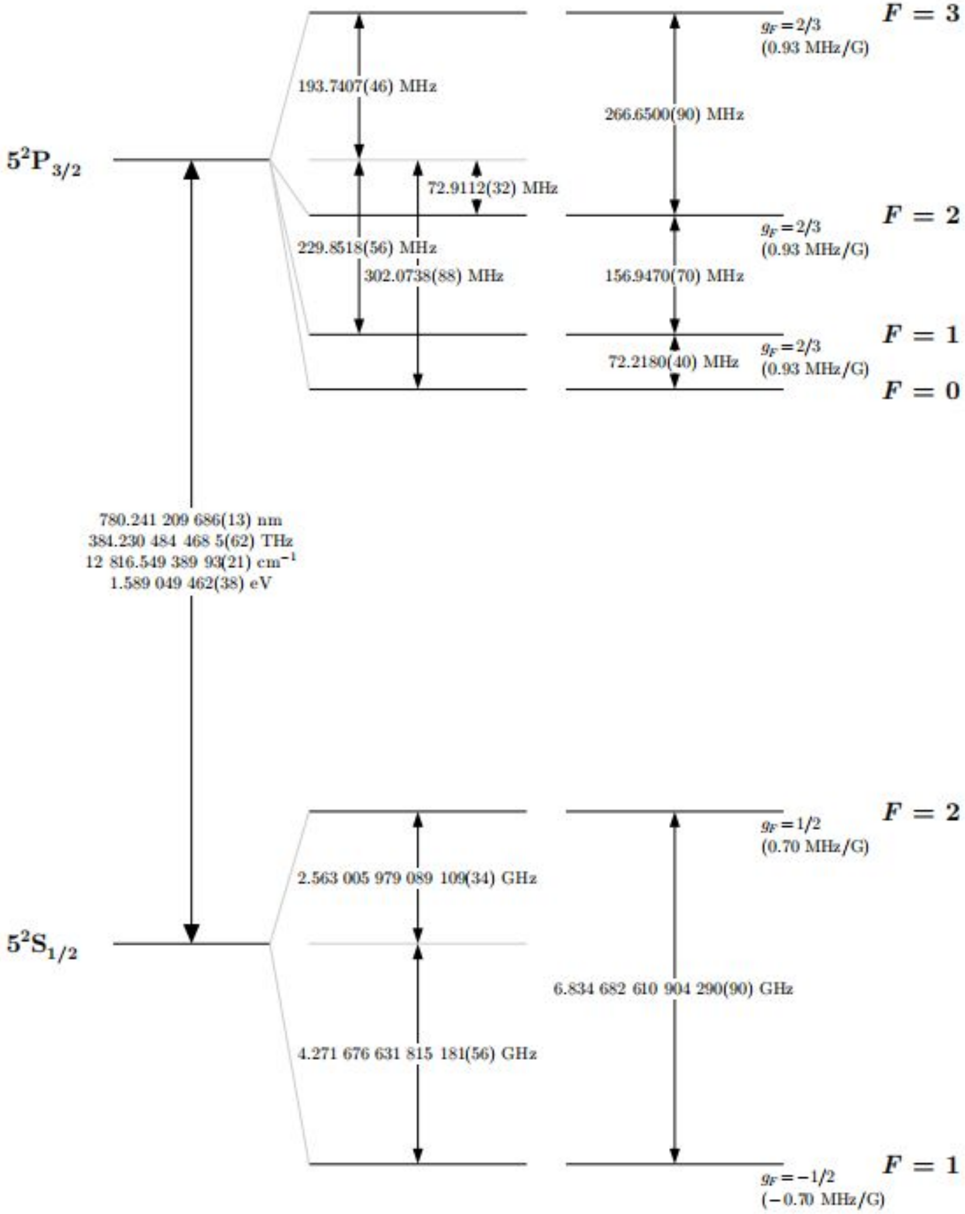
$$\begin{aligned} \mathrm{Re}[\alpha(\omega)] &= 6\pi\varepsilon_0 c^3 \frac{(\omega_0^2-\omega^2)\Gamma/\omega_0^2}{(\omega_0^2-\omega^2)^2 + \Gamma^2(\omega^3/\omega_0^2)^2}, \\ \mathrm{Im}[\alpha(\omega)] &= 6\pi\varepsilon_0 c^3 \frac{\Gamma^2\omega^3/\omega_0^4}{(\omega_0^2-\omega^2)^2 + \Gamma^2(\omega^3/\omega_0^2)^2}. \end{aligned}$$



Fine structure: J

$$\alpha(\omega) = \sum_{j=1}^{n} f_j \alpha_j(\omega).$$

$$\mathrm{Re}[\alpha(\omega)] \quad = \quad 2\pi\varepsilon_0 c^3 \sum_i \frac{2J'+1}{2J+1} \frac{(\omega_j^2-\omega^2)\Gamma_j/\omega_j^2}{(\omega_j^2-\omega^2)^2+\Gamma_j^2(\omega^3/\omega_j^2)^2},$$



Hyperfine structure

$$H_{\text{int}} = V^{\text{hfs}} + V^{EE}.$$

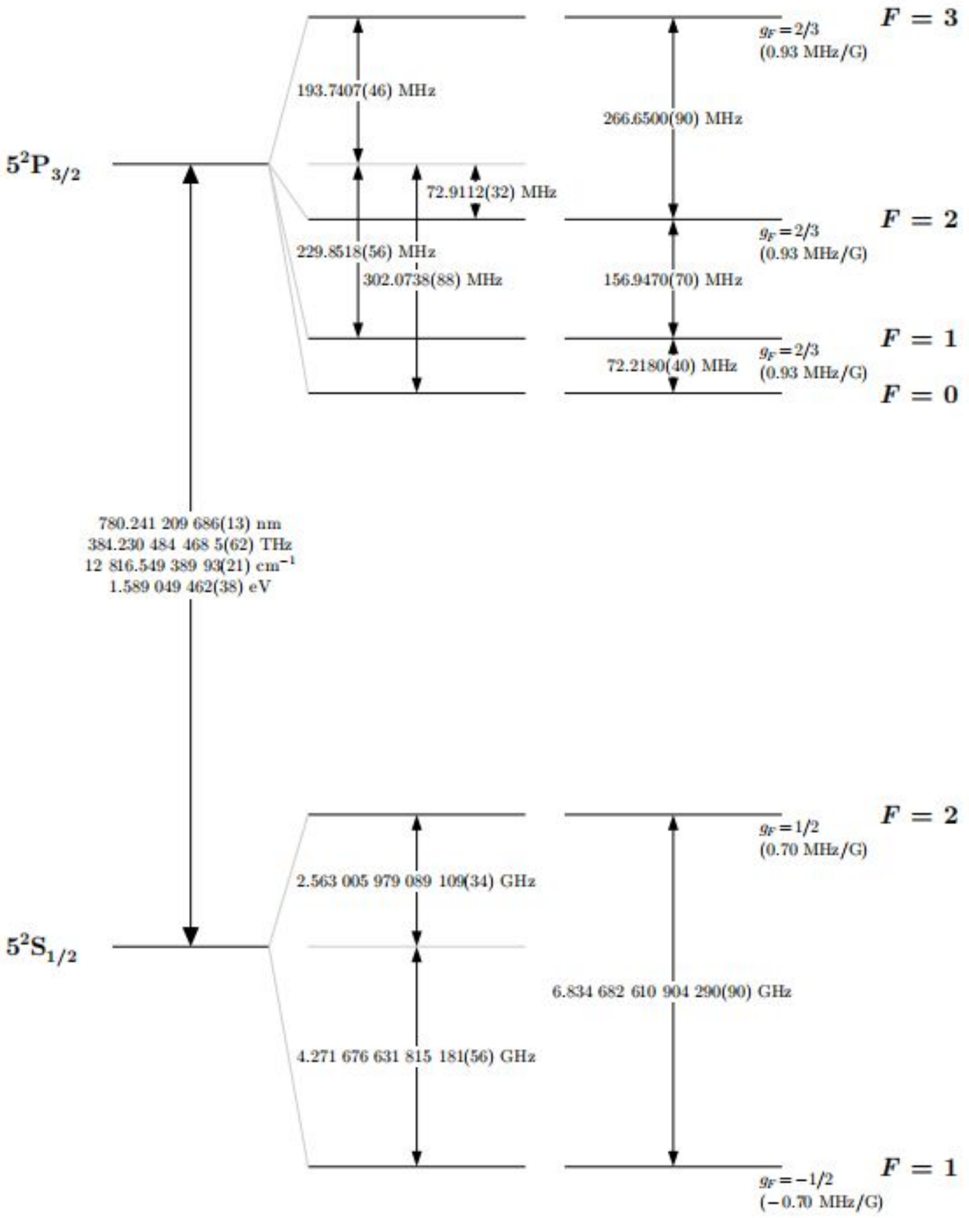
$$V^{\text{hfs}} = \hbar A_{\text{hfs}}\,\mathbf{I}\cdot\mathbf{J} + \hbar B_{\text{hfs}}\frac{6(\mathbf{I}\cdot\mathbf{J})^2 + 3\mathbf{I}\cdot\mathbf{J} - 2\mathbf{I}^2\mathbf{J}^2}{2I(2I-1)2J(2J-1)}.$$

$$\langle nJFM|V^{\text{hfs}}|nJFM\rangle = \frac{1}{2}\hbar A_{\text{hfs}}G + \hbar B_{\text{hfs}}\frac{\frac{3}{2}G(G+1) - 2I(I+1)J(J+1)}{2I(2I-1)2J(2J-1)},$$

where $G = F(F+1) - I(I+1) - J(J+1)$.

$$V^E = -\mathbf{E}\cdot\mathbf{d} = -\frac{1}{2}\mathcal{E}\mathbf{u}\cdot\mathbf{d}e^{-i\omega t} - \frac{1}{2}\mathcal{E}^*\mathbf{u}^*\cdot\mathbf{d}e^{i\omega t},$$

$$\mathbf{E} = \frac{1}{2}\boldsymbol{\mathcal{E}}e^{-i\omega t} + \text{c.c.} = \frac{1}{2}\mathcal{E}\mathbf{u}e^{-i\omega t} + \text{c.c.},$$



Second-order perturbation

$$V^E = -\mathbf{E}\cdot\mathbf{d} = -\frac{1}{2}\mathcal{E}\mathbf{u}\cdot\mathbf{d}e^{-i\omega t} - \frac{1}{2}\mathcal{E}^*\mathbf{u}^*\cdot\mathbf{d}e^{i\omega t},$$

$$\delta E_a = -\frac{|\mathcal{E}|^2}{4\hbar}\sum_b \mathrm{Re}\left(\frac{|\langle b|\mathbf{u}\cdot\mathbf{d}|a\rangle|^2}{\omega_b-\omega_a-\omega-i\gamma_{ba}/2} + \frac{|\langle a|\mathbf{u}\cdot\mathbf{d}|b\rangle|^2}{\omega_b-\omega_a+\omega+i\gamma_{ba}/2}\right).$$

$$V^{EE} = \frac{|\mathcal{E}|^2}{4}\left[(\mathbf{u}^*\cdot\mathbf{d})\mathcal{R}_+(\mathbf{u}\cdot\mathbf{d}) + (\mathbf{u}\cdot\mathbf{d})\mathcal{R}_-(\mathbf{u}^*\cdot\mathbf{d})\right]$$

with

$$\mathcal{R}_+ = -\frac{1}{\hbar}\sum_b \mathrm{Re}\left(\frac{1}{\omega_b-\omega_a-\omega-i\gamma_{ba}/2}\right)|b\rangle\langle b|,$$

$$\mathcal{R}_- = -\frac{1}{\hbar}\sum_b \mathrm{Re}\left(\frac{1}{\omega_b-\omega_a+\omega+i\gamma_{ba}/2}\right)|b\rangle\langle b|.$$

$$V_{EE}(\boldsymbol{\mathcal{E}},\omega) = \frac{1}{4}\mathcal{E}^2\sum_{K=0,1,2}\{\mathbf{e}^*\otimes\mathbf{e}\}_K\cdot\left[\{\mathbf{d}\otimes R_{E_0}(\omega)\mathbf{d}\}_K + (-1)^K\{\mathbf{d}\otimes R_{E_0}(-\omega)\mathbf{d}\}_K\right],$$

$$\{\mathbf{e}^*\otimes\mathbf{e}\}_{0,0} = -\frac{1}{\sqrt{3}}(\mathbf{e}^*\cdot\mathbf{e}) = -\frac{1}{\sqrt{3}}$$

$$\{\mathbf{e}^*\otimes\mathbf{e}\}_{1,\mu} = -\frac{1}{\sqrt{2}}(\mathbf{e}^*\times\mathbf{e})_\mu = -\frac{A}{\sqrt{2}}\delta_{\mu,0}$$

$$\{\mathbf{e}^*\otimes\mathbf{e}\}_{2,\mu} = -\frac{1}{\sqrt{6}}\delta_{\mu,0} + \frac{l}{\sqrt{2}}\delta_{\mu,\pm 2}.$$

polarizability and CG coefficients

$$V^{EE}_{FMF'M'} = \frac{1}{4}|\mathcal{E}|^2 \sum_{\substack{K=0,1,2 \\ q=-K,\ldots,K}} \alpha^{(K)}_{nJ} \{\mathbf{u}^* \otimes \mathbf{u}\}_{Kq}$$
$$\times (-1)^{J+I+K+q-M} \sqrt{(2F+1)(2F'+1)}$$
$$\times \begin{pmatrix} F & K & F' \\ M & q & -M' \end{pmatrix} \begin{Bmatrix} F & K & F' \\ J & I & J \end{Bmatrix}. \qquad (10)$$

$$\alpha^{(K)}_{nF}(\omega) = (-1)^{I-J+F} \sqrt{2K+1}\sqrt{2F+1}\sqrt{2F'+1} \begin{Bmatrix} F & K & F' \\ J & I & J \end{Bmatrix}$$
$$\times \sum_{n''J''} \begin{Bmatrix} J & K & J \\ 1 & J'' & 1 \end{Bmatrix} \langle nJ\|\mathbf{d}\|n''J''\rangle \langle n''J''\|\mathbf{d}\|nJ\rangle$$
$$\times \left[\frac{1}{E_{n,J} - E_{n'',J''} + \hbar\omega} + \frac{(-1)^K}{E_{n,J} - E_{n'',J''} - \hbar\omega} \right],$$

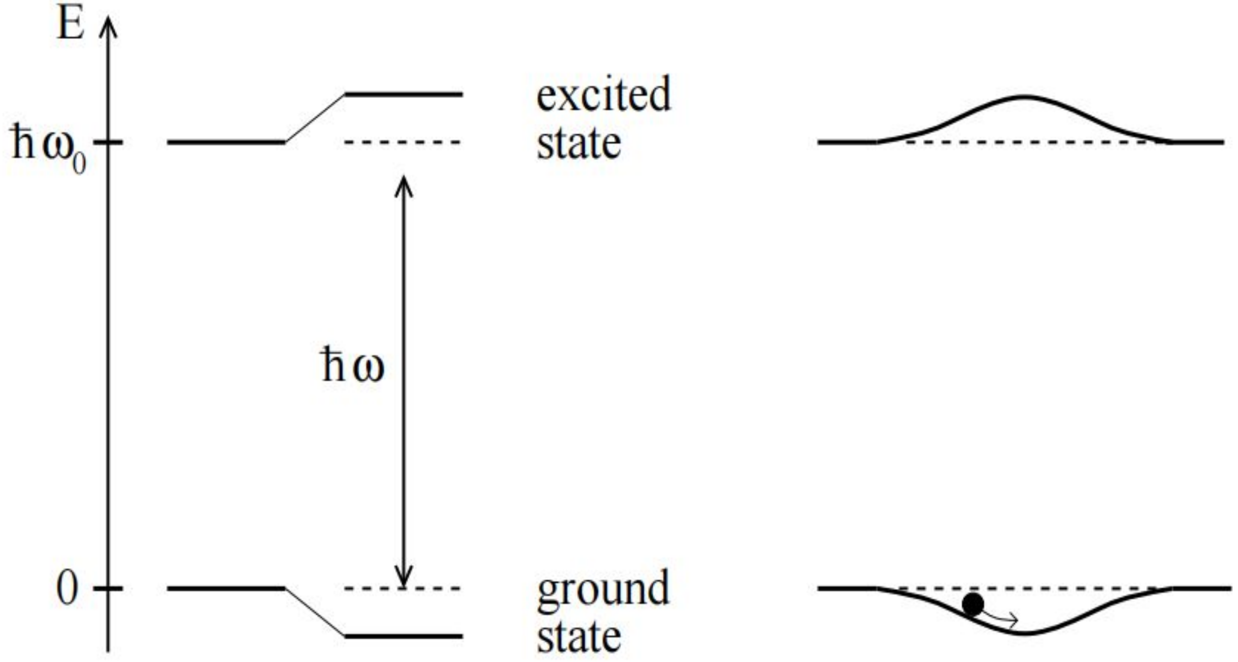
$$\Delta E_{\mathrm{ac}} = T_{M,M} = -\frac{1}{4}\mathcal{E}^2 \left[\alpha^s_{nF}(\omega) + A\alpha^a_{nF}(\omega)\frac{M_F}{2F} - \alpha^T_{nF}(\omega)\frac{3M_F^2 - F(F+1)}{2F(2F-1)} \right],$$

$$\alpha^s_{nF} = \frac{1}{\sqrt{3(2F+1)}}\alpha^{(0)}_{nF},$$
$$\alpha^a_{nF} = -\sqrt{\frac{2F}{(F+1)(2F+1)}}\alpha^{(1)}_{nF},$$
$$\alpha^T_{nF} = -\sqrt{\frac{2F(2F-1)}{3(F+1)(2F+1)(2F+3)}}\alpha^{(2)}_{nF}.$$

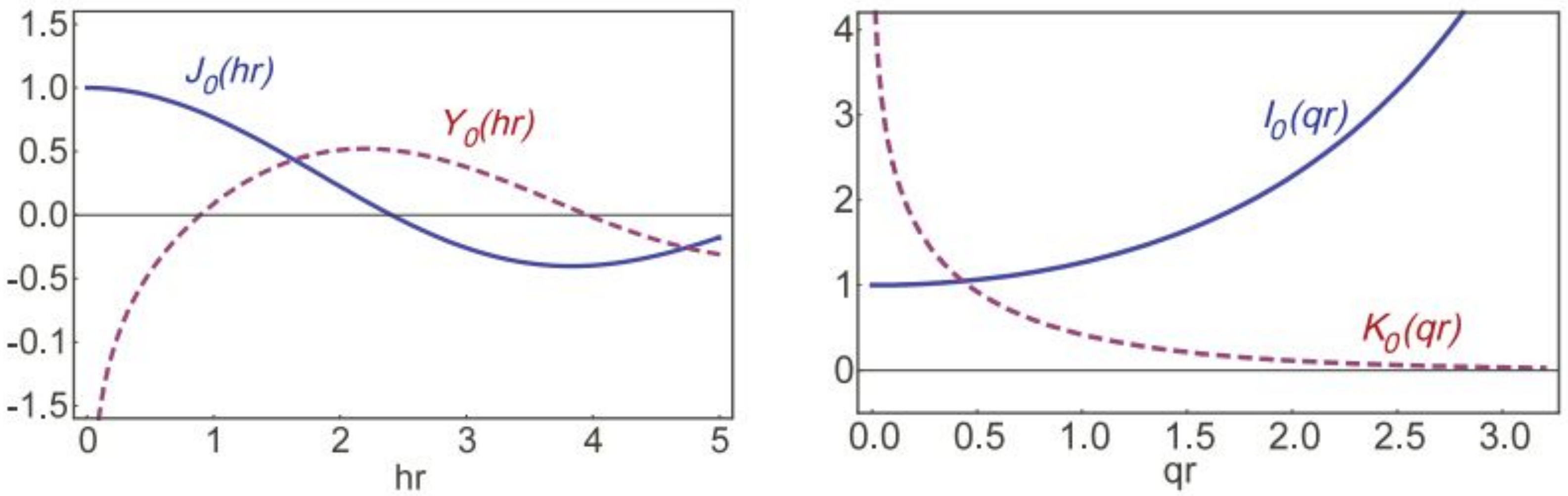
Large detuning AC stark shift

$$\Delta E_i = \left(\frac{|\langle j|\boldsymbol{d}\cdot\boldsymbol{\varepsilon}|i\rangle|^2}{E_i - E_j + \hbar\omega} + \frac{|\langle j|\boldsymbol{d}\cdot\boldsymbol{\varepsilon}|i\rangle|^2}{E_i - E_j - \hbar\omega}\right)|\frac{E_0}{2}|^2$$
$$= \left(\frac{|\boldsymbol{r}_{ij}|^2}{E_i - E_j + \hbar\omega} + \frac{|\boldsymbol{r}_{ij}|^2}{E_i - E_j - \hbar\omega}\right) e^2 a_{ij}^2 |\frac{E_0}{2}|^2$$

$$\Gamma_{ij} = \frac{e^2|\omega_{ij}|^3}{3\pi\hbar\varepsilon_0 c^3}|\boldsymbol{r}_{ij}|^2$$
$$\Delta E_i = \left(\frac{1}{\omega_{ij}+\omega} + \frac{1}{\omega_{ij}-\omega}\right) a_{ij}^2 \frac{3\pi c^2}{2|\omega_{ij}|^3}\Gamma_{ij}\boldsymbol{I}$$



Bessel function



$$\vec{\nabla}^2\vec{E} - \mu_0\varepsilon(r)\frac{\partial^2\vec{E}}{\partial t^2} = -\vec{\nabla}\left(\frac{\vec{E}}{\varepsilon(r)}\cdot\vec{\nabla}\varepsilon(r)\right)$$

$$\vec{\nabla}\times\vec{H} = \varepsilon(r)\frac{\partial\vec{E}}{\partial t}, \qquad \vec{\nabla}\cdot\vec{H} = 0$$

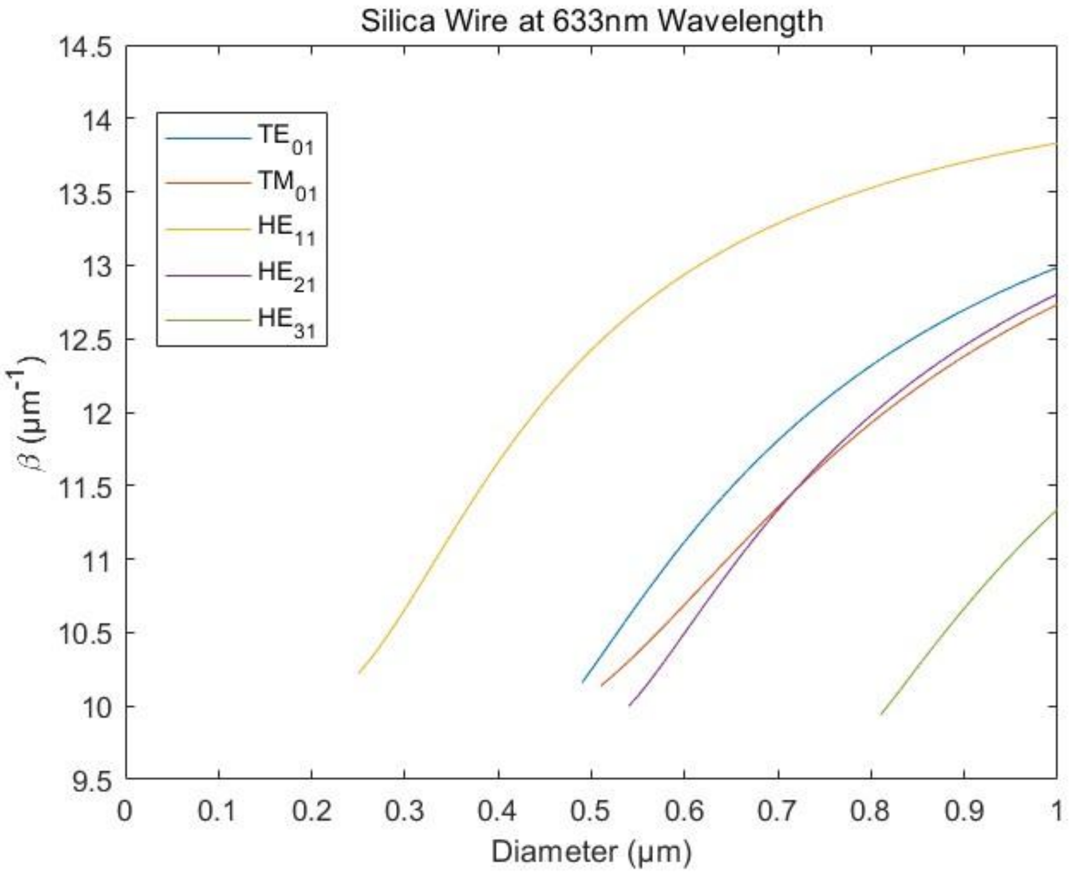
$$\vec{\nabla}\times\vec{E} = -\mu_0\frac{\partial\vec{H}}{\partial t}, \qquad \vec{\nabla}\cdot(\varepsilon(r)\vec{E}) = 0$$

$$\begin{bmatrix} E_z(\vec{r},t) \\ H_z(\vec{r},t) \end{bmatrix} = \begin{bmatrix} E_z(r,\phi) \\ H_z(r,\phi) \end{bmatrix} \exp[i(\omega t - \beta z)],$$

$$\begin{bmatrix} E_z(r,\phi) \\ H_z(r,\phi) \end{bmatrix} = R(r)\exp[\pm il\phi],$$

$$\left[\partial_r^2 + \frac{1}{r}\partial_r + (k^2 - \beta^2 - \frac{l^2}{r^2})\right]R(r) = 0.$$

Propagation constant: beta



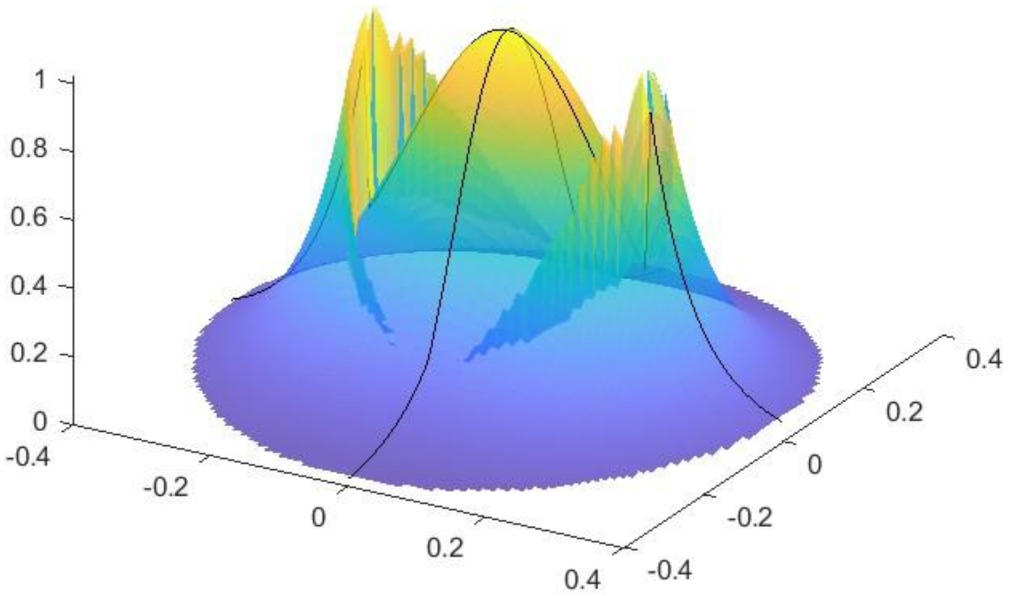
$$
\begin{aligned}
E_z(r,\phi,z,t) &= AJ_l(hr)\exp[i(\omega t \pm l\phi - \beta z)], \\
H_z(r,\phi,z,t) &= BJ_l(hr)\exp[i(\omega t \pm l\phi - \beta z)], \\
\text{with } h &= \sqrt{n_1^2 k_0^2 - \beta^2}
\end{aligned}
$$

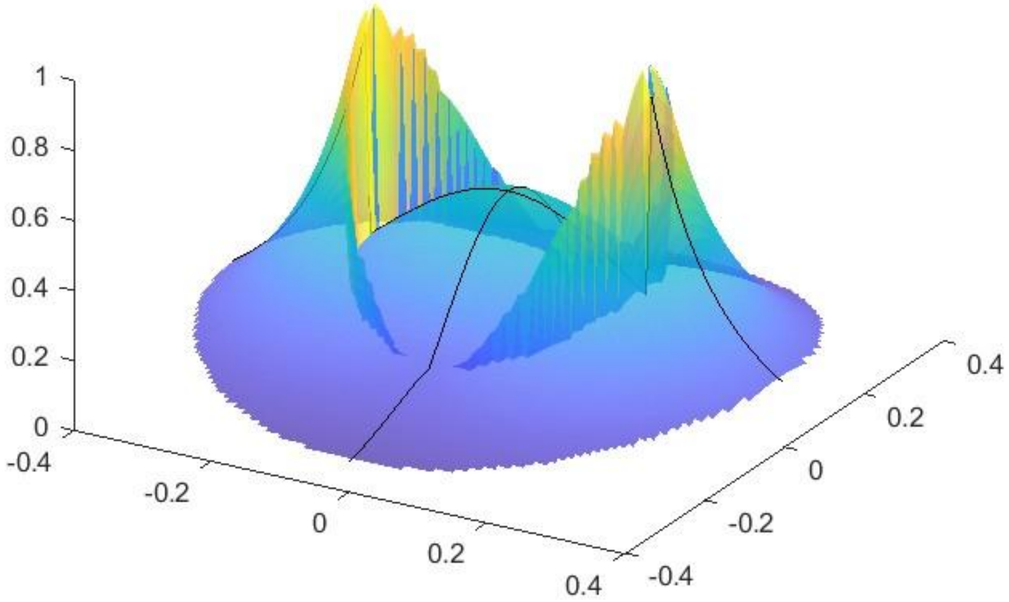
$(r < a)$, and

$$
\begin{aligned}
E_z(r,\phi,z,t) &= CK_l(qr)\exp[i(\omega t \pm l\phi - \beta z)], \\
H_z(r,\phi,z,t) &= DK_l(qr)\exp[i(\omega t \pm l\phi - \beta z)], \\
\text{with } q &= \sqrt{\beta^2 - n_2^2 k_0^2}
\end{aligned}
$$

core $(r > a)$.

Cross-section intensity



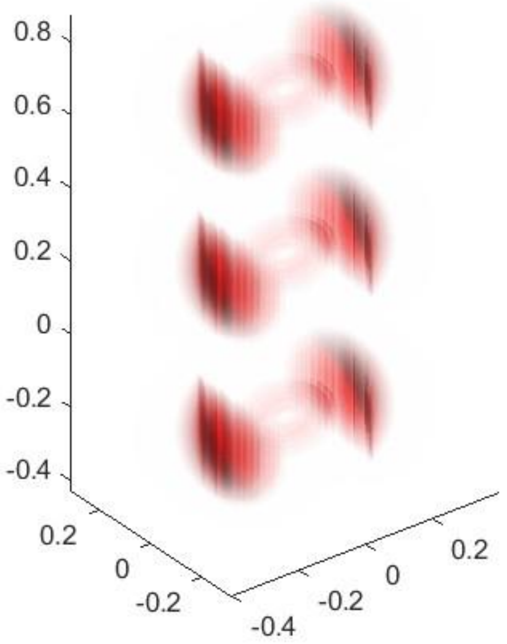


lam1 = 0.9352 um;

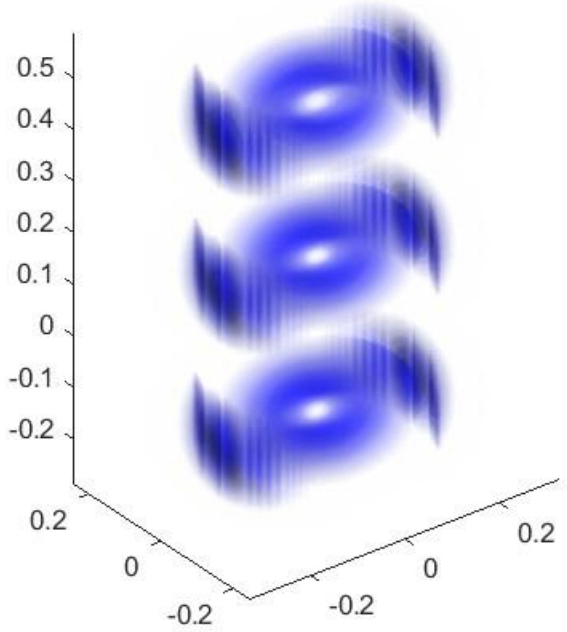
dia = 0.4 um, parameters from Ground group

lam2 = 0.6865 um;
lam3 = 0.6861 um;

3D distribution of intensity



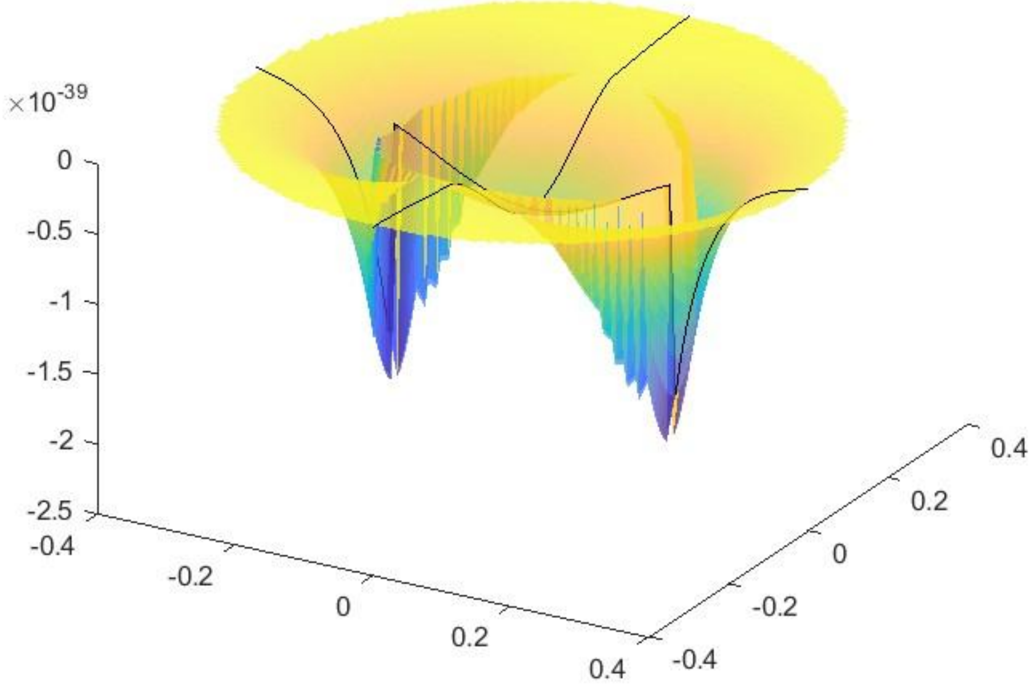
lam1 = 0.9352 um;

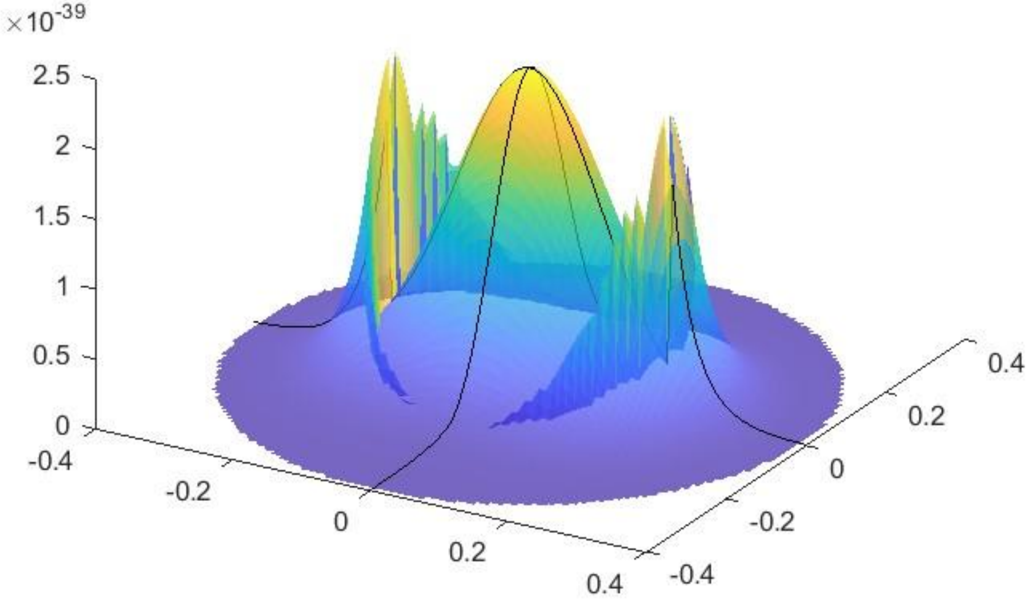


lam2 = 0.6865 um;
lam3 = 0.6861 um;

- dia = 0.4 um, parameters from Ground group
- z direction from -pi/beta to 2pi/beta

Cross-section potential





lam1 = 0.9352 um;

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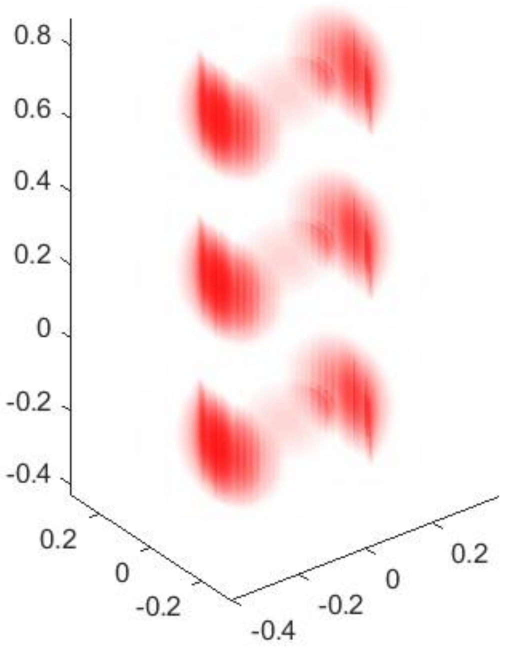
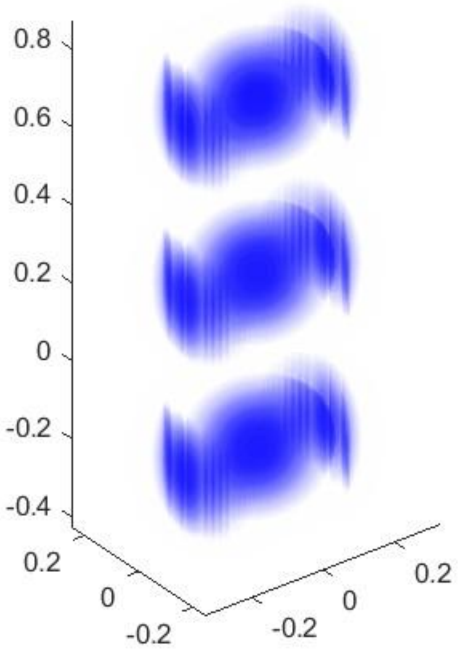
3D potential





lam1 = 0.9352 um;

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Total potential: $2U_{red} + U_{blue}$