Statistics and Machine Learning

Decision tree regression

Remote lecture Week 11 03/29 — 04/02

Contents

Review of linear regression algorithm

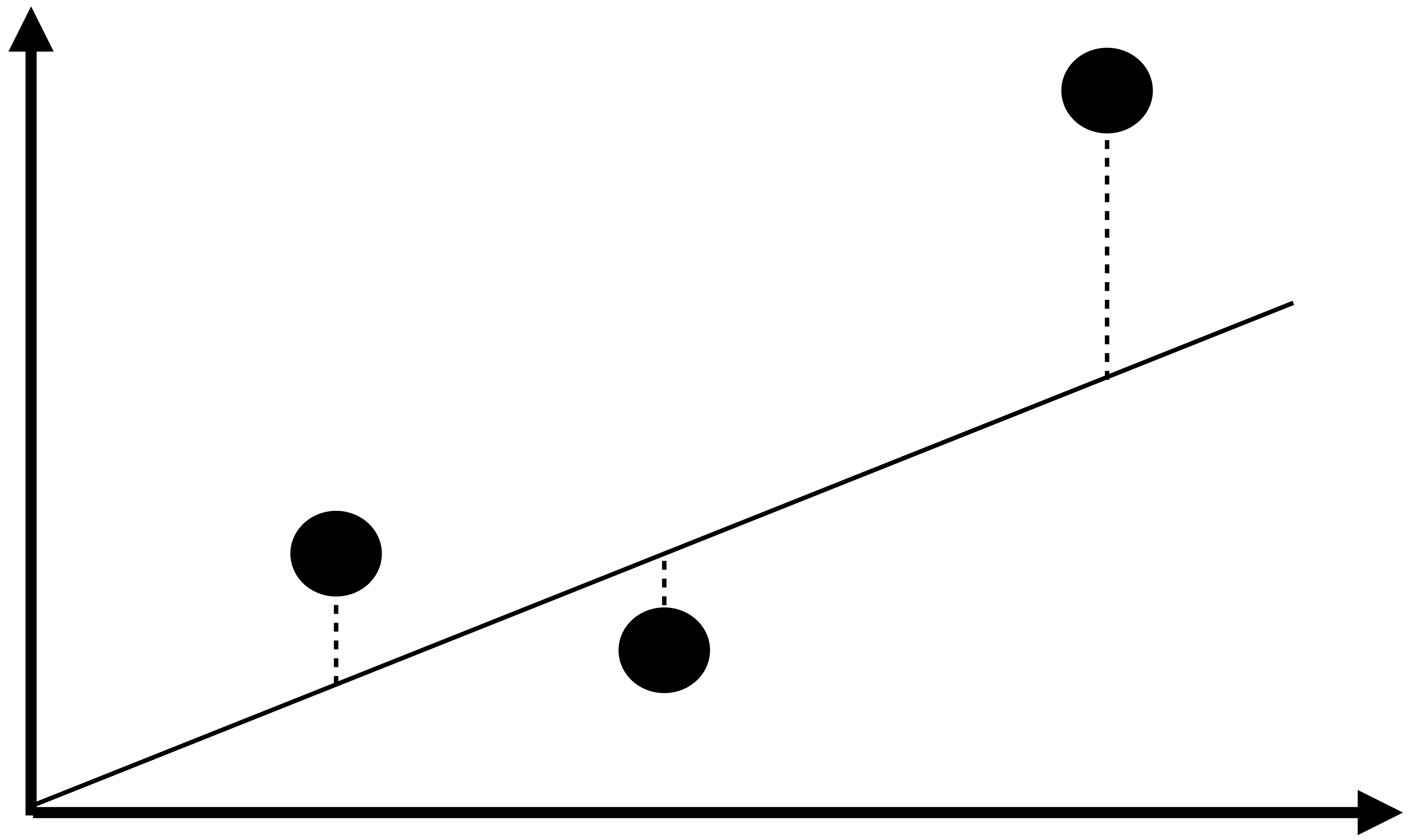
Why tree-based algorithm?

Tree-based regression algorithm (textbook 8.1.1)

Lab session: sk-learn library

Quiz

Recall what linear regression does!



- Assume the straight line function is $y = ax + b$
- Calculate the Loss = sum of all squared error
- Tune 'a' and 'b' so that the Loss is minimized!
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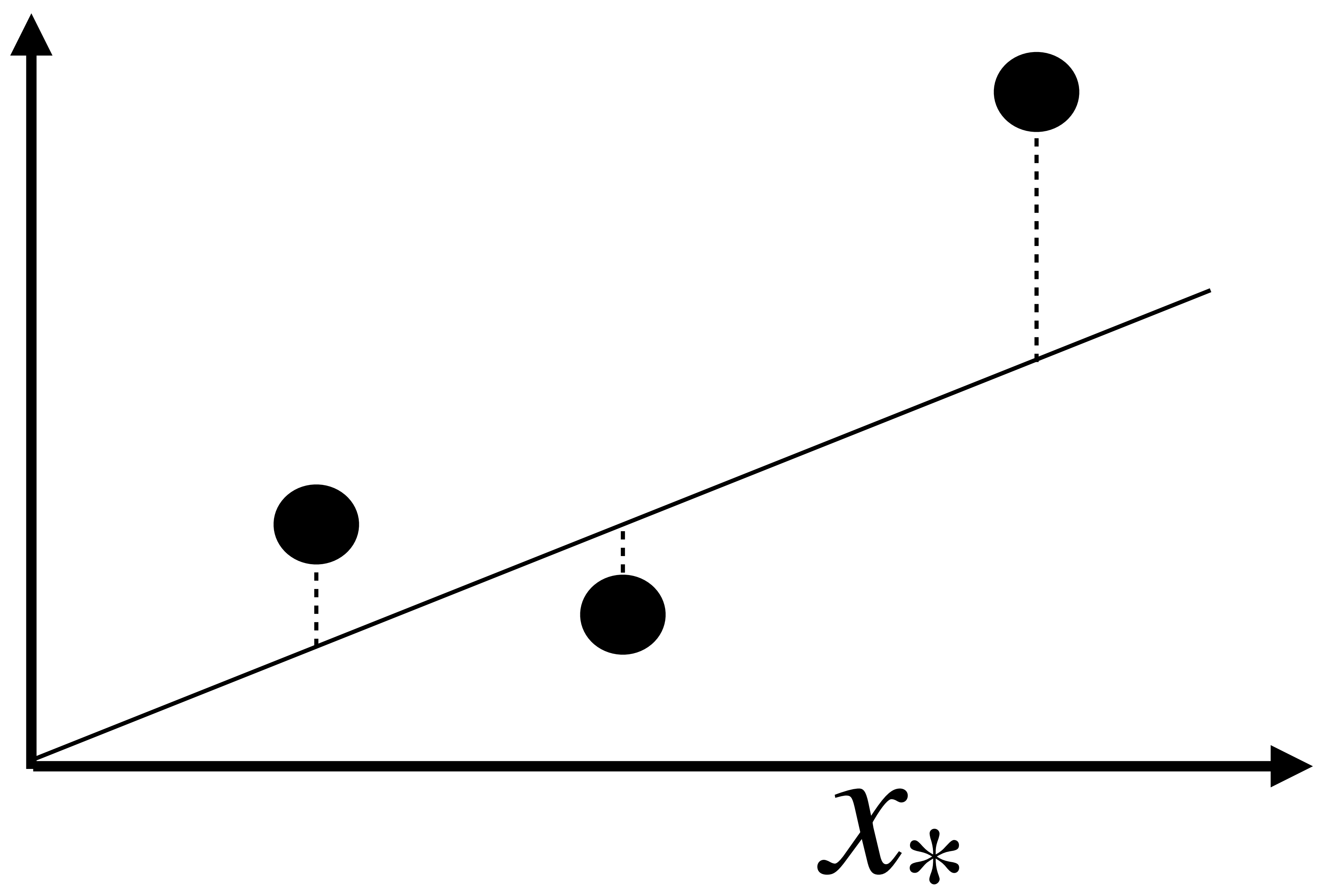

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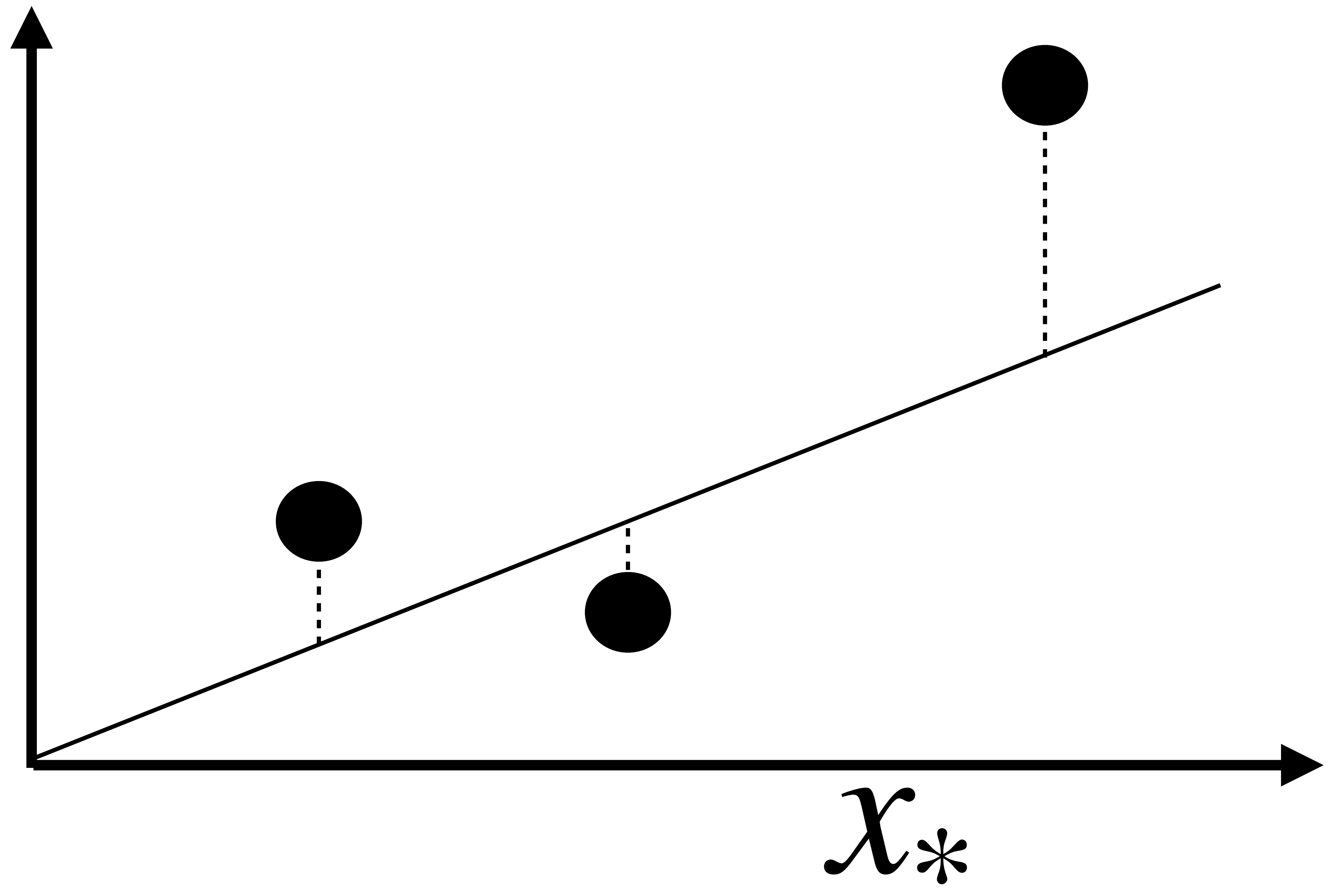

$$y_{pred} = ax_* + b$$

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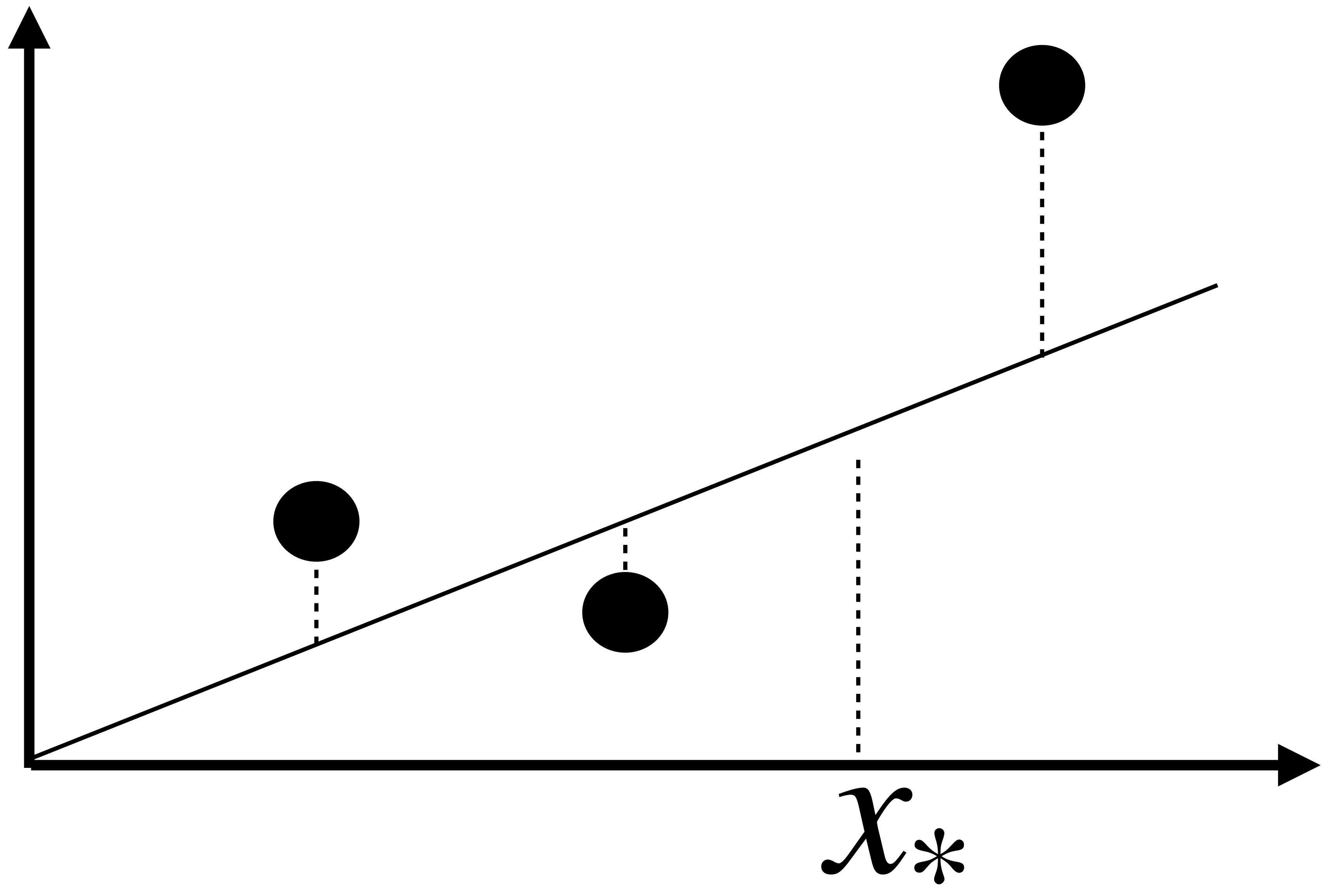

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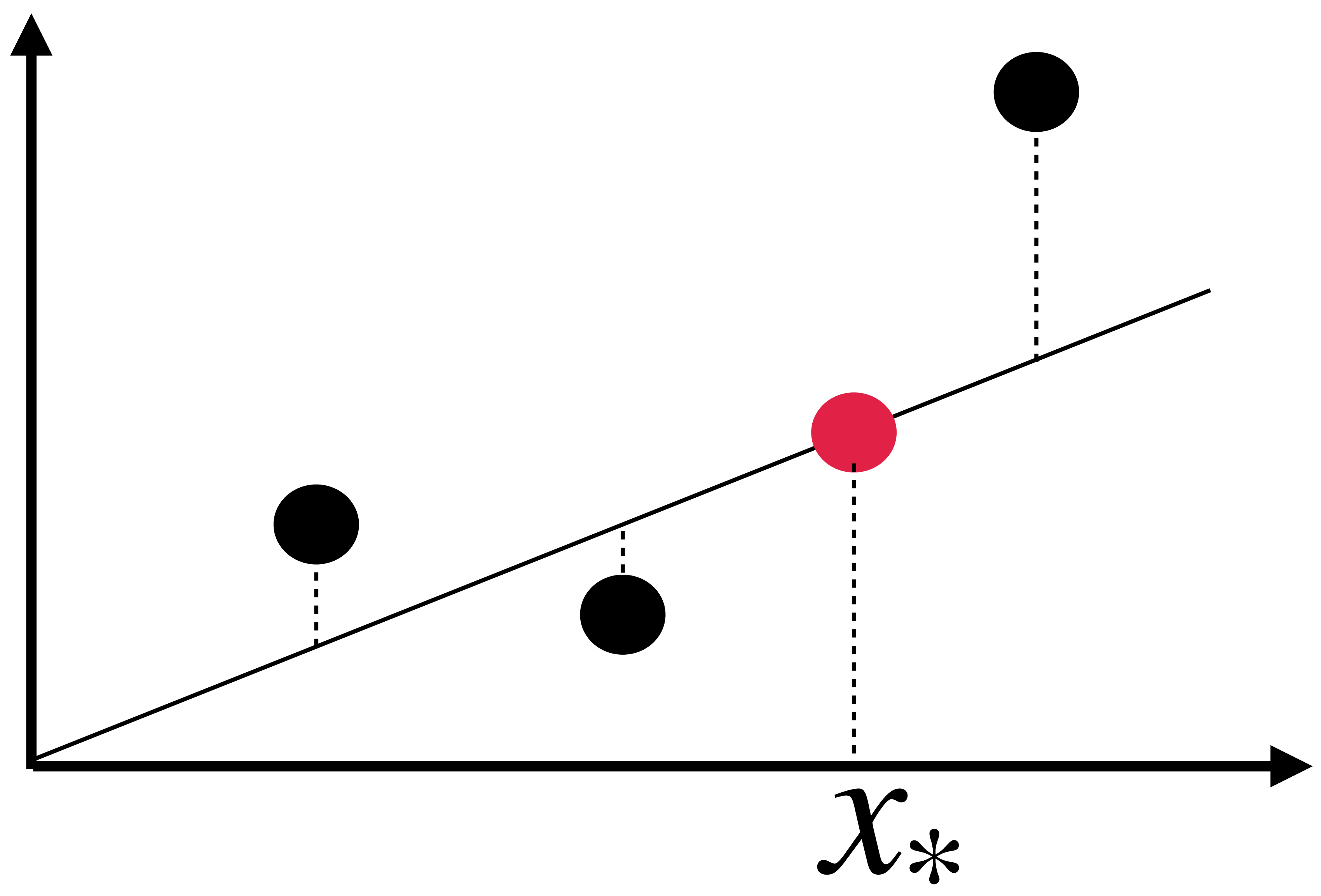

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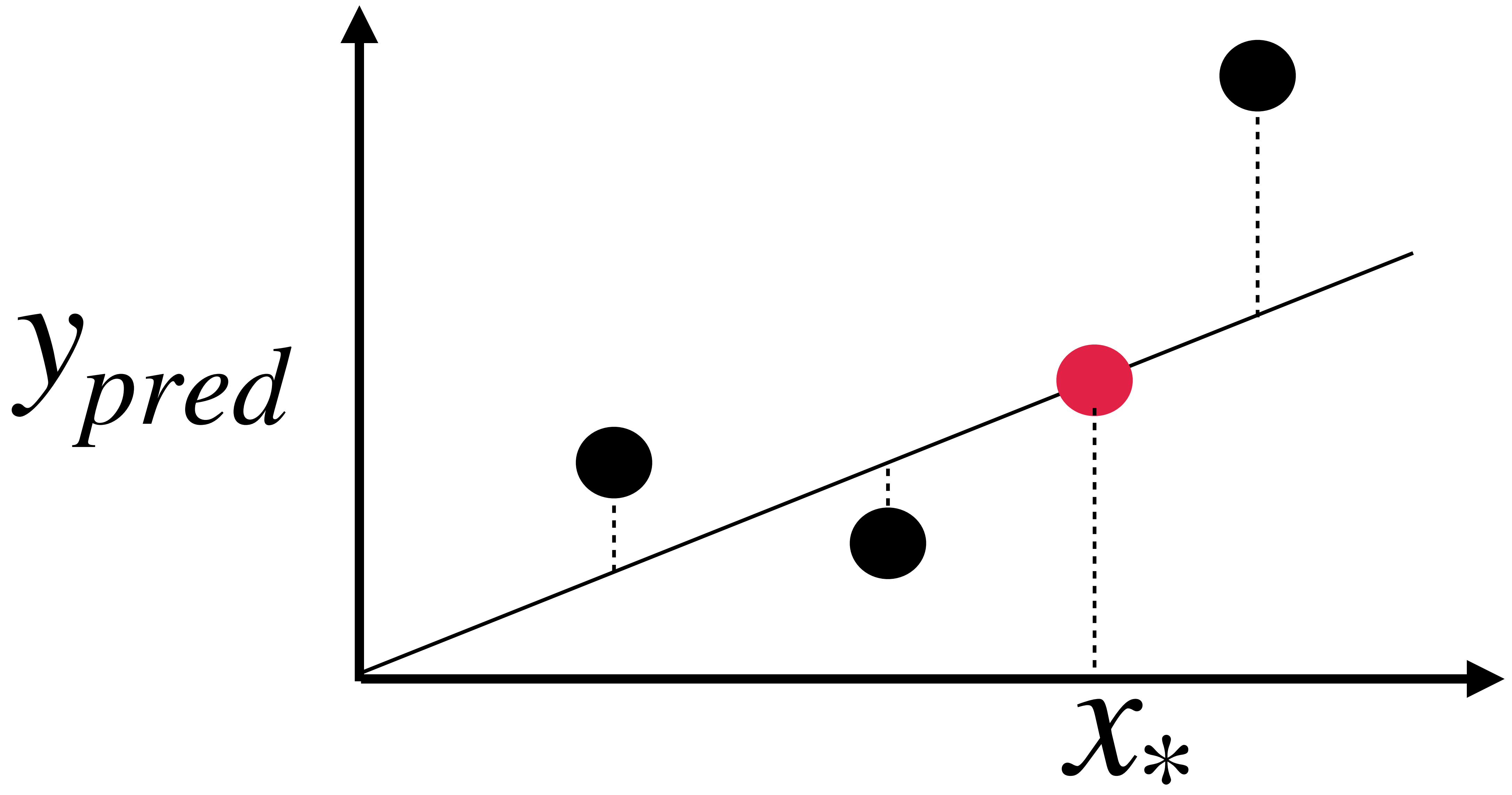

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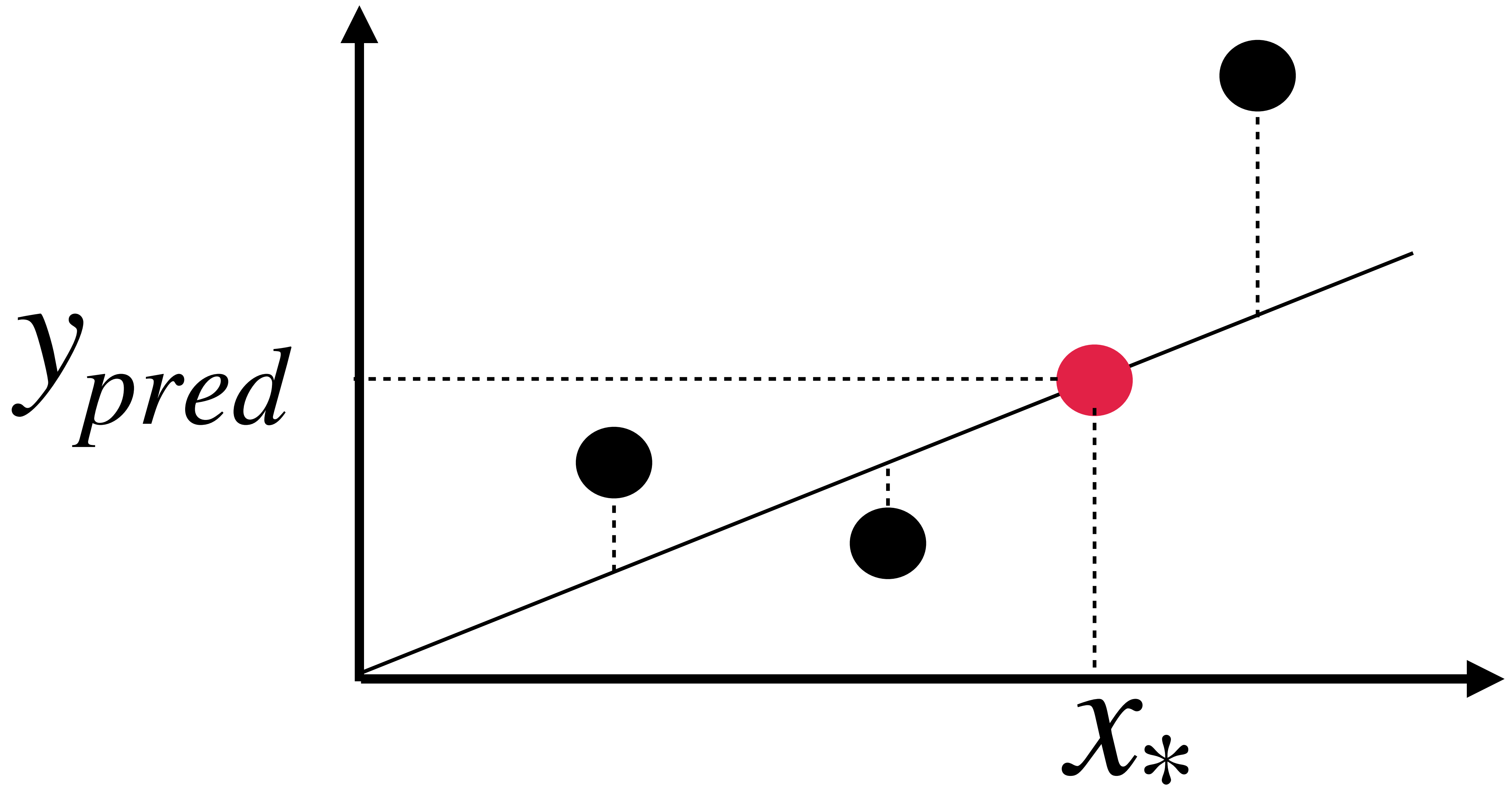

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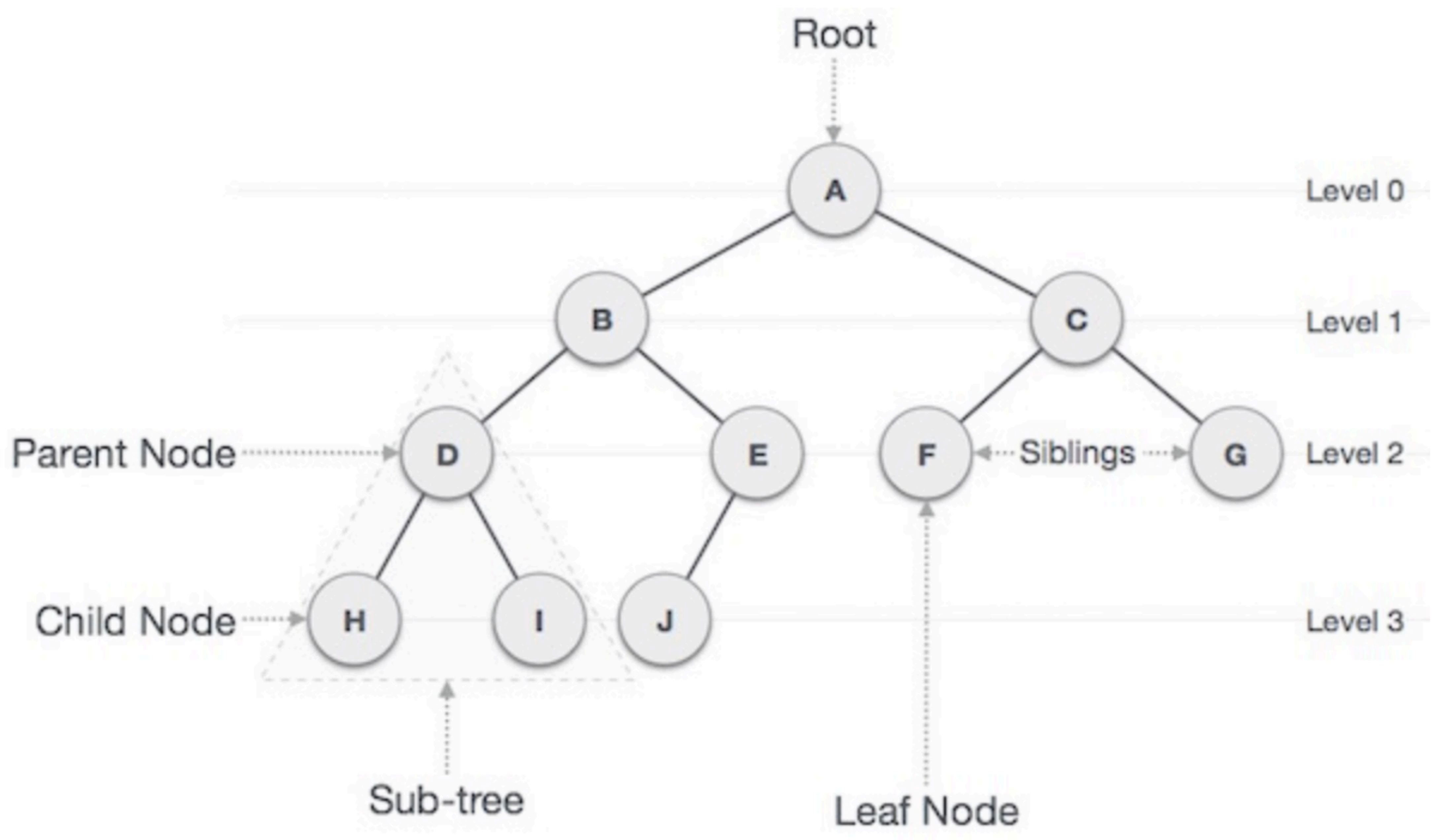
Why tree-based model?

Because it is very computer-sciencely!!

As a data structure, binary tree search, sort, etc...

An accompanying key phrase: divide and conquer



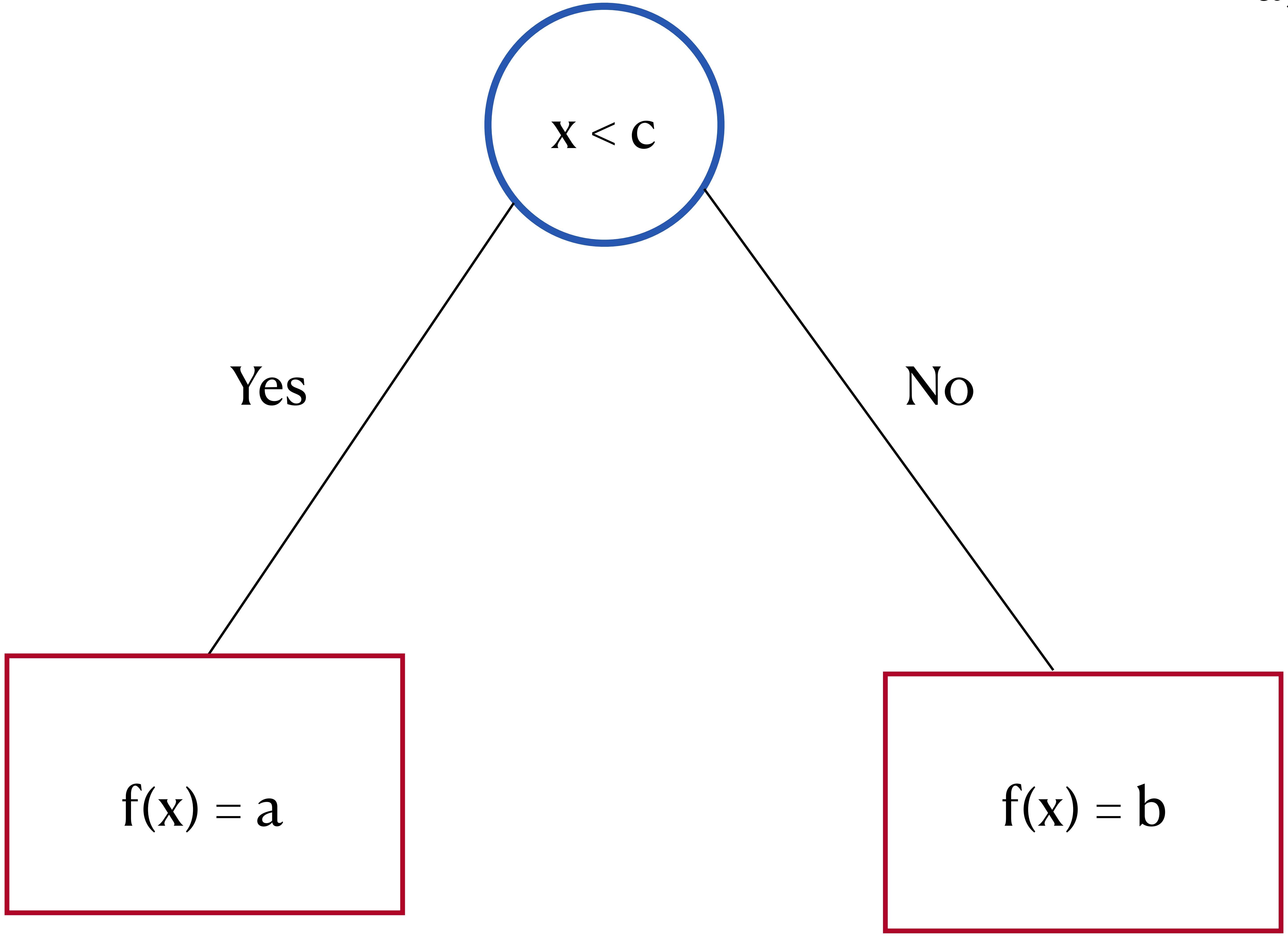
A simple numeric function of tree

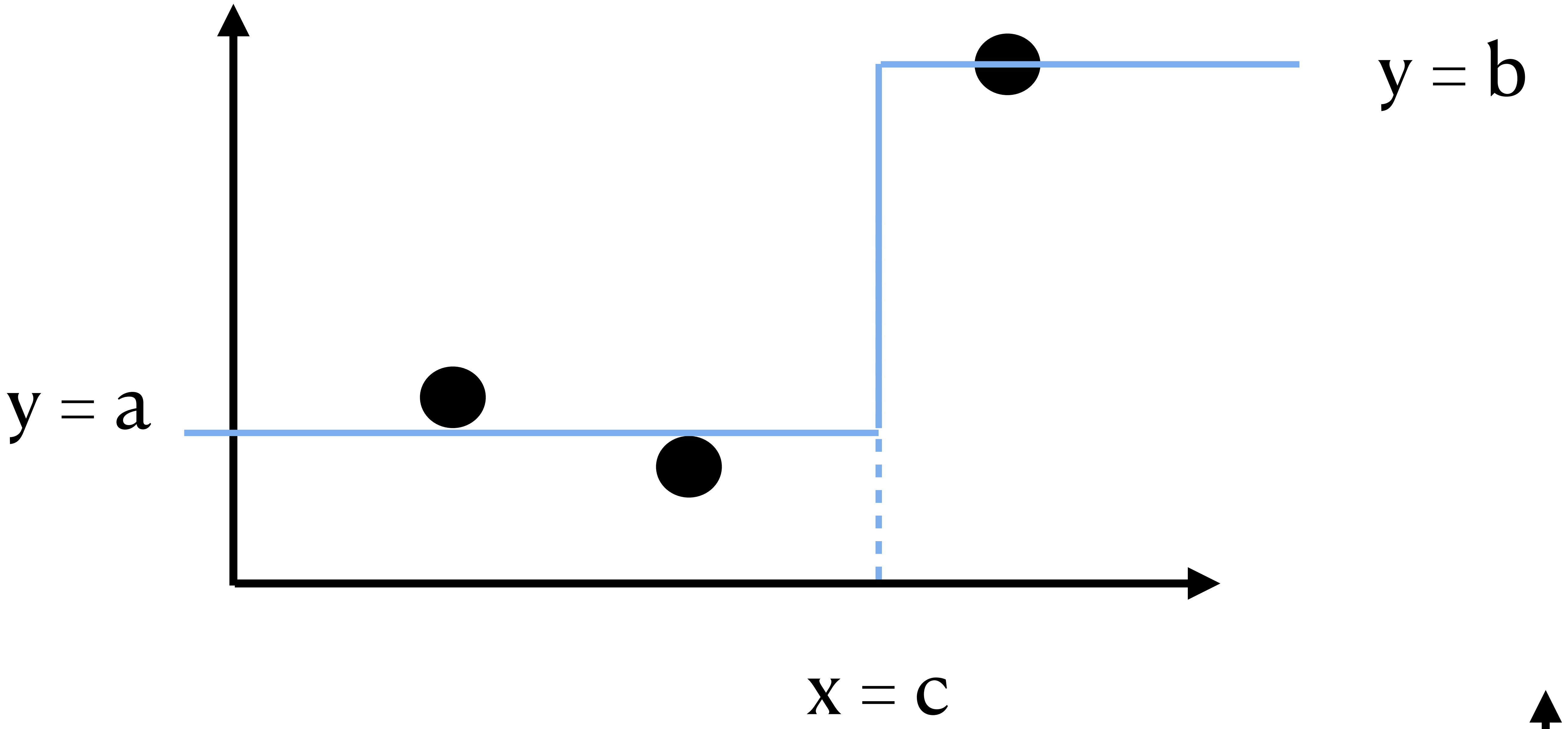
f(x):

The model has

'3' parameters:

a, b, c




y = b
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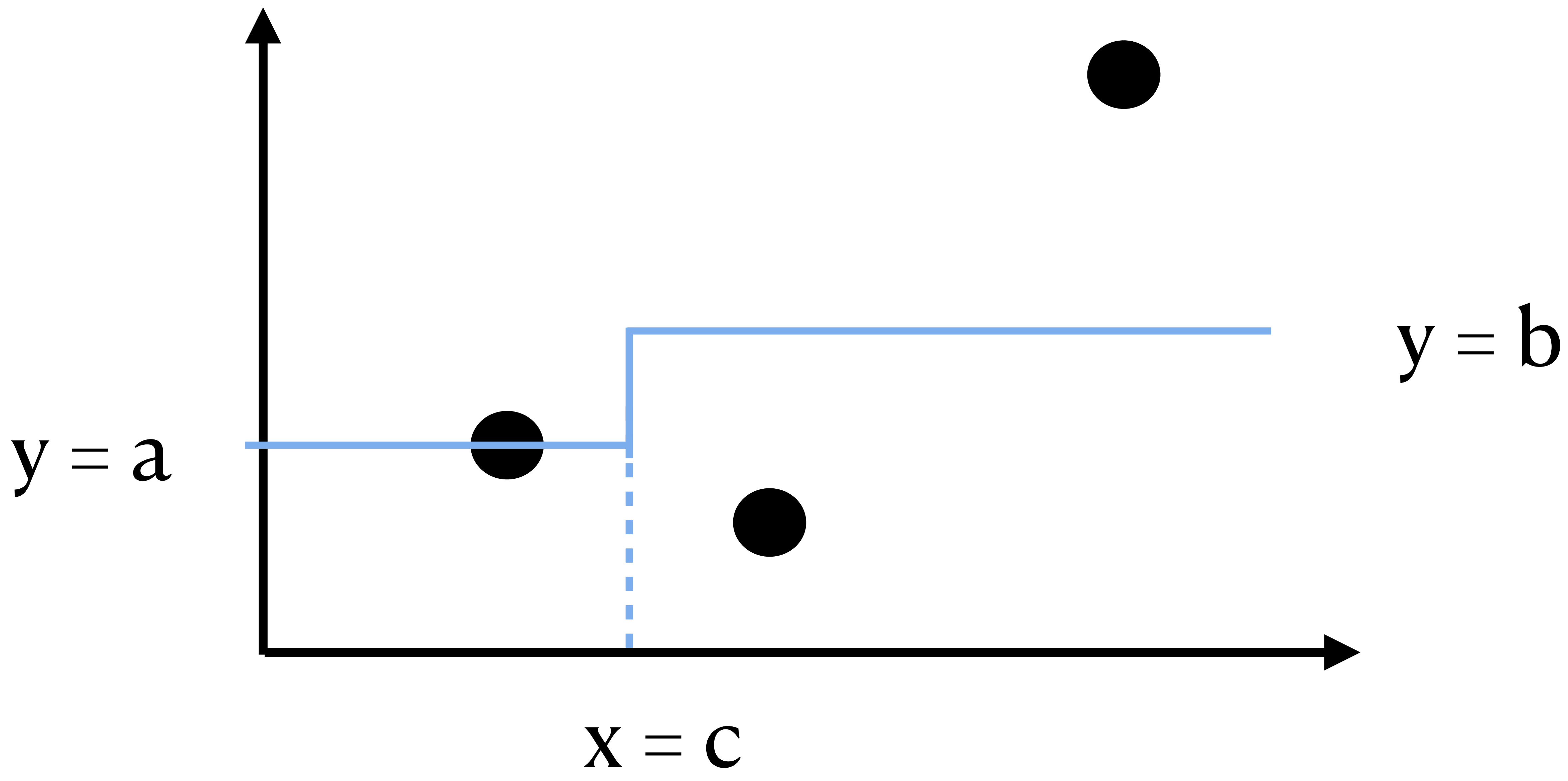
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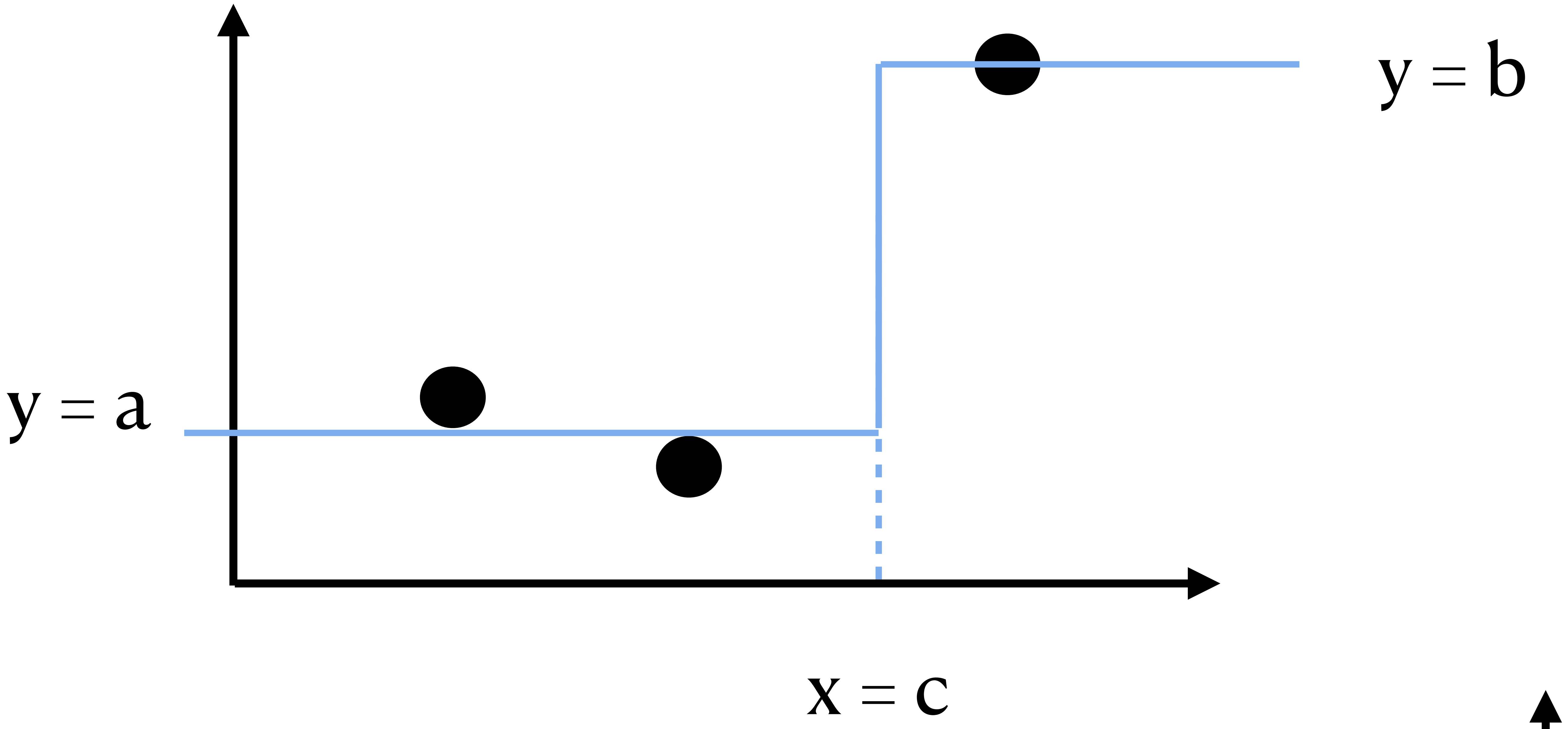

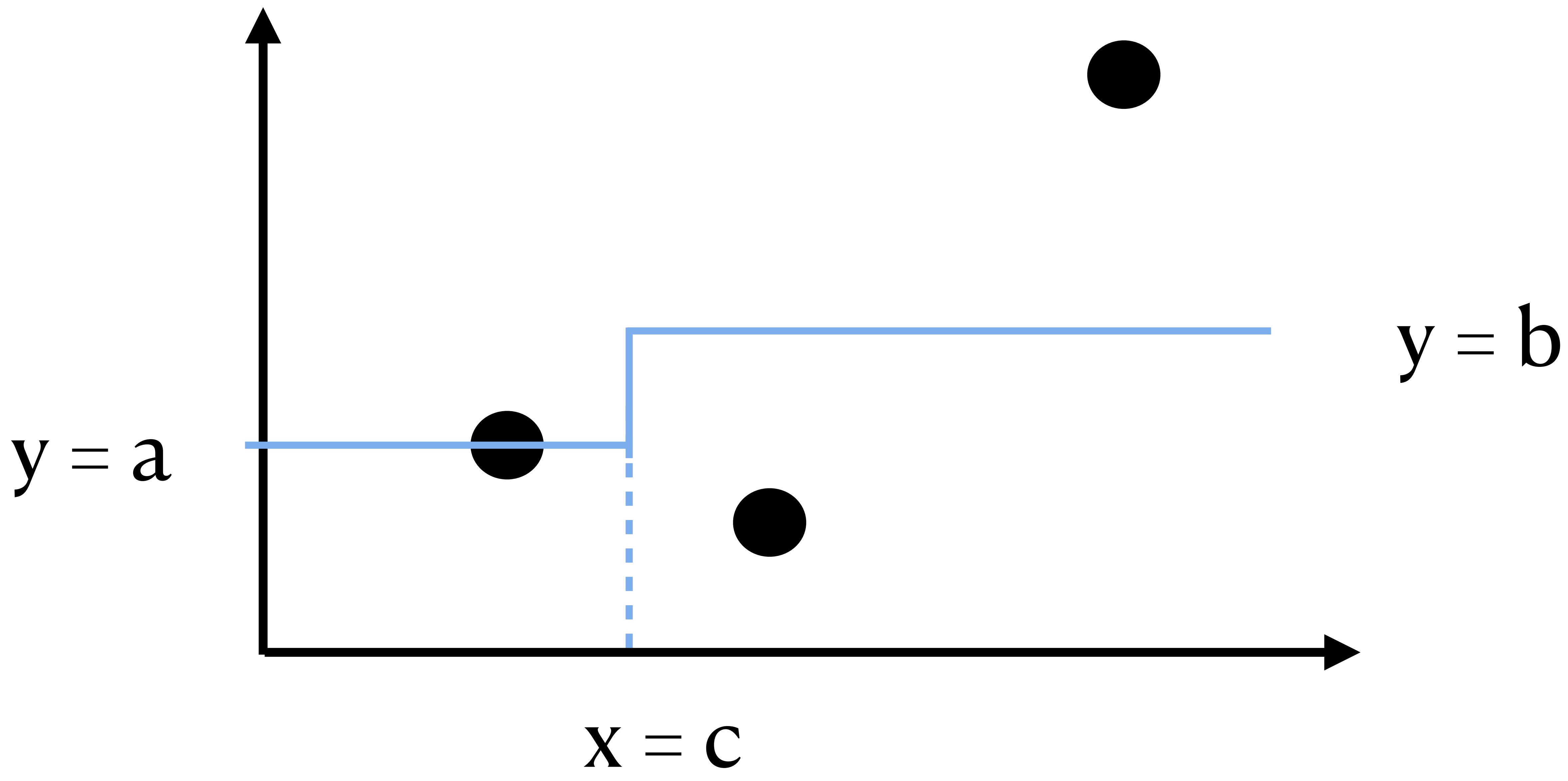

Which one has lower loss?

Baseball player salary data set

How player's salary is determined?

	Unnamed: 0	AtBat	Hits	HmRun	Runs	RBI	Walks	Years	CAtBat	CHits	...	CRuns	CRBI	CWalks	League	Division	PutOuts	Assists	Errors	Salary
1	-Alan Ashby	315	81	7	24	38	39	14	3449	835	...	321	414	375	N	W	632	43	10	475.0
2	-Alvin Davis	479	130	18	66	72	76	3	1624	457	...	224	266	263	A	W	880	82	14	480.0
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X_1

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Baseball player salary data set

How player’s salary is determined?

Y

X_2

X_1

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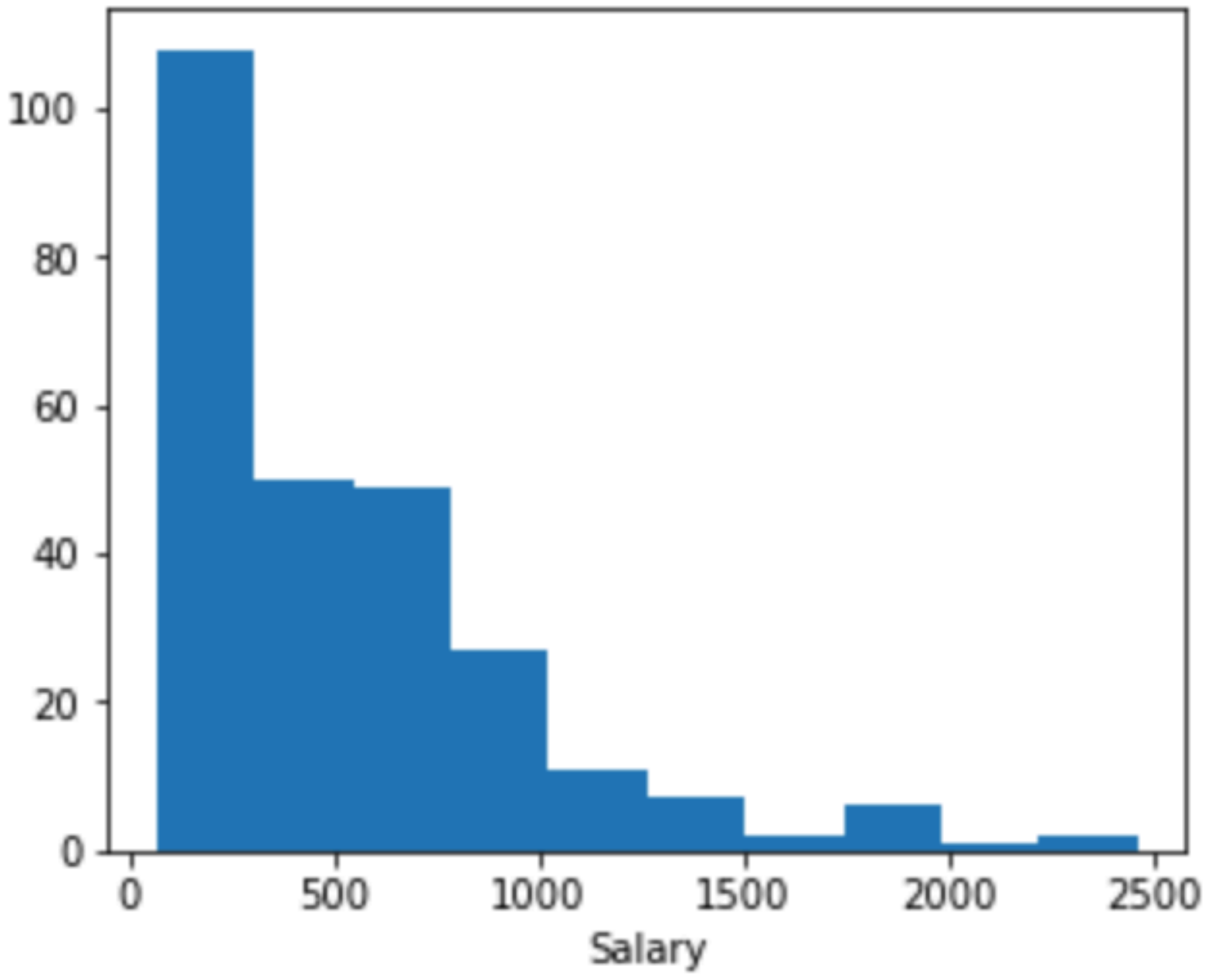
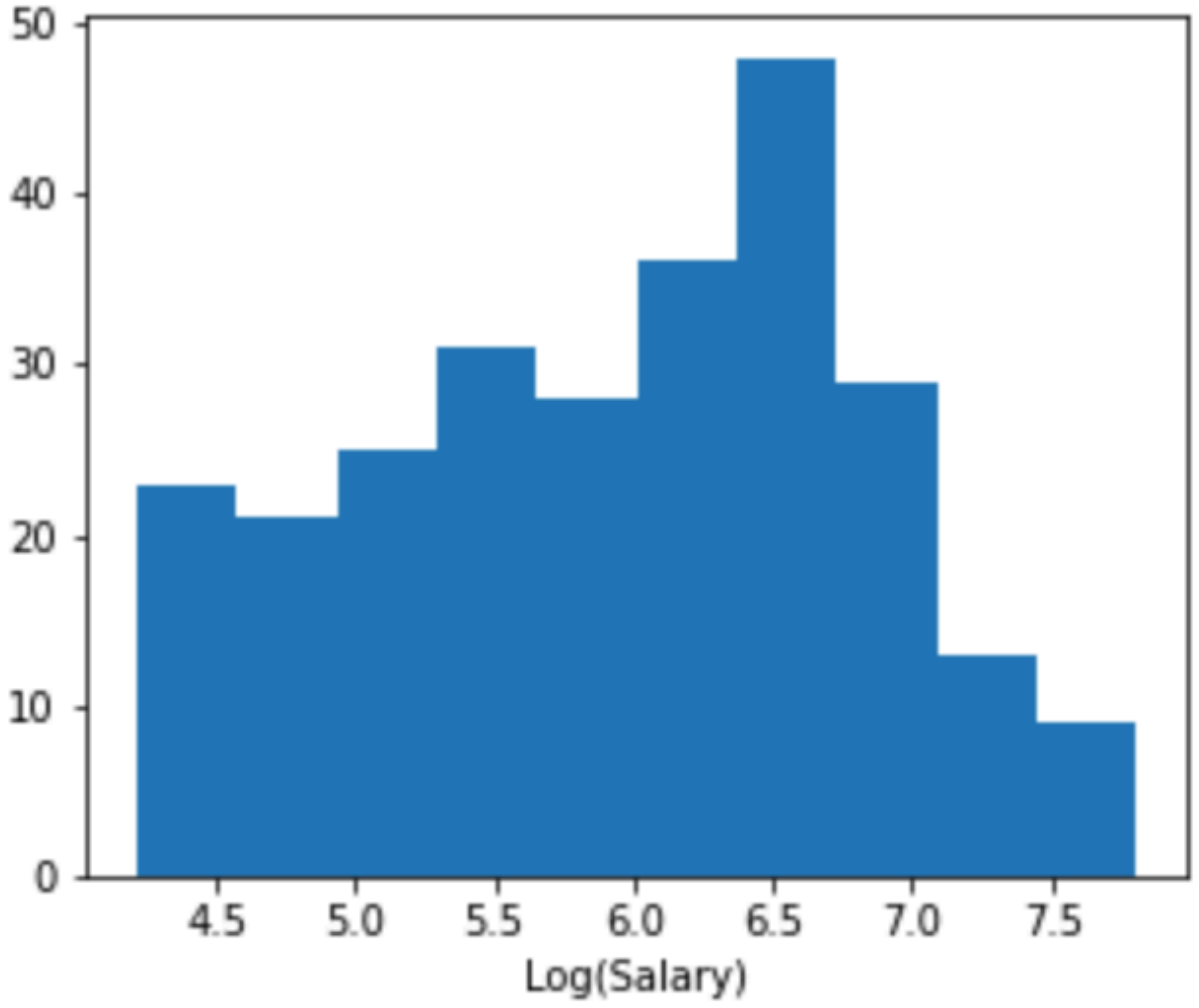
Pre-processing the data

Taking log of salary

$$salary \rightarrow \log salary$$





Tree-based model for Hitters data

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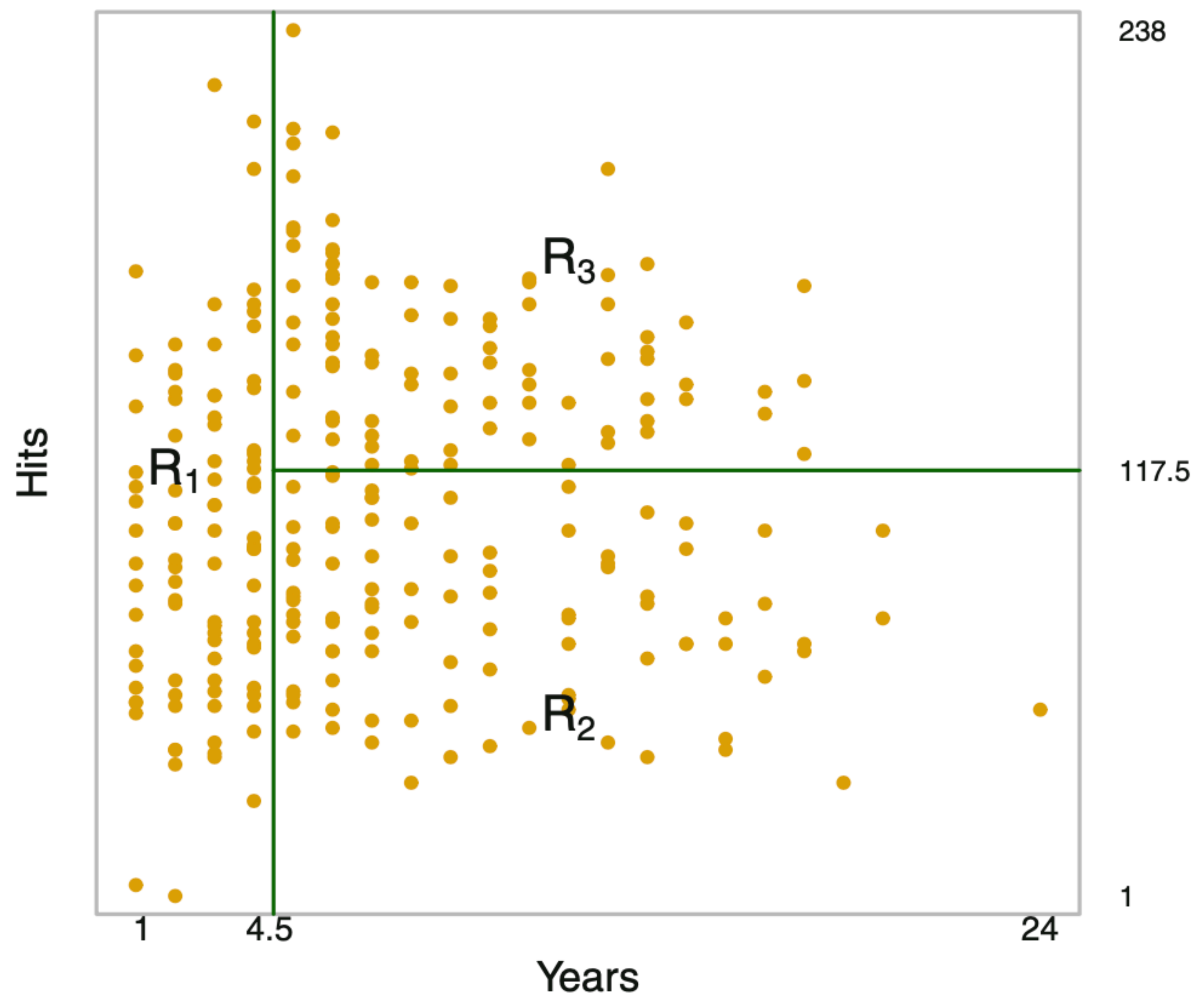

FIGURE 8.2. *The three-region partition for the* Hitters *data set from the regression tree illustrated in Figure 8.1.*

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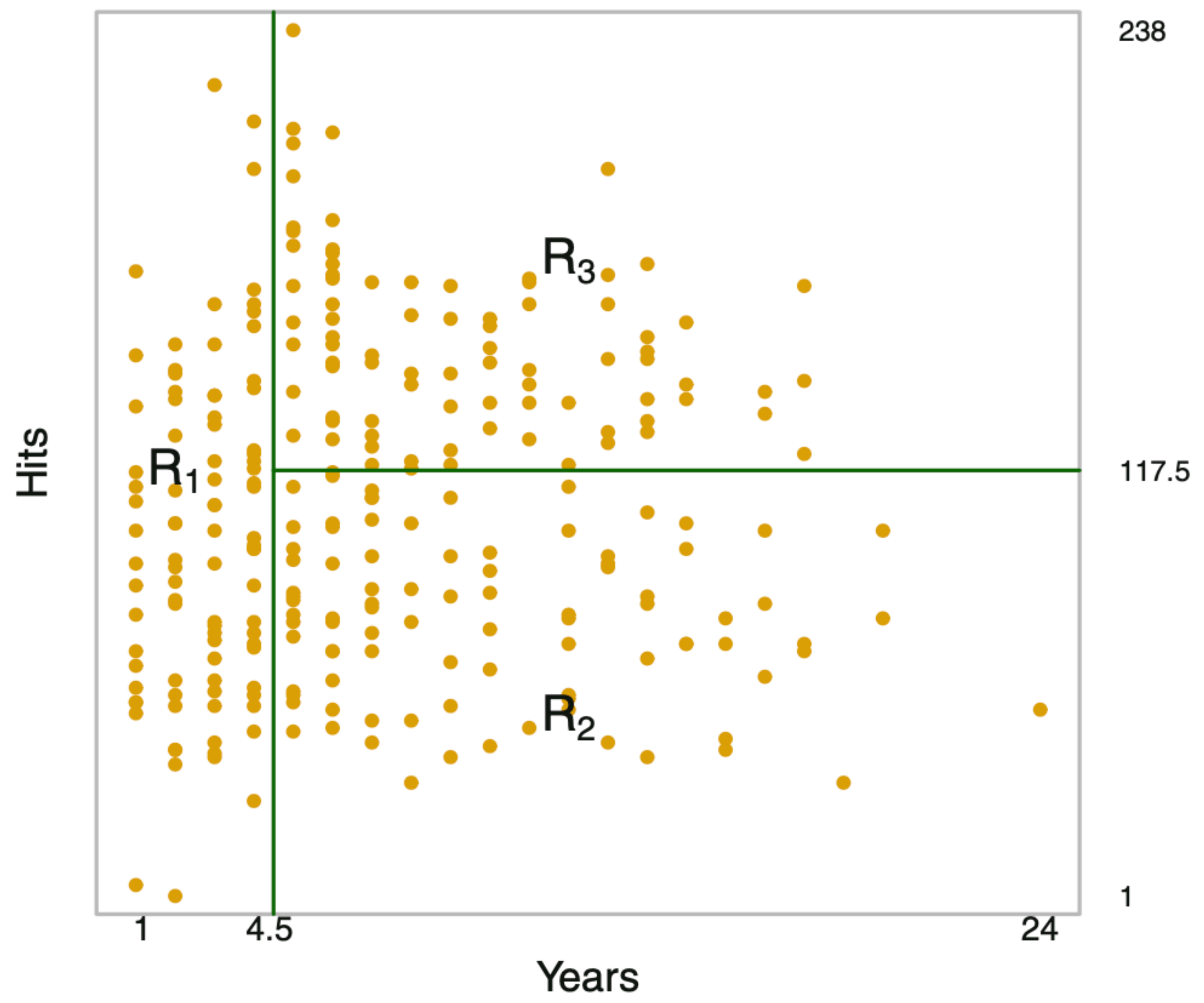

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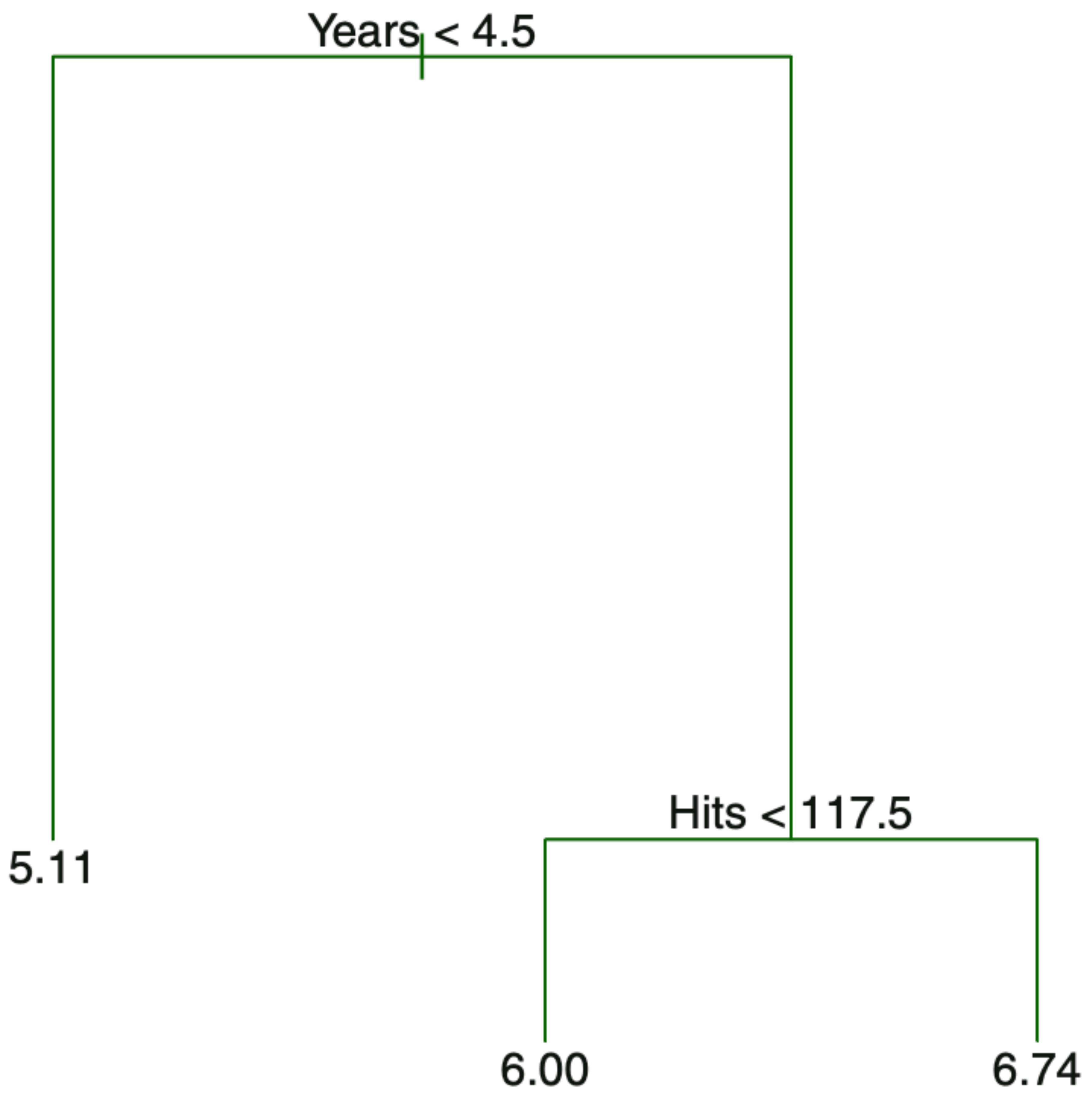

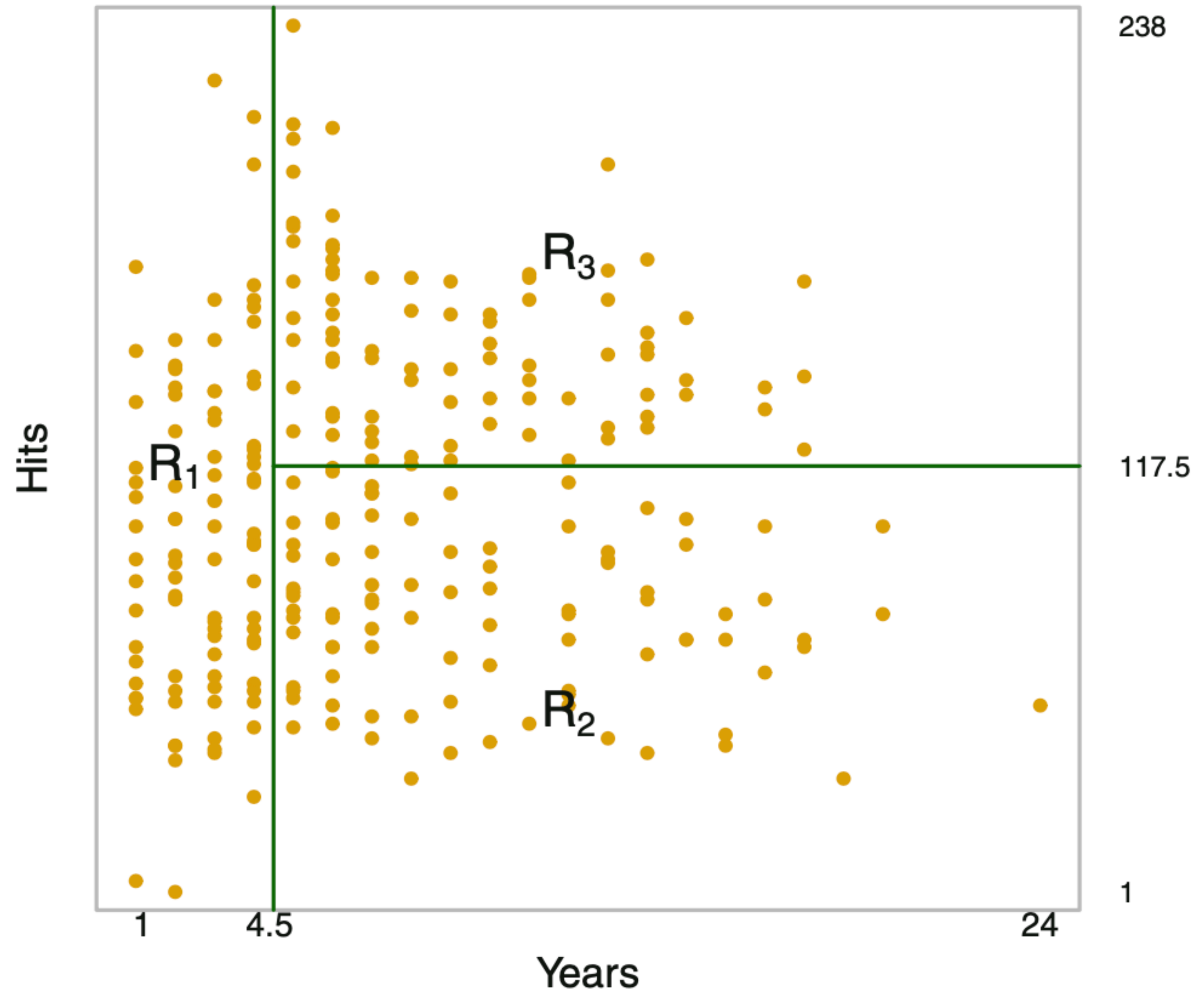

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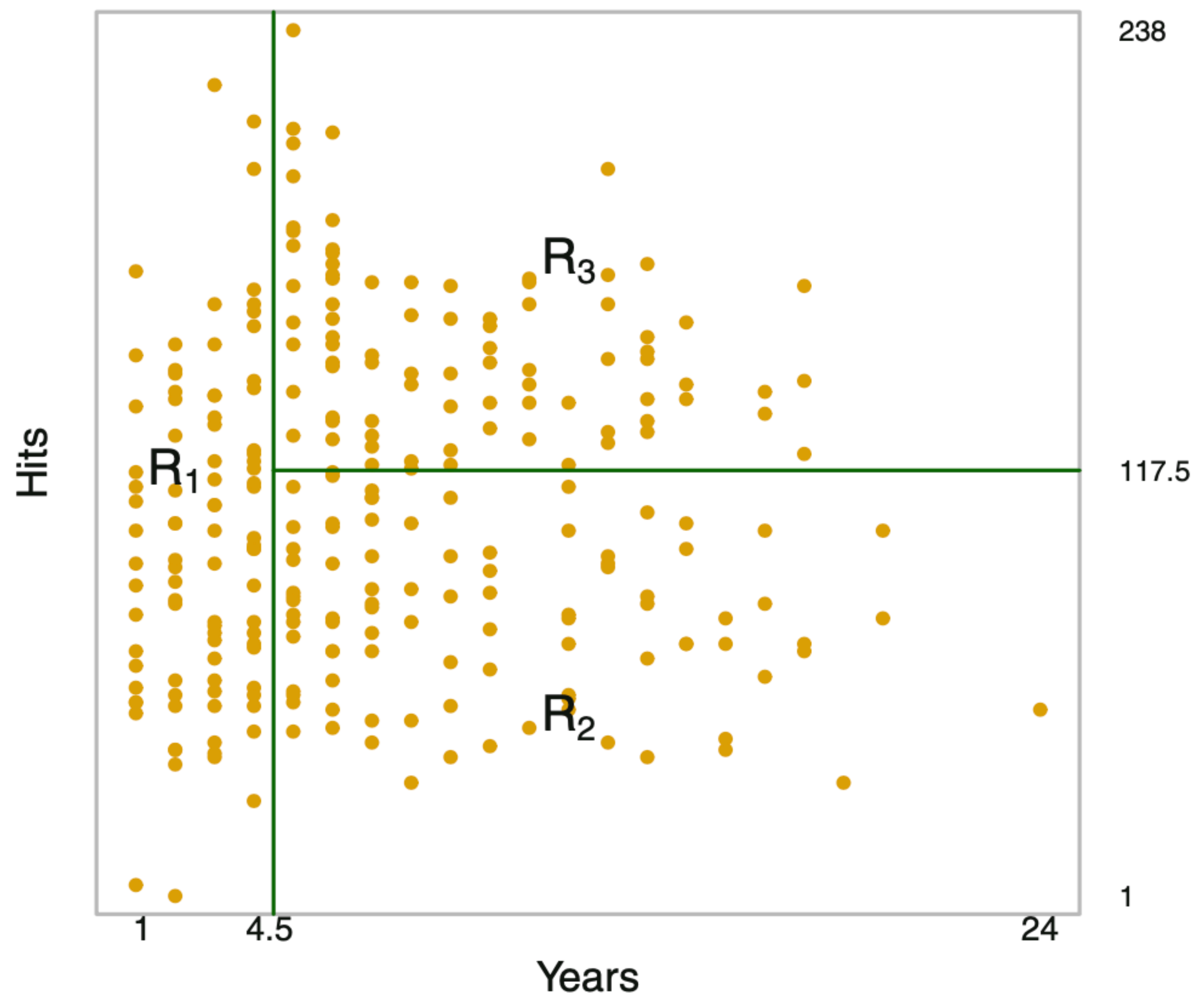

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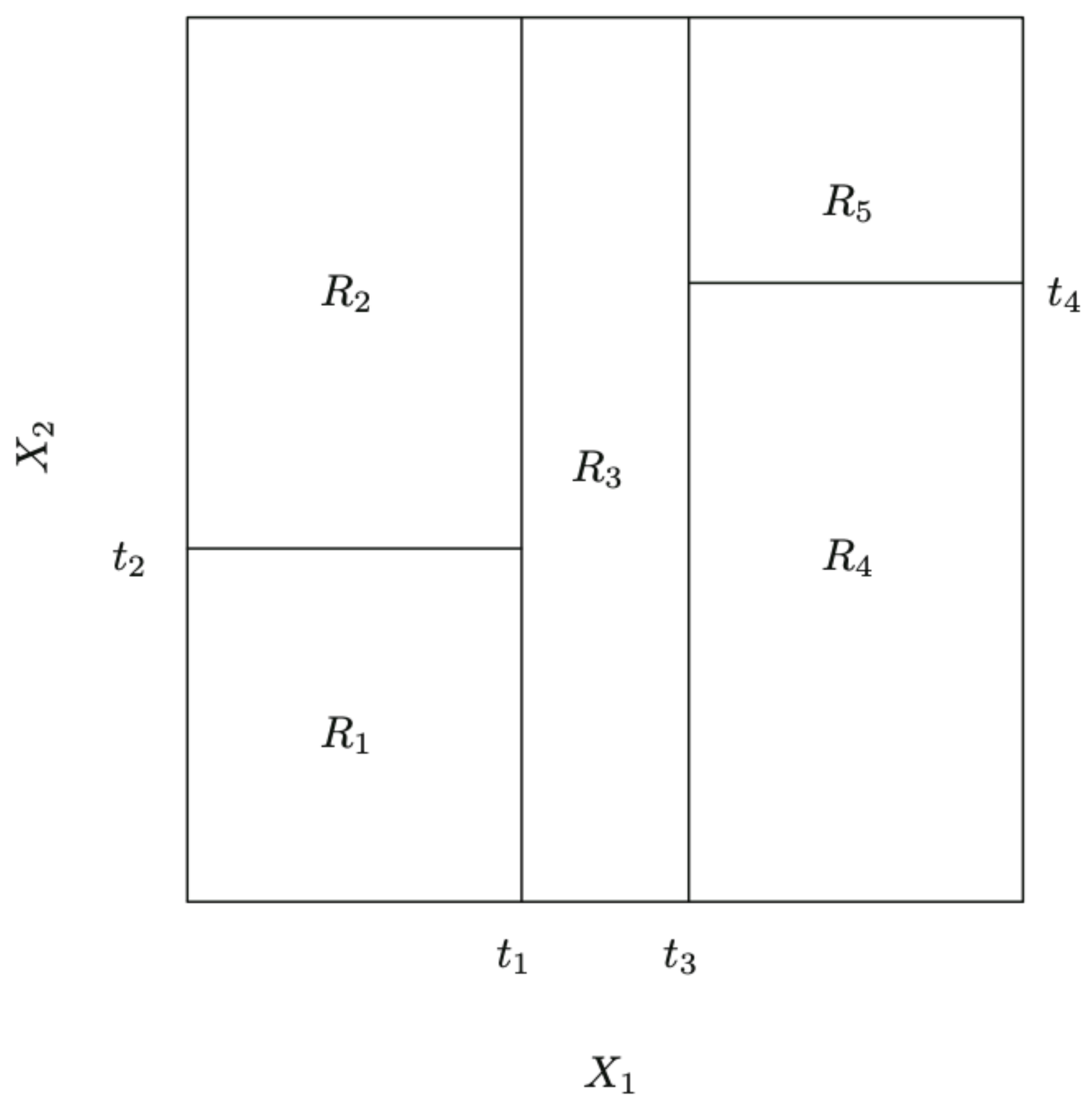
Deeper tree models



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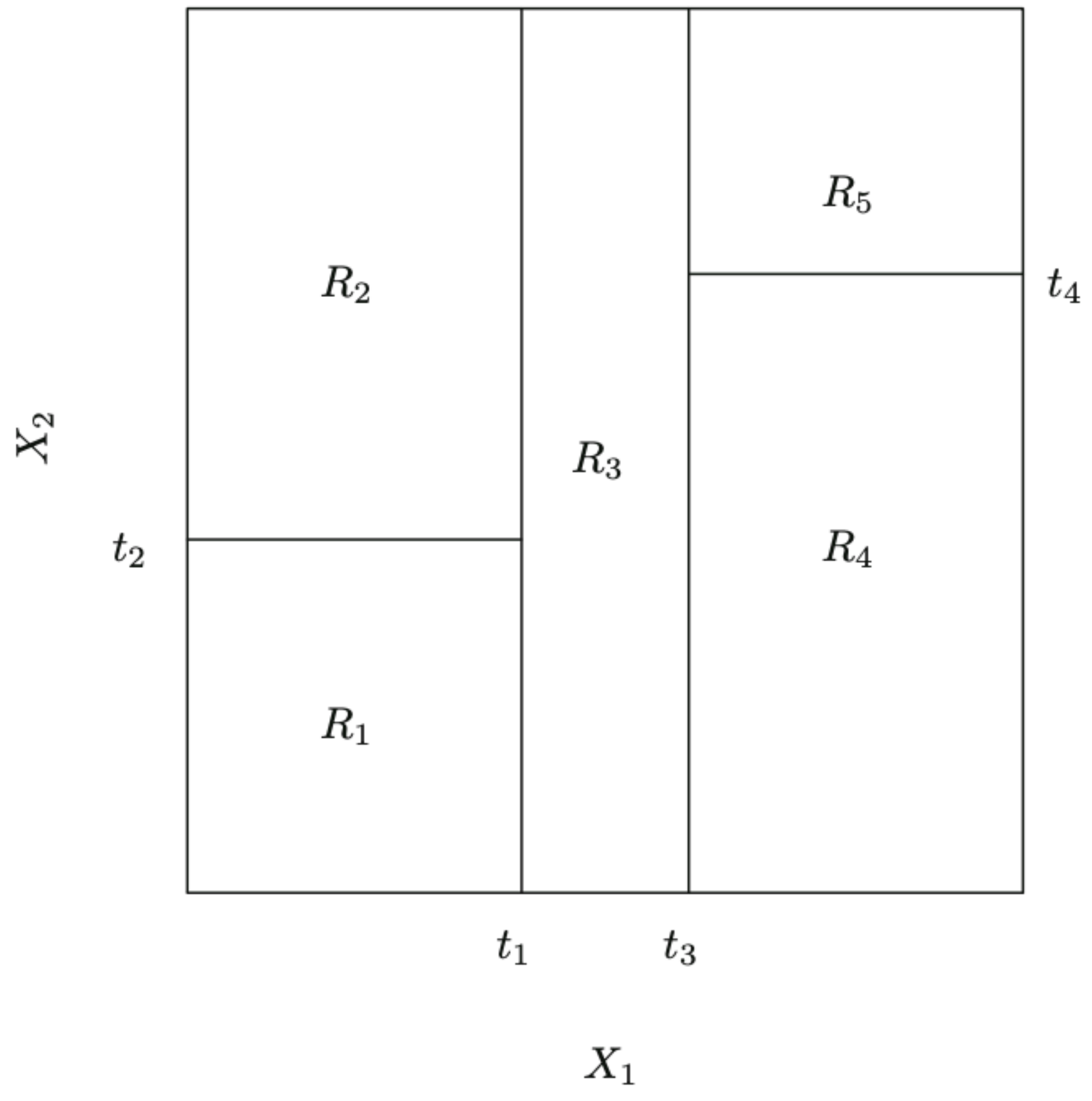

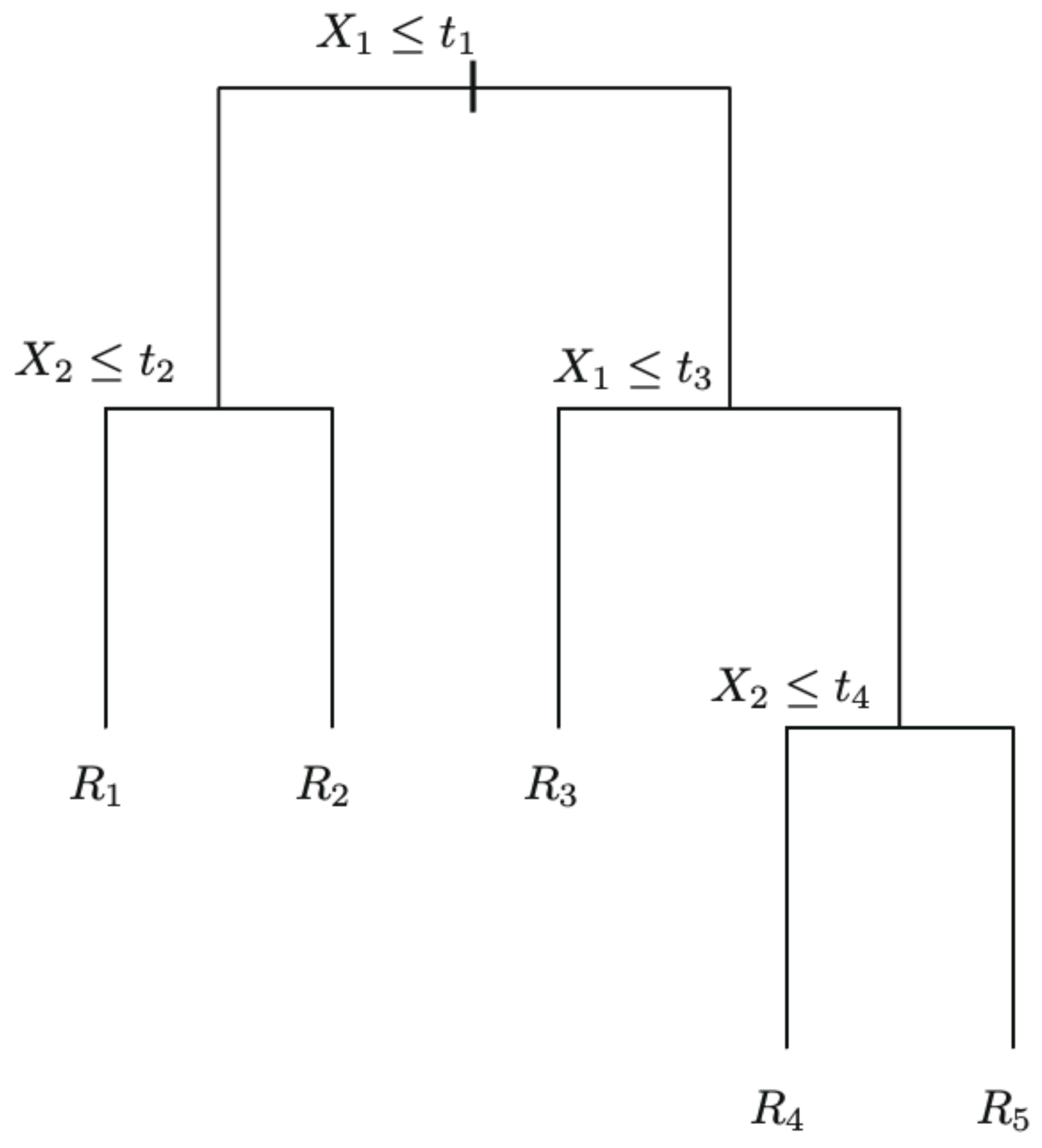

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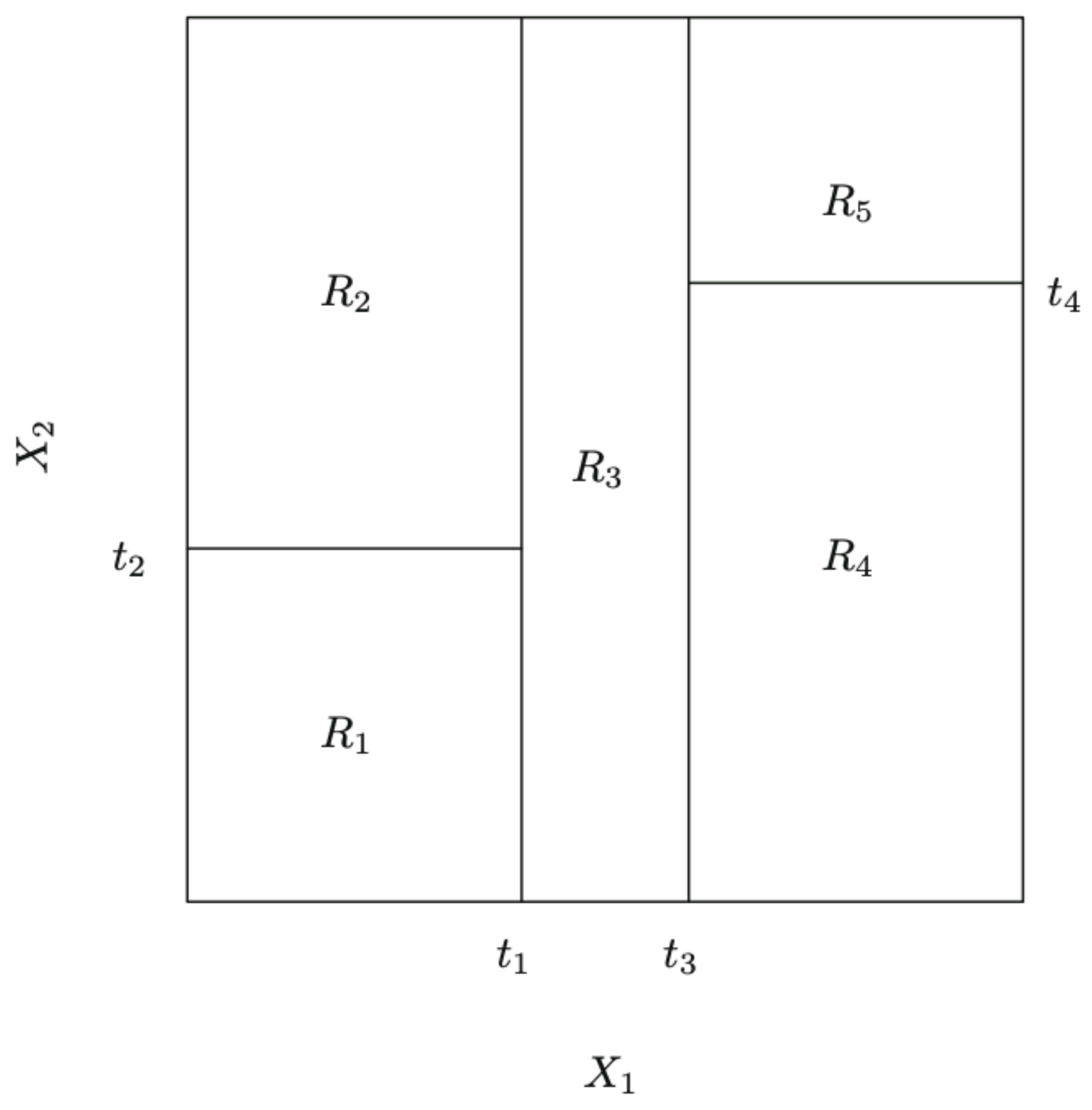

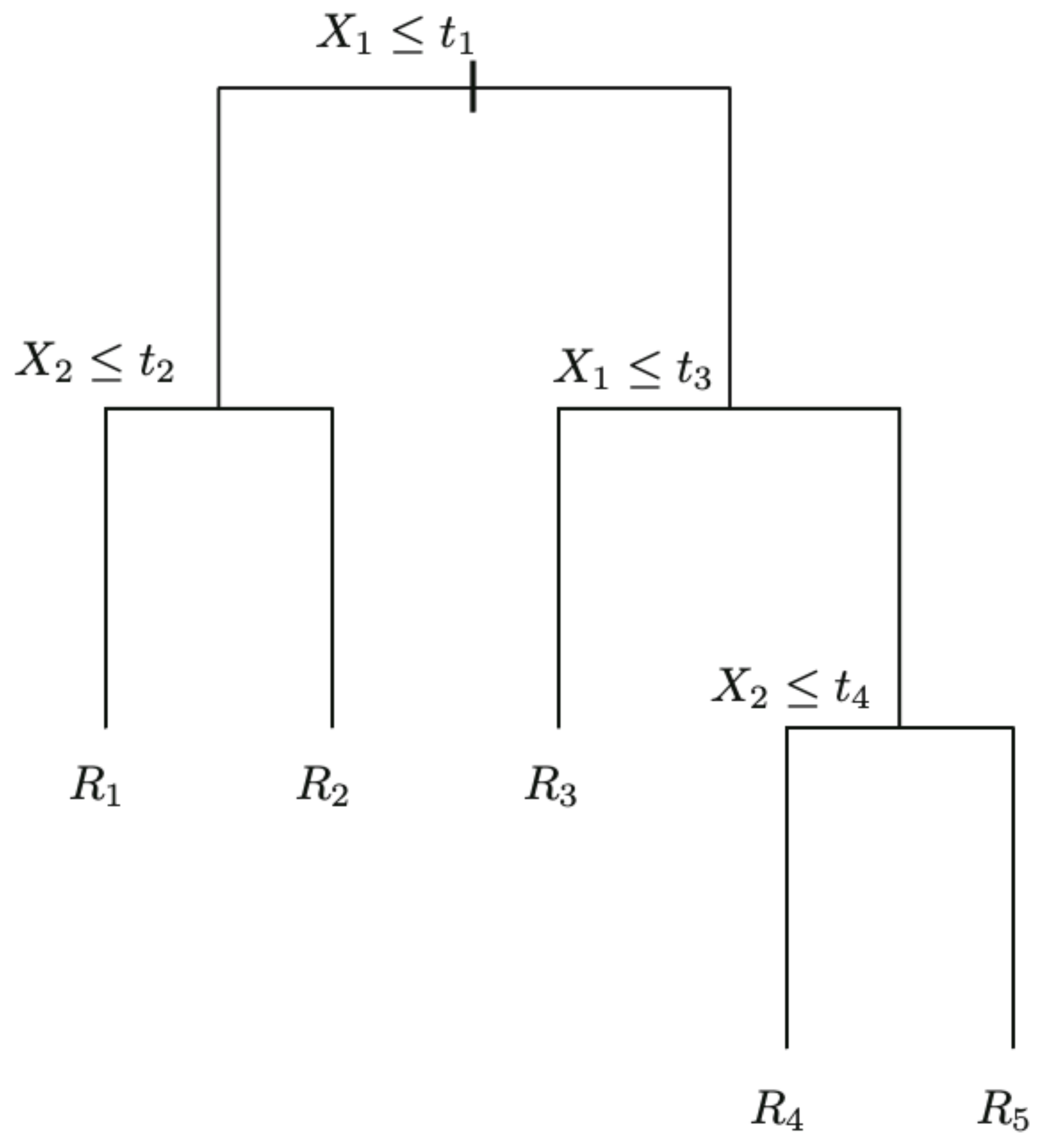

Convince yourself that the tree leads to the partition of X into the left diagram!!

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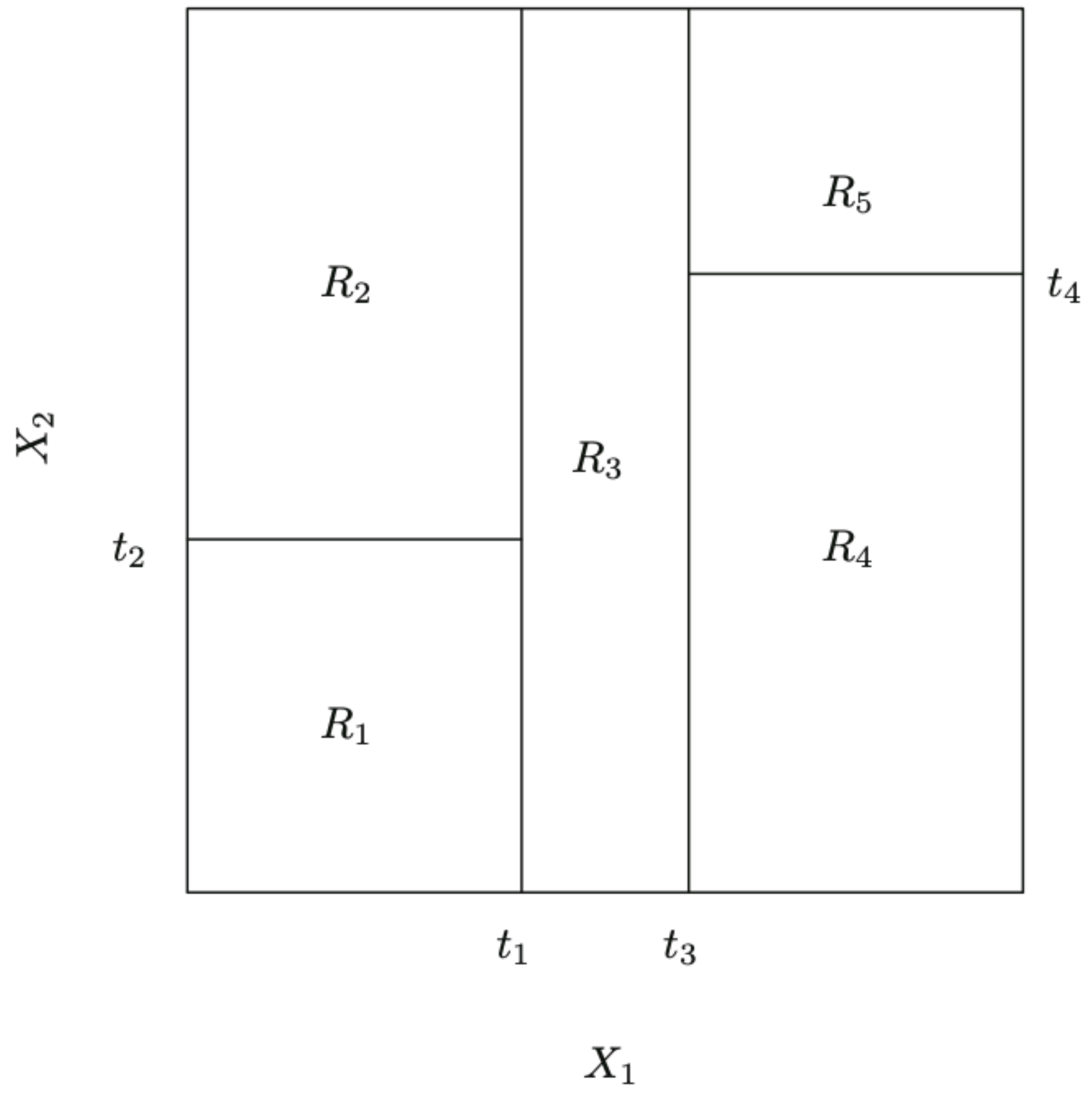

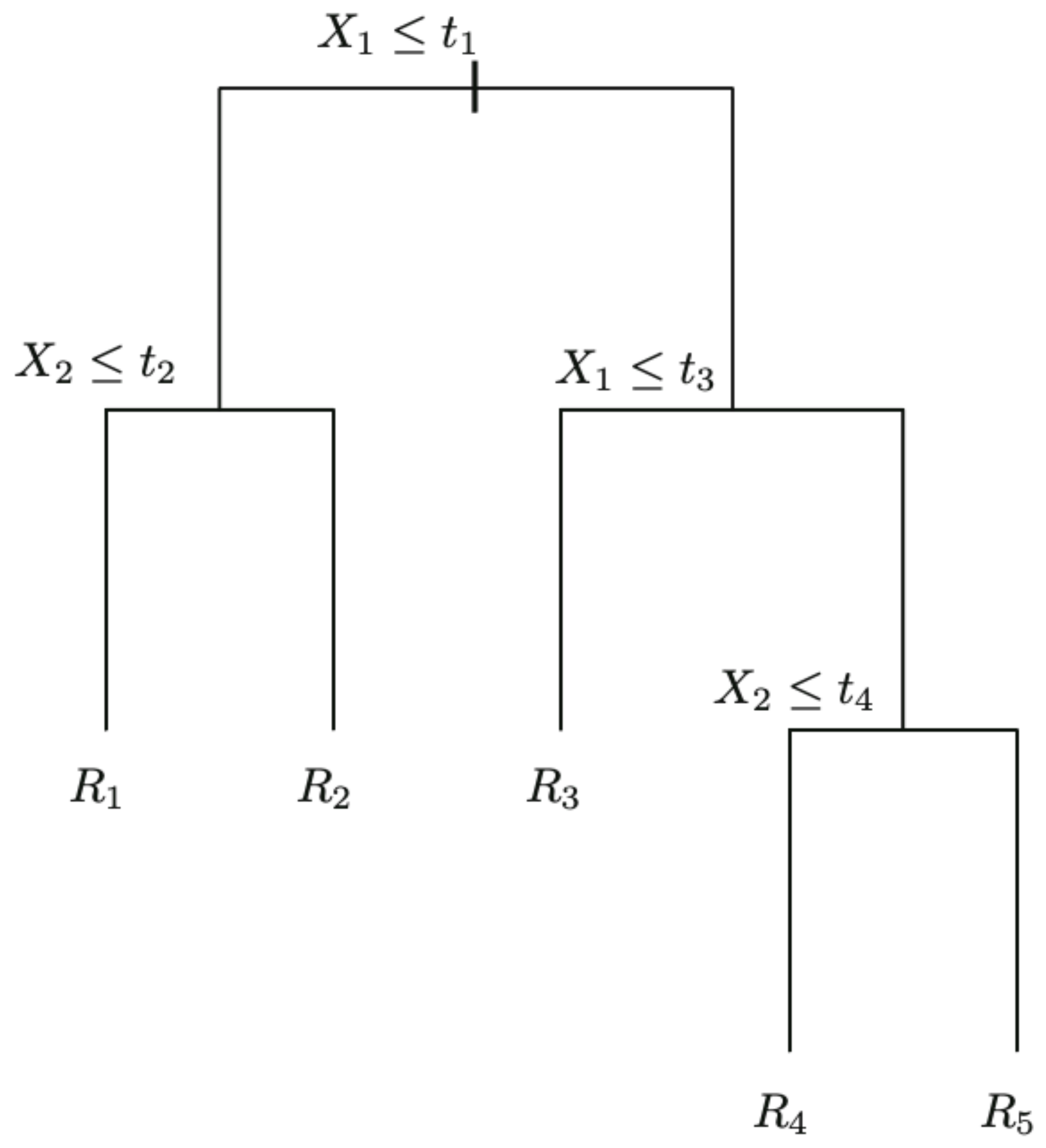

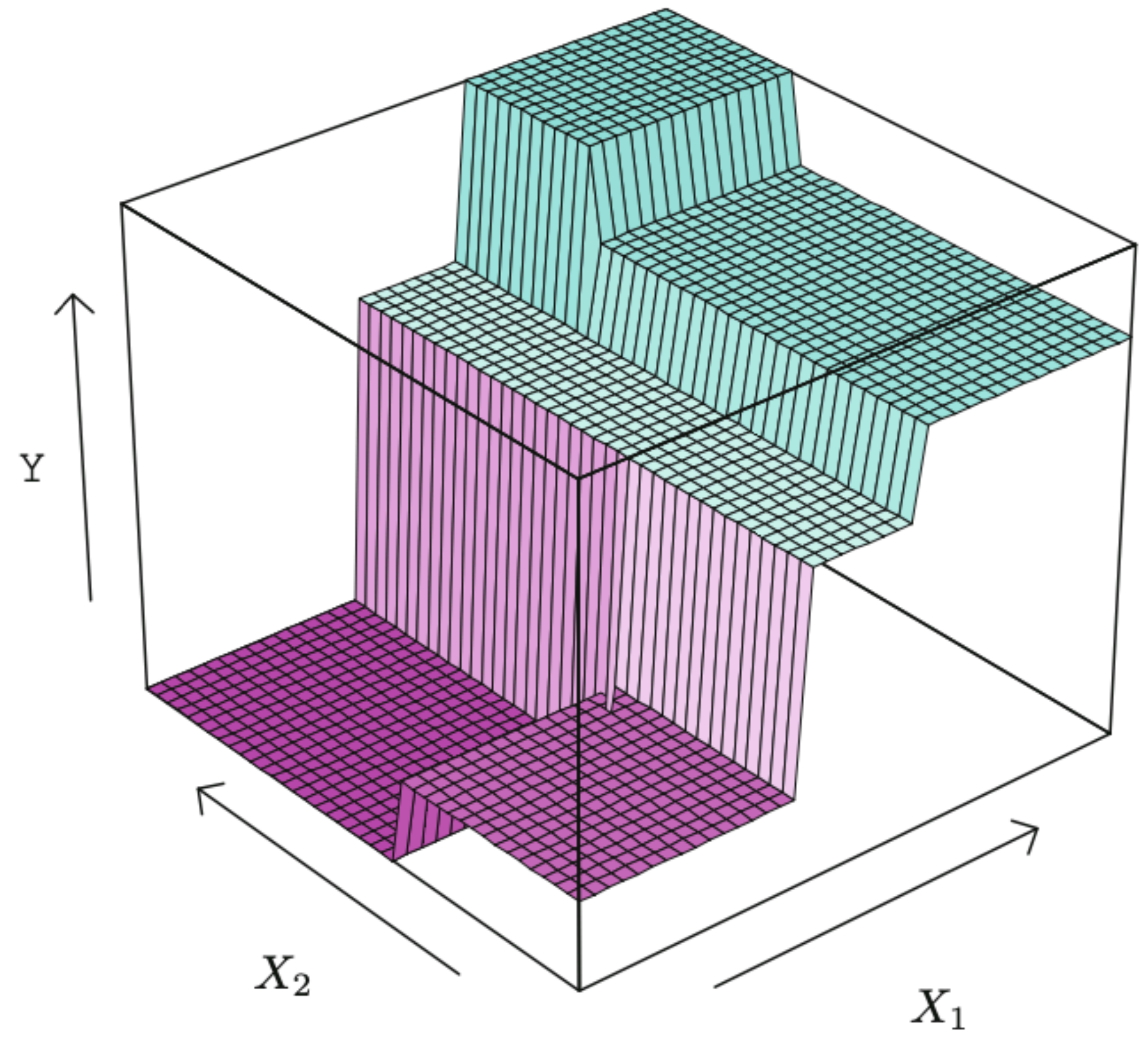

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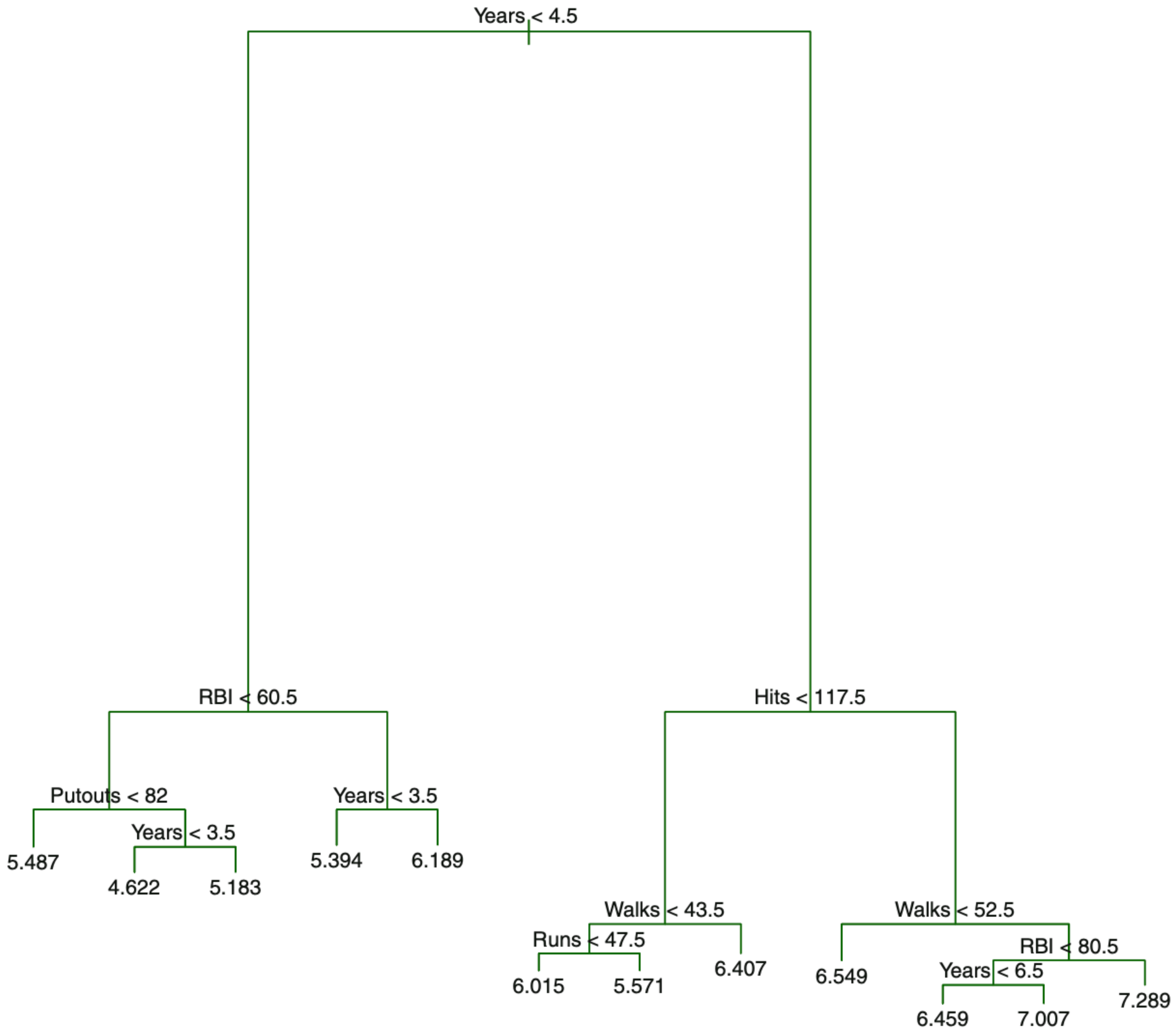
Deeper trees are overfitting

Deeper not always better



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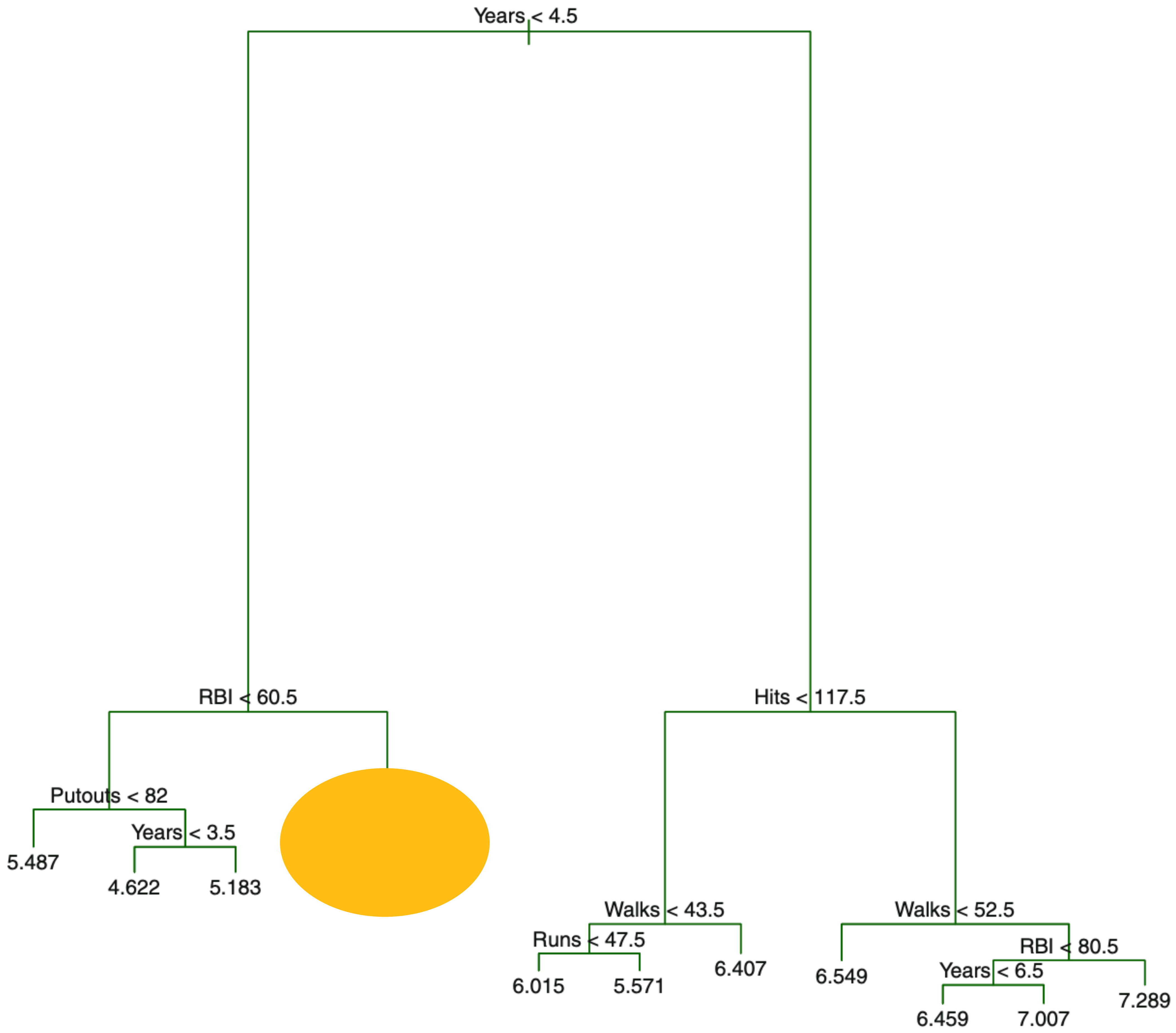

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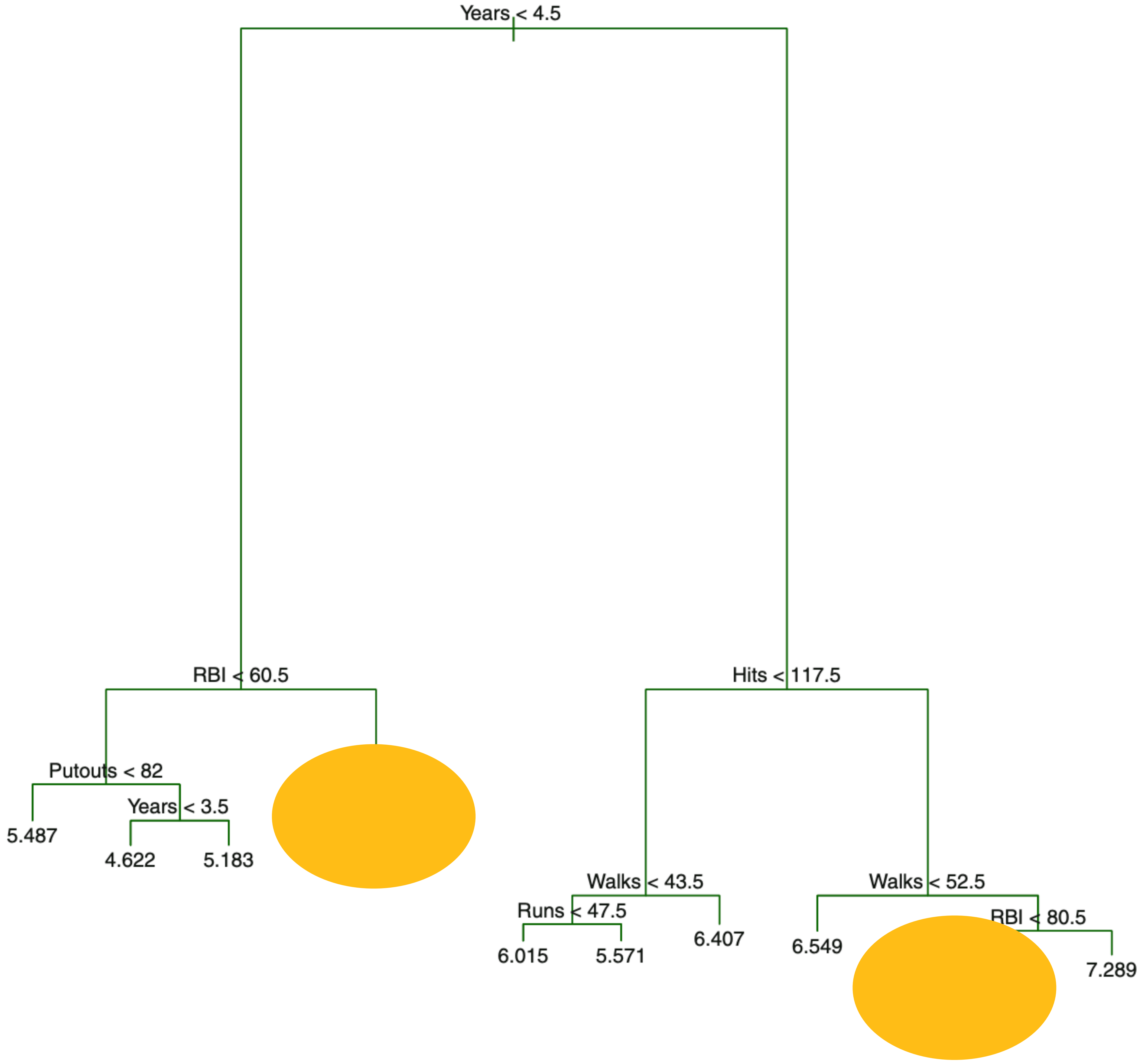

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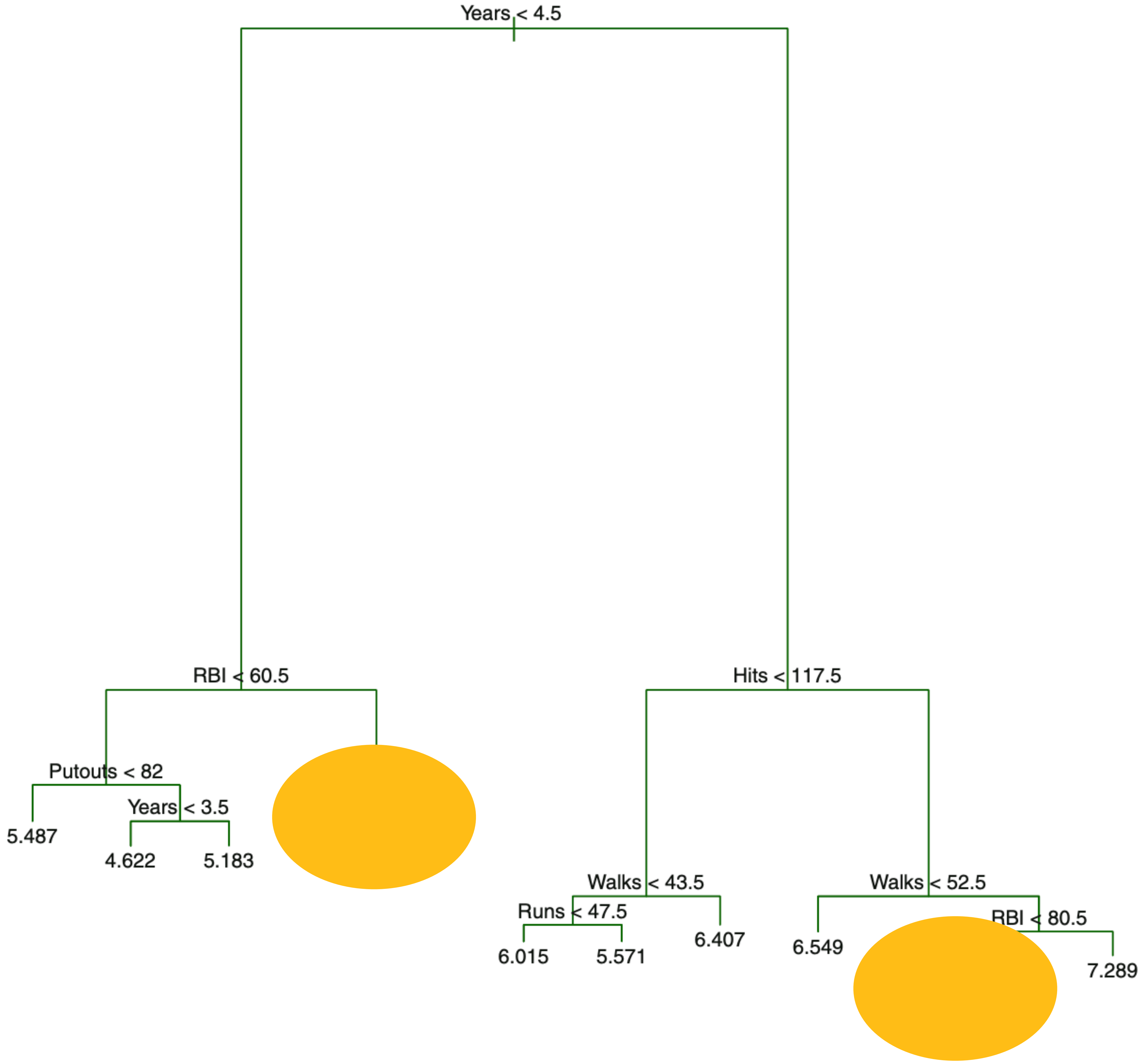

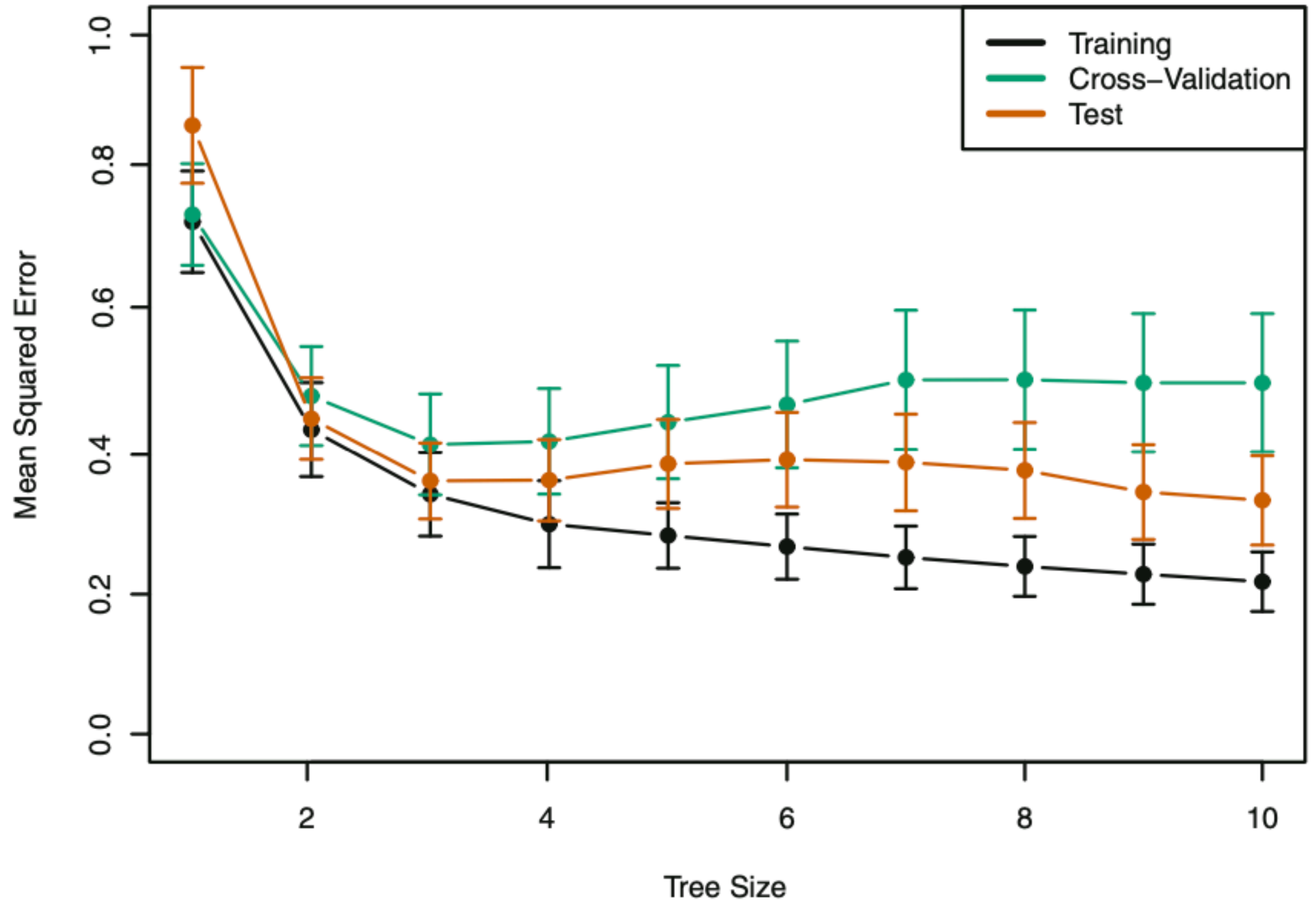

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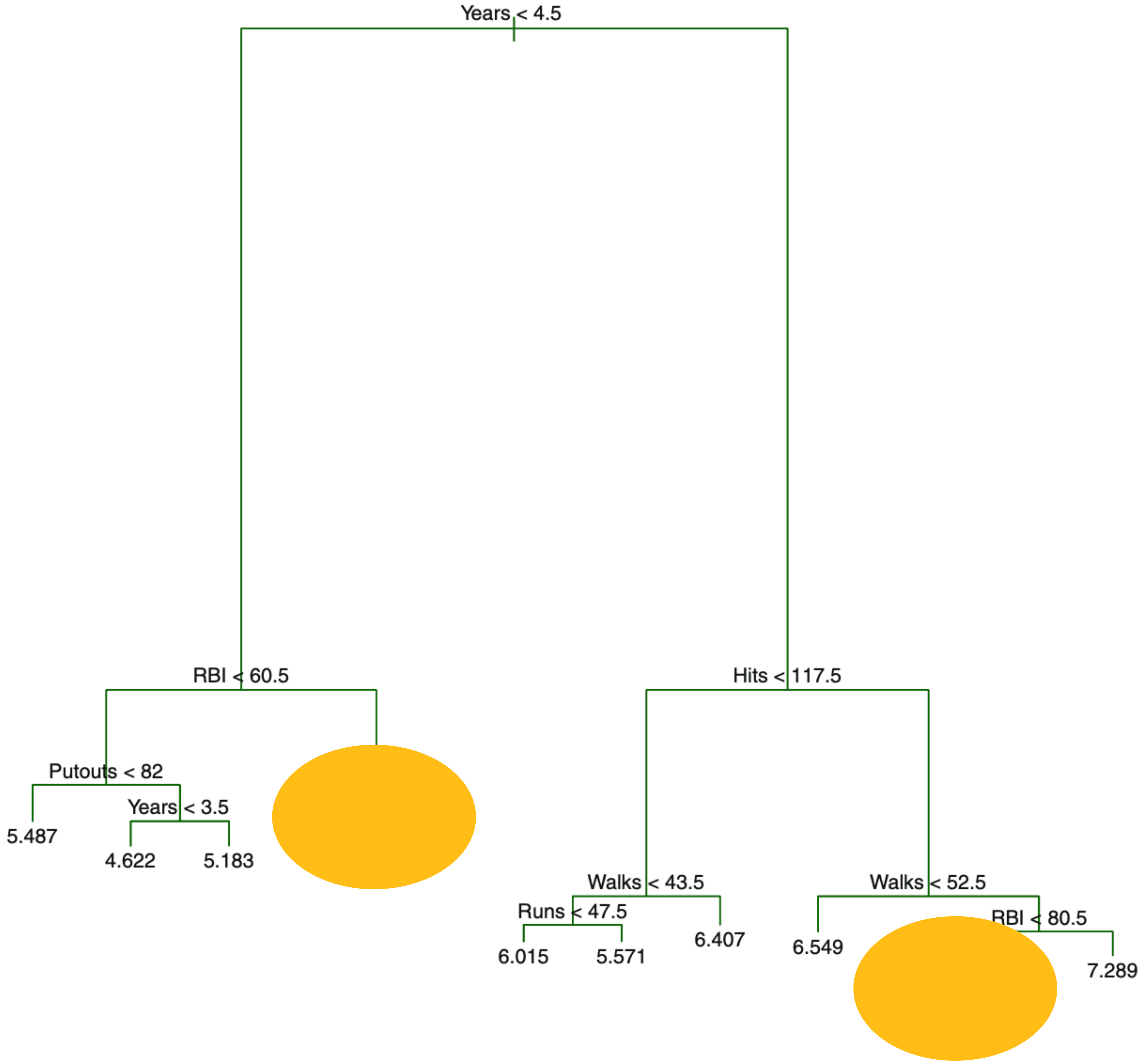

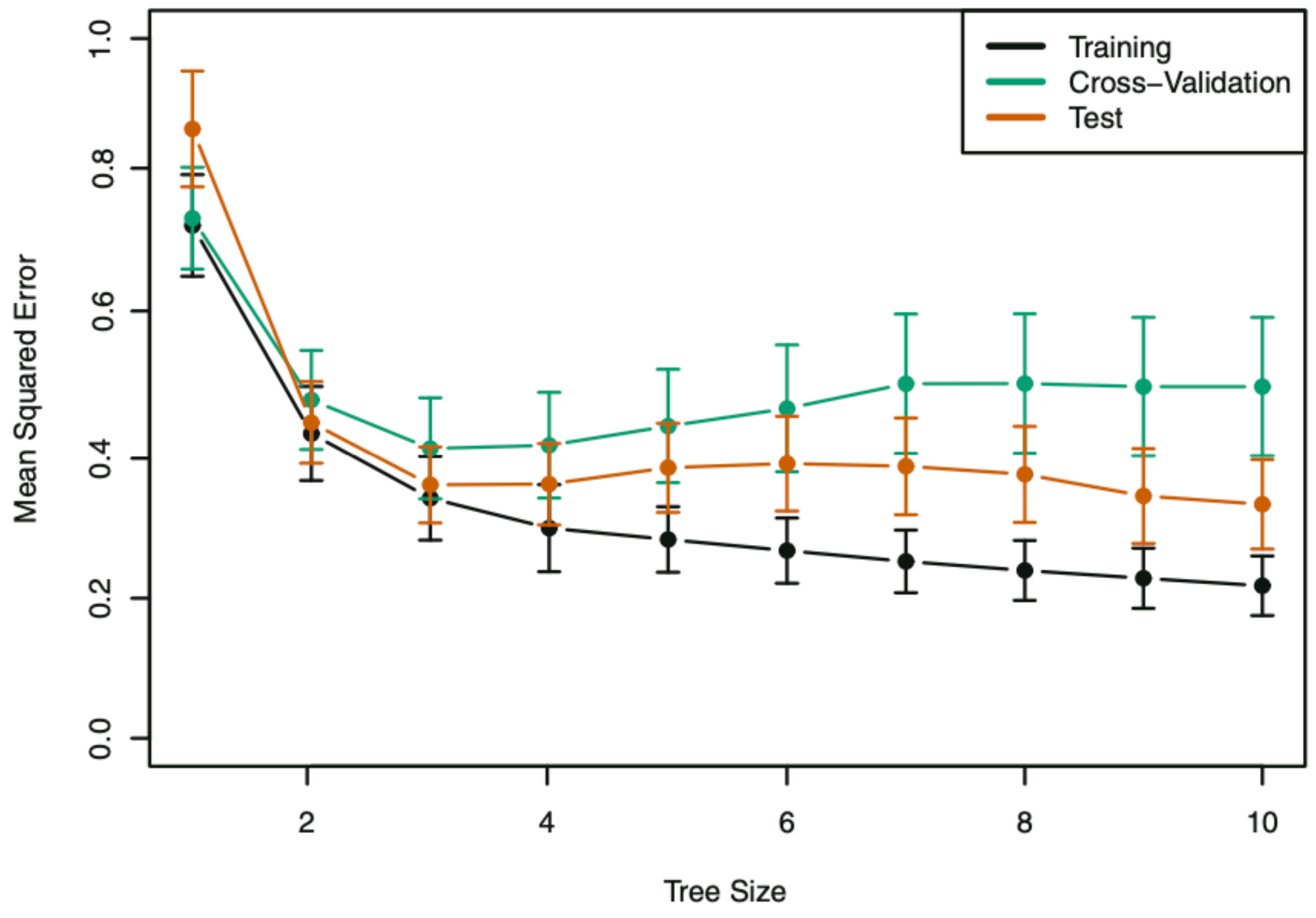

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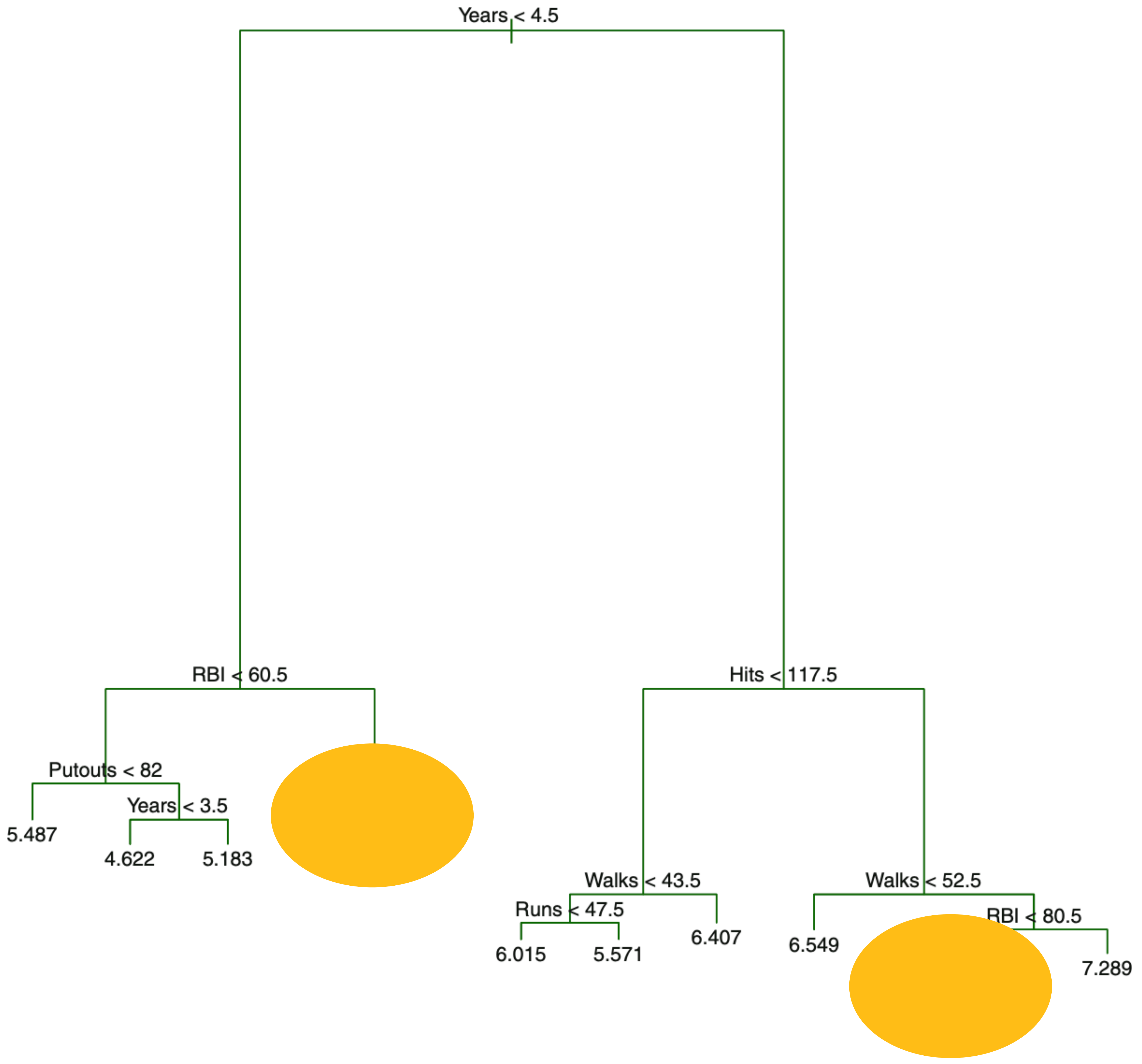

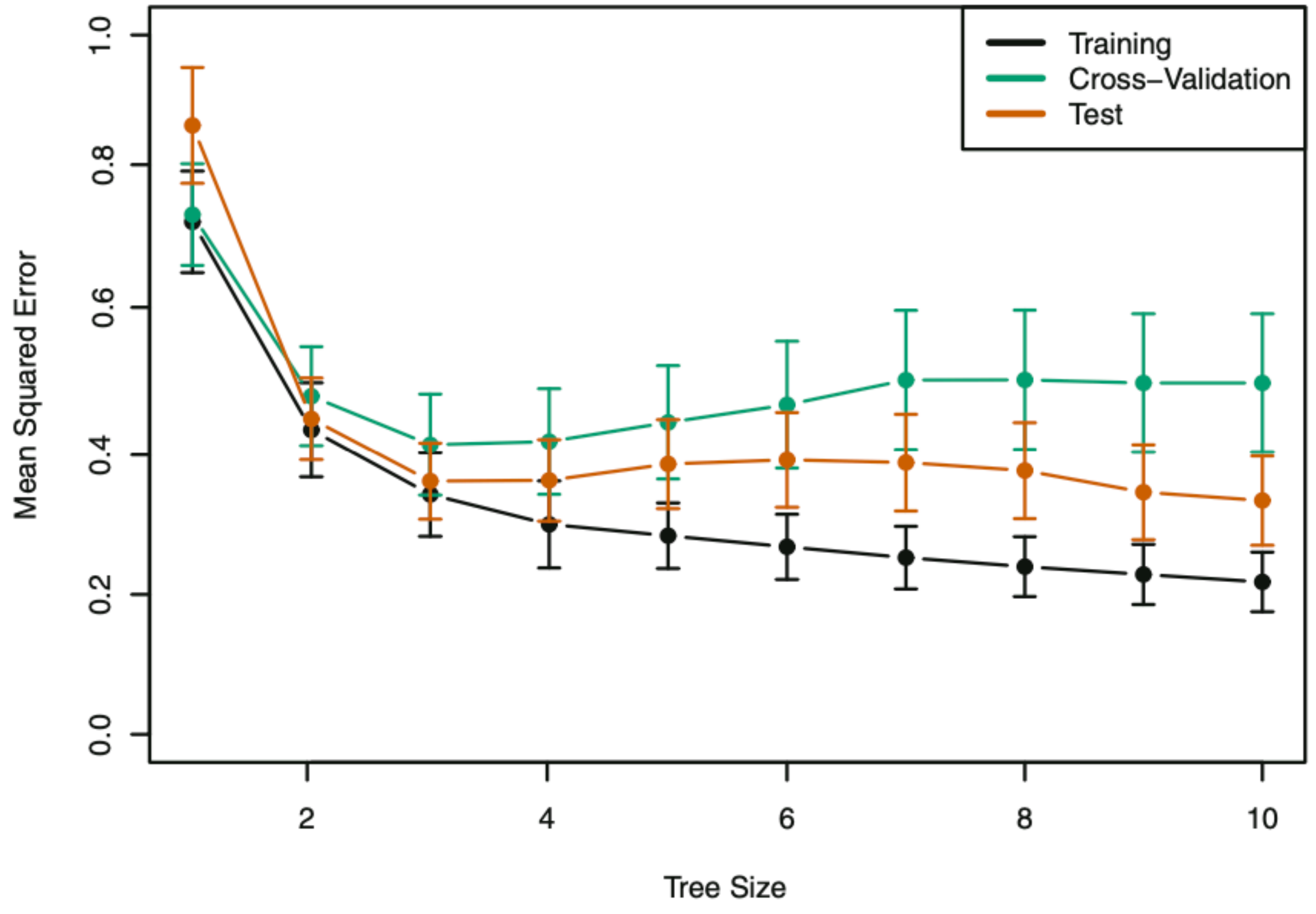

Pruning, less complex, less overfitting!

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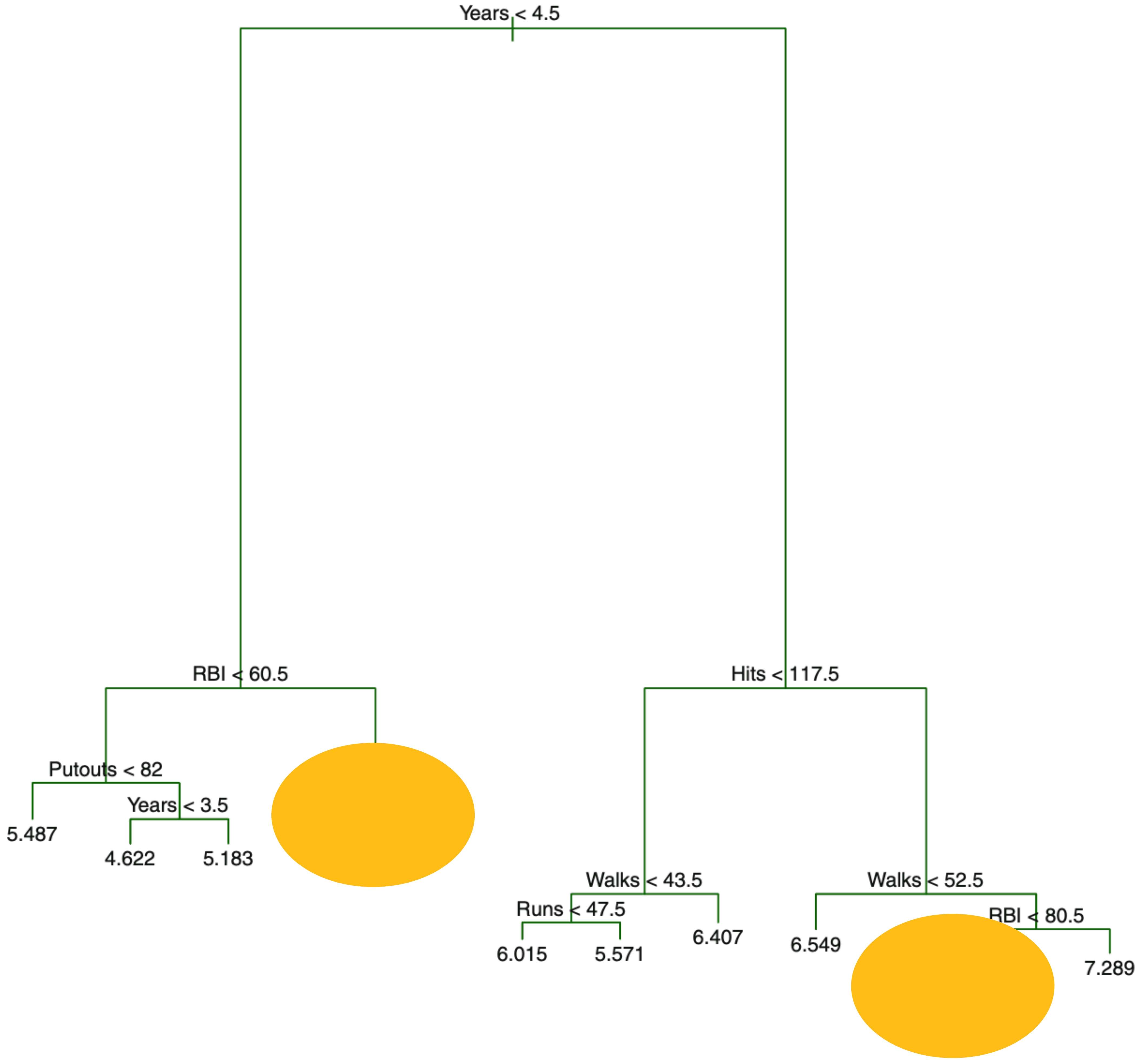

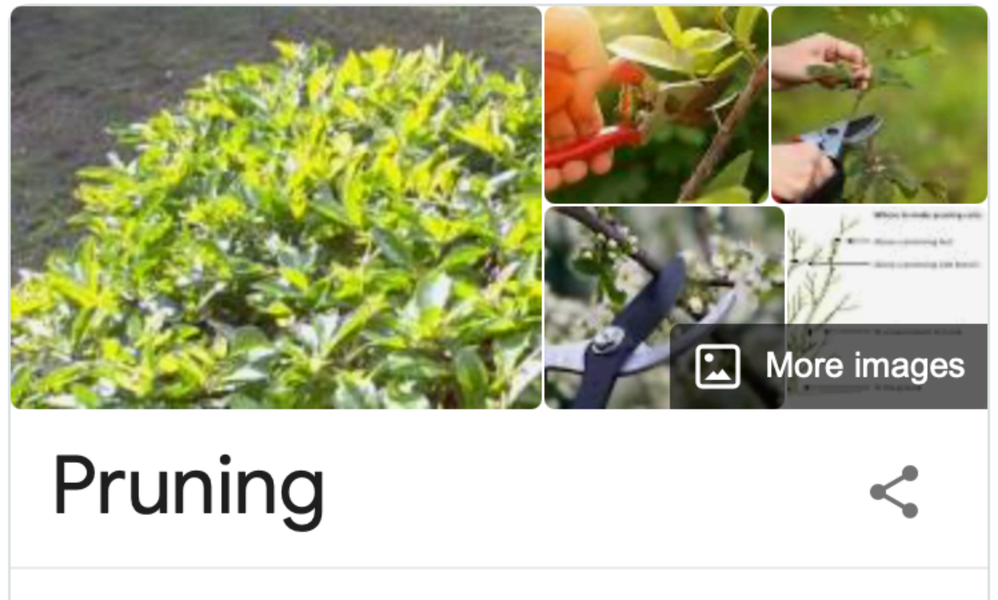

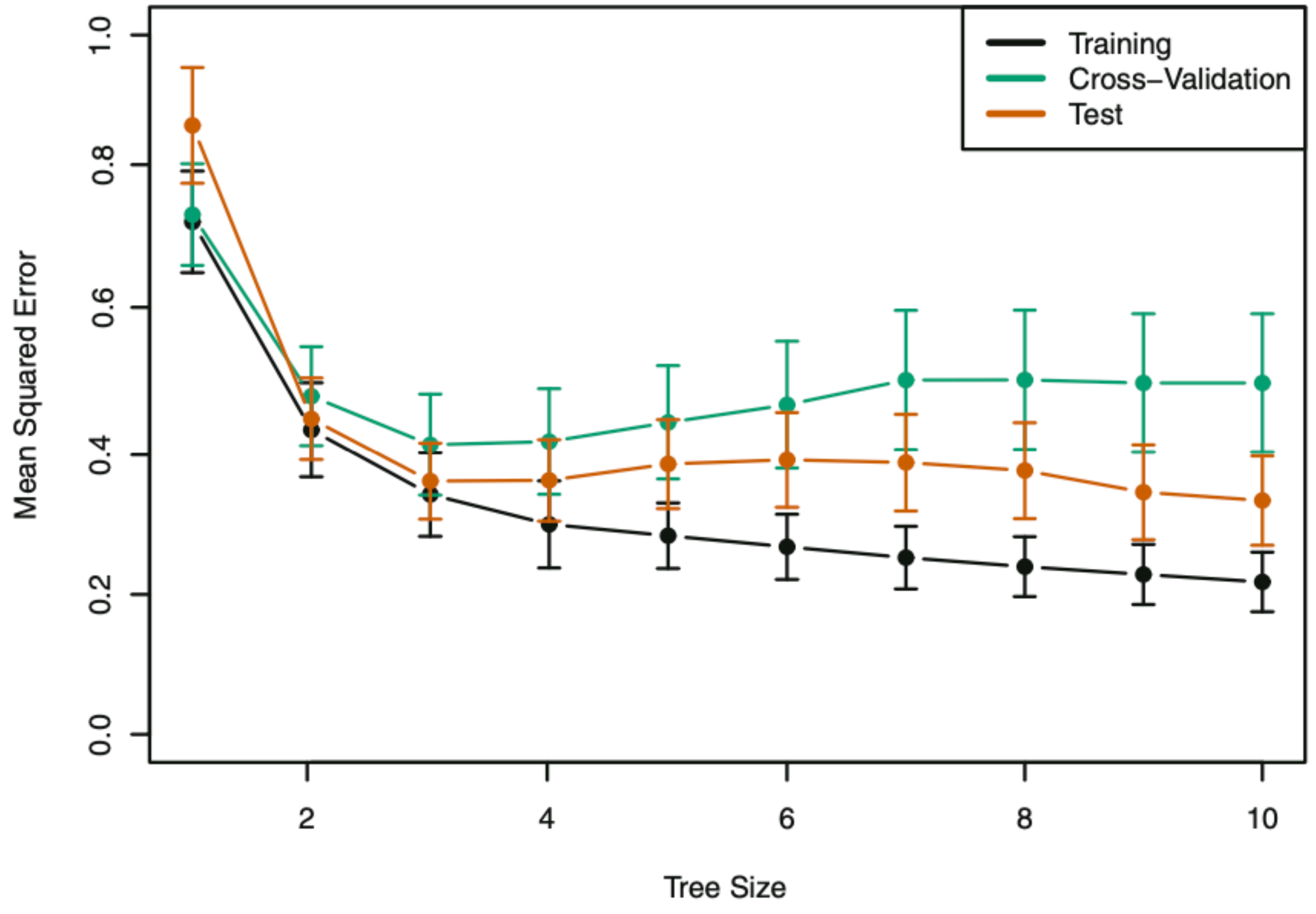

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