Single-Equation GMM

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Introduction - In this paper solutions to most exercises from Chapter 3 of Fumio Hayashi's *Econometrics* are provided

1 Questions for Review

1.1 Endogeneity Bias

1. Estimating demand and supply functions is imperative for econometricians. Nevertheless, a naive approach of blindly employing OLS for such tasks procures undesired results.

Suppose we seek to estimate the following Linear Regression Models:

$$q_i^D(p_i) = \alpha_0 + \alpha_1 p_i + u_i$$
$$q_i^S(p_i) = \beta_0 + \beta_1 p_i + v_i$$

Note that the same regressors are being used to estimate both Linear Regression Models. Thus, it is not clear whether the constant and price features estimate demand or supply. In fact, it can be shown that no function is being correctly estimated as the input variable price is not predetermined. Causal Inference cannot be performed directly as we are no longer dealing with exogeneous regressors.

Let us illustrate our claim by combining economic theory and mathematical logic. In market equilibrium: $q_i^D = q_i^S$

$$\alpha_0 + \alpha_1 p_i + u_i = \beta_0 + \beta_1 p_i + v_i$$
$$p_i = \frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{v_i - u_i}{\alpha_1 - \beta_1}$$

Then the covariance between price and error term v_i is expressed as:

$$\begin{aligned}
\operatorname{cov}(p_i, v_i) &= \operatorname{cov}\left(\frac{\beta_0 - \alpha_0}{\alpha_1 - \beta_1} + \frac{v_i - u_i}{\alpha_1 - \beta_1}, v_i\right) \\
&= \operatorname{cov}\left(\frac{v_i - u_i}{\alpha_1 - \beta_1}, v_i\right) \quad \text{(since the first fraction is composed of parameters)} \\
&= \frac{\mathrm{V}[v_i] - \operatorname{cov}(u_i, v_i)}{\alpha_1 - \beta_1} \quad (\neq 0)
\end{aligned}$$

Where $\alpha_1 < 0$ and $\beta_1 > 0$ from competitive market theory.

- If $\mathrm{V}[v_i] > \operatorname{cov}(u_i, v_i) \implies \operatorname{cov}(p_i, v_i) < 0$
- If $\mathrm{V}[v_i] < \operatorname{cov}(u_i, v_i) \implies \operatorname{cov}(p_i, v_i) > 0$
- If $\mathrm{V}[v_i] = \operatorname{cov}(u_i, v_i) \implies \operatorname{cov}(p_i, v_i) = 0$