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Model Predictive Control applied to Portfolio Management

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Contents

List of Figures iii

List of Tables v

1 Introduction 1

1.1 Problem definition and motivation . . . 1

1.2 Background, aim and objectives . . . 3

2 Literature review 6

2.1 Introduction . . . 6

2.2 Econophysics . . . 7

2.3 Return vs Risk: Optimisation . . . 9

2.4 The success of "Financial Robotics" . . . 10

3 Methodology 12

3.1 Model Parameters . . . 12

3.2 Trading . . . 14

3.3 Costs . . . 15

3.4 Self-financing constraint . . . 17

3.5 Investment Portfolio Dynamics . . . 18

3.6 General constraints . . . 20

4 Portfolio optimisation 23

4.1 Convex optimisation . . . 23

4.2 Portfolio performance metrics . . . 26

4.3 Investment policies . . . 29

5 Investment simulation 31

5.1 Data 31

5.1.1 Company profiles 32

5.1.2 Financial profiles 34

5.2 Software 37

5.3 Computation and results 38

6 Conclusions and further research 67

References 70

Alphabetical Index 78

List of Figures

1.1 Closed Loop System (Trodden, 2015). 1

1.2 Closed Loop Portfolio Management System (Brdys and Zubowicz, 2011). 2

1.3 MPC algorithm (Trodden, 2015). 4

4.1 Maximum drawdown in MSCI Emerging Markets Index, Sept 2001/2011 (Ramani, 2013). 28

4.2 Mean reverting & Momemtum trading strategies (Shal, 2017). 30

5.1 I.T. stocks. 34

5.2 Pharmaceutical/Healthcare stocks. 34

5.3 Entertainment & Shopping stocks. 35

5.4 Energy stocks. 36

5.5 Holding investment policy. 39

5.6 MPC investment policy. Forecast horizon test. 41

5.7 MPC investment policy. RAvs test1. 43

5.8 MPC investment policy. RAvs test2. 45

5.9 MPC investment policy. TAvs test1. 47

5.10 MPC investment policy. TAvs test2. 49

5.11 MPC investment policy. HAvs test1. 51

5.12 MPC investment policy. HAvs test2. 53

5.13 MPC investment policy. Levs test1. 55

5.14 MPC investment policy. Levs test2. 57

5.15 MPC investment policy. Long Only. RAvs test. 59

5.16 MPC investment policy. Long Only. Portfolio weights. 60

5.17 MPC investment policy. Dollar Neutral portfolio. RAvs test. 62

5.18 MPC investment policy. Dollar Neutral portfolio. Portfolio weights. . . . 63

5.19 MPC investment policy. Dollar Neutral portfolio. Levs test. 65

5.20 MPC investment policy. Dollar Neutral portfolio. Portfolio weights. . . . 65

List of Tables

5.1 Company profiles (Bloomberg, 2019). 32
5.2 Financial profiles (Bloomberg, 2019). 33
5.3 Holding investment policy. 38
5.4 MPC investment policy. Forecast horizon test. 40
5.5 MPC investment policy. RAvs test1. 42
5.6 MPC investment policy. RAvs test2. 44
5.7 MPC investment policy. TAvs test1. 46
5.8 MPC investment policy. TAvs test2. 48
5.9 MPC investment policy. HAvs test1. 50
5.10 MPC investment policy. HAvs test2. 52
5.11 MPC investment policy. Levs test1. 54
5.12 MPC investment policy. Levs test2. 56
5.13 MPC investment policy. Long Only. RAvs test. 58
5.14 MPC investment policy. Dollar Neutral portfolio. RAvs test. 61
5.15 MPC investment policy. Dollar Neutral portfolio. Levs test. 64

Abstract

This dissertation develops an investment framework based on MPC that offers an alternative solution (compared to traditional portfolio optimisation approaches) to the Portfolio Optimisation problem via convexity. Accordingly, interesting portfolio investment performance results have been extracted and can be very helpful for the development of new trading models and strategies based on Engineering Control Systems theory.

Acknowledgements

To my parents. Thank you for your unconditional love and support.

1 Introduction

1.1 Problem definition and motivation

While technology is evolving, techniques which were traditionally implemented in the industrial engineering sector have later been successfully applied to other fields. Such techniques have demonstrated high value in the social sciences sector over the last few years, more concretely within the finance industry. As an example, the following model illustrates a closed loop system designed to control engineering processes and later enforced in finance:

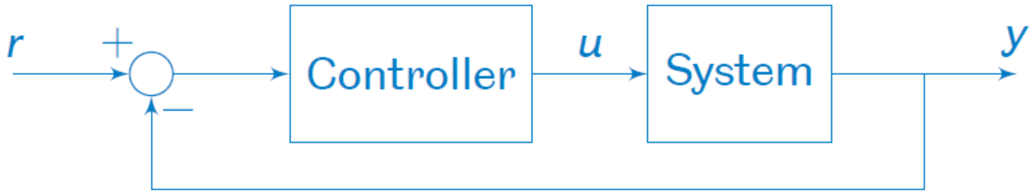

Figure 1.1: Closed Loop System (Trodden, 2015).

The figure 1.1 describes a closed loop system where **r** represents the track setpoint, **u** defines the controlled input and **y** describes the system output. The main objective of the controller is to provide an input signal that optimises the system's performance, making it feasible to achieve the track setpoint while satisfying some pre-determined constraints. Generally, optimisation implies operating near thresholds.

Portfolio optimisation can be defined as the process of finding the best asset allocation or asset distribution, maximising the returns and minimising the risks associated with the financial markets. Literature on distinct portfolio optimisation approaches is extensive, showing very profitable results as times are good. However, the recent global financial crisis illustrated how fragile these models can be while facing big challenges.

In probability theory, a stochastic process is defined or described by the behaviour of random variables (stochastic variables), which values are uncertain or unknown in the future, i.e. the variables are not deterministic. From a control engineering's perspective, an active portfolio management system can be considered a process that must be regulated with regards to the disturbances derived from the global financial market. Therefore, it is crucial to find a robust model that is able to analyse the current available information (historical data) and take decisions to achieve optimal portfolio performance.

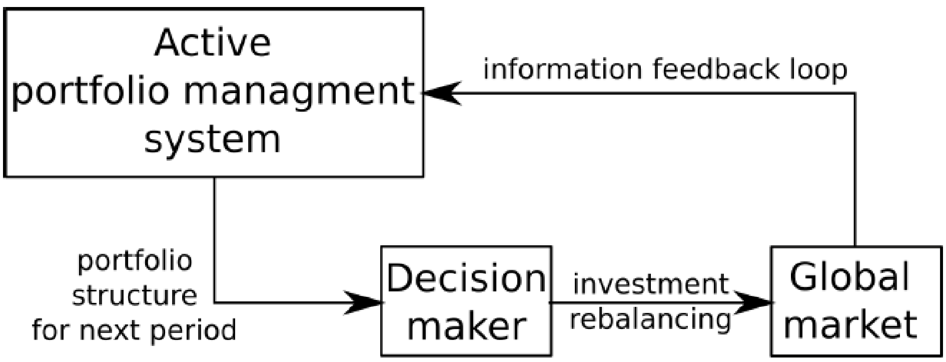

Figure 1.2: Closed Loop Portfolio Management System (Brdys and Zubowicz, 2011).

Following the figure 1.2, an active portfolio management system can be considered as a closed loop process, where the decision maker (controller) is fed by the portfolio structure for the next time period and produces controlled outputs (investment rebalancing) with the goal of achieving the optimal investment solution. Consequently, the system expresses the portfolio performance results obtained after applying such optimal solution as an information feedback loop.

The main motivation of this dissertation is thus to analyse the suitability and performance of the Engineering Control Systems theory in order to solve the Portfolio Optimisation problem.

1.2 Background, aim and objectives

Nowadays, the vast majority of the items which drive our lives work under rules derived by the control systems theory. Furthermore, there are plenty of control models such as PID's, neural networks or more recently machine learning systems, which take decisions over optimisation problems, always looking for the most suitable and effective solution. However, there is one control system model that has shown extraordinary efficiency and adaptability in the engineering field, compared to others: the Model Predictive Control, (MPC), also known as Stochastic Receding Horizon Control.

MPC was originally developed to meet the strict control requirements for gas and oil stations in the 1980's (Qin and Badgwell, 2003); however, nowadays it is present in many industrial applications such as missile guidance, spacecraft rendezvous or power generation systems (Maciejowski, 2000). Roughly speaking, a MPC algorithm measures the current system's state and generates an optimised control input signal which describes the most efficient path between an autonomous dynamic system and its target. The optimisation problem is discretely calculated at every time period while considering the horizon length. The following picture illustrates the MPC dynamics in an optimisation problem under constraints:

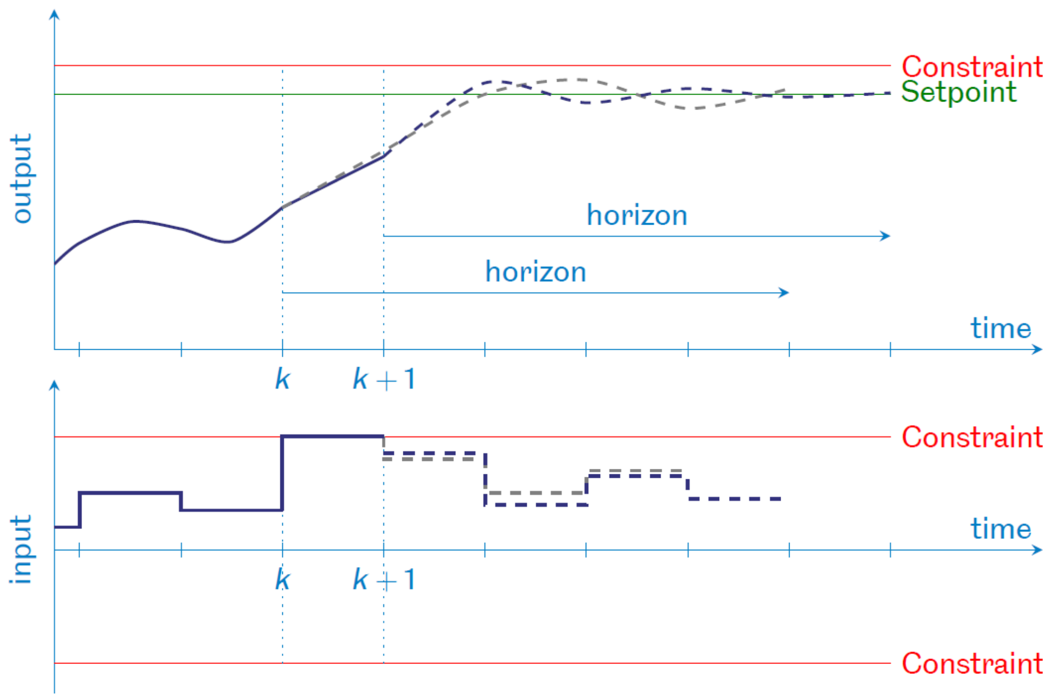

Figure 1.3: MPC algorithm (Trodden, 2015).

Furthermore, the figure 1.3 can be analysed from a portfolio manager's perspective. In that case, let the track setpoint be the portfolio investment policy. In addition, consider the constraint line as the restrictions imposed by the investment process (e.g. leverage limit). Moreover, let the controlled inputs be the portfolio transactions (weights) applied to every time period, taking into account both historical data and horizon predictions. Finally, the system output is derived from the portfolio performance's results in such financial market. Indeed, the above description perfectly illustrates the potential suitability of MPC as an Active Portfolio Management approach.

Consequently, the main contribution of this research will be to study and analyse the implementation of the MPC algorithm in order to solve the Portfolio Optimisation problem. Furthermore, we will develop and test a trading algorithm based on Boyd et al's research (2017) via Python, with the objective to estimate optimal investment decisions for several portfolios based on different investment policies and restricted by different thresholds.

In terms of contributing to existing literature, we will analyse several works that deal with portfolio optimisation and financial technology, and develop a framework that can be used for future research. On the other hand, we will analyse the computed results and try to illustrate new trends and patterns, derived from the addition and modification of model aversion parameters and advanced constraints, which can be priceless for investment and asset managers.

2 Literature review

2.1 Introduction

Capital allocation is a popular subject between institutional and private investors. Nowadays, literature is extensive on such traditional topic. Two of the most famous researches in the field are Markowitz (1952) and Merton (1969). The concepts of returns maximisation and risk minimisation can be considered as intrinsically assumed for all rational investors. Furthermore, Markowitz (1952) developed an investment framework based on these two concepts, simultaneously. Moreover, the main contribution of Markowitz's model was to highlight the benefits of portfolio diversification and the importance of the covariance relationships between its securities. Merton (1969) introduced the concept of expected utility maximisation, to make investors decide about consumption and wealth allocation in stocks and risk-free assets. This question was formulated and solved by Merton (1969) as a continuous-time problem. On the other hand, financial markets modelling is another subject that has been gaining popularity over the last two decades. However, the wide variety of financial crashes such as the Black Monday in 1987 or the Worldwide Financial Crisis in 2007 have demonstrated the inaccuracy and the inefficiency of financial models against unpredictable events. Colander et al (2009) points out the development of new financial products as one of the main factors which originated the most recent global financial crisis against the "over-investment" process in physical products (securities) from previous financial crashes. Moreover, Colander et al (2009) blame academic economics and researchers to ignore the consequences derived from the high level of connectivity between financial markets around the world. Farmer and Foley (2009) describes economy as a complex system and sets its approach in a middle point between the traditional economic approach and econometrics. Moreover, the paper points out the necessity of understanding the behavioural models and interactions within economy constituents in order to build more realistic and efficient models.

2.2 Econophysics

Over the last fifty years, numerous researchers and econometricians have observed a wide number of empirical patterns that appear regularly during the analysis of financial data. Pagan (1996) summarizes the financial econometrics techniques applied until the end of last century and points out that the origin of stylised facts cannot be fully explained and is still considered an open question from academics. Following Cont (2001), these patterns known as stylized facts, are intrinsic financial statistical properties that can be universally detected in the analysis of any financial data sample. Leptokurtosis, negative skewness, volatility clustering or leverage effect are four of the most popular properties of stocks return distribution. Lux (2008) points out that financial crisis are directly affected from stylised facts. Moreover, Sornette (2014) uses the term "Econophysics" to describe the relationship of statistical properties between the fields of economics and physics and remark the important role of individual interactions (interacting particles in physical systems) during the financial crisis. Sornette's research (2014) is a good example of an agent-based financial market model, although it is not the only one. Generally, the main objective of these models is to find the origins of stylised facts. Lux and Marchesi (1999) use the quantum-mechanical interference between quantum particles as a model to explain the universal characteristics of financial prices. Levy et al (1994) describe complex systems inspired on behavioural finance and physic models that use statistical tools such as the Monte Carlo simulation. However, particles models show an important disadvantage: their behaviour can only be empirically analysed under computer simulations. Several papers such as Hellthaler's (1996), Kohl's (1997), Egenter et al's (1999) and Zschischang and Lux's (2001) have brought to the table the fact that finite-size effects are the main reason for the stylised factor emerging in a wide number of agent-based models.

The use of kinetic models derived from partial differential equations has resulted in one of the most effective solutions to the finite-size effects (Trimborn et al, 2018). Bisi et al (2019) built a kinetic model for wealth distribution, including important market aspects such as taxes. Bouchaud and Mezard (2000) introduced an economic model based on random speculative trading where the exchange of monetary units (foreign exchange) follows a stochastic process. Burger et al (2013) presented a Boltzmann-type price formation model where trading events between buyers and vendors are considered as kinetic collisions. Chayes et al (2009) have developed a framework based on a dynamic evolving 1-Dimension diffusion equation to describe a scalar price evolution. Based on Cordier et al's research (2005), Che (2011) created a multi-dimensional kinetic model to illustrate the evolution over time of several portfolios consisting of different combinations of risky assets. Inspired by Levy et al (1994) and Levy et al (2000), Cordier et al (2009) designed a mesoscopic model which illustrates a financial market behaviour, where agents build their own portfolio combining a bond and a stock. Furthermore, the model is based on the kinetics theory and uses a linear Boltzman equation for wealth allocation. Similarly, Delitala and Lorenzi (2014) make use of integro-differential equations in order to model the dynamics of a market where agents make estimations to price the trading assets. During et al (2008) attempted to unify the use of kinetics equations in wealth allocation modelling. Therefore, they applied moment analysis on the Boltzman equation, making it possible to classify the fatness of the Pareto tail and its dynamic stability based on model parameters. Moreover, Kanazawa et al (2018) observed the dynamics of high-frequency traders (HFTs) in the FOREX market and derived a macroscopic model which describes financial Brownian motion. In addition, Kanazawa et al's second paper (2018) illustrates the parallelism between the kinetic theory in the field of statistical physics and the high-frequency trader model, based on a mathematical foundation. Maldarella and Pareschi (2012) described a simple financial market model composed by an individual stock and applied kinetic equations to analyse the price distributions and the asymptotic behaviour of the investment. Moreover, they illustrated and analysed the regimes of lognormal behaviour and the origin of power law tails. Matthes and Toscani (2008) performed a deep analysis of a kinetic model which describes wealth allocation in a simple economy and estimates rates of convergence for continuous and finite-agent model using the Monte Carlo simulation. Furthermore, they identified steady states to always be reached in the long run for a sub-exponential rate. Trimborn et al's research (2018) is very meaningful as to showing the mean field limit of the econophysical model, which Cross et al (2005) designed, being calculated. Moreover, Cross et al's kinetic model (2005) was able to reproduce some of the most popular stylised facts such as volatility clustering and leptokurtosis.

2.3 Return vs Risk: Optimisation

The analysis of all the above literature makes one point clear: the starting point of our framework must be to derive a model that is based on financial agents in order to optimise investment decisions. Mayne and Michalska (1990) and Camacho and Bordons (2004) applied Model Predictive Control (MPC) to facilitate the optimisation process on engineering systems (e.g. gas and oil plants, nuclear stations). In addition, MPC has been implemented into kinetic models over the last few years, e.g. Albi et al's (2014) and Pareschi et al's research (2015).

Generally, the analysis of the trade-off between return and risk in the portfolio investment optimisation problem is used to "ignore" the cost associated with the trading activity. However, these costs can have an important role when trading is realised over several time periods (Fabozzi et al, 2014). Furthermore, Goldsmith (1976) was one of the first authors who emphasised the importance of the costs associated with transactions. Moreover, Lobo et al (2007) and Moallemi and Saglam (2017) proposed a wide range of constraints and costs which can be included in the model for solving the portfolio optimisation problem. Based on Samuelson's (1969) and Merton's research (1971), Constantinides (1979) derived a discrete-time framework to solve portfolio optimisation problems with proportional transaction costs. Similarly, Davis and Norman (1990) and Dumas and Luciano (1991) illustrated a framework for a continuous-time model. Traditionally, the literature about dynamic portfolios, e.g. (Bellman, 1956) and (Bertsekas, 1995), is focused on dynamic programming, taking investment decisions based on the most updated information available (Gârleanu and Pedersen, 2013). Unfortunately, Powell (2007) and Boyd et al (2014) pointed out that 'the curse of dimensionality' makes the portfolio selection under dynamic programming infeasible. Therefore, this fact results in two possible solutions: relax objectives and constraints or omitting transaction costs (Campbell and Viceira, 2002).

2.4 The success of "Financial Robotics"

Recently, the number of portfolio optimisation approaches based on Control Systems, Machine Learning and Artificial Intelligence has undergone a dramatic increase. Shapcott (1992) developed an innovator portfolio investment framework based on a genetic algorithm combined with quadratic programming techniques. Herzog (2006) derived a dynamic solution for the portfolio optimisation based on MPC. Firstly, the paper illustrates the advantages of applying MPC on stochastic systems, given that the model will compute and evaluate the information flow in every time period, while an open-loop system cannot take advantage of the "fresh information" input. Finally, the results show portfolio performances with Sharpe ratios closed to unit. Marigo and Piccoli (2004) derived and analysed the costs associated to the MPC trajectories. Moreover, they remarked the strong relationship between sampling data and the length of the forecast horizon. Primbs (2007) applied MPC to solve a risk-adjusted wealth maximisation problem. In addition, he illustrated the optimality of MPC on tracking a pre-defined index of stocks. Brdys and Zubowicz (2011) developed a framework based on Monte Carlo simulation and State Space Wavelet Network. Moreover, it is proved how MPC algorithms can increase the portfolio expected returns. Trimborn et al (2018) applied MPC to solve the utility function optimisation problem. Furthermore, they built the first kinetic portfolio approach where the distribution of the stock price and wealth are jointly evaluated. Boyd and Vandenberghe's research (2004) focuses on the solution of convex optimisation problems and provides priceless software packages (CVX, CVXOPT and CVXPY) for several programming languages such as Python and MATLAB. In the same line, Meucci (2010) discusses the theoretical aspects of risk modelling and portfolio optimisation and gives examples based on MATLAB applications. Finally, Narang (2013) explains the foundations of high frequency trading and provides several examples where Artificial Intelligence plays a key role in the portfolio investment process.

3 Methodology

3.1 Model Parameters

Given a pre-defined time period (e.g. 3 or 5 years), we will develop, in this research, a dynamic portfolio model which will be the investment vehicle of the optimisation problem. The portfolio will be composed by **n** assets in addition to a cash account. The dynamics of the model over time will be discretely decomposed as follows:

$$t = 1, 2, 3...T \tag{3.1}$$

where **T** describes the total length of the time period analysed in the optimisation process. We will assign several values to T in order to explore the model behaviour and performance over time. In addition, the portfolio model will be composed by the following parameters:

1. **Asset prices**.

   Let $\boldsymbol{pr_t^i} \in \boldsymbol{R_+^n}$, for i=1,2,3...n, express the average of the bid/ask price of the ith asset at the beginning of the time period t.

2. **Asset positions**.

   Let $\boldsymbol{PP_t^i} \in \boldsymbol{R^{n+1}}$, for i=1,2,3...n, define the portfolio position on the asset i at the beginning of the time period t. Furthermore, it is possible to combine long positions ($\boldsymbol{PP_t^i > 0}$) and short positions ($\boldsymbol{PP_t^i < 0}$) in order to take advantage of the fluctuations of the market prices.

3. **Cash balance**.

   The cash balance is the last element of the portfolio positions vector ($\boldsymbol{PP_t^{n+1}}$) and expresses the amount of cash in the portfolio account. Note that the cash balance term can take value zero when all the money of the portfolio has already been invested into assets. Moreover, note that the cash balance can take negative values over certain time periods, which means that the cash amount is owed or borrowed.

4. **Portfolio value**.

   Let $\boldsymbol{PV_t}$ be the portfolio value at time t. Furthermore,

$$\boldsymbol{PV_t = [1]' * PP_t} \tag{3.2}$$

   where $[\mathbf{1}]$ and $\boldsymbol{PP_t}$ are the unit vector and the asset positions vector, respectively.

5. **Gross exposure**.

   The Gross exposure ($\boldsymbol{GE_t}$) is defined as the summation of the absolute values of the asset positions, thus:

$$\boldsymbol{GE_t = \sum_{i=1}^{n} |PP_t^i|} \tag{3.3}$$

6. **Portfolio Leverage**.

   Leverage expresses the act of borrowing money in order to increase the potential returns of an investment. Furthermore, the Portfolio Leverage at time t ($\boldsymbol{PL_t}$) can be mathematically defined as follows:

$$\boldsymbol{PL_t = \frac{\sum_{i=1}^{n} |PP_t^i|}{PV_t} = \frac{GE_t}{PV_t}} \tag{3.4}$$

   Moreover, if the portfolio is uniquely composed by long positions, then:

$$\boldsymbol{PL_t = \frac{GE_t}{PV_t} = 1} \tag{3.5}$$

7. **Portfolio weights**.

The weight of a portfolio is the percent that is held by a single asset. Therefore:

$$\boldsymbol{W_t = (w_t^1, w_t^2, w_t^3 ... w_t^{n+1})' = \frac{PP_t}{PV_t}} \tag{3.6}$$

In addition, the sum of the portfolio weights is always equal to unity:

$$\boldsymbol{W_t' * [1] = \sum_{i=1}^{n+1} w_t^i = 1} \tag{3.7}$$

Furthermore, the last component of the weights vector $\boldsymbol{w_t^{n+1}}$ expresses the portion of cash held in the portfolio at time t. Finally, the portfolio asset positions can be defined in terms of the weights as follows:

$$\boldsymbol{PP_t = PV_t * W_t} \tag{3.8}$$

3.2 Trading

Let $\boldsymbol{Tr_t^i \in R^{n+1}}$ express the value of the transaction of the asset i at time t. Note that a long position on the asset i at time t will imply $\boldsymbol{Tr_t^i > 0}$, while a short position on the same asset and time period will mean $\boldsymbol{Tr_t^i < 0}$. Furthermore, $\boldsymbol{Tr_t^{n+1}}$ articulates the amount of cash that is deposited into ($\boldsymbol{Tr_t^{n+1} > 0}$) or taken out of ($\boldsymbol{Tr_t^{n+1} < 0}$) the portfolio cash account.

Generally, trading occurs non-uniformly during full working days. However, in order to facilitate this research, we assume that transactions will take place at the beginning of every working day. Therefore, we have to define a new concept: the after-trading portfolio ($\boldsymbol{PP_t^{af}}$), which denotes the value of the portfolio asset positions after trading at time t and can be mathematically described as:

$$\boldsymbol{PP_t^{af} = PP_t + Tr_t \quad for\ t = 1, 2, 3 ... T} \tag{3.9}$$

Moreover,

$$\begin{aligned} \boldsymbol{PV_t^{af}} &= \boldsymbol{1' * PP_t^{af}} \\ \boldsymbol{PV_t^{af} - PV_t} &= \boldsymbol{1' * PP_t^{af} - 1' * PP_t = 1' * Tr_t} \end{aligned} \tag{3.10}$$

Finally,

$$\boldsymbol{\frac{PP_t^{af}}{PV_t^{af}} = W_t + NTr_t} \tag{3.11}$$

where $\boldsymbol{NTr_t}$ is the normalized transaction vector and it is defined as

$$\boldsymbol{NTr_t = \frac{Tr_t}{PV_t}} \tag{3.12}$$

3.3 Costs

In term of costs, we identify two categories: transaction costs and holding costs. Generally, asset prices periodically fluctuate during trading days. Therefore, portfolio managers have to constantly adjust their positions in order to achieve the desired performance. Every time portfolio positions are modified, we incur in transaction costs. These costs depend on parameters such as traded volume, bid-ask spread, broker fees, etc. In the literature we can find several attempts to model these costs, e.g. (Bershova and Rakhlin, 2013), (Lillo et al, 2003), (Almgren and Chrissn, 2001). In this research, we will hence use a transaction cost model inspired by Grinold and Kahn's research (1999) because of its capacity of implementation into code but also its good approximation to reality. In addition, this cost model is supported and validated by similar models such as Moro et al's (2009) and Gomes and Waelbroeck's (2015). Therefore, the transaction cost of the asset i at time t, $\boldsymbol{Tc_t^i}$, is defined as follows:

$$\boldsymbol{Tc_t^i = \alpha_t^i * |ta_t^i| + \beta * \sigma * \frac{ta_t^{3/2(i)}}{Vol_t^{1/2(i)}} + \gamma * ta_t^i} \tag{3.13}$$

where,

1. $\boldsymbol{\alpha_t^i} =$ bid/ask spread of the asset i divided by two, at time t.

2. $\boldsymbol{ta_t^i} =$ cash trade amount of the asset i at time t.

3. $\boldsymbol{\beta} =$ positive constant.

4. $\boldsymbol{Vol_t^i} =$ total market traded volume of the ith asset at time t.

5. $\boldsymbol{\sigma} =$ recent price volatility.

Note that $\boldsymbol{\gamma}$ expresses the asymmetry between the costs of sale and buy, given that, generally, selling is cheaper than buying. Moreover, we can illustrate the previous equation in terms of the portfolio value ($\boldsymbol{PV_t}$) such as:

$$\boldsymbol{Tc_t^i = \alpha_t^i * |NTr_t^i| + \beta * \sigma * \frac{NTr_t^{3/2(i)}}{(Vol_t^i/PV_t)^{1/2}} + \gamma * NTc_t^i} \tag{3.14}$$

Holding costs refer to the amount of cash that derives from holding the after-trading portfolio ($\boldsymbol{PP_t^{af}}$) from the time period t to $t+1$. Furthermore, this amount must be multiplied by three when the time period occurs from Friday to Monday. A potential holding cost model can be defined as follows:

$$\boldsymbol{Hc_t[PP_t^{af}] = Bfees_t' * SPP_t^{af}} \tag{3.15}$$

where $\boldsymbol{Bfees_t'}$ and $\boldsymbol{SPP_t^{af}}$ are the borrowing fees and the shorting positions of the after-trading portfolio at time t, respectively.

3.4 Self-financing constraint

We assume that during the whole investment simulation process, no additional cash is added into the cash balance $\boldsymbol{PP_t^{n+1}}$. Therefore, the costs derived from holding and transactions tasks will be self-financed by the portfolio's performance. This constraint is mathematically defined as follows:

$$\boldsymbol{Tc_t[Tr_t] + Hc_t[PP^{af}] = -[1]' * PP_t} \tag{3.16}$$

Self-financing condition implies that the portfolio obtained after subtracting costs ($\boldsymbol{Hc_t} +$ $\boldsymbol{Tc_t}$) is equal to the after-trading portfolio value. Therefore,

$$\boldsymbol{PV_t^{af} = PV_t - (Tc_t[Tr_t] + Hc_t[PP_t^{af}])} \tag{3.17}$$

Furthermore, we can express the self-financing constraint in terms of normalised trades and portfolio weights by dividing the previous equation into the portfolio value, resulting as follows:

$$\boldsymbol{\frac{Tc_t[PV_t * NTr_t]}{PV_t} + \frac{Hc_t[PV_t * (W_t + NTr_t)]}{PV_t} = -[1]' * NTr_t} \tag{3.18}$$

where

$$\begin{aligned} \boldsymbol{Tr_t} &= \boldsymbol{PV_t * NTr_t} \\ \boldsymbol{PP_t^{af}} &= \boldsymbol{PV_t * (W_t + NTr_t)} \end{aligned} \tag{3.19}$$

Finally,

$$\boldsymbol{Tc_t[NTr_t] + Hc_t[W_t + NTr_t] = -[1]' * NTr_t} \tag{3.20}$$

3.5 Investment Portfolio Dynamics

Let $\boldsymbol{R_t} \in \boldsymbol{R^{n+1}}$ define the asset and cash returns between two successive time periods (e.g. t and t+1). The return derived from the asset i during the time period t is mathematically defined as follows:

$$\boldsymbol{R_t^i = \frac{pr_{t+1}^i - pr_t^i}{pr_t^i} \quad for\ i = 1, 2, 3...n} \tag{3.21}$$

Alternatively, given that asset returns are small in comparison with unity, log-return representation can be used as follows:

$$\boldsymbol{log(\frac{pr_{t+1}}{pr_t}) = log(1 + R_t^i) \quad for\ i = 1, 2, 3...n} \tag{3.22}$$

Furthermore, the portfolio return $\boldsymbol{PR_t}$ is defined as the proportional increment of its value over a time period:

$$\boldsymbol{PR_t = \frac{PV_{t+1} - PV_t}{PV_t}} \tag{3.23}$$

Note that the last term of the portfolio return vector, $\boldsymbol{PR_t^{n+1}}$,expresses the cash return or risk-free interest rate. The asset and cash positions between two periods is resulting as follows:

$$\begin{aligned} \boldsymbol{PP_{t+1}} &= \boldsymbol{PP_t^{af} + R_t \circ PP_t^{af}} \\ &= \boldsymbol{PP_t^{af} \circ ([1] + R_t) \quad for\ t = 1, 2, 3...T - 1} \end{aligned} \tag{3.24}$$

Moreover, the value of the portfolio shows the following dynamics:

$$
\begin{aligned}
\boldsymbol{PV_{t+1}} =& [\mathbf{1}]' * \boldsymbol{PP_{t+1}} \\
=& \boldsymbol{PP_t^{af}} * ([\mathbf{1}] + \boldsymbol{R_t})' \\
=& \boldsymbol{PV_t} + ([\mathbf{1}] + \boldsymbol{R_t})' * \boldsymbol{Tr_t} + \boldsymbol{R_t'} * \boldsymbol{PP_t} \\
=& \boldsymbol{PV_t} + \boldsymbol{R_t'} * \boldsymbol{Tr_t} + \boldsymbol{R_t'} * \boldsymbol{PP_t} - (\boldsymbol{Tc_t}[\boldsymbol{Tr_t}] + \boldsymbol{Hc_t}[\boldsymbol{PP_t^{af}}])
\end{aligned}
\tag{3.25}
$$

Therefore, the portfolio return can be alternatively defined as follows:

$$
\boldsymbol{PR_t} = \boldsymbol{R_t'} * \boldsymbol{NTr_t} + \boldsymbol{R_t'} * \boldsymbol{W_t} - (\boldsymbol{Tc_t}[\boldsymbol{NTr_t}] + \boldsymbol{Hc_t}[\boldsymbol{W_t} + \boldsymbol{NTr_t}]) \tag{3.26}
$$

As we can observe, the portfolio return equation is clearly composed by three terms:

1. $\boldsymbol{R_t'} * \boldsymbol{NTr_t}$ is the trading returns.
2. $\boldsymbol{R_t'} * \boldsymbol{W_t}$ is the gross portfolio return (without costs).
3. $\boldsymbol{Tc_t}[\boldsymbol{NTr_t}] + \boldsymbol{Hc_t}[\boldsymbol{W_t} + \boldsymbol{NTr_t}]$ are the transaction and holding costs, respectively.

Finally, portfolio weight dynamics will result as follows:

$$
\boldsymbol{W_{t+1}} = \frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{PR_t}} * ([\mathbf{1}] + \boldsymbol{R_t}) \circ (\boldsymbol{W_t} + \boldsymbol{NTr_t}) \tag{3.27}
$$

Given that returns between periods are small in comparison to unity, we assume that $\boldsymbol{W_{t+1}} \approx \boldsymbol{W_t} + \boldsymbol{NTR_t}$ to simplify the weight dynamics equation.

3.6 General constraints

One of the main features of this research is to apply several constraints to the portfolio model in order to find potential conclusions about the profitability and risk of different portfolio configurations. In this section we will mention and describe some of these above mentioned constraints.

1. **Maximum trading size**.

   Generally, company capitalisations are restricted to avoid single investors owning a too large fraction of the assets and taking decisions that can negatively affect its interests (e.g. asset price) without any control from the director board. Let $\boldsymbol{ACap_t}$ and $\boldsymbol{MaxTr}$ be the asset capitalisations of a company and the maximum trading size respectively, and maximum trading size constraint can be defined as follows:

$$\boldsymbol{W_t^i + NTr_t^i \leq MaxTr \circ \frac{ACap_t}{PV_t}} \tag{3.28}$$

2. **Long Only portfolio**.

   This constraint implies that just long (buy) positions will be taken into the portfolio. The Long Only portfolio constraint is mathematically defined as follows:

$$\boldsymbol{W_t^i + NTr_t^i \geq 0 \quad for\ i = 1, 2, 3 ... n} \tag{3.29}$$

3. **Maximum leverage**.

   Portfolio leverage can be restricted to a certain value, in order to limit risk.

Therefore,

$$|\boldsymbol{W_t^i} + \boldsymbol{NTr_t^i}| \leq \boldsymbol{MaxLev} \quad \boldsymbol{for\ i = 1, 2, 3...n} \tag{3.30}$$

Unfortunately, portfolio returns over a period can produce leverage levels slightly higher than their limit in the following period.

4. **Dollar neutral long/short strategy**.

   The main idea behind this strategy is to take equal short and long positions, resulting in the summation of these to be neutral or equal to zero. Furthermore,

$$\boldsymbol{W_t^i} + \boldsymbol{NTr_t^i} = \boldsymbol{0} \quad \boldsymbol{for\ i = 1, 2, 3...n} \tag{3.31}$$

5. **Long cash only**.

   This constraint avoids the cash account or cash balance to display negative values.

$$\boldsymbol{W_t^{n+1}} + \boldsymbol{NTr_t^{n+1}} = \boldsymbol{0} \tag{3.32}$$

6. **Maximum/minimum portfolio weight limit**.

   Portfolio composition can be customised by applying restrictions on the maximum and/or minimum holdings for every single asset. This can be implemented as follows:

$$\begin{aligned} \boldsymbol{W_t^i} &\leq \boldsymbol{Wmax^i} \\ \boldsymbol{W_t^i} &\geq \boldsymbol{Wmin^i} \quad \boldsymbol{for\ i = 1, 2, 3...n} \end{aligned} \tag{3.33}$$

Note that if $\boldsymbol{Wmax^i > 0}$, we are effectively limiting the long positions on the asset i. Furthermore, if $\boldsymbol{Wmin^i < 0}$, we are effectively limiting the short positions on the asset i.

4 Portfolio optimisation

4.1 Convex optimisation

The first time that the portfolio optimisation problem was formulated as a convexity problem was in 1952 by Markowitz. The key to our trading algorithm is that it reviews all the information made available at the beginning of the time period t (including or not forecast information) and estimate the trading vector $\boldsymbol{Tr_t^i}$ for i=1,2,3...n, while taking into account the pre-defined constraints and risk aversion function (defined below). Furthermore, the cash account will be calculated based on the self-financing constraint (3.20). Given that $\boldsymbol{PR_t}$ and the cost functions ($\boldsymbol{Tc_t[NTr_t]}$ and $\boldsymbol{Hc_t[W_t + NTr_t]}$) are unknown at the beginning of time period t, we will have to estimate them (Campbell et al, 1997). Therefore, the estimated portfolio return will be:

$$\begin{aligned} \widehat{\boldsymbol{PR}}_t = \widehat{\boldsymbol{PR}}_t^i * \boldsymbol{W_t} + \widehat{\boldsymbol{PR}}_t^i * \boldsymbol{NTr_t} - (\widehat{\boldsymbol{Tc}}_t[\boldsymbol{NTr_t}] + \widehat{\boldsymbol{Hc}}_t[\boldsymbol{W_t} + \boldsymbol{NTr_t}]) \\ \boldsymbol{for\ i = 1, 2, 3...n} \end{aligned} \tag{4.1}$$

The optimisation problem is determined as follows:

$$\begin{aligned} \boldsymbol{maximise} \quad & \widehat{\boldsymbol{PR}}_t - \boldsymbol{RAv} * \boldsymbol{Rf}[\boldsymbol{W_t} + \boldsymbol{NTr_t}] \\ \boldsymbol{subject\,to} \quad & \widehat{\boldsymbol{Tc}}_t[\boldsymbol{NTr_t}] + \widehat{\boldsymbol{Hc}}_t[\boldsymbol{W_t} + \boldsymbol{NTr_t}] = -[\mathbf{1}]' * \boldsymbol{NTr_t} \end{aligned} \tag{4.2}$$

where $\boldsymbol{RAv}$ and $\boldsymbol{Rf}$ are the risk aversion factor ($\boldsymbol{RAv} > \mathbf{0}$) and the risk function ($\boldsymbol{Rf} \in \boldsymbol{R^{n+1}}$), respectively. Moreover, by substituting (4.1) into (4.2) and by considering just the terms that depend on $\boldsymbol{NTr_t}$, the result is as follows:

$$\begin{aligned} \boldsymbol{maximise} \quad & \widehat{\boldsymbol{PR}}_t^i * \boldsymbol{NTr_t} - (\widehat{\boldsymbol{Tc}}_t[\boldsymbol{NTr_t}] + \widehat{\boldsymbol{Hc}}_t[\boldsymbol{W_t} + \boldsymbol{NTr_t}] \\ & \qquad + \boldsymbol{RAv} * \boldsymbol{Rf_t}[\boldsymbol{W_t} + \boldsymbol{NTr_t}]) \\ \boldsymbol{subject\,to} \quad & \widehat{\boldsymbol{Tc}}_t[\boldsymbol{NTr_t}] + \widehat{\boldsymbol{Hc}}_t[\boldsymbol{W_t} + \boldsymbol{NTr_t}] = -[\mathbf{1}]' * \boldsymbol{NTr_t} \end{aligned} \tag{4.3}$$

As can be seen, the optimisation problem is divided into four parts:

1. $\boldsymbol{\widehat{PR}_t^i * NTr_t}$ = estimated trading returns.

2. $\boldsymbol{\widehat{Tc}_t[NTr_t]}$ = estimated transaction cost function.

3. $\boldsymbol{\widehat{Hc}_t[W_t + NTr_t]}$ = estimated holding cost function.

4. $\boldsymbol{RAv * Rf_t[W_t + NTr_t]}$ = after-trading portfolio risk.

The optimal trading vector $\boldsymbol{NTr_t^{opt}}$ for i=1,2,3...n, is the solution to the optimisation problem, expressed as (4.3). In addition, the cash trade term $\boldsymbol{NTr_t^{n+1}}$ is calculated from the self-financing constraint (3.20).

Generally, cost terms are small in comparison to the total value of the portfolio. Therefore, the self-financing constraint can be simplified as: $\boldsymbol{[1]' * NTr_t = 0}$. It is important to note that this assumption does not mean we are ignoring the costs derived from holding and transactions, as these terms are still present in the objective. Thus, the optimisation problem results as follows:

$$\begin{aligned} \boldsymbol{maximise} \quad & \boldsymbol{\widehat{PR}_t^i * NTr_t - (\widehat{Tc}_t[NTr_t] + \widehat{Hc}_t[W_t + NTr_t]} \\ & \qquad\qquad \boldsymbol{+ RAv * Rf_t[W_t + NTr_t])} \\ \boldsymbol{subject\,to} \quad & \boldsymbol{[1]' * NTr_t = 0} \end{aligned} \tag{4.4}$$

Considering that we are interested in finding the after-trade portfolio weights, the optimisation problem (4.4) can be reformulated as follows:

$$\begin{aligned} \boldsymbol{maximise} \quad & \boldsymbol{\widehat{PR}_t^i * W_{t+1} - (\widehat{Tc}_t[W_{t+1} - W_t] + \widehat{Hc}_t[W_{t+1}]} \\ & \qquad\qquad \boldsymbol{+ RAv * Rf_t[W_{t+1}])} \\ \boldsymbol{subject\,to} \quad & \boldsymbol{[1]' * W_{t+1} = 1} \end{aligned} \tag{4.5}$$

Multiple ways to define the risk function $\boldsymbol{Rf_t}$ can be found in existing literature, e.g. (Frittelli and Gianin, 2002), (Busseti et al, 2016) . Furthermore, the risk function is generally obtained from the variance-covariance matrix of the asset returns (cf. Markowitz, 1952 and Fabozzi et al, 2010). Let $\boldsymbol{\Sigma_t} \in \boldsymbol{R^{(n+1)*(n+1)}}$ be the variance covariance matrix at the time period t. Assuming that asset returns follow a stochastic process (not predictable), the quadratic risk function $\boldsymbol{Rf_t}[\boldsymbol{x}]$ is defined as follows:

$$\boldsymbol{Rf}_t[\boldsymbol{x}] = \boldsymbol{x}' * \Sigma_t * \boldsymbol{x} \tag{4.6}$$

Furthermore, the variance of the portfolio return at the time period t will result in:

$$\boldsymbol{Var}[\boldsymbol{PR}_t] = (\boldsymbol{W}_t + \boldsymbol{NTr}_t)' * \Sigma_t * (\boldsymbol{W}_t + \boldsymbol{NTr}_t) \tag{4.7}$$

The main requirement in order to solve the portfolio optimisation problem is to make sure that the risk and cost functions are convex (Nesterov and Nemirovskii, 1994; O'Donoghue et al, 2016). Moreover, any additional constraint imposed to the model must be convex. However, one of the previously discussed conditions is not convex in its definition: the self-financing constraint (3.20). Consequently, in order to make the optimisation problem feasible, the self-financing constraint can be relaxed and expressed as one of the following alternatives:

1. $\boldsymbol{Tc}_t[\boldsymbol{NTr}_t] + \boldsymbol{Hc}_t[\boldsymbol{W}_t + \boldsymbol{NTr}_t] \leq -[\mathbf{1}]' * \boldsymbol{NTr_t}$. (Inequality expression).

2. $[\mathbf{1}]' * \boldsymbol{NTr_t} = \mathbf{0}$. (Simplified expression).

On the other hand, we can increase the level of flexibility of our algorithm by assigning aversion factors to the cost functions. For example, if we assign a large value to the holding cost factor effectively, we are sending a message to our algorithm to avoid too large holding short positions. Let $\boldsymbol{TAv}$ and $\boldsymbol{HAv}$ be positive parameters to define the transaction aversion costs and the holding aversion costs, respectively. Therefore, the portfolio optimisation problem will finally be defined as follows:

$$\begin{aligned}
\boldsymbol{maximise} \quad & \widehat{\boldsymbol{PR}}_t^{\boldsymbol{i}} * \boldsymbol{NTr_t} - (\boldsymbol{TAv} * \widehat{\boldsymbol{Tc}}_t[\boldsymbol{NTr_t}] + \boldsymbol{HAv} * \widehat{\boldsymbol{Hc}}_t[\boldsymbol{W_t} + \boldsymbol{NTr_t}] \\
& + \boldsymbol{RAv} * \boldsymbol{Rf_t}[\boldsymbol{W_t} + \boldsymbol{NTr_t}]) \\
\boldsymbol{subject\,to} \quad & [\mathbf{1}]' * \boldsymbol{NTr_t} = \mathbf{0}
\end{aligned} \tag{4.8}$$

4.2 Portfolio performance metrics

In order to evaluate and analyse the performance of the simulated portfolios under the different constraints and conditions previously described, we will define the following portfolio performance metrics:

1. **Annual Expected Portfolio Return (%).**

   The expected return of a portfolio is mathematically defined as follows:

   $$\boldsymbol{E}[\boldsymbol{PR}] = \boldsymbol{W}' * \boldsymbol{E}[\boldsymbol{R}] = \sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{W^i} * \boldsymbol{E}[\boldsymbol{R^i}] \tag{4.9}$$

   where $\boldsymbol{E}[\boldsymbol{R^i}]$ is the expected return for the ith asset. Furthermore, considering 250 working days per year, we can determine the annualised expected portfolio return in percentage (%) to be:

   $$\boldsymbol{Annual}\,\boldsymbol{E}[\boldsymbol{PR}](\%) = \boldsymbol{W}' * \boldsymbol{E}[\boldsymbol{R}] * \mathbf{250} * \mathbf{100} \tag{4.10}$$

2. **Annual Excess Return (%).**

   The portfolio excess return expresses quantitatively the advantage of investing in the current portfolio against theoretical risk free products (e.g. United States bonds). Moreover, the annual excess return is calculated as follows:

$$\boldsymbol{Annual\,Excess\,Return(\%) = (E[PR] - Risk\,Free\,Rate) * 250 * 100} \tag{4.11}$$

3. **Annual Excess Risk (%).**

   Similarly to the annual excess return, the annual excess risk describes the difference in terms of risk between investing in the current portfolio and theoretical risk free products. The risk of any portfolio is defined as the standard deviation of its expected portfolio returns ($\boldsymbol{Std[PR]}$). Thus,

$$\boldsymbol{Annual\,Excess\,Risk(\%) = (Std[PR] - Risk\,Free\,Rate) * 250 * 100} \tag{4.12}$$

4. **Sharpe ratio.**

   Sharpe ratio was defined by Sharpe (1994) and articulates how well the portfolio compensates its owner(s) under the risk taken. Hence, the Sharpe ratio is mathematically expressed as follows:

$$\boldsymbol{Sharpe\,ratio = \frac{E[PR] - Risk\,Free\,Rate}{Std[PR]} = \frac{Excess\,Return}{Std[PR]}} \tag{4.13}$$

5. **Maximum drawdown.**

   The maximum drawdown is determined by the accumulated loss from a long investment position at its highest price peak and a short position at its lowest price peak, expressed in percentage. The following figure shows the maximum drawdown of the MSCI Emerging Markets Index achieved from September 2001 to September 2011.

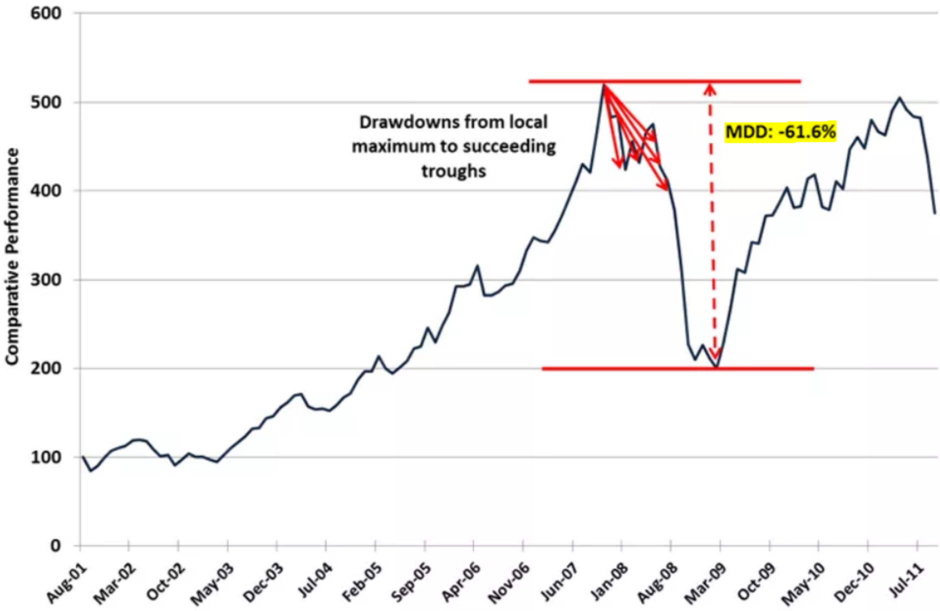

Figure 4.1: Maximum drawdown in MSCI Emerging Markets Index, Sept 2001/2011 (Ramani, 2013).

6. **Annual Portfolio Turnover.**

   The portfolio turnover quantifies how frequently portfolio managers adjust long or short positions. Moreover, portfolio turnover at time t is given by:

$$\boldsymbol{PTurnover_t = \frac{\sum_{i=1}^{n} |Tr_t^i|}{PV_t}} \tag{4.14}$$

   Finally, the Annual Portfolio Turnover is calculated as follows:

$$\boldsymbol{Annual\, Portfolio\, Turnover\,(\%) = E[PTurnover] * 250 * 100} \tag{4.15}$$

4.3 Investment policies

1. **Hold original portfolio configuration.**

   The original portfolio allocation is held from the beginning to the end of the investment simulation period. Therefore,

$$\boldsymbol{Tr_t = 0 \quad for\ t = 1, 2, 3...T} \tag{4.16}$$

2. **Model predictive control**.

   In terms of financial forecasting, the two most popular trends in the existing literature are the mean reverting strategy and the momentum strategy. Roughly speaking, the mean reverting trading strategy assumes that the current asset prices trend will revert back towards the mean in the future (Campbell, 2000; Franke, 1991; Do et al, 2006). On the other hand, momentum strategy is based on the assumption that current asset prices trend (positive or negative) will hold in the future (Menkhoff et al, 2011; Bae and Elkamhi, 2011; Filippou et al, 2016; Grobys and Heinonen, 2016). These two trading strategies are graphically represented in figure 4.2.

   Based on its flexibility and friendly computation features, we will apply in this research the mean-reverting strategy, where the optimisation problem is calculated at every time period, while taking into account the historical data plus a variable horizon length forecast. Moreover, the trading algorithm assumes that the current asset prices trend will revert in the following time periods, as showed in figure 4.2.

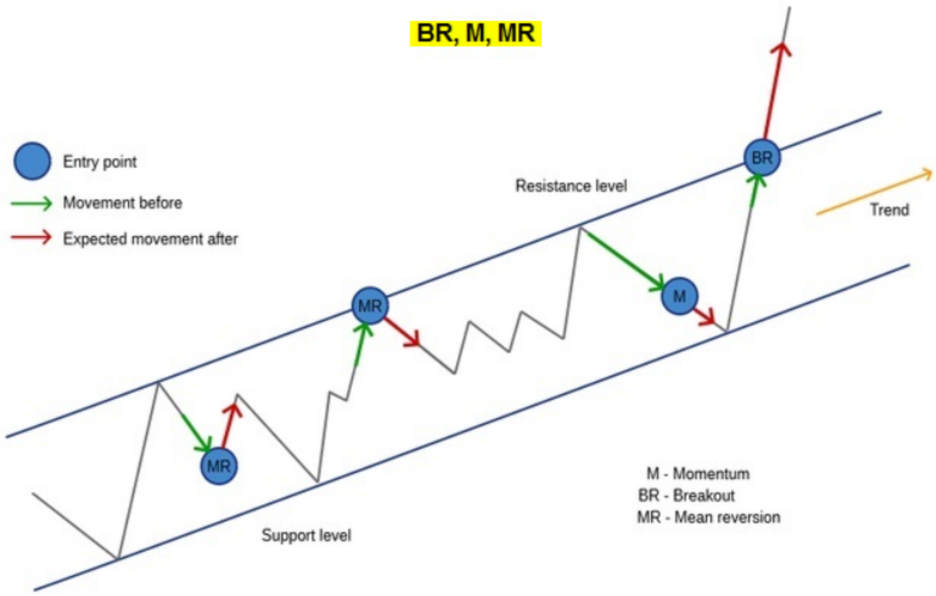

Figure 4.2: Mean reverting & Momemtum trading strategies (Shal, 2017).

5 Investment simulation

5.1 Data

Given the large amount of data and different configurations of the investment algorithm, we will work with fixed sample periods of 5 years. The stock data is obtained from the open-source financial database (Quandl, 2010), which provides daily closed market stock prices from Standard & Poor's 500 index (S&P 500). Moreover, the size of the total portfolio will consist of 20 different stocks, which is a reasonable amount that makes the simulation strongly realistic. Furthermore, in an attempt to build a diversified stock portfolio, we will pick several groups of 5 stocks based on different business sectors. In the following subsections, we introduce the selected stocks and show its main features.

5.1.1 Company profiles

COMPANY	FOUNDED	HEADQUARTERS	EMPLOYEES
Information Technology			
Akamai Technologies	1988	Cambridge, MA	8,000
Juniper Networks	1996	Sunnyvale, CA	9,100
Motorola Solutions	2011	Chicago, IL	16,000
Accenture	1989	Dublin, Ireland	447,000
Adobe	1982	San Jose, CA	21,428
Pharmaceutical/Healthcare			
Abbot Laboratories	1888	San Jose, CA	100,000
HCP	1985	Irvine, CA	201
Baxter International	1931	Deerfield, IL	48,000
Patterson Companies	1875	Mendota Heights, MN	7,500
Alexion Pharmaceuticals	1992	Boston, MA	2,525
Entertainment/Shopping			
Viacom	1952	Manhattan, NY	11,200
Konami	1969	Tokyo, Japan	4,606
Urban Outfitters	1970	Philadelphia, PA	16,330
Activision Blizzard	2008	Santa Monica, CA	9,900
Amazon.com	1994	Seattle, WA	647,000
Energy			
PPL Corporation	1920	Allentown, PA	12,512
Ameren Corporation	1997	St. Louis, MO	8,615
National Oilwell Varco	1841	Houston, TX	37,000
AES Corporation	1981	Arlington, VA	10,500
American Electric Power	1906	Columbus, OH	17,666

Table 5.1: Company profiles (Bloomberg, 2019).

COMPANY	REVENUE	NET INCOME	TOTAL ASSETS
Information Technology			
Akamai Technologies	\$2.72 billion	\$298 million	\$3.18 billion
Juniper Networks	\$5.03 billion	\$308 million	\$4.82 billion
Motorola Solutions	\$7.35 billion	\$966 million	\$9.42 billion
Accenture	\$41.61 billion	\$4.07 billion	\$24.45 billion
Adobe	\$9.04 billion	\$3.63 billion	\$18.77 billion
Pharmaceutical/Healthcare			
Abbot Laboratories	\$30.58 billion	\$477 million	\$76.27 billion
HCP	\$1.85 billion	\$415 million	\$12.72 billion
Baxter International	\$10.56 billion	\$717 million	\$17.11 billion
Patterson Companies	\$3.24 billion	\$213 million	\$2.45 billion
Alexion Pharmaceuticals	\$3.55 billion	\$444 million	\$13.58 billion
Entertainment/Shopping			
Viacom	\$12.94 billion	\$1.69 billion	\$23.16 billion
Konami	\$2.21 billion	\$280 million	\$3.02 billion
Urban Outfitters	\$3.45 billion	\$283 million	\$2.22 billion
Activision Blizzard	\$7.51 billion	\$1.82 billion	\$17.84 billion
Amazon.com	\$232.89 billion	\$10.07 billion	\$162.65 billion
Energy			
PPL Corporation	\$7.52 billion	\$1.91 billion	\$43.41 billion
Ameren Corporation	\$6.08 billion	\$659.11 million	\$24.71 billion
National Oilwell Varco	\$7.31 billion	\$236.21 million	\$20.21 billion
AES Corporation	\$10.53 billion	\$1.16 billion	\$33.12 billion
American Electric Power	\$15.36 billion	\$1.48 billion	\$56.42 billion

Table 5.2: Financial profiles (Bloomberg, 2019).

5.1.2 Financial profiles

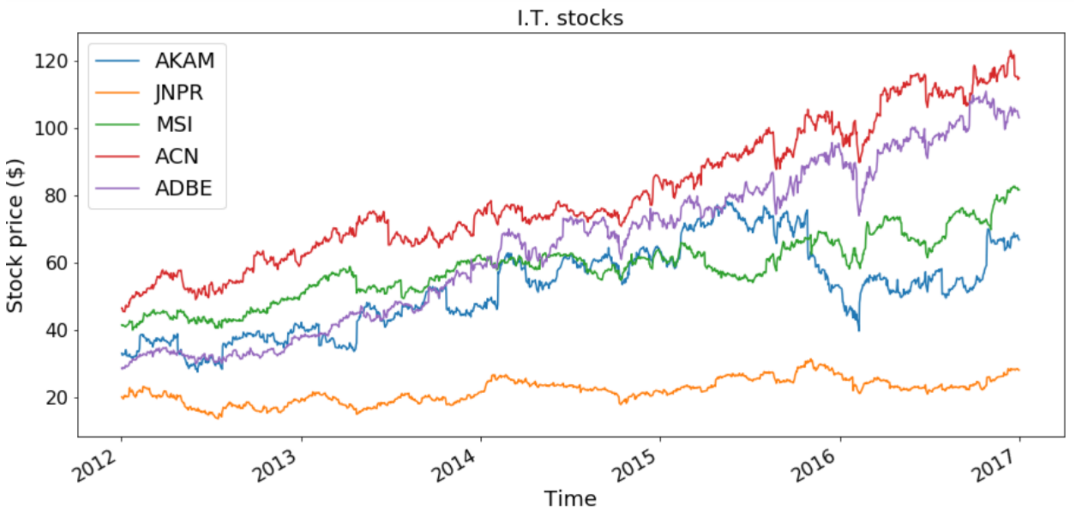

Figure 5.1: I.T. stocks.

The figure 5.1 illustrates an important increment in the stock price of Adobe (ADBE), Motorola Solutions (MSI) and Accenture (ACN), with the last one achieving the highest peak (of 120 dollars) at the beginning of 2017. Furthermore, Akamai Technologies (AKAM) displays an irregular development, showing jumps and falls from 2012 to 2017. On the other hand, Juniper Networks' stock price has been notably stable during the same period.

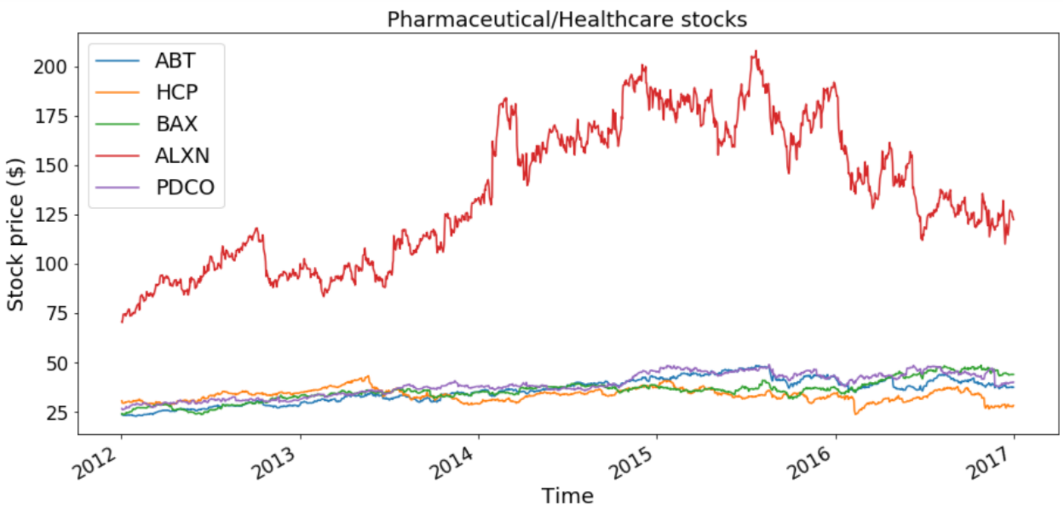

Figure 5.2: Pharmaceutical/Healthcare stocks.

The figure 5.2 shows that Alexion Pharmaceuticals' stock (ALXN) underwent a turbulent path from 2012 to 2017, achieving a maximum peak of 200 dollars during the third quarter of 2016. On the other hand, the rest of Pharmaceutical/Healthcare stocks (Abbot Laboratories (ABT), HCP (HCP), Baxter International (BAX) and Patterson Companies (PDCO)) developed an important stage of stability over the same period.

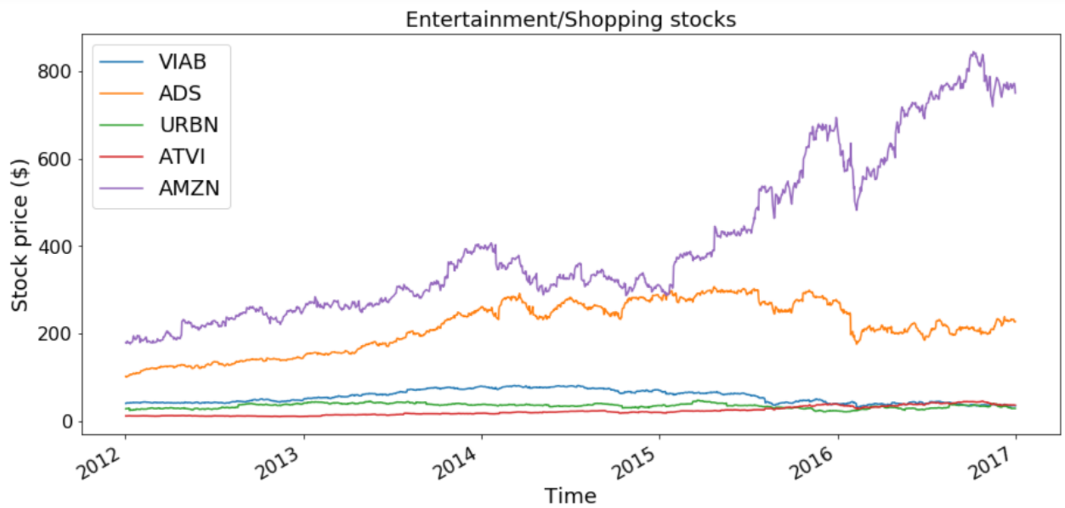

Figure 5.3: Entertainment & Shopping stocks.

In terms of the Entertainment & Shopping sector, the figure 5.3 illustrates the dramatic improvement developed by Amazon (AMZN) over the last few years, as it almost quadrupled its stock value from 250 dollars at the beginning of 2015 to 800 dollars at the end of 2016. In addition, Konami (ADS) exhibited a positive period from 2012 until the beginning of 2016, when it suffered an important fall of its stock value. Viacom's (VIAB), Urban Outfitters' (URBN) and Activision Blizzard's (ATVI) stocks did not undergo important changes over the last few years.

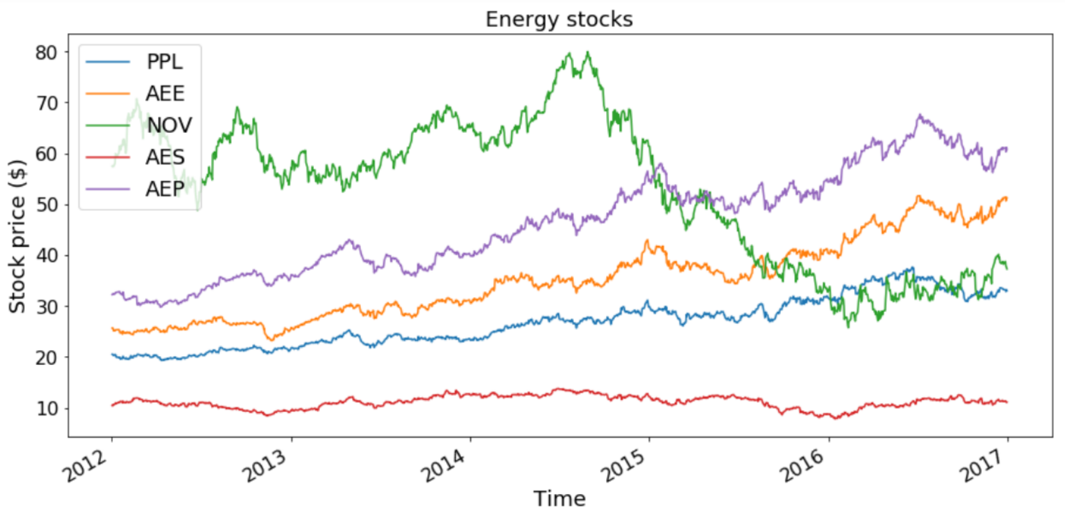

Figure 5.4: Energy stocks.

The figure 5.4 illustrates the positive trends displayed by American Electric Power (AEP), PPL Corporation (PPL) and Ameren Corporation (AEE), with American Electric Power being the one to achieve the highest peak (of 65 dollars) during the second quarter of 2016. On the other hand, National Oilwell Varco (NOV) displayed a turbulent trend from 2012 to the end of 2014, followed by a massive fall of 50 dollars per stock at the beginning of 2016. Finally, AES Corporation (AES) did not show significant changes on its stock price.

5.2 Software

The portfolio investment simulation will thus be computed through a customised Python trading algorithm based on the open-source package CVXPortfolio (Boyd et al, 2017). Moreover, the model optimisation tasks rely on the open-source package CVXPY (Diamond and Boyd, 2016). It is important to highlight that the numerical results reached by our algorithm could not be highly accurate, given the approximations, estimations and assumptions needed to build the model. However, the following simulations will be helpful in showing the big picture of the portfolio optimisation problem, in which several factors play a critical role in the investment process.

Investment Policy: HOLD	
Data Inputs	
Start date	1st January 2012
End date	31st December 2016
Historical data	N/A
Forecast horizon	N/A
Initial portfolio budget	$200,000
Aversion Parameters	
Risk Aversion	N/A
Trading Aversion	N/A
Holding Aversion	N/A
Cost Parameters	
Bid/ask half spread	N/A
Borrow costs	N/A
General constraints	
Leverage limit	N/A
Results	
Annual Expected Portfolio Return	12.379 %
Annual Excess Return	12.264 %
Annual Excess Risk	14.199 %
Sharpe ratio	0.864
Maximum Drawdown	17.759 %
Annual Portfolio Turnover	0 %

Table 5.3: Holding investment policy.

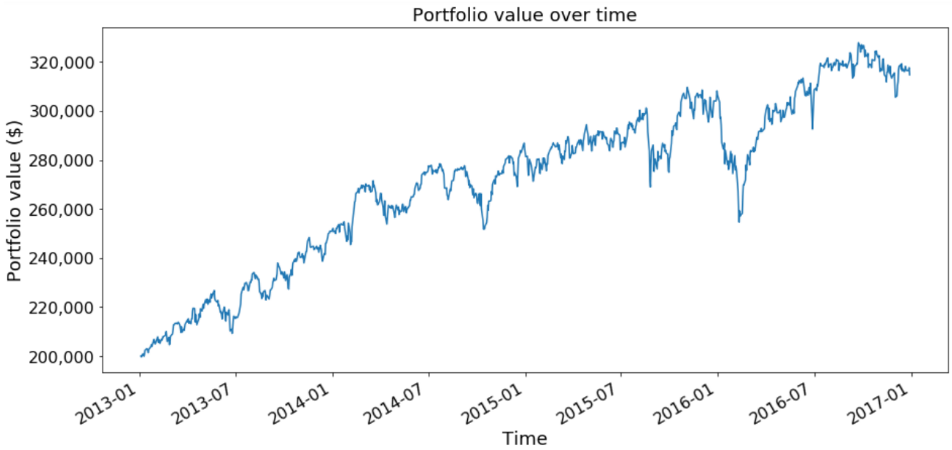

Figure 5.5: Holding investment policy.

The table 5.3 displays the results obtained under the Holding investment policy. Note that many of the parameters and constraints are not applicable. The initial portfolio budget is of $200,000 ($ 10,000 dollars per stock). Banning asset transactions makes the portfolio exclusively dependent on the asset prices. The Annual Excess risk is higher than the Annual Excess Return, resulting in a Sharpe ratio weaker than unity. Furthermore, we can observe that the Annual Portfolio Turnover is of 0% as trading is not allowed in this investment policy. Moreover, the figure 5.5 illustrates an increase of the Portfolio value over time due to the positive overall performance of the portfolio stocks over the investment period.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	0	10	50	100	250
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	5				
Trading Aversion	1				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	10.94	31.80	36.93	32.75	23.14
Annual Excess Return (%)	10.82	31.69	36.82	32.64	23.03
Annual Excess Risk (%)	27.37	25.73	22.27	20.40	12.69
Sharpe ratio	0.40	1.23	1.65	1.60	1.81
Maximum Drawdown (%)	33.03	20.48	21.10	21.07	14.55
Annual Portfolio Turnover (%)	428.6	354.1	215.9	148.2	52.95

Table 5.4: MPC investment policy. Forecast horizon test.

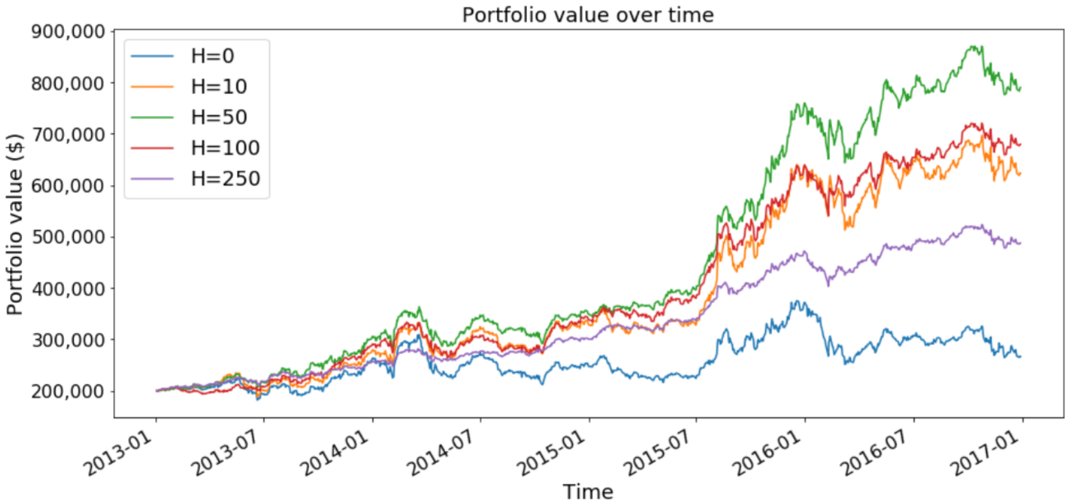

Figure 5.6: MPC investment policy. Forecast horizon test.

The table 5.4 illustrates the results obtained under the MPC investment policy. The algorithm takes into account the stock performance over the last 250 days (1 financial year) plus several forecast horizons (0, 10, 50, 100 and 250 days) to calculate the optimisation point. Holding other factors fixed, the table 5.4 shows that the performance of the MPC investment policy strongly depends on the horizon length. The portfolio displays extremely poor results when only historical data is taken into account. However, the portfolio's performance improves as the forecast horizon increases, achieving the highest Sharpe ratio (1.81) at a 250-day length. On the other hand, the figure 5.6 illustrates that the highest forecast horizon does not account for the highest portfolio value, where the maximum ($\sim$ \$900,000) is achieved at a 50-day length in the third quarter of 2016.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	10				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1	5	10	20	50
Trading Aversion	1				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	71.31	31.81	18.56	10.89	5.74
Annual Excess Return (%)	71.19	31.69	14.27	10.77	5.63
Annual Excess Risk (%)	51.92	25.73	14.27	8.11	3.99
Sharpe ratio	1.37	1.23	1.29	1.33	1.41
Maximum Drawdown (%)	50.51	20.48	12.52	7.13	4.05
Annual Portfolio Turnover (%)	1,331	354.2	172.1	101.3	60.20

Table 5.5: MPC investment policy. RAvs test1.

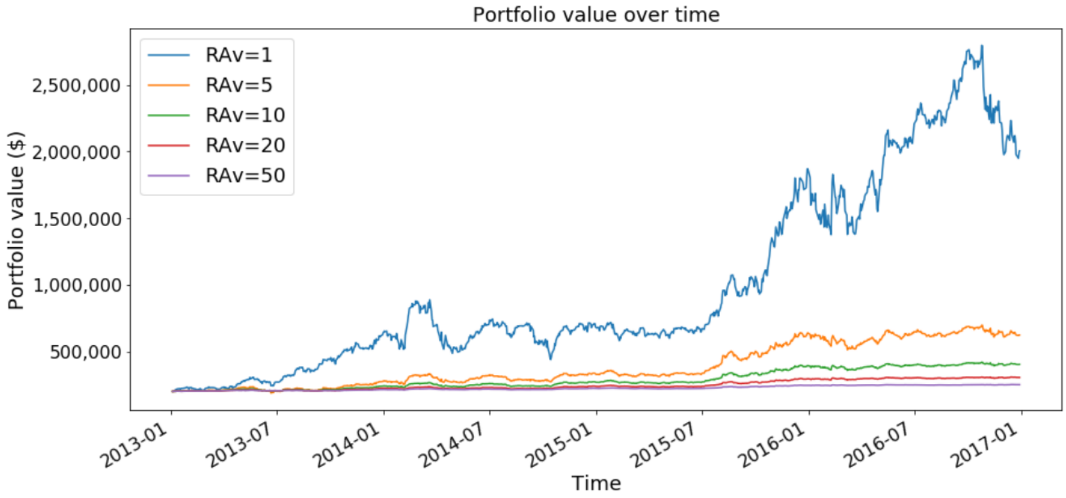

Figure 5.7: MPC investment policy. RAvs test1.

The table 5.5 demonstrates the results obtained under the MPC investment policy. The algorithm considers the stock performance of the past 250 days (1 financial year) plus a fixed forecast horizon of 10 days. In this case, we have applied several values to the risk aversion factor RAv (1, 5, 10, 20, 50) to explore its role into the investment process. Holding other factors fixed, the table 5.5 does not show a clear relationship between the increase of the risk aversion factor and the Sharpe ratio, perhaps only a very slightly positive trend as the risk aversion factor increases. Furthermore, the figure 5.7 illustrates a significant increase in the portfolio's value over time when the risk aversion factor is settled to its minimum value (1). Finally, the highest portfolio value, with a maximum of $\sim$ \$2,500,000, is achieved during third quarter of 2016.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	100				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1	5	10	20	50
Trading Aversion	1				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	87.34	32.76	18.22	10.71	5.62
Annual Excess Return (%)	87.22	32.64	18.11	10.60	5.51
Annual Excess Risk (%)	51.24	20.40	11.49	6.94	3.61
Sharpe ratio	1.71	1.60	1.58	1.53	1.53
Maximum Drawdown (%)	49.52	21.07	11.91	6.88	2.89
Annual Porfolio Turnover (%)	952.4	148.2	81.59	55.99	42.72

Table 5.6: MPC investment policy. RAvs test2.

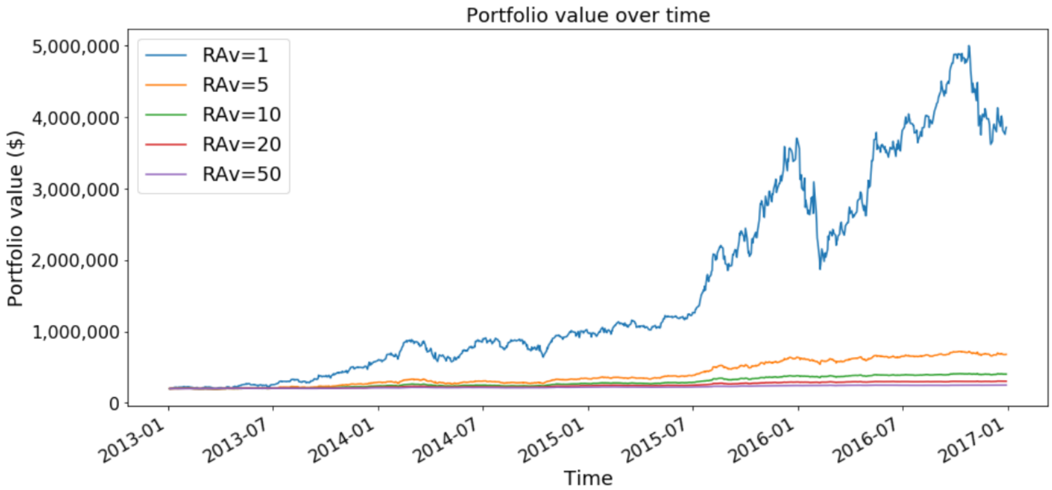

Figure 5.8: MPC investment policy. RAvs test2.

Following the previous simulation, the table 5.6 exhibits the results obtained under the MPC investment policy with a fixed forecast horizon of 100 days. In this case, the results illustrate a contradictory trend against the results obtained in table 5.5. Holding other factors fixed, the table 5.6 shows a slightly negative performance trend as the risk aversion factor increases. However, the overall Sharpe ratio has significantly improved and the overall annual portfolio turnover has decreased. Furthermore, the figure 5.8 illustrates a similar trend with respect to the figure 5.7, showing an explosive increase of the portfolio value over time as the risk aversion factor is settled to its minimum value (1). Again, the highest portfolio value, with a maximum of $\sim$ $5,000,000, is achieved during the third quarter of 2016.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	10				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1				
Trading Aversion	0.5	0.7	1	1.2	1.5
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	84.22	75.10	71.31	70.34	51.37
Annual Excess Return (%)	84.10	74.97	71.19	70.22	51.25
Annual Excess Risk (%)	50.10	51.44	51.92	51.76	50.23
Sharpe ratio	1.65	1.46	1.37	1.36	1.02
Maximum Drawdown (%)	44.52	45.41	50.50	48.72	48.01
Annual Porfolio Turnover (%)	1,667	1,538	1,331	1,085	683.8

Table 5.7: MPC investment policy. TAvs test1.

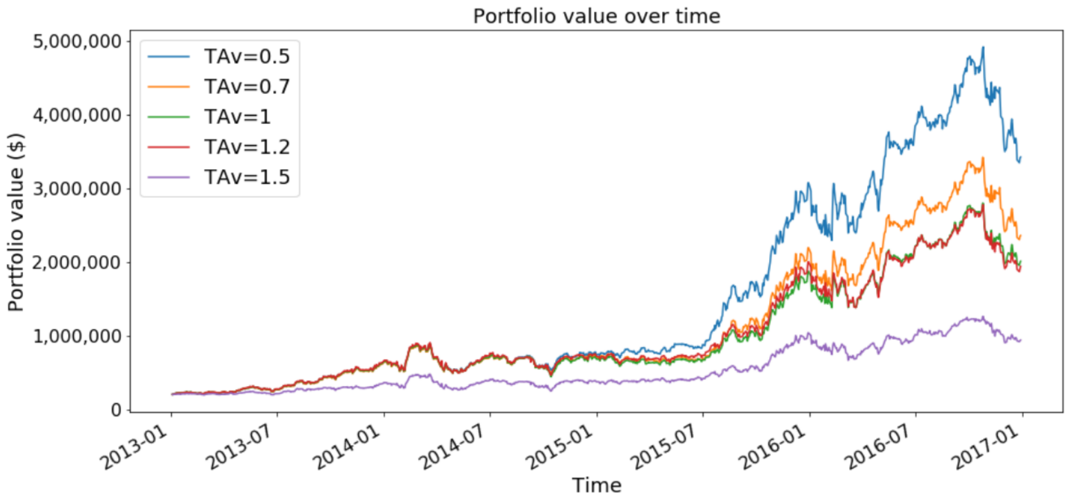

Figure 5.9: MPC investment policy. TAvs test1.

The table 5.7 demonstrates the results obtained under the MPC investment policy. The algorithm considers the stock performance of the past 250 days (1 financial year) plus a fixed forecast horizon of 10 days. In this case, we have applied several values into the trading aversion factor TAv (0.5, 0.7, 1, 1.2, 1.5) to explore its role into the investment process. Holding other factors fixed, the table 5.7 shows a clear relationship between the increase of the trading aversion factor and the Sharpe ratio, describing a rational negative trend as the trading aversion factor increases. This shown feature is logical, since avoiding trading reduces the chances of generating more substantial profits. Furthermore, the figure 5.9 shows a similar trend of the portfolio value over time for the different trading aversion factors, achieving its maximum values when the trading factor is settled to its minimum value (0.5). Finally, the highest portfolio value of $\sim$ \$5,000,000 is achieved during the third quarter of 2016.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	100				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1				
Trading Aversion	0.5	0.7	1	1.2	1.5
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	96.81	97.40	87.34	77.18	64.63
Annual Excess Return (%)	96.70	97.28	87.23	77.05	64.51
Annual Excess Risk (%)	51.41	51.13	51.24	49.76	45.52
Sharpe ratio	1.88	1.90	1.70	1.54	1.42
Maximum Drawdown (%)	48.94	48.94	49.52	51.97	44.83
Annual Porfolio Turnover (%)	1,502	1,393	952.4	616.6	326.4

Table 5.8: MPC investment policy. TAvs test2.

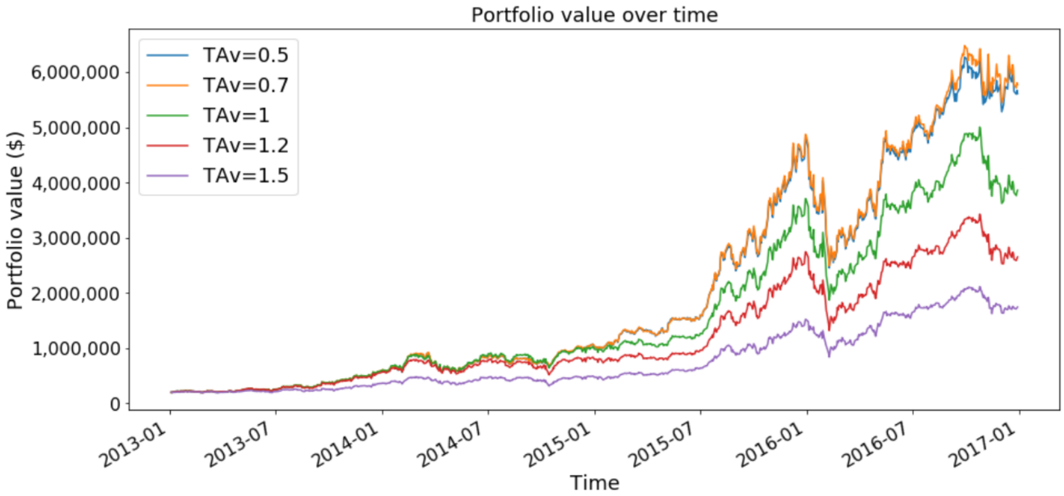

Figure 5.10: MPC investment policy. TAvs test2.

Following the previous simulation, the table 5.8 illustrates the results obtained under the MPC investment policy with a fixed forecast horizon of 100 days. In this case, the results illustrate an irregular trend against the results obtained in the table 5.6. Holding other factors fixed, the table 5.8 shows an aggressive negative performance trend as the trading aversion factor increases, starting from 0.7. However, the overall Sharpe ratio has significantly improved and the overall annual portfolio turnover has decreased. Moreover, the figure 5.10 shows a peculiar similar trend for trading aversion factors equal to 0.5 and 0.7. In addition, the rest of the portfolio values clearly decrease as TAv increases. Note that the highest portfolio value, with a maximum of $\sim$ \$6,000,000, is achieved during the third quarter of 2016, for a TAv value equal to 0.7. Interestingly, this value was not reached however when the trading factor was settled to its minimum (0.5).

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	10				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1				
Trading Aversion	0.7				
Holding Aversion	1	5	10	20	50
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	75.09	70.57	66.39	67.06	67.79
Annual Excess Return (%)	74.43	70.45	66.28	66.95	67.67
Annual Excess Risk (%)	51.43	52.28	54.29	58.01	58.85
Sharpe ratio	1.46	1.35	1.22	1.15	1.15
Maximum Drawdown (%)	45.41	45.26	52.56	64.37	65.84
Annual Porfolio Turnover (%)	1,538	1,445	1,360	1,287	1,260

Table 5.9: MPC investment policy. HAvs test1.

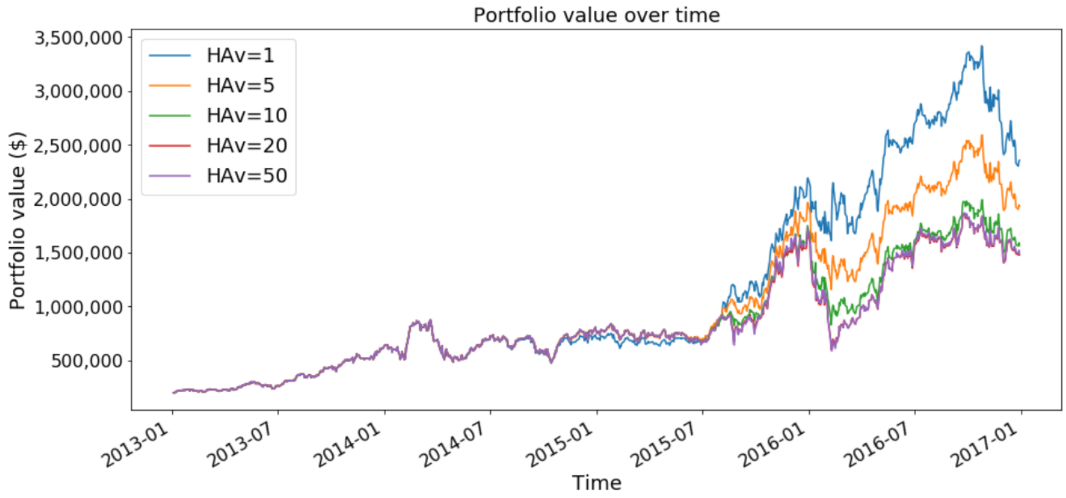

Figure 5.11: MPC investment policy. HAvs test1.

The table 5.9 exhibits the results obtained under the MPC investment policy. The algorithm considers the stock performance of the past 250 days (1 financial year) plus a fixed forecast horizon of 10 days. In this case, we have applied several values into the holding aversion factor HAv (1, 5, 10, 20, 50) to explore its role into the investment process. Holding other factors fixed, the table 5.9 displays a relatively clear relationship between the increase of the holding aversion factor and the Sharpe ratio, describing a slow negative performance trend as the holding aversion factor increases. In addition, the figure 5.11 shows a similar trend of the portfolio value over time for the different holding aversion factors, achieving its maximum values when the holding factor is settled to the minimum (1). Finally, the highest portfolio value is of $\sim$ \$3,500,000 and it is achieved during the third quarter of 2016, following the trend of the previous simulations.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	100				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1				
Trading Aversion	0.7				
Holding Aversion	1	5	10	20	50
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	97.40	97.32	95.53	92.58	92.08
Annual Excess Return (%)	97.28	97.21	95.41	92.44	91.96
Annual Excess Risk (%)	51.13	53.71	56.47	58.47	58.73
Sharpe ratio	1.90	1.81	1.69	1.58	1.57
Maximum Drawdown (%)	48.94	53.96	64.27	65.60	65.42
Annual Porfolio Turnover (%)	1,393	1,287	1,100	964.1	937.8

Table 5.10: MPC investment policy. HAvs test2.

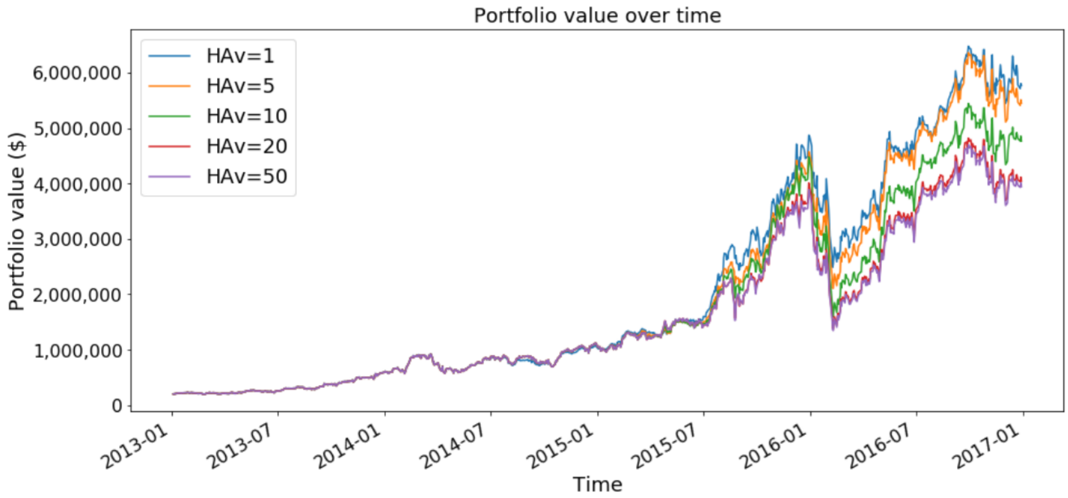

Figure 5.12: MPC investment policy. HAvs test2.

Following the previous investment portfolio process, the table 5.10 shows the results obtained under the MPC investment policy with a fixed forecast horizon of 100 days. Holding other factors fixed, the table 5.10 displays a negative performance trend as the holding aversion factor increases. Again, as in the previous simulations, the increase of the horizon length drives an increase of the overall Sharpe ratio. Furthermore, the overall annual portfolio turnover has decreased. Moreover, the figure 5.12 illustrates a very similar performance trend for all the holding aversion factors. The highest portfolio value with a maximum of $\sim$ \$6,000,000 is achieved during the third quarter of 2016, for HAvs equal to 1 and 5, simultaneously.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	10				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1				
Trading Aversion	0.7				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	1	1.5	2	2.5	3
Results					
Annual Expected Portfolio Return (%)	20.43	32.05	44.75	59.34	75.09
Annual Excess Return (%)	20.31	31.93	44.64	59.22	74.97
Annual Excess Risk (%)	19.53	28.35	36.69	44.30	51.44
Sharpe ratio	1.04	1.13	1.22	1.34	1.46
Maximum Drawdown (%)	19.26	27.05	32.05	40.26	45.41
Annual Porfolio Turnover (%)	246.3	438.6	732.3	1,099	1,538

Table 5.11: MPC investment policy. Levs test1.

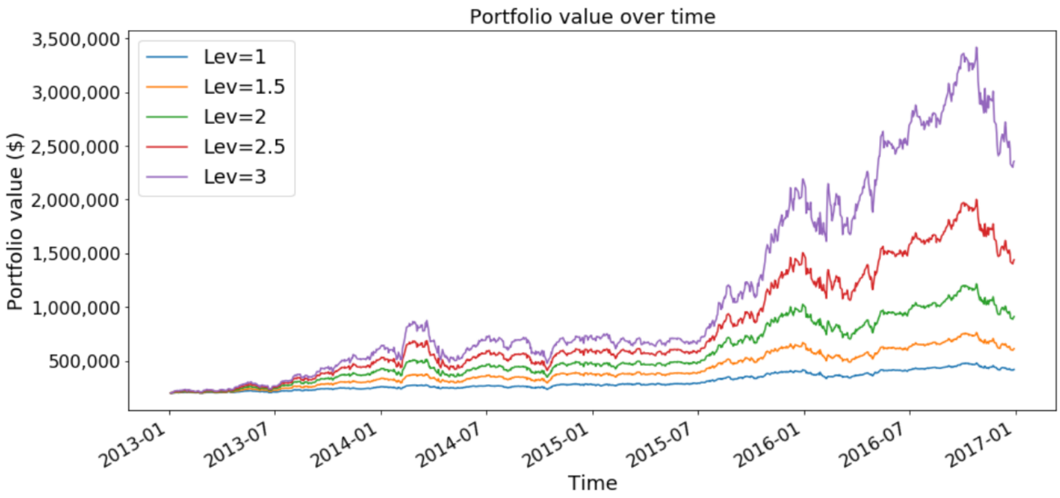

Figure 5.13: MPC investment policy. Levs test1.

The table 5.11 illustrates the results obtained under the MPC investment policy. The algorithm considers the stock performance over the past 250 days (1 financial year) plus a fixed forecast horizon of 10 days. In this case, we have applied several leverage limits (1, 1.5, 2, 2.5, 3) to explore its role into the investment process. Holding other factors fixed, the table 5.11 illustrates a clear and rational relationship between the increase of the leverage and the Sharpe ratio, describing a positive performance trend as leverage increases, which is logical in terms of taking advantage of borrowing to maximise profits. Furthermore, the figure 5.13 displays the same trend as previously mentioned, with higher portfolio values over time for higher leverage limits, achieving its maximum values when the leverage limit is settled to the maximum one (3). Finally, the highest portfolio value is of $\sim$ \$3,500,000 and it is achieved during the third quarter of 2016, as usual.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	100				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1				
Trading Aversion	0.7				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Leverage limit	1	1.5	2	2.5	3
Results					
Annual Expected Portfolio Return (%)	28.95	45.42	62.11	80.04	97.40
Annual Excess Return (%)	28.83	45.31	61.99	79.92	97.28
Annual Excess Risk (%)	21.49	30.13	37.83	44.79	51.13
Sharpe ratio	1.34	1.50	1.64	1.78	1.90
Maximum Drawdown (%)	22.64	32.77	43.23	46.81	48.94
Annual Porfolio Turnover (%)	189.6	368.8	646.5	992.8	1,393

Table 5.12: MPC investment policy. Levs test2.

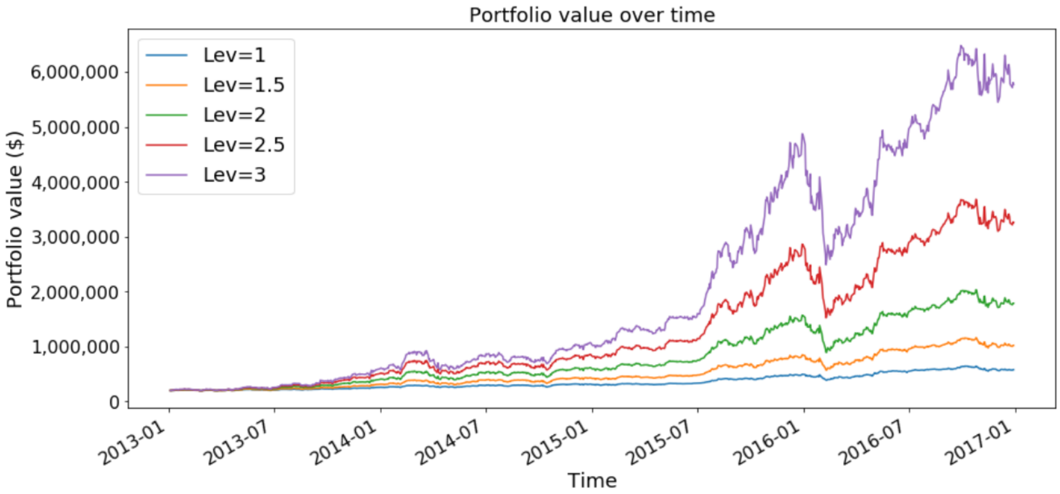

Figure 5.14: MPC investment policy. Levs test2.

Following the previous simulation, the table 5.12 demonstrates the results obtained under the MPC investment policy with a fixed forecast horizon of 100 days. The results illustrate an similar trend to the results obtained in table 5.11. Holding other factors fixed, the table 5.12 shows a clear positive trend as the leverage limit increases. Again, the overall Sharpe ratio has significantly improved, although the overall annual portfolio turnover has not substantially decreased in this case. Moreover, the figure 5.14 shows a very similar portfolio performance trend to figure 5.13, where portfolio values clearly increased as the leverage limit increased. Note that the highest portfolio value, with a maximum of $\sim$ \$6,000,000, is achieved again during the third quarter of 2016 for the maximum leverage limit (3).

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	100				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1	5	10	20	50
Trading Aversion	0.7				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Long Only portfolio					
Results					
Annual Expected Portfolio Return (%)	21.93	18.86	12.26	6.63	3.17
Annual Excess Return (%)	21.82	18.64	12.14	6.52	3.06
Annual Excess Risk (%)	22.06	16.81	11.16	5.87	2.56
Sharpe ratio	0.99	1.12	1.09	1.11	1.19
Maximum Drawdown (%)	30.15	21.80	11.95	5.92	2.76
Annual Porfolio Turnover (%)	152.3	153.7	116.4	68.03	40.87

Table 5.13: MPC investment policy. Long Only. RAvs test.

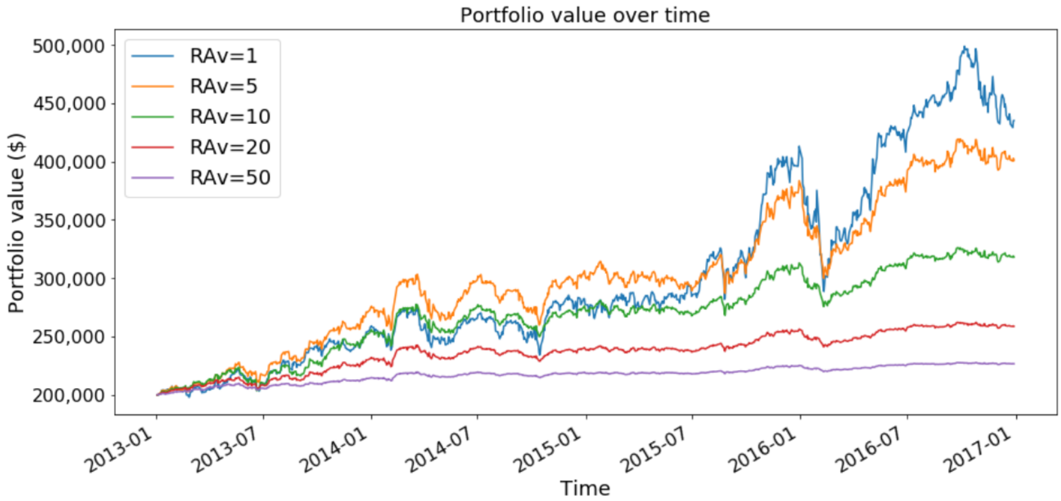

Figure 5.15: MPC investment policy. Long Only. RAvs test.

The table 5.13 exhibits the results obtained under the MPC investment policy under the Long Only portfolio constraint. In this case, we have applied several values to the risk aversion factor RAv (1, 5, 10, 20, 50) to explore its role into the investment process. The algorithm considers the stock performance over the past 250 days (1 financial year) plus a fixed forecast horizon of 100 days. Furthermore, the displayed results in the table 5.13 illustrate a very poor portfolio investment performance, which is logical as limiting a portfolio to take only long positions is a strong constraint that has an important negative impact on profits. Moreover, the figure 5.15 displays a similar but less aggressive performance trend to the figures 5.7 and 5.8, where higher portfolio values over time were achieved by decreasing the value of the risk aversion factor. Finally, the highest portfolio value is $\sim$ \$500,000 and it is achieved during the third quarter of 2016, following the trend of the previous simulations.

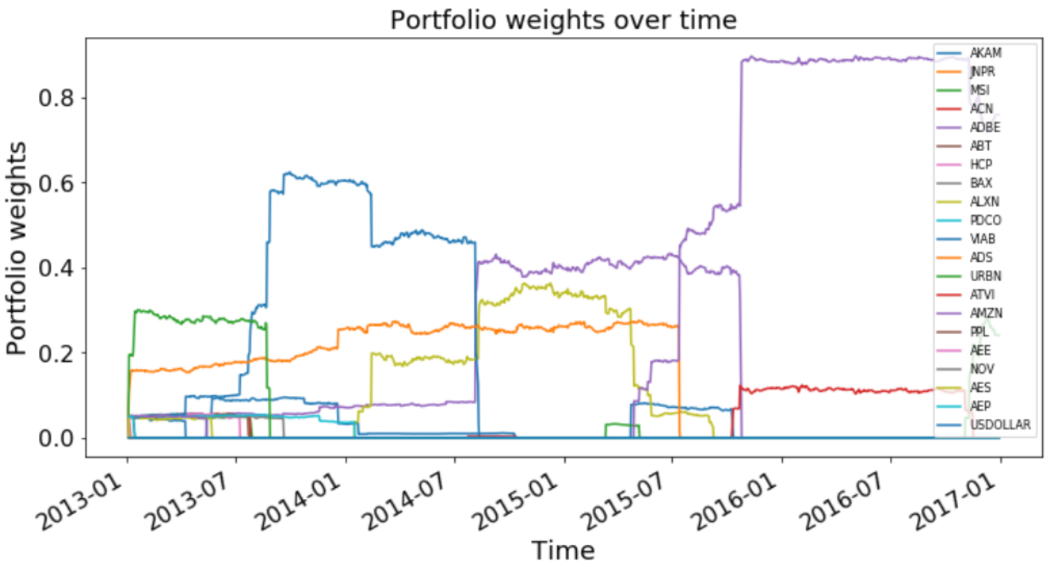

Figure 5.16: MPC investment policy. Long Only. Portfolio weights.

The figure 5.16 illustrates the constraint described by the equation 3.29. This constraint implies that just long (buy) positions will be taken into the portfolio investment process.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	100				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1	5	10	20	50
Trading Aversion	0.7				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Dollar Neutral portfolio					
Leverage limit	3				
Results					
Annual Expected Portfolio Return (%)	60.20	34.77	19.01	10.07	4.5
Annual Excess Return (%)	60.09	34.66	18.90	9.96	4.39
Annual Excess Risk (%)	40.23	21.03	12.91	6.92	3.03
Sharpe ratio	1.49	1.65	1.46	1.44	1.45
Maximum Drawdown (%)	40.20	21.14	13.13	7.55	3.19
Annual Porfolio Turnover (%)	1,524	428.7	233.8	135.1	74.54

Table 5.14: MPC investment policy. Dollar Neutral portfolio. RAvs test.

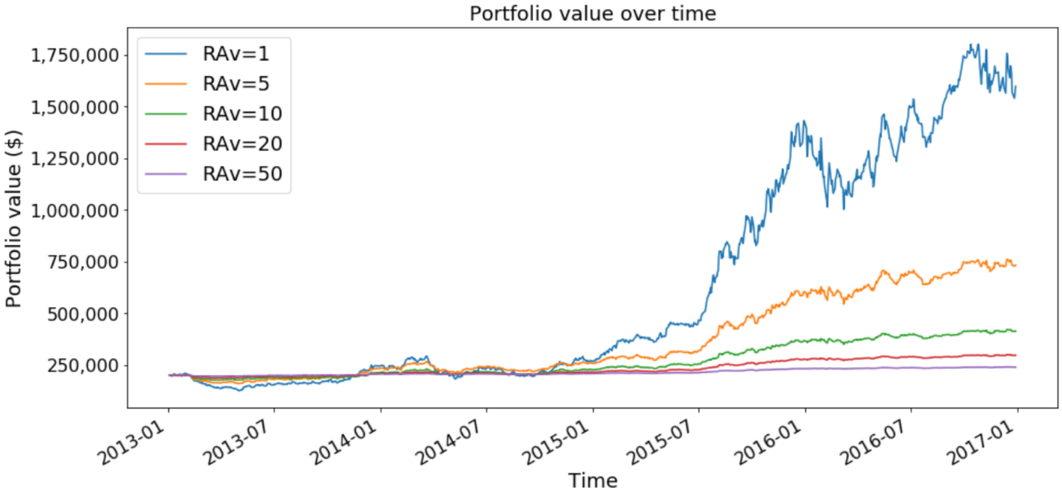

Figure 5.17: MPC investment policy. Dollar Neutral portfolio. RAvs test.

The table 5.14 displays the results obtained by the MPC investment policy under the Dollar Neutral portfolio constraint (cf. equation 3.31). In this case, we have applied several values to the risk aversion factor RAv (1, 5, 10, 20, 50) to explore its role into the investment process. The algorithm considers the stock performance over the past 250 days (1 financial year) plus a fixed forecast horizon of 100 days. Furthermore, the described results in the table 5.14 show a very stable portfolio investment performance, with Sharpe ratios describing a minimal variability. Moreover, the figure 5.17 illustrates a clear performance trend, where higher portfolio values over time are achieved by decreasing the value of the risk aversion factor. Finally, the highest portfolio value is $\sim$ \$1,750,000 and it is achieved during the third quarter of 2016, as in the previous simulations.

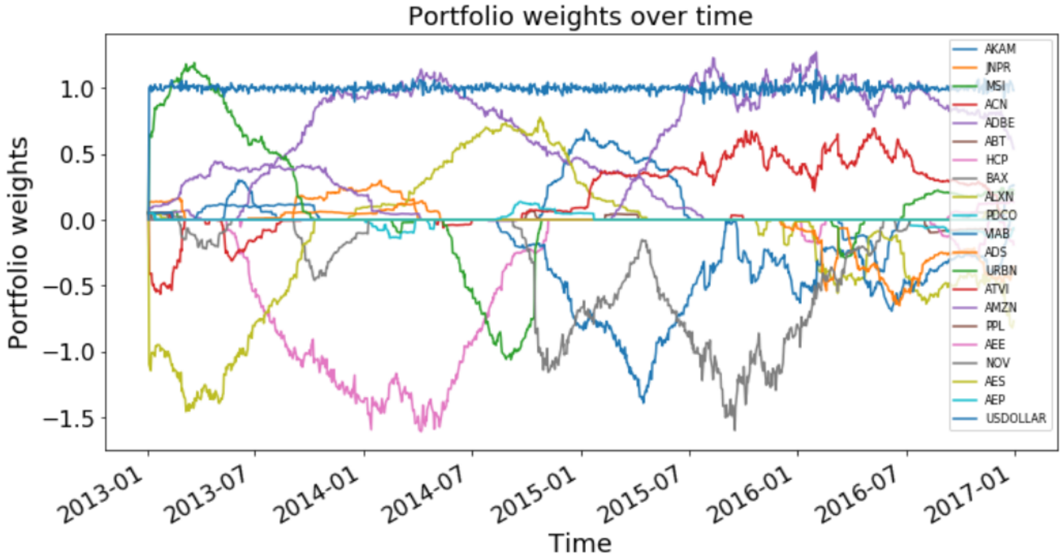

Figure 5.18: MPC investment policy. Dollar Neutral portfolio. Portfolio weights.

The figure 5.18 shows the constraint described by the equation 3.31. The main idea behind this strategy is to take equal short and long positions, resulting in the summation of these to be neutral or equal to zero.

Investment Policy: MPC					
Data Inputs					
Start date	1st January 2012				
End date	31st December 2016				
Historical data (days)	250				
Forecast horizon (days)	100				
Initial portfolio budget ($)	200,000				
Aversion Parameters					
Risk Aversion	1				
Trading Aversion	0.7				
Holding Aversion	1				
Cost Parameters					
Bid/ask half spread (%)	0.05				
Borrow costs (%)	0.01				
General constraints					
Dollar Neutral portfolio					
Leverage limit	1	1.5	2	2.5	3
Results					
Annual Expected Portfolio Return (%)	18.21	27.21	38.61	48.54	60.20
Annual Excess Return (%)	18.10	27.09	38.49	48.43	60.09
Annual Excess Risk (%)	15.42	22.29	28.84	34.78	40.23
Sharpe ratio	1.17	1.22	1.34	1.39	1.49
Maximum Drawdown (%)	14.38	21.17	28.67	35.42	40.20
Annual Porfolio Turnover (%)	373.9	600.9	880.1	1,200	1,524

Table 5.15: MPC investment policy. Dollar Neutral portfolio. Levs test.

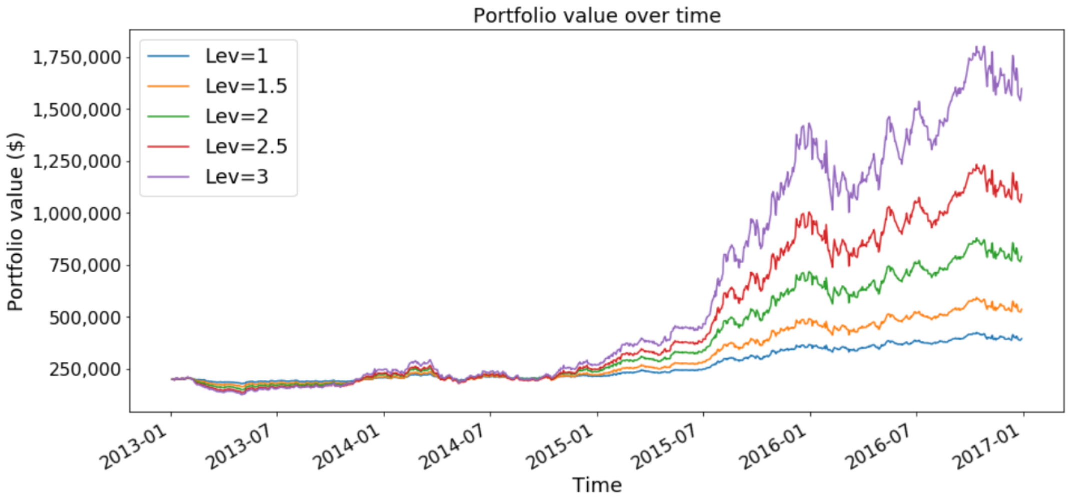

Figure 5.19: MPC investment policy. Dollar Neutral portfolio. Levs test.

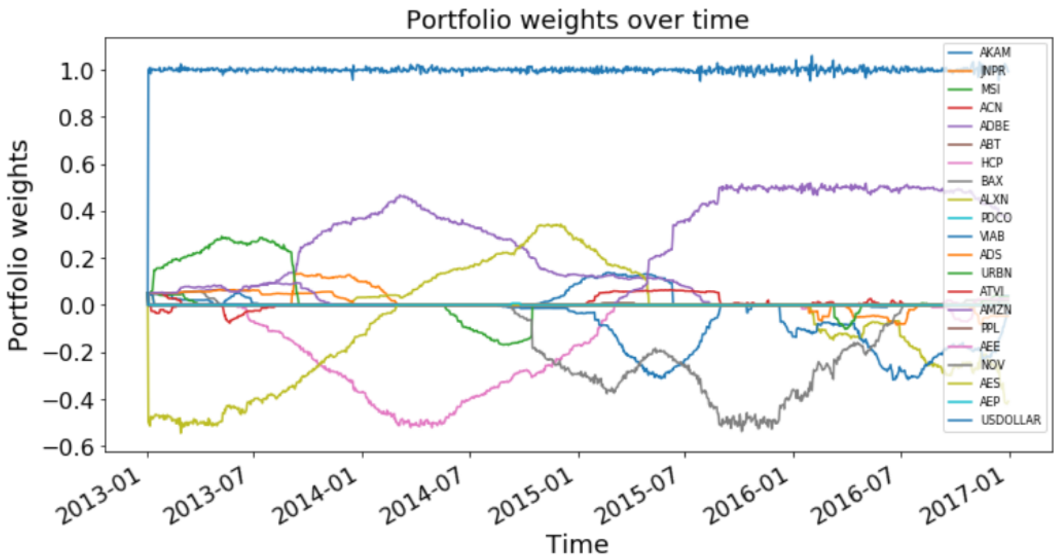

Figure 5.20: MPC investment policy. Dollar Neutral portfolio. Portfolio weights.

The table 5.15 demonstrates the results obtained by the MPC investment policy under the Dollar Neutral portfolio constraint (cf. equation 3.31). The algorithm considers the stock performance over the past 250 days (1 financial year) plus a fixed forecast horizon of 100 days. In this case, we have applied several leverage limits (1, 1.5, 2, 2.5, 3) to explore its role into the investment process. Holding other factors fixed, the table 5.15 illustrates a clear and rational relationship between the increase of the leverage and the Sharpe ratio, describing a positive trend as leverage increases, which is logical in terms of taking advantage of borrowing to maximise profits. Furthermore, the figure 5.19 displays the same trend as previously mentioned, of higher portfolio values over time for higher leverage limits, achieving its maximum values when the leverage limit is settle to the maximum one (3). Finally, the highest portfolio value is of $\sim$ \$1,750,000 and it is achieved during the third quarter of 2016, as usual.

6 Conclusions and further research

The main achievement of this dissertation has been to develop a framework based on MPC that offers an alternative solution (compared to the traditional portfolio optimisation approaches) to the Portfolio Optimisation problem via convexity. Firstly, we have build a portfolio of 20 stocks, composed by four groups of stocks related to different industries in order to take advantage of the benefits of portfolio diversification. Furthermore, we have defined a range of portfolio performance metrics to evaluate the simulation results from different perspectives. This practice allows portfolio managers to analyse and compare diverse investment strategies under various possible scenarios. Moreover, we have computed several investment simulations (~ 90) under a wide range of model parameters, investment policies and constraints. Finally, we have displayed the results using numerical tables and graphs to illustrate the details but also the big picture of investment scenario under analysis.

Accordingly, we have extracted interesting conclusions that can be very helpful for the development of new trading models and strategies based on Engineering Control Systems theory. Firstly, the results emphasise the addition of a forecast horizon under the mean-reverting strategy to improve the performance of the investment process. Furthermore, the optimal length of the horizon might be between 50 and 100 days. Secondly, risk aversion factors might affect the investment process, but it is not clear to what point its contribution is beneficial. Similarly, trading aversion factors may limit investment profits as one is effectively limiting the portfolio capacity to take advantage of the arising market opportunities. However, the holding aversion factors do not seem to have a big impact on the investment portfolio process. On the other hand, the increase of the leverage limit has potentially a considerable effect on the portfolio investment returns, but also on its risk. Finally, the computation of Long Only portfolio or more advanced constraints, such as Dollar Neutral portfolio, do not demonstrate additional features in terms of portfolio performance.

The positive performance feedback obtained in this research is just another step in the path that Robotics and Finance have walked together over the last decades. Following the trends illustrated by Marigo and Piccoli (2004), the application of MPC has demonstrated to have brought to the table several potential advantages compared to other traditional portfolio investment approaches. Moreover, the results can be considered more optimistic than the ones shown by Herzog (2006), obtaining slightly higher Sharpe ratios. Furthermore, the addition of the Holding and Risk Aversion parameters into the system can be considered as an "advanced version" of the framework developed by Primbs (2007). On the other hand, following Narang's research (2013), the algorithm presented in this dissertation can be customised to deal with several trading frequencies. However, note that the quality of the data that feeds the system (eg. accuracy, missing data) will play a critical role for High Frequency Trading (HFT) or Ultra High Frequency Trading (UHFT).

Every investment professional knows the sentence "All the good models work very well until they no longer do". As the rest of other trading algorithms, this model has its inaccuracies and limitations derived from the approximations, estimations and assumptions needed to build it. Its computation also requires important loading times, making it infeasible for high frequency trading. In addition, given the assumption of mean-reverting trends, the absence of peaks in the stock prices significantly reduces the performance. However, these simulations are helpful in showing the big picture of the portfolio optimisation problem, where several factors play a critical role in the investment process.

There are several features of real trading systems that have not been modelled in this dissertation, but that we encourage to be included in future research. Some of these features are listed as follows:

1. Dividends.
2. Trading timing constraint.
3. External income and/or output cash flow.
4. Merger & acquisitions.
5. Trading freeze.
6. Bankruptcy.

Finally, there are two additional investment policies that might be interesting to be included in future frameworks:

1. **Full short policy**.

   Sell all non-cash assets. This investment policy is based on taking exclusively short positions on all non-cash assets. Thus,

$$\begin{aligned} \boldsymbol{PP_t^i} \quad \boldsymbol{for\ t = 1, 2, 3...T} \\ \boldsymbol{i = 1, 2, 3...n} \end{aligned} \tag{6.1}$$

2. **Fixed trading configuration**.

   The portfolio simulation follows a customised trading vector.

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Alphabetical Index

A

algorithm, 3, 4, 25, 31, 37, 41, 43, 47, 51, 55, 59, 62, 65

C

closed loop, 1, 2

constraint, 1, 5, 10, 17, 20, 21, 23–26, 38, 39, 59, 60, 62, 63, 65, 67–69

convexity, 23, 67

D

Dollar Neutral, 21, 61–65, 68

E

engineering, 1–3, 9, 67

H

hold, 29, 39, 41, 43, 45, 47, 49, 51, 53

holding aversion, 26, 51, 53, 67

I

input, 1, 3, 4

investment, 4, 6, 9, 10, 12, 13, 17, 18, 27, 29, 31, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 60, 62, 65, 67–69

L

Long Only, 20, 58–60, 67

M

MATLAB, 11

Model Predictive Control, 9, 29

MPC, 3, 4, 9, 41, 43, 45–47, 49, 51, 53, 55, 57, 59, 62, 65, 67

O

optimisation, 1, 3, 9, 10, 12, 23–25, 29, 37, 41, 68

output, 1

P

portfolio optimisation, 2, 4, 23, 25, 26, 37

portfolio weights, 14, 17, 24, 60, 63, 65

Python, 4, 11, 37

R

return, 7, 9, 13, 18, 19, 23–27, 39

risk aversion, 23, 43, 45, 59, 62, 67

S

setpoint, 1, 4

Sharpe ratio, 27, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 62, 66

T

trading algorithm, 4, 23, 29, 37

trading aversion, 47, 49, 67

Appendix

[1]:
```
import os
import sys
sys.path.insert(0, os.path.abspath('..'))
```

[2]:
```
%matplotlib inline
import numpy as np
import pandas as pd
import quandl
import cvxportfolio as cp
import matplotlib.pyplot as plt
import matplotlib.ticker as mtick

#Quandl financial database user key.
quandl.ApiConfig.api_key = "##########LfHLgiKau2"
```

[3]:
```
#Portfolio stocks.
stocks = ['AKAM', 'JNPR', 'MSI', 'ACN','ADBE','ABT','HCP','BAX','ALXN',
          ␣
 ↪'PDCO','VIAB','ADS','URBN','ATVI','AMZN','PPL','AEE','NOV','AES','AEP']

#Investment time period.
start_date='2012-01-01'
end_date='2016-12-31'

#Gettind financial data from Quandl database.
Portfolio_returns = pd.DataFrame(dict([(ticker, quandl.get('WIKI/'+ticker,
start_date=start_date,
end_date=end_date)['Adj. Close'].pct_change())
for ticker in stocks]))

#Risk free rate.
Portfolio_returns[["USDOLLAR"]]=quandl.get('FRED/DTB3',
                                           start_date=start_date,
                                           end_date=end_date)/(250*100)

Portfolio_returns = Portfolio_returns.fillna(method='ffill').iloc[1:]
```

[4]:
```python
#Expected returns and standar deviations of stocks.
historical_data = 250
expected_returns = returns.rolling(window=historical_data,
                                   min_periods=historical_data).mean().shift(1).
 ↪dropna()
std_returns = returns.rolling(window=historical_data,
                              min_periods=historical_data,
                              closed='neither').cov().dropna()
```

[83]:
```python
#Cost models.
Transaction_costs_model=cp.TcostModel(half_spread=10E-4)
Holding_costs_model=cp.HcostModel(borrow_costs=1E-4)

#Risk model.
Risk_model = cp.FullSigma(std_returns)
```

[84]:
```python
#Aversion parameters.

#Risk aversion parameters.
RAv1 =1.
RAv2 =5.
RAv3 =10.
RAv4 =20.
RAv5 =50.

#Trading aversion parameters.
TAv1 =0.5
TAv2 =0.7
TAv3 =1.
TAv4 =1.2
TAv5 =1.5

#Holding aversion parameters.
HAv1 =1.
HAv2 =5.
HAv3 =1.
HAv4 =10.
HAv5 =50.

#Leverage limits.
Lev1 = cp.LeverageLimit(1)
Lev2 = cp.LeverageLimit(1.5)
Lev3 = cp.LeverageLimit(2)
Lev4 = cp.LeverageLimit(2.5)
Lev5 = cp.LeverageLimit(3)
```

[ ]:
```
#Investment policies configuration.
MPC_policy1 = cp.MultiPeriodOpt(return_forecast=expected_returns,
                                costs=[RAv1*Risk_model,
                                       TAv1*Transaction_costs_model,
                                       HAv1*Holding_costs_model],
                                constraints=[Lev1,cp.LongOnly(),
                                             cp.DollarNeutral()])

MPC_policy2 = cp.MultiPeriodOpt(return_forecast=expected_returns,
                                costs=[RAv2*Risk_model,
                                       TAv2*Transaction_costs_model,
                                       HAv2*Holding_costs_model],
                                constraints=[Lev2,cp.LongOnly(),
                                             cp.DollarNeutral()])

MPC_policy3 = cp.MultiPeriodOpt(return_forecast=expected_returns,
                                costs=[RAv3*Risk_model,
                                       TAv3*Transaction_costs_model,
                                       HAv3*Holding_costs_model],
                                constraints=[Lev3,cp.LongOnly(),
                                             cp.DollarNeutral()])

MPC_policy4 = cp.MultiPeriodOpt(return_forecast=expected_returns,
                                costs=[RAv4*Risk_model,
                                       TAv4*Transaction_costs_model,
                                       HAv4*Holding_costs_model],
                                constraints=[Lev4,cp.LongOnly(),
                                             cp.DollarNeutral()])

MPC_policy5 = cp.MultiPeriodOpt(return_forecast=expected_returns,
                                costs=[RAv5*Risk_model,
                                       TAv5*Transaction_costs_model,
                                       HAv5*Holding_costs_model],
                                constraints=[Lev5,cp.LongOnly(),
                                             cp.DollarNeutral()])
```

[ ]:
```
#Market simulation process.
Investment_simulation=cp.MarketSimulator(expected_returns,
                                         [Transaction_costs_model,
                                         Holding_costs_model],
                                         cash_key='USDOLLAR')

#Initial budget: $200,000 ($10,000 per stock).
Initial_portfolio = pd.Series(index=returns.columns, data=10000.)

#Self-funding portfolio.
Initial_portfolio.USDOLLAR = 0
```

```
[ ]: #Investment simulation process.
Simulation_results = market_sim.run_multiple_backtest(Initial_portfolio,
                                start_time='2013-01-03', end_time='2016-12-29',
                               policies=[cp.Hold(),
                                         MPC_policy1,
                                         MPC_policy2,
                                         MPC_policy3,
                                         MPC_policy4,
                                         MPC_policy5])
```

```
[ ]: #Display numerical results
Simulation_results[0].summary()
Simulation_results[1].summary()
Simulation_results[2].summary()
Simulation_results[3].summary()
Simulation_results[4].summary()
Simulation_results[5].summary()
```

```
[ ]: #Display graphical results. Portfolio value over time.
results[0].v.plot(figsize=(15,7),label='Lev=1')
results[1].v.plot(figsize=(15,7),label='Lev=1.5')
results[2].v.plot(figsize=(15,7),label='Lev=2')
results[3].v.plot(figsize=(15,7),label='Lev=2.5')
results[4].v.plot(figsize=(15,7),label='Lev=3')

#Plot configuration
plt.gca().tick_params(axis='both',labelsize=16)
plt.xlabel('Time',fontsize=18)
plt.title('Portfolio value over time',fontsize=18)
plt.axis('tight')
plt.ylabel('Portfolio value ($)',fontsize=18)

ax = plt.gca()
ax.get_yaxis().set_major_formatter(plt.FuncFormatter(lambda x,
                                                     loc: "{:,}".
 ↪format(int(x))))
plt.legend(loc='upper left',fontsize=18)
plt.savefig('MPC.png')
```

[ ]:

```
#Display graphical results. Portfolio weights over time.
results[0].w.plot(figsize=(15,7),label='Lev=1')
results[1].w.plot(figsize=(15,7),label='Lev=1.5')
results[2].w.plot(figsize=(15,7),label='Lev=2')
results[3].w.plot(figsize=(15,7),label='Lev=2.5')
results[4].w.plot(figsize=(15,7),label='Lev=3')


#Plot configuration
plt.gca().tick_params(axis='both',labelsize=16)
plt.xlabel('Time',fontsize=18)
plt.title('Portfolio weight over time',fontsize=18)
plt.axis('tight')
plt.ylabel('Portfolio weight',fontsize=18)

ax = plt.gca()
ax.get_yaxis().set_major_formatter(plt.FuncFormatter(lambda x,
                                                     loc: "{:,}".
 ↪format(int(x))))
plt.legend(loc='upper left',fontsize=18)
plt.savefig('MPC2.png')
```