Capitulo X

Fasores

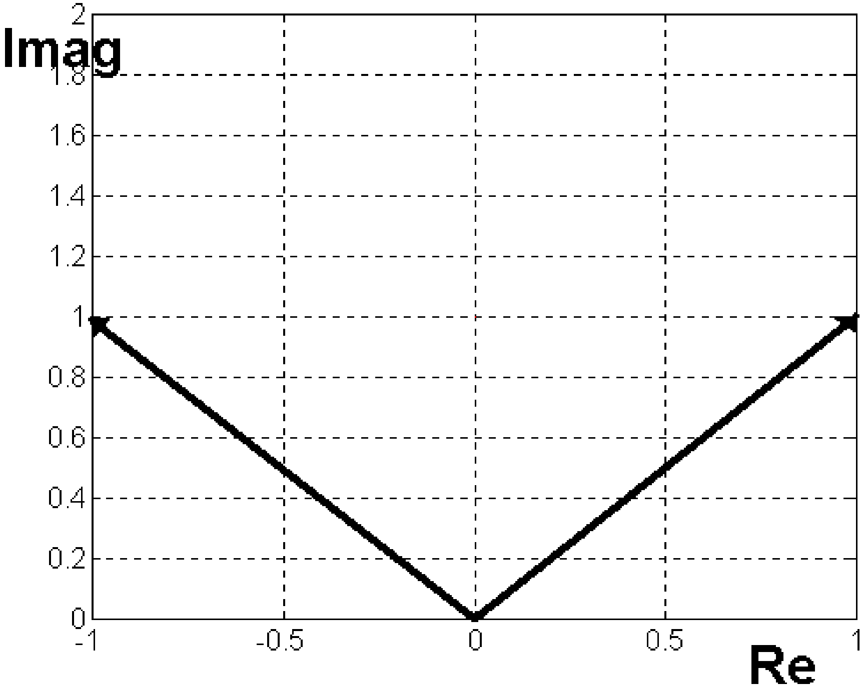
Plano Complejo

- Los numeros en el plano complejo vienen definidos por dos componentes una real y otra imaginaria

$$\sigma + j\omega$$

$$1 + j$$

$$-1 + j$$



Plano Complejo

- Debido a su forma vectorial la suma se hace vectorialmente

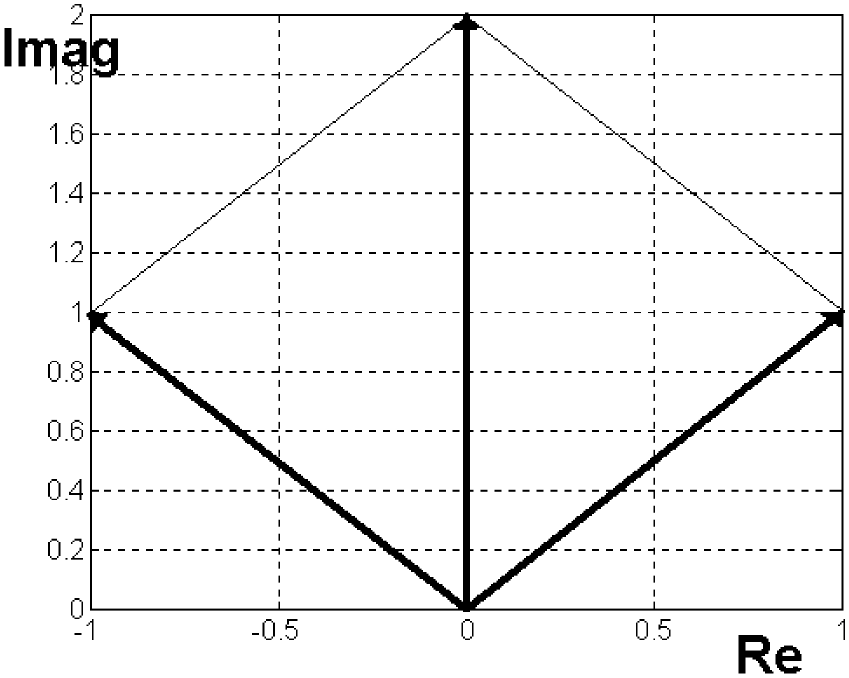
$$V_1 = (1, j)$$

$$V_2 = (-1, j)$$

$$V_T = V_1 + V_2$$

$$V_T = (1-1, j+j)$$

$$V_T = (0, 2j)$$



Plano Complejo

- Conversion rectangular a polar.
- El producto es mas simple en polar.

$$V = (x, jy) = \left|(x, yj)\right| \underline{\lfloor \theta} = \sqrt{x^2 + y^2} \underline{\left\lfloor \arctan(\frac{y}{x}) \right.}$$

$$V_1 = (1, j) = \sqrt{2} \underline{\lfloor 45^0}$$

$$V_2 = (-1, j) = \sqrt{2} \underline{\lfloor -45^0} \equiv \sqrt{2} \underline{\lfloor 180^0 - 45^0} = \sqrt{2} \underline{\lfloor 135^0}$$

$$V_T = V_1 * V_2 = \sqrt{2} * \sqrt{2} \underline{\lfloor 135^0 + 45^0} = 2 \underline{\lfloor 180^0}$$

Plano Complejo

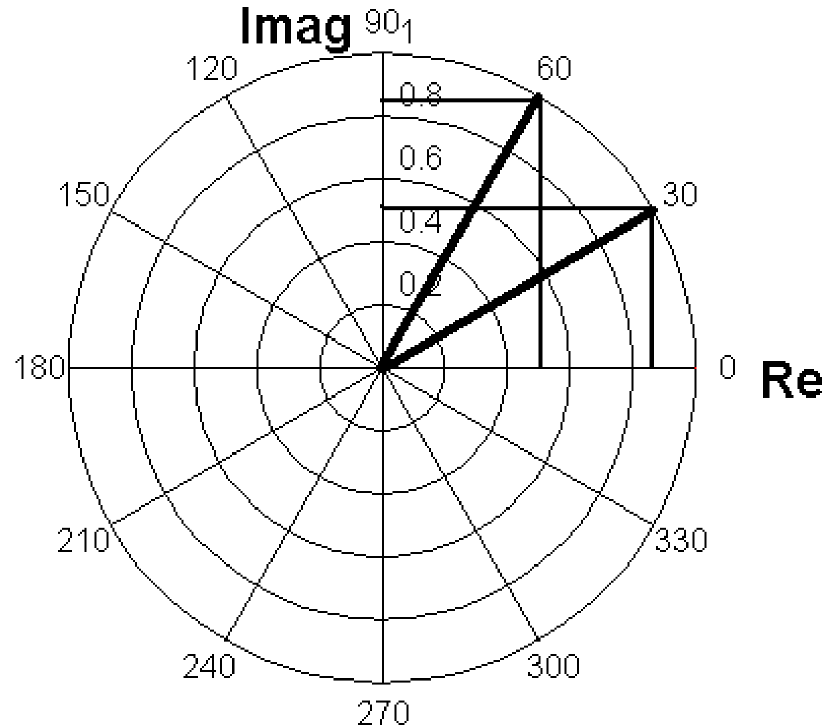
- Conversion polar a rectangular.

$$V = r\angle\theta = \left(r\cos(\theta), r\sin(\theta)\, j\right)$$

$$V_1 = \sqrt{2}\angle 45^0 = \left(\sqrt{2}\cos(45), \sqrt{2}\, j\sin(45)\right)$$

Fasores

- El fasor se define como un vector en representación polar que cambia el angulo en función del tiempo



$$\mathrm{Re} = r\cos\omega t$$

$$\mathrm{Im}\,ag = r\sin\omega t$$

$$x_1(t) = r\cos\left(\omega t + 30^o\right) \quad x_2(t) = r\cos\left(\omega t + 60^o\right)$$

$$y_1(t) = r\sin\left(\omega t + 30^o\right) \quad y_2(t) = r\sin\left(\omega t + 60^o\right)$$

Fasores

- Como tal los fasores pueden ser operados: suma, resta, multiplicación y división

$$V_1(t) = r\cos(\omega t + 30^o) \quad V_2(t) = r\cos(\omega t + 60^o)$$

$$V_T(t) = V_1(t) + V_2(t)$$

$$V_T\big|_{t=0} = r\underline{\big|\ 30^o} + r\underline{\big|\ 60^o}$$

$$x_T\big|_{t=0} = r(\cos(30^o) + \cos(60^o)) = 0.866 + 0.5 = 1.366$$

$$y_T\big|_{t=0} = r(\sin(30^o) + \sin(60^o)) = 0.5 + 0.866 = 1.366$$

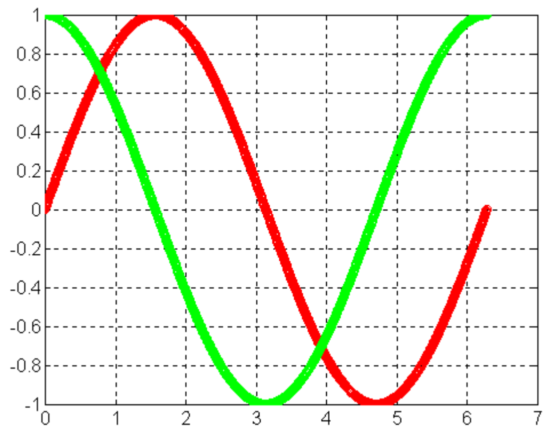
$$V_T = 1.366 + 1.366j = 1.52\underline{\big|\ 45^o}$$

$$V_T(t) = 1.52\cos(\omega t + 45^o) \qquad r = 1$$

Impedancia Inductiva Adelanto

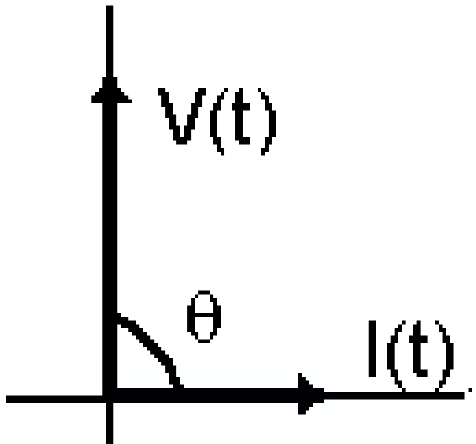
- Estudiaremos la respuesta del circuito, como fasores:

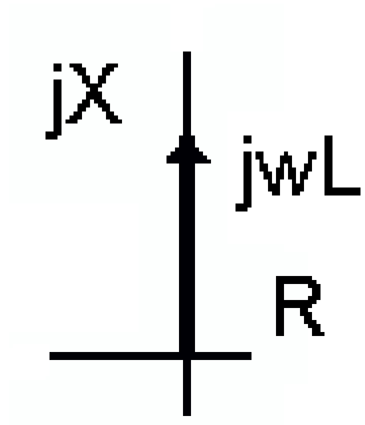
$$I(t)=e^{j\omega t}=\cos(\omega t)+j\sin(\omega t)$$

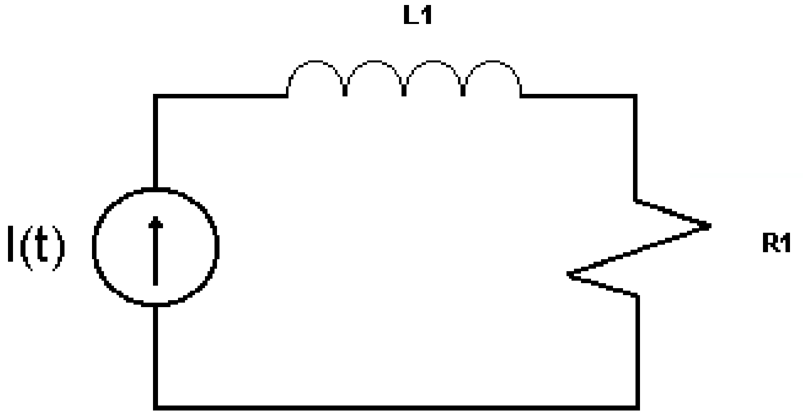


$$V_L(t)=L\frac{dI(t)}{dt}=\omega jLe^{j\omega t}$$

$$Z_L(t)=\frac{V_L(t)}{I_L(t)}=\frac{\omega jLe^{j\omega t}}{e^{j\omega t}}=Lj\omega$$







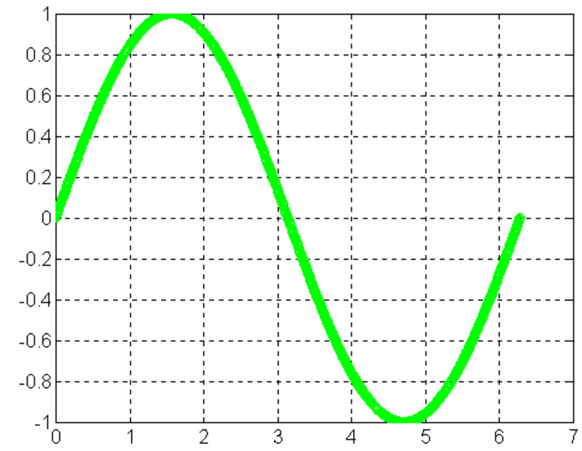
Impedancia Resistiva
Fase

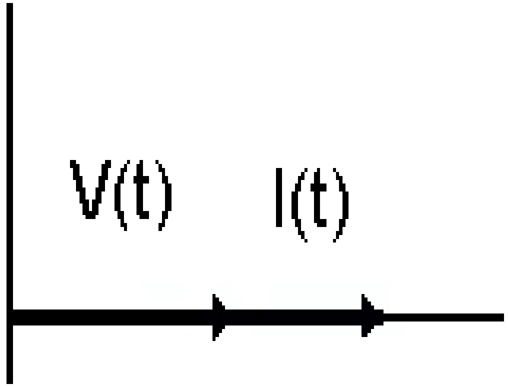
- Estudiaremos la respuesta del circuito, como fasores:

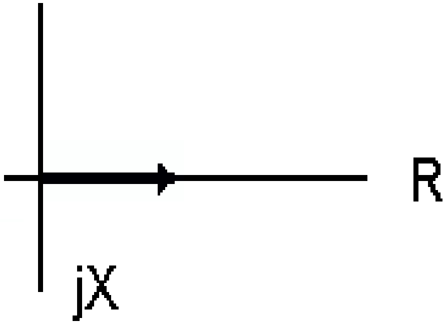
$$I(t) = e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

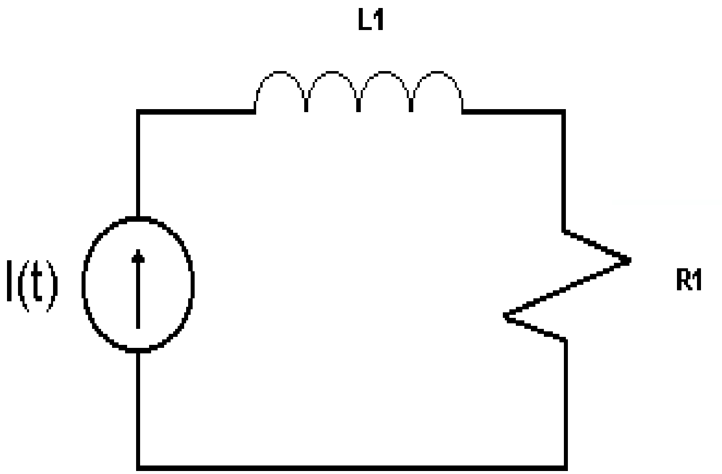
$$V_R(t) = RI(t) = e^{j\omega t}R$$

$$Z_R(t) = \frac{V_R(t)}{I_L(t)} = \frac{e^{j\omega t}R}{e^{j\omega t}} = R$$





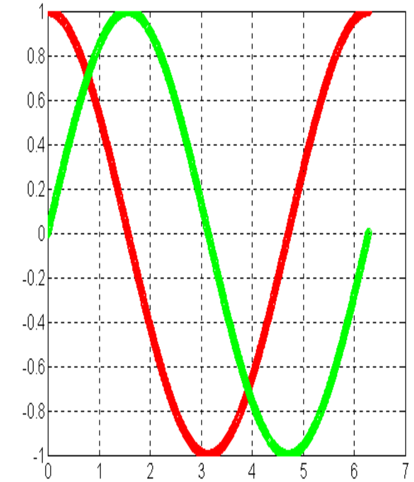




Impedancia Capacitiva Atraso

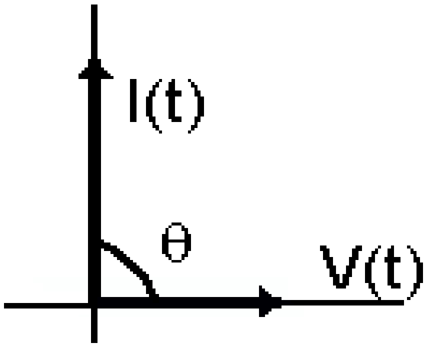
- Estudiaremos la respuesta del circuito, como fasores:

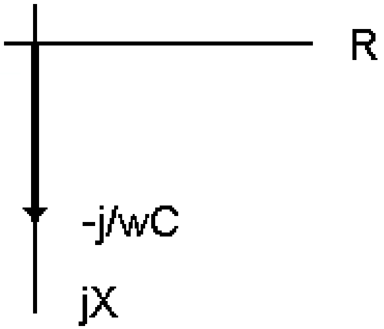
$$I(t)=e^{j\omega t}=\cos(\omega t)+j\sin(\omega t)$$

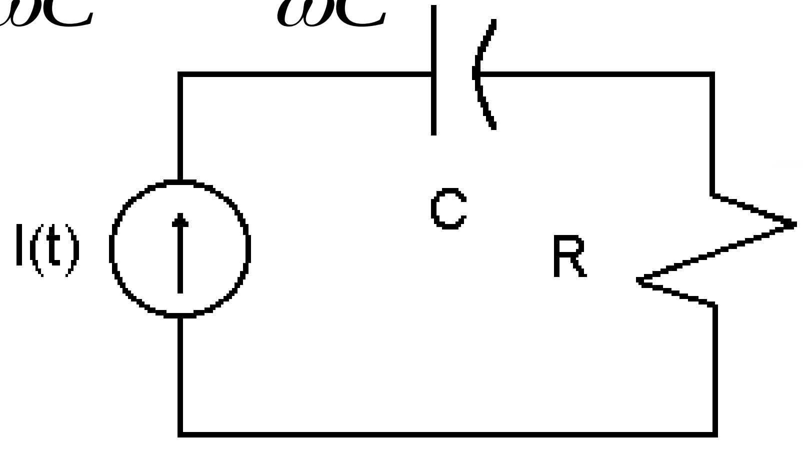


$$V_C(t)=\frac{1}{C}\int I(t)dt=\frac{1}{C}\int e^{j\omega t}=\frac{1}{j\omega C}e^{j\omega t}$$

$$Z_C(t)=\frac{V_C(t)}{I_L(t)}=\frac{\frac{1}{j\omega C}e^{j\omega t}}{e^{j\omega t}}=\frac{1}{j\omega C}=-\frac{j}{\omega C}$$

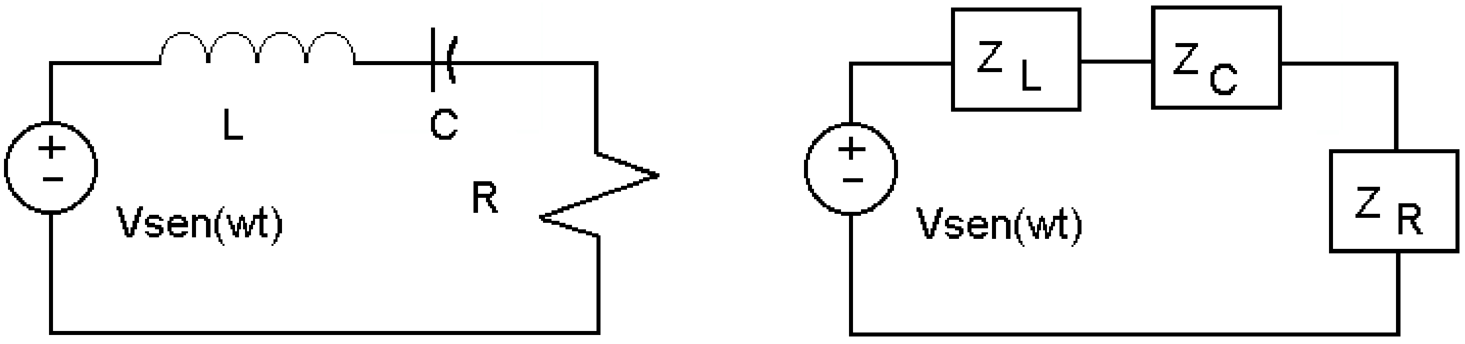






Analisis Fasorial Impedancias

- Supongamos que tenemos es siguiente circuito, por fasores o Laplace:



$$C = 1mF \quad L = 10mH \quad R = 1\Omega \quad f = 60\,Hz$$

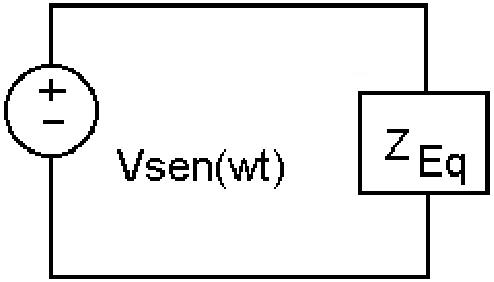
$$Z_C = -\frac{j}{\omega C} = -\frac{j}{2\pi 60 C} = -2.65j = 2.65\underline{\big|\,-90^o}$$

$$Z_L = j\omega L = 2\pi 60 Lj = 3.77j = 3.77\underline{\big|\,90^o}$$

$$Z_R = 1 = 1\underline{\big|\,0^o}$$

Analisis Fasorial
Impedancias

- Aplicaremos el concepto de impedancias



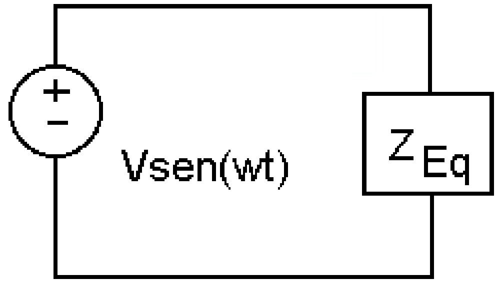
$$Z_{Eq} = Z_C + Z_L + Z_R$$

$$Z_{Eq} = 1 + 3.77j - 2.65j = 1 + 1.12j$$

$$Z_{Eq} = 1.50\angle 48.23^o$$

Analisis Fasorial
Corrientes

- Calculemos la corriente en el siguiente circuito



$$V(t) = V\sin(\omega t) \rightarrow V\cos(\omega t) = V\angle 0^o = 10\angle 0^o$$

$$I_T(t) = \frac{V(t)}{Z_{Eq}} = \frac{10\angle 0^o}{1.5\angle 48.23^o} = 6.66\angle -48.23^o$$

Analisis Fasorial
Tensiones

- Podemos calcular la tension en cada impedancia

$$V_L = I_T Z_L = \left(6.66\angle -48.23^o\right)\left(3.77\angle 90^o\right)$$

$$V_L = 25.1\angle 41.77^o$$

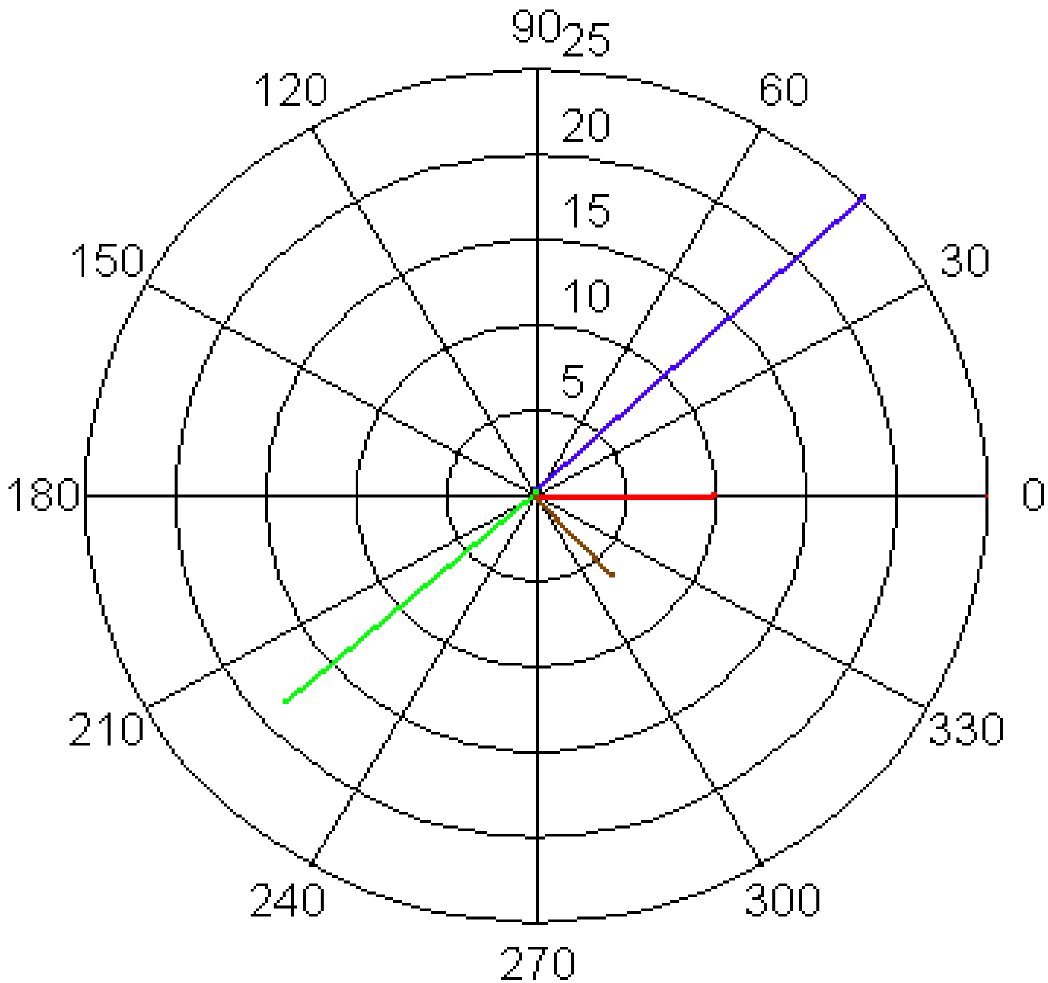
$$V_C = I_T Z_C = \left(6.66\angle -48.23^o\right)\left(2.65\angle -90^o\right)$$

$$V_C = 17.64\angle -138.23^o$$

$$V_R = I_T Z_R = \left(6.66\angle -48.23^o\right)\left(1\angle 0^o\right)$$

$$V_R = 6.66\angle -48.23^o$$

Analisis Fasorial
Tensiones



Analisis Fasorial Demostración

- Apliquemos LTK, para demostrar el resultado:

$$V_T = V_L + V_C + V_R$$

$$V_T = 25.1\underline{\left| \, 41.77^o \right.} + 17.64\underline{\left| \, -138.23^o \right.} + 6.66\underline{\left| \, -48.23^o \right.}$$

$$V_T = 18.72 + 16.72j - 13.15 - 11.75j + 4.43 - 4.96j$$

$$V_T = 10 + 0j = 10\underline{\left| \, 0^o \right.}$$

Analisis Fasorial
Potencias

- Podemos calcular la potencia en cada impedancia

$$P_L = V_L I_T = (25.1\angle 41.77^o)(6.66\angle -48.23^o)$$

$$P_L = 167.16\angle -6.46^o$$

$$P_C = V_C I_T = (17.64\angle -138.23^o)(6.66\angle -48.23^o)$$

$$P_C = 117.48\angle -186.46^o$$

$$P_R = V_R I_T = (6.66\angle -48.23^o)(6.66\angle -48.23^o)$$

$$P_R = 44.35\angle -96.46^o$$

Analisis Fasorial
Potencias

- La potencia absorbida por los elementos será:

$$P_L = 167.16\angle{-6.46^o} = 166.1 - 18.81j$$

$$P_C = 117.48\angle{-186.46^o} = -116.73 + 13.22j$$

$$P_R = 44.35\angle{-96.46^o} = -4.99 - 44.07j$$

$$P_{Absorbida} = P_L + P_C + P_R = 44.38 - 49.66j = 66.6\angle{-48.21^o}$$

$$P_{Entregada} = V_T I_T = \left(10\angle{0^o}\right)\left(6.66\angle{-48.23^o}\right) = 66.66\angle{-48.23^o}$$

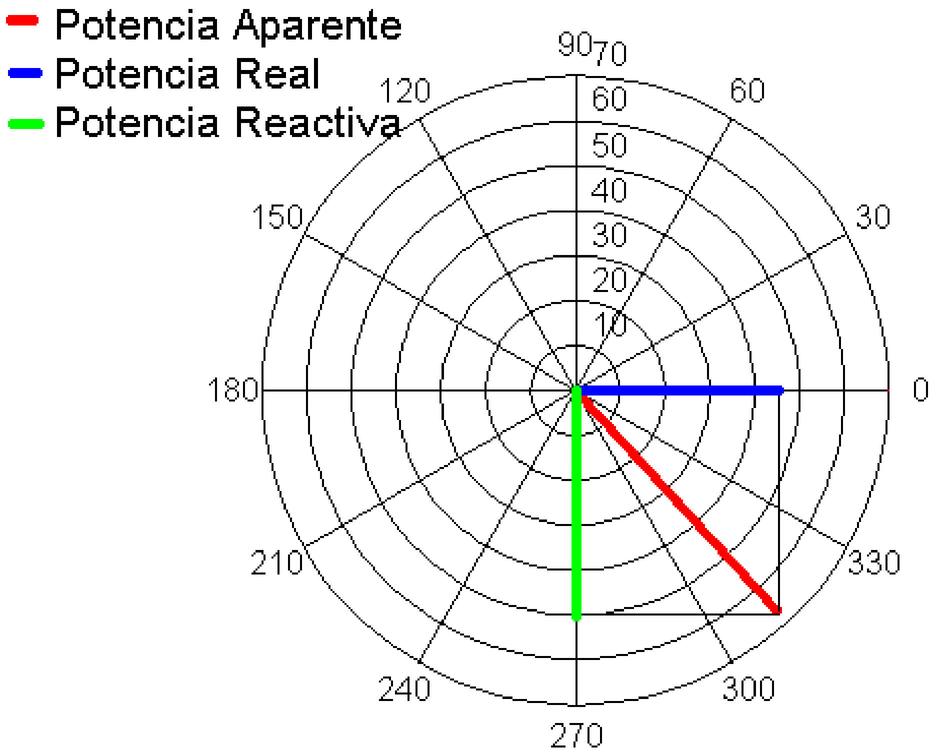
Triangulo dePotencias

$$P_{Entregada} = 44.38 - 49.66j = 66.6\angle -48.21^{o}$$

$$P_{Aparente} = 66.66VA(Voltio - Amperios)$$

$$P_{\mathrm{Re}\,al} = 44.38Watt\,(Vatios\,)$$

$$P_{\mathrm{Re}\,activa} = 49.66VAr\,(Voltio - Amperios - \mathrm{Re}\,activos)$$



Analisis Fasorial
Triangulo de Potencias

- En esencia, siempre se cumplira el triangulo de potencia, formado por:
  - Potencia aparente (S(VA)) formara la hipotenusa del triangulo.
  - Potencia Real será uno de los catetos (P(W)).
  - Potencia Reactiva será el otro catetos (Q(VAR)).
  - Y θ sera el angulo de la potencia total.