Strategic Portfolio Construction and Risk Evaluation

Final Report – Financial Data Science

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1 Introduction

In an era marked by heightened uncertainty and volatility, a key question emerges: can a portfolio be designed to perform robustly across both boom and bust cycles? This report explores that challenge by constructing a strategic portfolio of six carefully selected assets — MSFT, DE, COST, BYDDY, AMD, and GLD — chosen for their sectoral diversification and resilience under stress. Research shows that diversification benefits plateau beyond 6 to 10 assets, making our selection both sufficient and efficient. Our analysis spans from January 2018 to May 2025, encompassing market calm, the COVID-19 crisis, and subsequent recovery — a comprehensive test of long-term durability.

2 Data Analysis

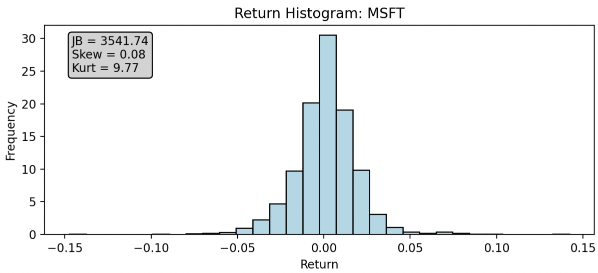
We begin by analyzing two fundamental properties of asset returns: their statistical distribution and interdependence.

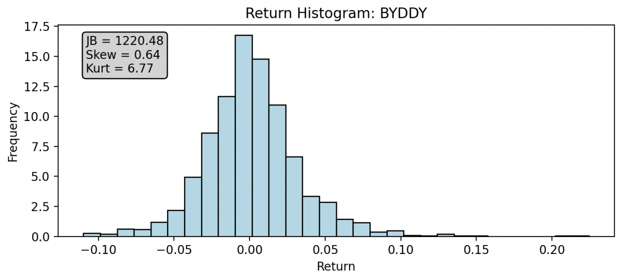
2.1 Normality: Are Returns Really "Normal"?

If asset returns followed a perfect normal distribution, risk modeling would be far simpler. However, financial markets are shaped by memory, behavioral biases, and unexpected shocks. To assess the validity of the normality assumption, we apply the Jarque-Bera (JB) test, which evaluates whether a distribution's skewness (S) and kurtosis (K) deviate from those of a Gaussian distribution (S = 0, K = 3). The JB statistic is computed as:

$$JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$$

A value close to zero supports the null hypothesis of normality; large values indicate significant deviations. In our dataset, none of the selected assets pass the test. For example, COST exhibits a JB statistic exceeding 5,000, reflecting heavy tails and noticeable skewness. The implication is clear: assuming normality underestimates risk.





2.2 Correlation: The Architecture of Diversification

Diversification is not just about holding multiple assets — it's about combining them effectively. This effectiveness is governed by correlation: the lower the correlation between assets, the greater the risk-reduction potential. Figure 3 illustrates this with investment opportunity sets under different correlation levels. As correlation falls, the efficient frontier expands, enabling portfolios that dominate individual assets in terms of risk-return trade-off. In particular, low or negative correlation allows for the construction of portfolios with lower volatility for the same level of expected return. Our portfolio was designed with this principle at its core, emphasizing sectoral, geographical, and structural diversity. The resulting correlation matrix highlights this, with GLD (Gold) standing out for its remarkably low correlationswith all other assets:

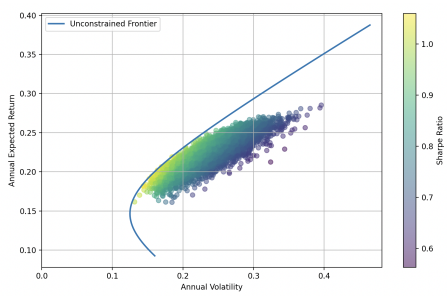
GLD acts as a strategic counterweight — largely uncorrelated with market-driven assets, it enhances portfolio resilience and helps shift the efficient frontier leftward, lowering risk without sacrificing return. In this sense, correlation is not just a statistic, but a design tool for robust portfolio construction.

	AMD	BYDDY	COST	DE	GLD	MSFT
AMD	1.00	0.29	0.40	0.32	0.11	0.56
BYDDY	0.29	1.00	0.20	0.26	0.08	0.32
COST	0.40	0.20	1.00	0.32	0.10	0.56
DE	0.32	0.26	0.32	1.00	0.07	0.39
GLD	0.11	0.08	0.10	0.07	1.00	0.07
MSFT	0.56	0.32	0.56	0.39	0.07	1.00

Table 1: Correlation matrix

3 Simulation and the Efficient Frontier: Pursuing Markowitz's Legacy

With the assets selected and their interactions understood, we now turn to a core exercise in modern portfolio theory: identifying the efficient frontier. Inspired by the pioneering work of Harry Markowitz (1952), we simulate and optimize portfolios to find those combinations that maximize expected return per unit of risk. To visualize the investment opportunity set (IOS), we generate 10,000 random portfolios. Each point represents a different combination of weights across the six assets. When plotted in the risk-return space, an enveloping curve emerges toward the upper-left edge — this is the empirical efficient frontier.



However, beyond visual intuition, we seek analytical precision. Therefore, we formally set up the Markowitz optimization problem as follows:

$$\begin{aligned} \text{Minimize:} \quad & \sigma_p^2 = \mathbf{w}^\top \Sigma \mathbf{w} \\ \text{Subject to:} \quad & \mathbf{w}^\top \mathbf{r} = \mu_{\text{target}} \\ & \mathbf{w}^\top \mathbf{1} = 1 \end{aligned}$$

Where:

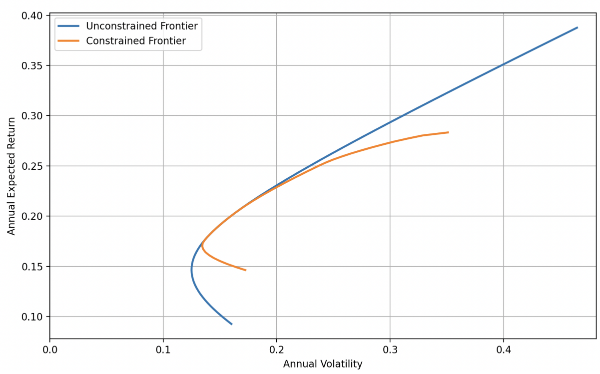
- $\mathbf{w}$ is the vector of portfolio weights,
- Σ is the covariance matrix of asset returns,
- $\mathbf{r}$ is the vector of expected returns,
- μ_{target} is the target portfolio return.

This formulation minimizes portfolio variance for a desired level of expected return, while ensuring full investment—that is, the weights sum to one.

To reflect realistic constraints faced by portfolio managers, we add two additional conditions:

1. **No short-selling:** $w_i \geq 0$ for all i
2. **Forced diversification:** $w_i \leq 0.5$ for all i

These constraints give rise to a second, constrained frontier—narrower, but more representative of real-world investment conditions.

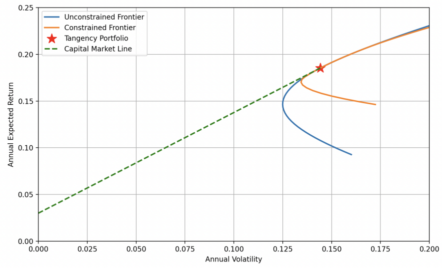


Efficient Frontier under Real-World Constraints

Comparing the unconstrained and constrained efficient frontiers enables us to quantify the cost of constraints while appreciating the power of diversification. In essence, this section pays tribute to Markowitz's central insight: diversification is not only desirable—it is mathematically and visually compelling.

4 Sharpe Ratio and the Tangency Portfolio: When Efficiency Meets Return

Identifying portfolios along the efficient frontier is essential — but selecting the best among them requires a clear metric. The Sharpe Ratio serves this purpose, measuring the excess return per unit of risk:



$$\text{Sharpe Ratio} = \frac{\mu_p - r_f}{\sigma_p}$$

Where:

- μ_p: expected portfolio return,
- r_f: risk-free rate (assumed at 2% annually),
- σ_p: portfolio standard deviation.

Maximizing this ratio leads us to the tangency portfolio — the portfolio that lies on the efficient frontier and is tangent to the Capital Market Line (CML), offering the best risk-return trade-off. Figure 6 and Figure 7 illustrate this: the red star marks the tangency portfolio, where the CML intersects the efficient frontier. This point not only represents optimality from a risk-return standpoint but also serves as a robust benchmark for investment strategies. Although we do not construct investor-specific portfolios (which would require modeling individual risk preferences), the tangency portfolio provides a universal reference point from which tailored decisions can be developed.

5 CAPM Analysis

5.1 CAPM Regression: Uncovering the DNA of Portfolio Returns

After identifying the tangency portfolio, a natural question arises: where do its returns come from? To dissect this, we apply the Capital Asset Pricing Model (CAPM) — a classical framework that decomposes returns into market-driven and strategy-specific components:

$$R_p = \alpha + \beta R_m + \varepsilon$$

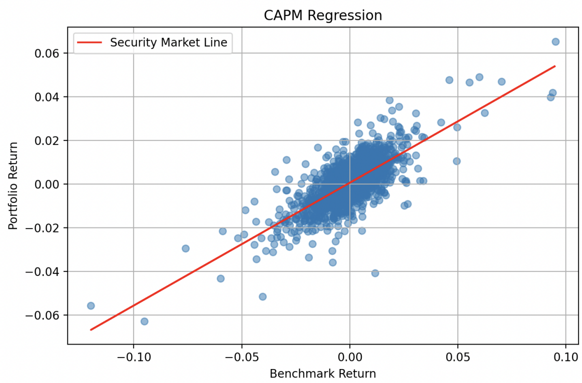
Where:

- R_p: portfolio return,
- R_m: market return (proxied by the S&P 500),
- α: excess return not explained by the market (value added),
- β: sensitivity to the market,
- ε: idiosyncratic component.

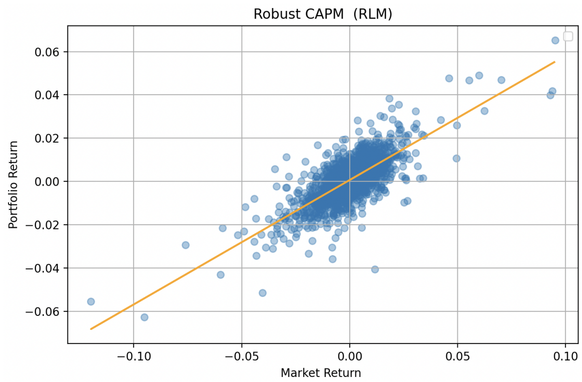
Using Ordinary Least Squares (OLS) regression on daily returns, we find:

- $R^2 = 0.54$: over half of the variance is explained by the market,
- $\beta = 0.53$: moderate market exposure,
- $\alpha > 0$: evidence of outperformance beyond market return.

The figures above show the linear fit using OLS and confirm its robustness via Robust Linear Modeling (RLM), which yields nearly identical estimates — validating the stability of our beta.



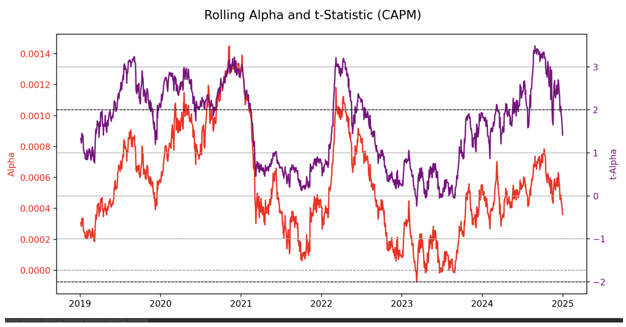
OLS



Robust Linear Modeling

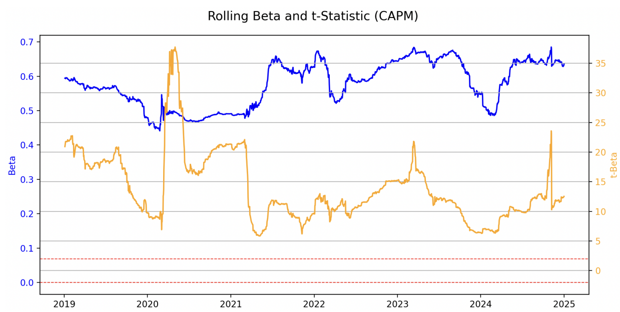
5.2 Rolling CAPM Analysis: Dynamic Alpha and Beta Estimation

Recognizing that the portfolio's exposure to the market evolves over time, we employ a rolling regression approach with 252-day windows to dynamically estimate the CAPM coefficients — alpha (α) and beta (β) — along with their statistical significance.



Rolling Alpha and t-Statistic

The graph shows the time-varying alpha (red) and its t-statistic (purple). Several periods (e.g., late 2019, 2021, early 2024) exhibit significantly positive α ($t > 2$), indicating real value added by the portfolio. In contrast, early 2020 and mid-2022 reflect phases of insignificant or negative alpha, highlighting the market-dependence of returns. These shifts underscore the importance of dynamic monitoring.



Rolling Beta and t-Statistic

The graph presents the evolution of beta (blue) and its t-statistic (orange). Beta fluctuates between 0.2 and 0.7, indicating varying but persistent market exposure. The t-statistics consistently exceed standard thresholds, confirming that market risk is a dominant driver of portfolio returns across time.

Key Takeaways

- **Market dependence is persistent:** Beta remains statistically significant over time, confirming stable exposure to systematic risk.
- **Alpha is intermittent:** Periods of significant positive alpha suggest added value by the strategy, though this is not consistent.
- **Parameters are dynamic:** The evolving nature of both α and β highlights the importance of ongoing performance diagnostics.

In summary, the rolling CAPM framework provides a more realistic and adaptive lens for analyzing portfolio behavior over time — revealing not only *if* a strategy works, but also *when*, *how*, and *why*.

6 Expected vs. Realized Portfolio Performance

A robust evaluation of portfolio quality requires comparing theoretical expectations with actual market outcomes. Below, we present the key performance metrics and corresponding analysis.

Key Metrics

- **Expected Annual Return:** $E[R_p] = \mathbf{w}^\top \boldsymbol{\mu} = 22.88\%$
- **Realized Annualized Return:** $R_{\text{real}} = \left(\prod_{t=1}^{T}(1 + r_{p,t})\right)^{\frac{252}{T}} - 1 = 24.23\%$
- **Compound Annual Growth Rate (CAGR):** $\text{CAGR} = \left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right)^{1/n} - 1 = 24.18\%$

Performance Chart Analysis

The chart below tracks cumulative portfolio growth from a normalized base value of 1 (in 2018) through 2025. The solid blue curve reflects actual portfolio performance, while the dashed red line represents the expected trajectory assuming constant 22.88% annual growth.

Throughout the period, the portfolio consistently outperforms expectations, ultimately growing to more than five times the initial capital. This strong result underscores the impact of optimized asset allocation and the inclusion of high-growth assets such as **AMD** and **BYDDY**.

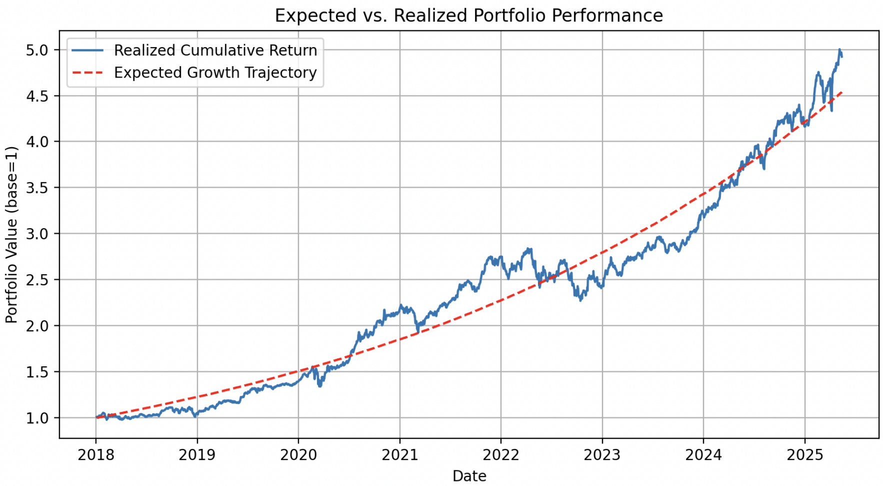


Figure 1: Expected vs. Realized Portfolio Performance (2018–2025)

Conclusion

The close alignment between expected and realized returns—alongside:

- A CAGR exceeding 24%, and
- Strong performance during crisis periods—

confirms that the strategy:

- Aligns with theoretical efficiency, and
- Demonstrates empirical robustness in real-world market conditions.

This harmony between model and reality positions the portfolio as a compelling option for investors seeking both sustained growth and disciplined risk management.