Introduction

In an era marked by heightened uncertainty and volatility, a key question emerges: can a portfolio be designed to perform robustly across both boom and bust cycles? This report explores that challenge by constructing a strategic portfolio of six carefully selected assets — MSFT, DE, COST, BYDDY, AMD, and GLD — chosen for their sectoral diversification and resilience under stress.  
Research shows that diversification benefits plateau beyond 6 to 10 assets, making our selection both sufficient and efficient. Our analysis spans from January 2018 to May 2025, encompassing market calm, the COVID-19 crisis, and subsequent recovery — a comprehensive test of long-term durability.

**2. Data Analysis: Normality and Diversification**

We begin by analyzing two fundamental properties of asset returns: their statistical distribution and interdependence.

**Normality: Are Returns Really “Normal”?**

If asset returns followed a perfect normal distribution, risk modeling would be far simpler. However, financial markets are shaped by memory, behavioral biases, and unexpected shocks. To assess the validity of the normality assumption, we apply the **Jarque-Bera (JB) test**, which evaluates whether a distribution's **skewness (S)** and **kurtosis (K)** deviate from those of a Gaussian distribution (S = 0, K = 3). The JB statistic is computed as:

JB=n6(S2+(K−3)24)JB=6n​(S2+4(K−3)2​)

A value close to zero supports the null hypothesis of normality; large values indicate significant deviations. In our dataset, **none of the selected assets** pass the test. For example, **COST** exhibits a JB statistic exceeding 5,000, reflecting **heavy tails** and **noticeable skewness**. The implication is clear: assuming normality underestimates risk.

*Figure 2: Histograms of monthly returns with skewness, kurtosis, and JB statistics.*

**Correlation: The Architecture of Diversification**

Diversification is not just about holding multiple assets — it’s about combining them effectively. This effectiveness is governed by **correlation**: the lower the correlation between assets, the greater the risk-reduction potential.

*Figure 3* illustrates this with investment opportunity sets under different correlation levels. As correlation falls, the **efficient frontier expands**, enabling portfolios that dominate individual assets in terms of risk-return trade-off. In particular, **low or negative correlation** allows for the construction of portfolios with lower volatility for the same level of expected return.

Our portfolio was designed with this principle at its core, emphasizing **sectoral, geographical, and structural diversity**. The resulting **correlation matrix** highlights this, with **GLD (Gold)** standing out for its **remarkably low correlations**with all other assets:

|  | **AMD** | **BYDDY** | **COST** | **DE** | **GLD** | **MSFT** |
| --- | --- | --- | --- | --- | --- | --- |
| **AMD** | 1.00 | 0.29 | 0.40 | 0.32 | 0.11 | 0.56 |
| **BYDDY** | 0.29 | 1.00 | 0.20 | 0.26 | 0.08 | 0.32 |
| **COST** | 0.40 | 0.20 | 1.00 | 0.32 | 0.10 | 0.56 |
| **DE** | 0.32 | 0.26 | 0.32 | 1.00 | 0.07 | 0.39 |
| **GLD** | 0.11 | 0.08 | 0.10 | 0.07 | 1.00 | 0.07 |
| **MSFT** | 0.56 | 0.32 | 0.56 | 0.39 | 0.07 | 1.00 |

GLD acts as a **strategic counterweight** — largely uncorrelated with market-driven assets, it enhances **portfolio resilience** and helps **shift the efficient frontier leftward**, lowering risk without sacrificing return. In this sense, **correlation is not just a statistic, but a design tool** for robust portfolio construction.

**3. Simulation and the Efficient Frontier: Pursuing Markowitz’s Legacy**

With the assets selected and their interactions understood, we now turn to a core exercise in modern portfolio theory: identifying the efficient frontier. Inspired by the pioneering work of Harry Markowitz (1952), we simulate and optimize portfolios to find those combinations that maximize expected return per unit of risk.

To visualize the investment opportunity set (IOS), we generate 10,000 random portfolios. Each point represents a different combination of weights across the six assets. When plotted in the risk-return space, an enveloping curve emerges toward the upper-left edge — this is the empirical efficient frontier.

However, beyond visual intuition, we seek analytical precision. Therefore, we formally set up the Markowitz optimization problem as follows:

Minimize:σp2=w⊤ΣwSubject to:w⊤r=μtarget,w⊤1=1Minimize:σp2​=w⊤ΣwSubject to:w⊤r=μtarget​,w⊤1=1

Where:

* ww is the vector of portfolio weights,
* ΣΣ is the covariance matrix of asset returns,
* rr is the vector of expected returns,
* μtargetμtarget​ is the target portfolio return.

This formulation minimizes portfolio variance for a desired level of expected return, while ensuring full investment — that is, the weights sum to one.

To reflect realistic constraints faced by portfolio managers, we add two additional conditions:

1. **No short-selling**: wi≥0wi​≥0 for all ii
2. **Forced diversification** (no asset exceeds 50% of the portfolio): wi≤0.5wi​≤0.5 for all ii

These constraints give rise to a second, constrained frontier — narrower, but more representative of real-world investment conditions.

Comparing the unconstrained and constrained efficient frontiers enables us to quantify the cost of constraints while appreciating the power of diversification. In essence, this section pays tribute to Markowitz’s central insight: diversification is not only desirable — it is mathematically and visually compelling.

**4. Sharpe Ratio and the Tangency Portfolio: When Efficiency Meets Return**

Identifying portfolios along the efficient frontier is essential — but selecting the best among them requires a clear metric. The **Sharpe Ratio** serves this purpose, measuring the excess return per unit of risk:

Sharpe Ratio=μp−rfσpSharpe Ratio=σp​μp​−rf​​

Where:

* μpμp​: expected portfolio return,
* rfrf​: risk-free rate (assumed at 2% annually),
* σpσp​: portfolio standard deviation.

Maximizing this ratio leads us to the **tangency portfolio** — the portfolio that lies on the efficient frontier and is tangent to the **Capital Market Line (CML)**, offering the best risk-return trade-off.

*Figure 6* and *Figure 7* illustrate this: the red star marks the tangency portfolio, where the CML intersects the efficient frontier. This point not only represents optimality from a risk-return standpoint but also serves as a robust benchmark for investment strategies.

Although we do not construct investor-specific portfolios (which would require modeling individual risk preferences), the tangency portfolio provides a universal reference point from which tailored decisions can be developed.

**5. CAPM Regression: Uncovering the DNA of Portfolio Returns**

After identifying the tangency portfolio, a natural question arises: where do its returns come from? To dissect this, we apply the **Capital Asset Pricing Model (CAPM)** — a classical framework that decomposes returns into market-driven and strategy-specific components:

Rp=α+βRm+εRp​=α+βRm​+ε

Where:

* RpRp​: portfolio return
* RmRm​: market return (proxied by the S&P 500)
* αα: excess return not explained by the market (value added)
* ββ: sensitivity to the market
* εε: idiosyncratic component

Using **Ordinary Least Squares (OLS)** regression on daily returns, we find:

* R2=0.54R2=0.54: over half of the variance is explained by the market,
* β=0.53β=0.53: moderate market exposure,
* α>0α>0: evidence of outperformance beyond market return.

*Figure 8* shows the linear fit under OLS, while *Figure 9* confirms robustness via **Robust Linear Modeling (RLM)**, which yields nearly identical estimates — validating the stability of our beta.

Recognizing that markets evolve, we extend our analysis using **rolling regressions** with 252-day windows to track changes in αα and ββ over time.

* *Figure 10*: **Alpha** is occasionally positive and significant (t-statistic > 2), indicating genuine value added; at other times, it declines, reflecting market-dependent performance.
* *Figure 11*: **Beta** fluctuates between 0.2 and 0.7, confirming that market sensitivity is not constant — likely influenced by sector shifts or macro conditions.

These findings lead to three main insights:

1. The tangency portfolio is consistently influenced by market dynamics (high ββ and R2R2).
2. Periods of significant αα indicate moments of strategy-driven outperformance.
3. Both αα and ββ are dynamic, warranting continuous monitoring.

In summary, CAPM reveals not just whether the portfolio generates returns, but how — and rolling analysis transforms it into a powerful, adaptive tool for portfolio evaluation.

**6. Expected vs. Realized Portfolio Performance**

A robust evaluation of portfolio quality requires comparing theoretical expectations with actual market outcomes. Below, we present the key performance metrics and corresponding analysis.

**📐 Key Metrics**

* **Expected Annual Return**:  
  E[Rp]=w⊤μ=22.88%E[Rp​]=w⊤μ=22.88%
* **Realized Annualized Return**:  
  Rreal=(∏t=1T(1+rp,t))252/T−1=24.23%Rreal​=(∏t=1T​(1+rp,t​))252/T−1=24.23%
* **Compound Annual Growth Rate (CAGR)**:  
  CAGR=(VfinalVinitial)1/n−1=24.18%CAGR=(Vinitial​Vfinal​​)1/n−1=24.18%

**📈 Performance Chart Analysis**

The chart tracks cumulative portfolio growth from a normalized value of 1 (2018) through 2025. The **solid blue curve**reflects actual performance, while the **dashed red line** represents the expected trajectory assuming constant 22.88% annual growth.

Throughout the period, the portfolio **consistently outperforms expectations**, ultimately growing to more than five times the initial capital. This result underscores the impact of optimized asset allocation and the inclusion of high-growth names like AMD and BYDDY.

**🦠 COVID-19 Resilience**

During the March 2020 market crash:

* The portfolio experienced only a **moderate drawdown**,
* **Recovered rapidly**, within months, and
* **Resumed upward momentum**, indicating structural resilience.

This stress-test performance validates the portfolio’s robustness during exogenous shocks.

**📌 Conclusion**

The close alignment between expected and realized returns, alongside:

* A **CAGR exceeding 24%**, and
* Strong crisis behavior,

confirms that the strategy:

* Aligns with theoretical efficiency, and
* Demonstrates **empirical robustness** in real markets.

This harmony between model and reality positions the portfolio as a compelling choice for investors seeking both growth and disciplined risk management.