The mass of a finite particle

This script will show that a finite particle oposes to be acelerated i.e., it has a mass.

The first finite size particle model is very simple:

- Two point elements with charge Q/2 on the horizontal axis separated by a distance of 2*r_o
- The two elements are bounded by a nuclear force
- The particle is acelerated with constant acceleration

```
% Solving for the arrival time of a particle at constant aceleration
% a   : Aceleration
% r_o : Particle radious
% c   : Speed of light
% K   : eletrostatic constant
% t   : time to reach a target
% Q   : Particle charge
syms a Q r_o c K t real;
assumeAlso(a > 0)
assumeAlso(r_o > 0)
assumeAlso(c > 0)
assumeAlso(K > 0)
assumeAlso(t > 0)
assumeAlso(Q > 0)
```

Solving for the arrival time

I'll solve for the forces that the particle experiment at time equal zero.

```
eq1 = -a/2*t^2-c*t+2*r_o==0 % right corpuscle moving towards the left element
```

eq1 =

$$2\,r_o - c\,t - \frac{a\,t^2}{2} = 0$$

```
t_1=solve(eq1,t) % The time it takes to reach the left element
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

t_1 =

$$-\frac{c - \sqrt{c^2 + 4\,a\,r_o}}{a}$$

```
t_1 = t_1(1)
```

t_1 =

$$-\frac{c-\sqrt{c^2+4\,a\,r_o}}{a}$$

```
eq2 = -a/2*t^2+c*t-2*r_o==0 % The equation of the corpuscle traveling from left to right element
```

eq2 =

$$-2\,r_o+c\,t-\frac{a\,t^2}{2}=0$$

```
t_2= solve(eq2,t) % The time it takes to reach the right element
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

t_2 =

$$\begin{pmatrix} \frac{c+\sqrt{c^2-4\,a\,r_o}}{a} \\ \frac{c-\sqrt{c^2-4\,a\,r_o}}{a} \end{pmatrix}$$

```
t_2=t_2(2) % I only take the shortest time
```

t_2 =

$$\frac{c-\sqrt{c^2-4\,a\,r_o}}{a}$$

```
vpa(subs(t_1,[a,r_o,c],[1,1.0e-15,1]))-vpa(subs(t_2,[a,r_o,c],[1,1.0e-15,1]))
```

ans = $-0.00000000000000000000000000000040000000001946799263974006163732$

```
D_a=simplify(1/(t_2^2)-1/(t_1^2)) % The force is due to differences in arrival time
```

D_a =

$$\frac{a^2}{\left(c-\sqrt{c^2-4\,a\,r_o}\right)^2}-\frac{a^2}{\left(c-\sqrt{c^2+4\,a\,r_o}\right)^2}$$

```
D_a= taylor(D_a,a,Order=4) % Lets take an aproximation
```

D_a =

$$-\frac{a}{r_o}-\frac{a^3\,r_o}{c^4}$$

```
F=K*Q^2/(4*c^2)*D_a % The reaction force experimented by the acelerated particle
```

F =

$$-\frac{K\,Q^2\left(\frac{a}{r_o}+\frac{a^3\,r_o}{c^4}\right)}{4\,c^2}$$

Hence if you acelerate a electically charged finite-sized particle, it will opose the aceleration. In other words particles have intertia and is origen is the fact that electric charges have dimensions.

Energy and Mass

```
U= -K*Q^2/4*int(1/t^2,t,Inf,2*r_o) % The energy required to bound the two elements
of the particle
```

U =

$$\frac{K\,Q^2}{8\,r_o}$$

```
syms U
F_net=expand(simplify(subs(F,Q^2,8*U*r_o/K))) % The reaction force in units of the
energy
```

F_net =

$$-\frac{2\,U\,a}{c^2}-\frac{2\,U\,a^3\,{r_o}^2}{c^6}$$

where r_o is very small and c is very large.

The first term of the total reactive force is:

```
F_net = taylor(F_net,Order=2) % The first term
```

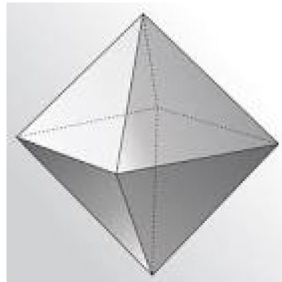
F_net =

$$-\frac{2\,U\,a}{c^2}$$

The sum of all the forces is equal zero, and $F_{net}+ma=0$, then $m=\frac{2U}{c^2}$ or the $U\approx mc^2$ the famus Einstein equation.

The expanded model of the particle

Let use a more complex model of the particle this time lets use 6 elements to model the finite-sized paticle. Each element is in the corner of a regular octahedron:

.

The particle resides in the origen, and each one of the six elements is at a distance r_o form the origen. The constant aceleration is in the positve x direction. The analysis involve estimating the time of arrival of each corpuscle to the other elements.

```
eq3 = (c*t)^2 - r_o^2 - (r_o-1/2*a*t^2)^2 == 0 % corpuscle moving from off x-axis left
```

eq3 =

$$c^2\,t^2 - {r_o}^2 - \left(r_o - \frac{a\,t^2}{2}\right)^2 = 0$$

```
t_dL= solve(eq3,t) % The time it takes for corpuscles to reach diagonal left element
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

t_dL =

$$\begin{pmatrix} \sqrt{\dfrac{2\left(a\,r_o + \sqrt{c^4 + 2\,a\,c^2\,r_o - a^2\,{r_o}^2} + c^2\right)}{a^2}} \\ \sqrt{\dfrac{2\left(a\,r_o - \sqrt{c^4 + 2\,a\,c^2\,r_o - a^2\,{r_o}^2} + c^2\right)}{a^2}} \end{pmatrix}$$

```
t_dL = t_dL(2)
```

t_dL =

$$\sqrt{\frac{2\left(a\,r_o - \sqrt{c^4 + 2\,a\,c^2\,r_o - a^2\,{r_o}^2} + c^2\right)}{a^2}}$$

```
eq4 = (c*t)^2 - r_o^2 - (r_o + 1/2*a*t^2)^2 == 0 % diagnoal corpuscle to right element
```

eq4 =

$$c^2\,t^2 - {r_o}^2 - \left(r_o + \frac{a\,t^2}{2}\right)^2 = 0$$

```
t_dR= solve(eq4,t) % Diagonal Time to reach the right element
```

```
Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.
t_dR =
```

$$\begin{pmatrix} \sqrt{\frac{2\left(\sqrt{c^4 - 2\,a\,c^2\,r_o - a^2\,{r_o}^2} - a\,r_o + c^2\right)}{a^2}} \\ \sqrt{-\frac{2\left(a\,r_o + \sqrt{c^4 - 2\,a\,c^2\,r_o - a^2\,{r_o}^2} - c^2\right)}{a^2}} \end{pmatrix}$$

```
t_dR = t_dR(2) % The shortest time
```

```
t_dR =
```

$$\sqrt{-\frac{2\left(a\,r_o + \sqrt{c^4 - 2\,a\,c^2\,r_o - a^2\,{r_o}^2} - c^2\right)}{a^2}}$$

```
eq5= (1/2*a*t^2)^2-(c*t)^2 + (2*r_o)^2==0 % The equation from off x-axis to the opposite element
```

```
eq5 =
```

$$4\,{r_o}^2 - c^2\,t^2 + \frac{a^2\,t^4}{4} = 0$$

```
t_5= solve(eq5,t)
```

```
Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.
t_5 =
```

$$\begin{pmatrix} \sqrt{\frac{2\left(c^2 + \sqrt{-(c^2 + 2\,a\,r_o)\,(-c^2 + 2\,a\,r_o)}\right)}{a^2}} \\ \sqrt{\frac{2\left(c^2 - \sqrt{-(c^2 + 2\,a\,r_o)\,(-c^2 + 2\,a\,r_o)}\right)}{a^2}} \end{pmatrix}$$

```
t_5=t_5(2) % The shortest time
```

```
t_5 =
```

$$\sqrt{\frac{2\left(c^2 - \sqrt{-(c^2 + 2\,a\,r_o)\,(-c^2 + 2\,a\,r_o)}\right)}{a^2}}$$

```
eq6= (1/2*a*t^2)^2-(c*t)^2 + 2*r_o^2 == 0 % Off axis to the next off axis
```

```
eq6 =
```

$$2\,r_o^2 - c^2\,t^2 + \frac{a^2\,t^4}{4} = 0$$

```
t_6= solve(eq6,t)
```

```
Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the
solution, specify the 'ReturnConditions' value as 'true'.
t_6 =
```

$$\begin{pmatrix} \sqrt{\frac{2\left(\sqrt{c^4 - 2\,a^2\,r_o^2} + c^2\right)}{a^2}} \\ \sqrt{-\frac{2\left(\sqrt{c^4 - 2\,a^2\,r_o^2} - c^2\right)}{a^2}} \end{pmatrix}$$

```
t_6=t_6(2) % The shortest time
```

```
t_6 =
```

$$\sqrt{-\frac{2\left(\sqrt{c^4 - 2\,a^2\,r_o^2} - c^2\right)}{a^2}}$$

Now, I'll compute the reactive forces for each one of the element to element interactions

```
syms d v real positive
cos_L = cos(pi/4);
cos_R = cos(pi/4);
cos_5 = (1/2*a*t_5^2)/sqrt((2*r_o)^2+(1/2*a*t_5^2)^2);
cos_6 = (1/2*a*t_6^2)/sqrt(2*r_o^2+(1/2*a*t_6^2)^2);

F_1 = -1/(c*t_1)^2 - 4/(c*t_dL)^2*cos_L; % Left element
F_2 = +1/(c*t_2)^2 + 4/(c*t_dR)^2*cos_R; % Right element
F_3 = 1/(c*t_5)^2*cos_5; % The two off axis elements

F_3 = F_3 + 2/(c*t_6)^2*cos_6; % The  two sides of the off-axis elements
F_3 = F_3 - 1/(c*t_dL)^2*cos_L; % add the right element to the off-axis
F_3 = F_3 + 1/(c*t_dR)^2*cos_R; % add the left element to off-axis

F_netp = simplify(F_1 + F_2 + 4*F_3,steps=100); % One left, one right plus the four
off-axis
F_net = K*(Q/6)^2*simplify(taylor(F_netp,a,Order=2))
```

```
F_net =
```

$$-\frac{\sqrt{2}\,K\,Q^2\,a}{18\,c^2\,r_o}$$

```
% The Binding energy of the regular octahedron
U_1 = K*(Q/6)^2/(2*r_o) % Right to left
```

U_1 =

$$\frac{K\,Q^2}{72\,r_o}$$

```
U_2 = K*(Q/6)^2/(sqrt(2)*r_o) % off diagonal
```

U_2 =

$$\frac{\sqrt{2}\,K\,Q^2}{72\,r_o}$$

```
U_net = simplify(3*U_1 + 12*U_2)
```

U_net =

$$\frac{K\,Q^2\,\left(4\,\sqrt{2}+1\right)}{24\,r_o}$$

```
syms U_net Q2
qeU = U_net - (K*Q2*(4*sqrt(sym(2)) + 1))/(24*r_o) == 0
```

qeU =

$$U_{\mathrm{net}}-\frac{K\,Q_2\,\left(4\,\sqrt{2}+1\right)}{24\,r_o}=0$$

```
simplify(solve (qeU,Q2))
```

ans =

$$\frac{24\,U_{\mathrm{net}}\,r_o}{K\,\left(4\,\sqrt{2}+1\right)}$$

```
F_net
```

F_net =

$$-\frac{\sqrt{2}\,K\,Q^2\,a}{18\,c^2\,r_o}$$

```
F_u=simplify(subs(F_net,Q^2,(24*U_net*r_o)/(K*(4*sqrt(sym(2)) + 1))))
```

F_u =

$$\frac{U_{\mathrm{net}}\,a\,\left(\frac{4\,\sqrt{2}}{93}-\frac{32}{93}\right)}{c^2}$$

```
vpa(subs(F_u,[a,c,r_o],[a,1,1]))
```

ans = $-0.28325963172588838499777682906625\,U_{\mathrm{net}}\,a$

The inertial mass of the regular octahedron is lower that that of the two element particle.

Add the central element to the regular octahedron

```
eq1p = -a/2*t^2-c*t+r_o==0 % right corpuscle moving towards the left element
```

eq1p =

$$r_o - c\,t - \frac{a\,t^2}{2} = 0$$

```
t_1p=solve(eq1p,t) % The time it takes to reach the left element
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

t_1p =

$$-\frac{c - \sqrt{c^2 + 2\,a\,r_o}}{a}$$

```
t_1p = t_1p(1)
```

t_1p =

$$-\frac{c - \sqrt{c^2 + 2\,a\,r_o}}{a}$$

```
eq2p = -a/2*t^2+c*t-r_o==0 % The equation of the corpuscle traveling from left to right element
```

eq2p =

$$-r_o + c\,t - \frac{a\,t^2}{2} = 0$$

```
t_2p= solve(eq2p,t) % The time it takes to reach the right element
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

t_2p =

$$\begin{pmatrix} \frac{c + \sqrt{c^2 - 2\,a\,r_o}}{a} \\ \frac{c - \sqrt{c^2 - 2\,a\,r_o}}{a} \end{pmatrix}$$

```
t_2p=t_2p(2) % I only take the shortest time
```

t_2p =

$$\frac{c-\sqrt{c^2-2\,a\,r_o}}{a}$$

```
eq_c = (1/2*a*t^2)^2-(c*t)^2 + r_o^2==0 % Points off axis
```

eq_c =

$$r_o{}^2-c^2\,t^2+\frac{a^2\,t^4}{4}=0$$

```
t_c= solve(eq_c,t)
```

Warning: Solutions are only valid under certain conditions. To include parameters and conditions in the solution, specify the 'ReturnConditions' value as 'true'.

t_c =

$$\begin{pmatrix} \sqrt{\frac{2\left(c^2+\sqrt{-(c^2+a\,r_o)\,(-c^2+a\,r_o)}\right)}{a^2}} \\ \sqrt{\frac{2\left(c^2-\sqrt{-(c^2+a\,r_o)\,(-c^2+a\,r_o)}\right)}{a^2}} \end{pmatrix}$$

```
t_c=t_c(2) % The shortest time
```

t_c =

$$\sqrt{\frac{2\left(c^2-\sqrt{-(c^2+a\,r_o)\,(-c^2+a\,r_o)}\right)}{a^2}}$$

```
cosp = (1/2*a*t_c^2)/sqrt(r_o^2+(1/2*a*t_c^2)^2);

F_cp = -1/(c*t_1p)^2 + 1/(c*t_2p)^2 + 4/(c*t_c)^2*cosp;
F_1p = F_1 - 1/(c*t_1p)^2; % Left element
F_2p = F_2 + 1/(c*t_2p)^2; % Righ element
F_3p = F_3 + 1/(c*t_c)^2*cosp; %Off axis elements
F_netcp = simplify(F_cp + F_1p + F_2p + 4*F_3p,steps=100); % One left, one right
plus the four off-axis
F_netc = K*(Q/7)^2*simplify(taylor(F_netcp,a,Order=2))
```

F_netc =

$$-\frac{2\,\sqrt{2}\,K\,Q^2\,a}{49\,c^2\,r_o}$$

```
% The Binding energy of the regular octahedron with a central element
U_p = K*(Q/7)^2/(r_o) % Elements pairs separed by r_o
```

U_p =

$$\frac{K\,Q^2}{49\,r_o}$$

```
U_1 = K*(Q/7)^2/(2*r_o) % Elements  pairs separed by the sqrt(2)r_o
```

U_1 =

$$\frac{K\,Q^2}{98\,r_o}$$

```
U_2 = K*(Q/7)^2/(sqrt(2)*r_o) % Elements pairs separated by 2r_o
```

U_2 =

$$\frac{\sqrt{2}\,K\,Q^2}{98\,r_o}$$

```
U_net = simplify(3*U_1 + 12*U_2 + 6*U_p) % The 21 pairs of element to element
interactions.
```

U_net =

$$\frac{3\,K\,Q^2\,\left(4\,\sqrt{2}+5\right)}{98\,r_o}$$

```
syms U_net Q2
qeU = U_net - (3*K*Q2*(4*sqrt(sym(2)) + 5))/(98*r_o) == 0
```

qeU =

$$U_{\mathrm{net}}-\frac{3\,K\,Q_2\,\left(4\,\sqrt{2}+5\right)}{98\,r_o}=0$$

```
simplify(solve (qeU,Q2))
```

ans =

$$\frac{98\,U_{\mathrm{net}}\,r_o}{3\,K\,\left(4\,\sqrt{2}+5\right)}$$

```
F_netc
```

F_netc =

$$-\frac{2\,\sqrt{2}\,K\,Q^2\,a}{49\,c^2\,r_o}$$

```
F_uc=simplify(subs(F_netc,Q^2,(98*U_net*r_o)/(3*K*(4*sqrt(sym(2)) + 5))))
```

F_uc =

$$\frac{U_{\mathrm{net}}\,a\,\left(\frac{20\,\sqrt{2}}{21}-\frac{32}{21}\right)}{c^2}$$

```
vpa(subs(F_uc,[a,c,r_o],[a,1,1]))
```

```
ans = 
```
$$-0.17693946440657614399839169122886\, U_{\mathrm{net}}\, a$$

The inertial mass of the regular octahedron with a central element is even lower.

The implications is that it may be possible to find a charge distribution that will explain the oserved mass of charged particles.