1 Hydro-organic liquid Chromatography

The following chapter contains a detailed description of the mathematical expressions used to model retention in Hydro-organic liquid Chromatography. To properly model the retention, we need some previous data of retention, usually obtained in the laboratory by chromatographic experimentation. The data should have two types of variables: independents and dependents. The first corresponds to the solvents percentage in the dilution. Here we can have one or two solvents. The quantity of these solvents will mostly determine the mathematical model used to describe retention in each case. The dependent variable is the one we want to model, i.e, the retention, which varies according to changes in the solvents percentage. In this section, as we only intend to model retention in Hydro-organic liquid Chromatograpy, we will only be dealing with solvents, not additives. We will further discuss retention models in the presence of additives in section 2.

As we have stated, we refer to models which only handle one solvent φ as *univariant* mathematical models. In contrast, we call *bivariant* mathematical models to the ones dealing with two solvents, which we denote as φ_1, φ_2.

1.1 Univariant mathematical models used to describe retention

The following expressions are used to describe retention in the abscence of additives, i.e, mathematical models only using solvent information.

$$k(\varphi) = e^{\ln k_w - S\varphi} \ , \tag{1}$$

$$k(\varphi) = e^{\ln k_w - S\varphi + b\varphi^2} \ , \tag{2}$$

$$k(\varphi) = e^{\ln k_w - S\frac{\varphi}{1+b\varphi}} \ , \tag{3}$$

$$k(\varphi) = (1+b\varphi)^2 e^{\ln k_w - S\frac{\varphi}{1+b\varphi}} \ , \tag{4}$$

$$k(\varphi) = (a\varphi)^{-n} \ , \tag{5}$$

$$k(\varphi) = (a+b\varphi)^{-n} \ , \tag{6}$$

$$k(\varphi) = \frac{e^{\ln k_w}}{\left(1+(g-1)S\varphi e^{(g-1)\ln k_w}\right)^{\frac{1}{g-1}}} \ . \tag{7}$$

To optimize the parameters p_j in each expression, we intend to solve the following minimization problem:

$$\min_{p_j} \quad \sum_{i=1}^{n} \left[k_i - \hat{k}(\varphi_i, p_j)\right]^2 \tag{8}$$

in which k represents the raw data obtained during empirical experimentation, $\hat{k}(\varphi_i, p_j)$ is the retention prediction of one particular model for specific values of p_j.

Parameter optimization in previous models is carried out using two optimization methods: *Levenberg-Marquadt* algorithm and *Nelder-Mead* algorithm. The convergence of these algorithms requires a relatively accurate approximation of the parameters, i.e., a good initial guess, which is required as an input in each model. In the following section, we will show how we can estimate these initial parameters.

2 Liquid Chromatography in the presence of additives

In this section, similarly to how we proceeded previously, we will summarize the most used mathematical models to describe retention in liquid Chromatography in the presence of additives. Two kinds of additives are used in this type of chromatography: surfactants and ionic liquid, denoted by $\mathcal{A}_1$, $\mathcal{A}_2$ respectively. As a consequence, we now have several types of combinations between independent variables to provide some quantification of the retention. With this in mind, we will classify the models depending on the number independent variables used in each case, analogously to what we did before.

2.1 Univariant mathematical models used to describe retention

The following mathematical expression can be used to describe retention in the presence of one additive:

$$k = \frac{K_s}{1 + K_m[\mathcal{A}]} \ , \tag{9}$$

2.2 Bivariant mathematical models used to describe retention

These mathematical expressions correspond to the modelization of the retention depending in two additives or in the combination of additive and solvent.

$$k = \frac{K_s}{1 + K_m\varphi + a[\mathcal{A}] + b\varphi[\mathcal{A}]} \ , \tag{10}$$

$$k = \frac{K_s}{1 + a_1\varphi + a_2[\mathcal{A}] + a_3\varphi[\mathcal{A}] + a_4\varphi\sqrt{[\mathcal{A}]}} \ , \tag{11}$$

$$k = \frac{K_{as}\dfrac{1}{1 + K_{ad}[\mathcal{A}]}}{1 + K_{am}\dfrac{1 + K_{md}[\mathcal{A}]}{1 + K_{ad}[\mathcal{A}]}\varphi} \ , \tag{12}$$

$$k = \frac{K_{as}\dfrac{1 + K_{sd}[\mathcal{A}]}{1 + K_{ad}[\mathcal{A}]}}{1 + K_{am}\dfrac{1 + K_{md}[\mathcal{A}]}{1 + K_{ad}[\mathcal{A}]}\varphi} \ , \tag{13}$$

$$k = \frac{K_{as}\dfrac{1+K_{sd}[\mathcal{A}]}{1+K_1[\mathcal{A}]+K_2[\mathcal{A}]^2}}{1+K_{am}\dfrac{1+K_{md}[\mathcal{A}]}{1+K_1[\mathcal{A}]+K_2[\mathcal{A}]^2}\varphi} \ , \tag{14}$$

$$k = \frac{K_{as}}{1+K_{am}\dfrac{1+K_3[\mathcal{A}]+K_4[\mathcal{A}]^2}{1+K_1[\mathcal{A}]+K_2[\mathcal{A}]^2}\varphi} \ . \tag{15}$$