

# Northwestern University

## Baja SAE - Powertrain 2021

Mech\_Eng 399 - Independent Study - Summer 2020

Adviser: Q. Jane Wang

Written by: Jose Amador

### Introduction

Baja SAE is an intercollegiate design competition where students design, build, and race an off-road vehicle. The goal of Baja SAE is to challenge students to design a car capable of handling all sorts of rough terrain and dynamic events. However, industry is moving toward four-wheel drive (4WD) off-road vehicles which means Baja SAE must move with it. Originally, teams would be provided with extra competition points if they were capable of creating a 4WD car for the 2020 season before it became mandatory for all teams in the 2021 season.

Northwestern's team transitioned to a 4WD car late into the design process and created a functional, albeit redundant, powertrain system. This year, I took responsibility as powertrain lead to create a less redundant 4WD system that fits into the chassis of the car. This consists of removing redundant components, rotating the engine 90-degrees, and switching to a 1-stage gearbox. This report will cover the entire process from start to finish, including calculations, an overview of the chosen components, and the final system layout.

### Previous Year

The powertrain system for the 2020 season car named 'Lil Slurpie' was a last minute addition that was implemented once rule changes for 4WD were released. For this reason a 4WD system that fit inside the frame of the car was hastily thrown together. Figure 1 shows the general overview of this system with the original gearbox in red on the left side of the image. The input shaft was extended to accommodate a series of v-belt pulleys that transferred power to a 90-degree gearbox, changing the direction of power and transferring it to the front of the car.

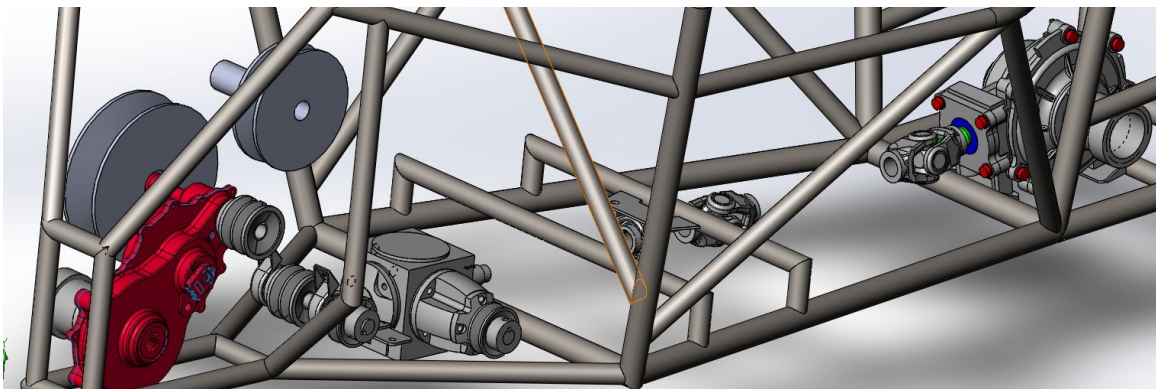


Figure 1: Lil Slurpie Powertrain Overview

The power flow diagram from the CVT (which connects to the engine) to the 90-degree gearbox can be seen in Figure 2.

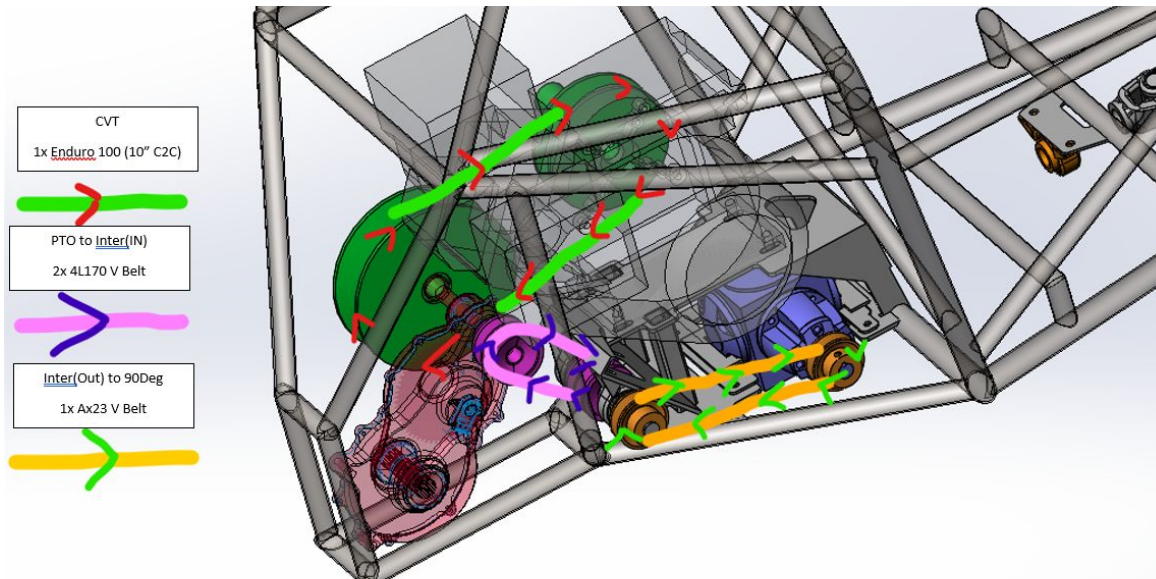


Figure 2: Lil Slupie Power Flow Diagram

Once power is transferred to the 90-degree gearbox a series of shafts (not pictured) and u-joints transfers power to the front of the car to a front differential as shown in Figure 3. The differential used in this system is from a Polaris Sportsman 500 ATV and is not a standard differential. These differentials have an “on-demand” feature which uses an electronically controlled variation of a sprag clutch to toggle the power to the wheels on/off. Furthermore, this feature improves upon the original differential design because it prevents wheel slip when one wheel loses traction while still allowing the outputs to rotate at different speeds similar to a conventional differential.

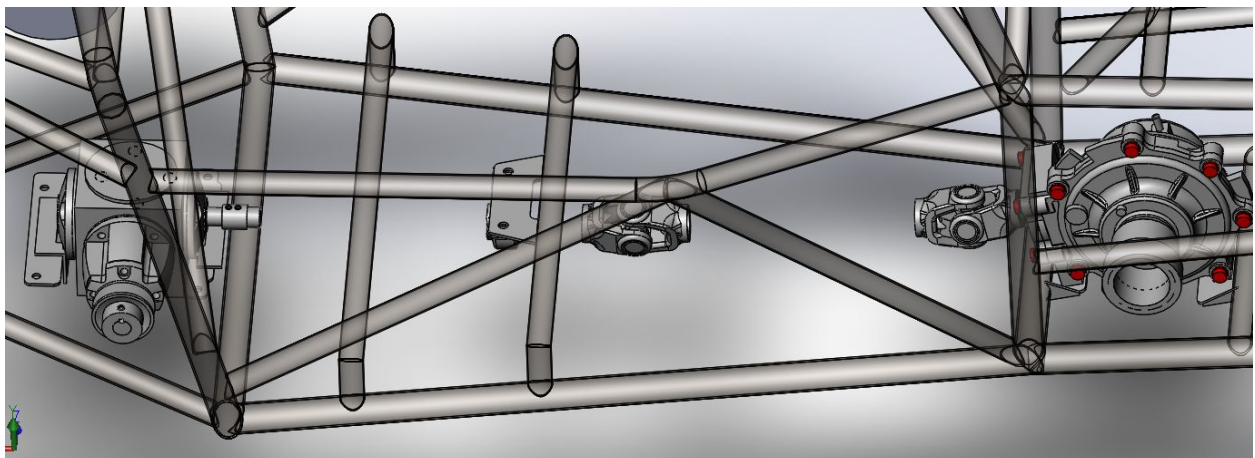


Figure 3: Lil Slupie Forward Shaft Image

## Goals for 2021 Season

One of the main goals for the 2021 season is to remove the redundancy in the powertrain system and keep it simple. This means removing the complicated v-belt system and 90-degree gearbox altogether which greatly reduces the amount of components that need to be purchased/fabricated and saving approximately 35 pounds. The 90-degree gearbox was necessary to change the direction of power, however if the engine was rotated 90 degrees such that the output shaft faced the front of the car rather than the side, the direction of power problem would be solved. This can be seen more clearly in Figure 4. Furthermore, turning has always been something the team has struggled with mainly due to the lack of a differential type system that allows the wheels to rotate at different speeds. To solve this problem two differentials can be used at the output to the wheels in both the front and back of the car to allow for better turning. The Sportsman 500 differential mentioned above has a gear reduction of 3.82 built in, allowing for the team-designed gearbox to use a much smaller ratio because it will be multiplied by 3.82 at the end.

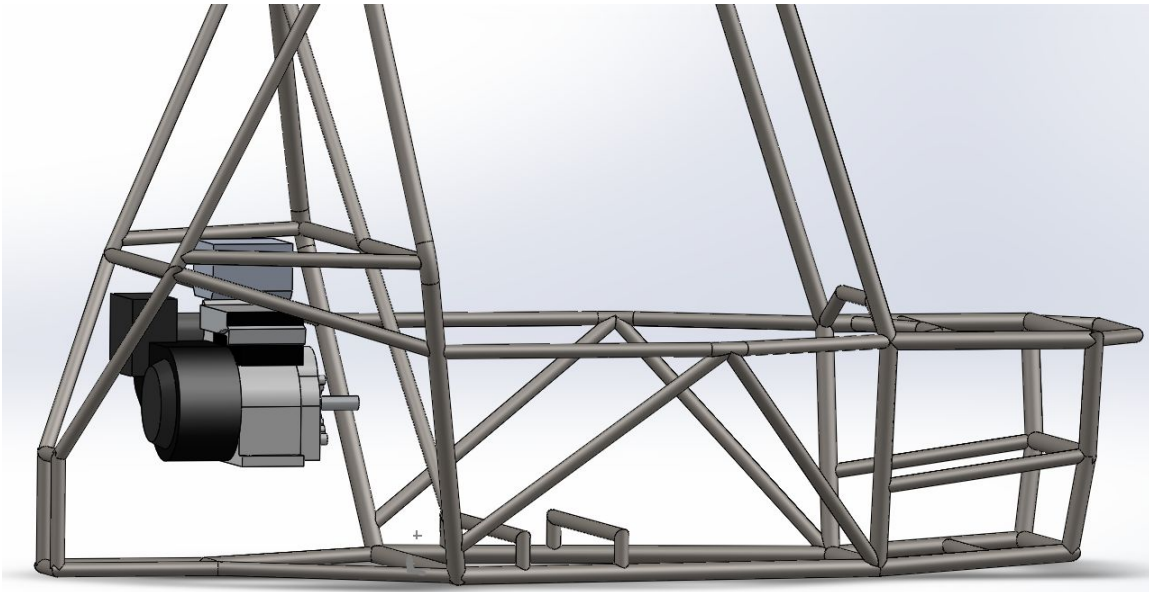


Figure 4: Rotated Engine

## Setup

For the 2021 car I chose to use three gears in the gear train: the pinion, an idler, and a gear. Originally, I wanted to use two gears, a pinion and gear, however due to geometry constraints with the CVT, the distance from input to output shaft had to be increased which was easily accomplished with idler gear. The following section will go through each of the calculations for the pinion (Gear 1), the idler (Gear 2), and the gear (Gear 3), along with the pinion shaft (Shaft 1), the idler shaft (Shaft 2), and the gear shaft (Shaft 3). These can be seen more clearly in the final design in Figure 5 which will be examined more closely later in the report.

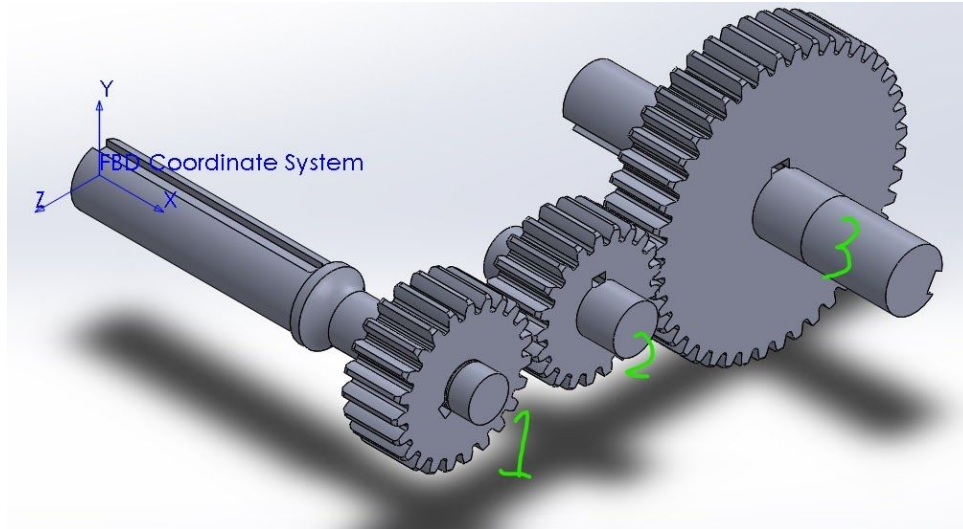


Figure 5: Gearbox Labels

## Calculations

For the calculations I created a spreadsheet located here:

<https://docs.google.com/spreadsheets/d/1Km9xtSvW50BVc4s5BhMeKERUTA9LFGQCN6k4-IJ7v3M/edit?usp=sharing>

On the main tab *Master 2021* all of my work for the gears/shafts and bearings can be seen and it is all parameterized. This spreadsheet takes in all the parameters of the design such as gear module, number of teeth, shaft diameter, and more and as the design changes, factors of safety will be automatically calculated and updated. To begin, I started by specifying all the general properties of the design such as material strengths, engine horsepower, engine rpm, etc at the top of the spreadsheet. The required engine is a 10 HP Briggs and Stratton that achieves max torque at 2800 RPM. The input shaft material is Ti 6AL-6V-2SN, a titanium alloy the team receives from a sponsor and the other shafts are 1144 steel. The gears are made of hardened carbon steel (S45C).

## Gears

The gear analysis depends on three parameters: the number of teeth( $N$ ), the module( $m$ ), and face width( $b_w$ ) of the gear. With these three parameters and engine horsepower/rpm I was able to calculate the max torque, tangential force( $W^t$ ), and radial force( $W^r$ ) in SI units on each of the gears using:

$$Torque = \frac{Horsepower * 5252}{RPM} * 1.356$$

$$W^t = \frac{Torque}{d/2}$$

$$W^r = W^t * \tan \phi$$



Where  $d$  is the pitch diameter and  $\phi$  is the pressure angle in radians. With the forces on the gears known I can use the following equations to calculate the resulting bending/contact stress:

$$\sigma_{bending} = \frac{W^t}{b_w m Y_j} K_a K_s K_m K_v K_i K_b$$

$$\sigma_{contact} = K_e \left( \frac{W^t}{b_w d_p} \frac{1}{I} K_a K_s K_m K_v \right)^{\frac{1}{2}}$$

For bending,  $Y_j$  is the geometry factor,  $K_a$  is the application factor,  $K_s$  is the size factor,  $K_m$  is the load distribution factor,  $K_v$  is the dynamic factor,  $K_i$  is the idler factor, and  $K_b$  is the rim thickness factor. For contact  $K_a$ ,  $K_s$ ,  $K_m$ , and  $K_v$  are the same as those in the bending stress equation,  $K_e$  is the elastic factor, and  $I$  is the geometry factor. The elastic factor can be calculated with:

$$K_e = \sqrt{\frac{2}{\frac{1-v_p^2}{E_p} + \frac{1-v_g^2}{E_g}}}$$

Where  $v$  is poisson's ratio and  $E$  is the elastic modulus of the gear material. The geometry factor for contact can be calculated with:

$$I = \frac{\pi \cos \phi \sin \phi}{1 + \frac{d_p}{d_g}}$$

The geometry factor for bending can be found with the following diagram from the Shigley's Mechanical Design Textbook:

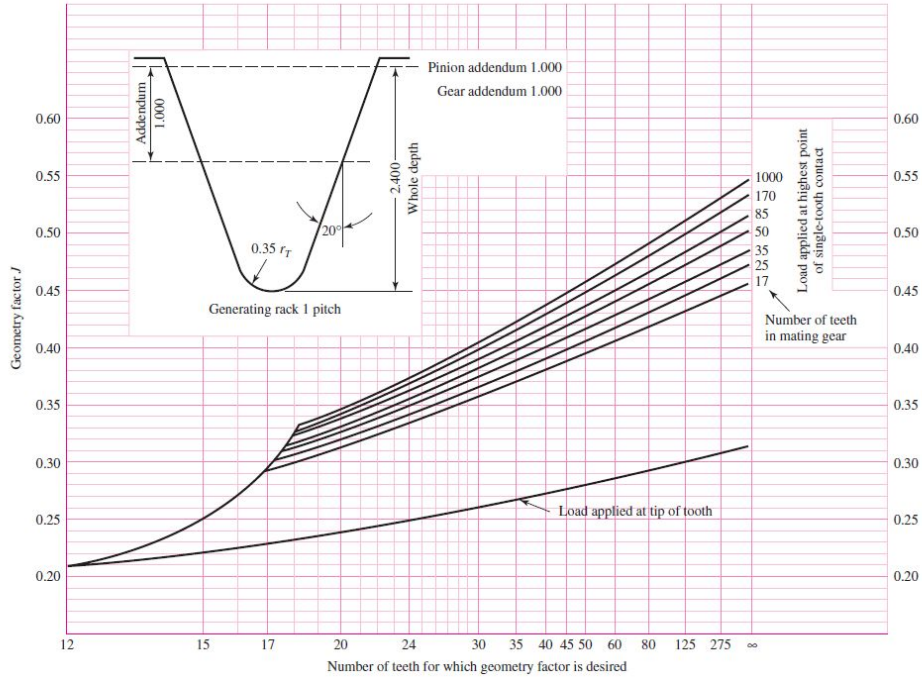


Figure 6: Geometry Factor for Bending

Each of the K-factors and geometry factors modify the stresses to consider various environmental conditions such as gear size, loading smoothness, loading distribution, and more. All of these K-factors and their values can be seen below.

-Application Factor,  $K_a$

The application factor considers the effects of speed variation and shock. The car is powered by an internal combustion engine and therefore a high  $K_a$  value of 1.3 was chosen.

-Size Factor,  $K_s$

The size factor considers the non-uniformity of material properties due to size. The gears chosen are general purpose and are not extremely large in relation to other gears so  $K_s = 1$ .

-Load Distribution Factor,  $K_m$

The load distribution factor considers how the load is distributed along the teeth and increases with the face width of the gear.  $K_m$  can be calculated with the following equation:

$$K_m = 1.0 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

$C_{mc}$  is the lead correlation factor and for uncrowned teeth,  $C_{mc} = 1$ .

$C_{pf}$  is the pinion proportion factor and for a face width of less than 25mm it is equal to:

$$C_{pf} = \frac{b_w}{10d} - 0.025$$

$C_{pm}$  is the pinion proportion modifier which is affected by the mounting location of the gear. For a gear mounted near the center of the shaft,  $C_{pm} = 1$ .

$C_{ma}$  is the mesh alignment factor and can be calculated with the following equation:

$$C_{ma} = A + Bb_w + Cb_w^2$$

The coefficients for a precision gear are  $A = 0.0675$ ,  $B = 5.04(10)^{-4}$ ,  $C = -1.44(10)^{-7}$ .

$C_e$  is the mesh alignment correlation factor and if the gearing is not adjusted at assembly,  $C_e = 1$ .

-Dynamic Factor,  $K_v$

The dynamic factor takes into account the impact loading that the gears experience. It can be calculated with the following equations:

$$K_v = \left( \frac{A + \sqrt{200V}}{A} \right)^B$$

$$B = \frac{(12 - Q_v)^{\frac{2}{3}}}{4}$$

$$A = 50 + 56(1 - B)$$

Where  $V$  is the tangential velocity in m/s and  $Q_v$  is the accuracy number of the gears.

Refer to the calculation spreadsheet for more information.

-Idler Factor,  $K_i$

This factor changes if a specific gear is an idler gear or not.  $K_i = 1.42$  if it is an idler gear and 1.0 if not.

-Rim Thickness Factor,  $K_b$

The gears are solid disks so  $K_b$  is equal to 1.

Using all of the factors from above the bending and contact stresses can be calculated for each of the gears which can be seen more in detail in the calculation spreadsheet. To calculate the factors of safety, the allowable bending and contact stresses must also be calculated. They can be found with the following equations:

$$\sigma_{bending,allowable} = \frac{S_b Y_N}{K_T K_R}$$
$$\sigma_{contact,allowable} = \frac{S_c Z_N C_H}{K_T K_R}$$

Here  $S_b$  and  $S_c$  are for the bending and contact strengths of the material,  $K_T$  is the temperature factor,  $K_R$  is the reliability factor,  $Y_N$  is the stress cycle factor for bending,  $Z_N$  is the stress cycle factor for contact, and  $C_H$  the hardness ratio factor.

-Bending Strength,  $S_b$

The bending strength is dependent on the Brinell Hardness of the gears and can be found with the graph in Figure 7 from Shigley's Mechanical Design Textbook.

### Figure 14-2

Allowable bending stress number for through-hardened steels. The SI equations are  $S_t = 0.533H_B + 88.3$  MPa, grade 1, and  $S_t = 0.703H_B + 113$  MPa, grade 2.

(Source: ANSI/AGMA 2001-D04 and 2101-D04.)

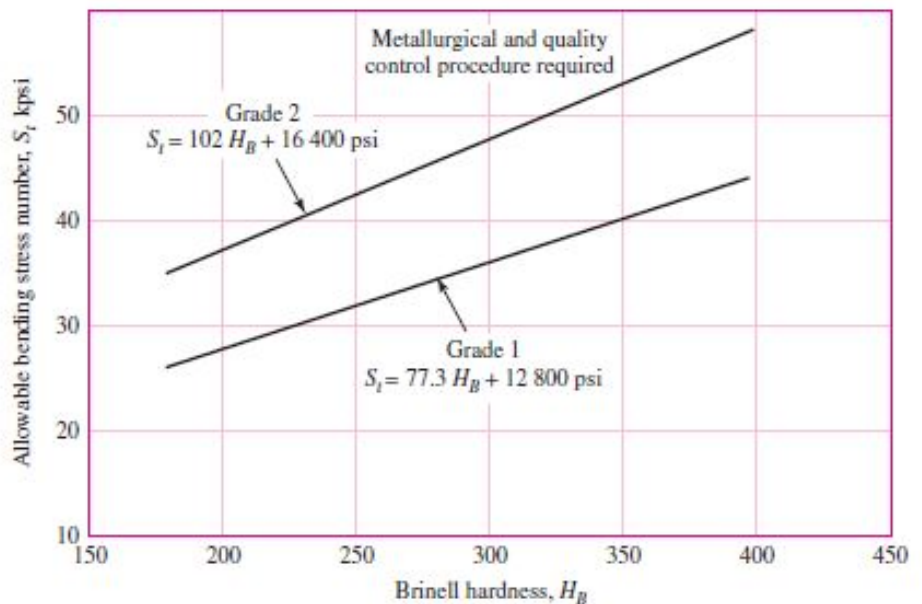


Figure 7: Allowable Bending Strength

### -Contact Strength, $S_c$

The contact strength is dependent on the Brinell Hardness of the gears and can be found with the graph in Figure 8 from Shigley's Mechanical Design Textbook.

### Figure 14-5

Contact-fatigue strength  $S_c$  at  $10^7$  cycles and 0.99 reliability for through-hardened steel gears. The SI equations are  $S_c = 2.22H_B + 200$  MPa, grade 1, and  $S_c = 2.41H_B + 237$  MPa, grade 2. (Source: ANSI/AGMA 2001-D04 and 2101-D04.)

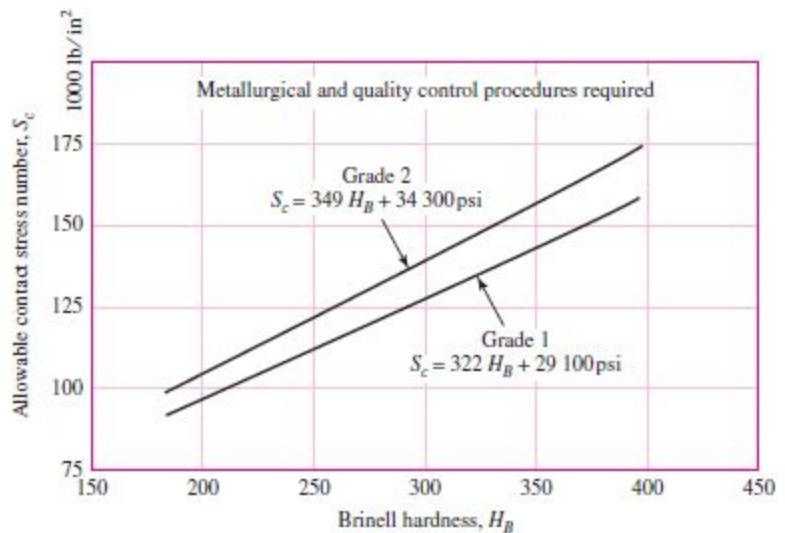


Figure 8: Allowable Contact Strength

### -Temperature Factor, $K_T$

The temperature factor considers the effects of operating temperature on material strength and  $K_T = 1$  if the temperature is not higher than 125 degrees celsius.

### -Reliability Factor, $K_R$



The reliability factor considers how reliable the system should be designed for with  $K_R$  being higher if the reliability requirement is higher. For 99% reliability of survival  $K_R = 1$ .

-Stress Cycle Factor for Bending,  $Y_N$

The stress cycle factor for bending considers the number of cycles the gear undergoes and can be found with the graph in Figure 9 from Shigley's Mechanical Design Textbook.

**Figure 14-14**

Repeatedly applied bending strength stress-cycle factor  $Y_N$ . (ANSI/AGMA 2001-D04.)

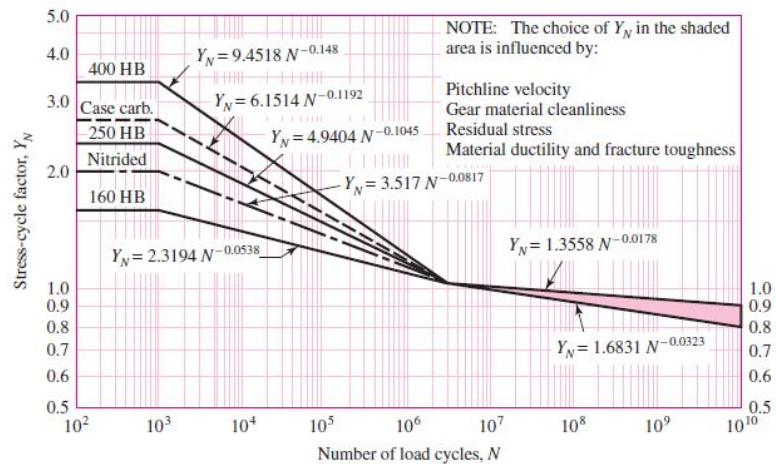


Figure 9: Stress Cycle Factor  $Y_N$

-Stress Cycle Factor for Contact,  $Z_N$

The stress cycle factor for contact considers the number of cycles the gear undergoes and can be found with the graph in Figure 10 from Shigley's Mechanical Design Textbook.

**Figure 14-15**

Pitting resistance stress-cycle factor  $Z_N$ . (ANSI/AGMA 2001-D04.)

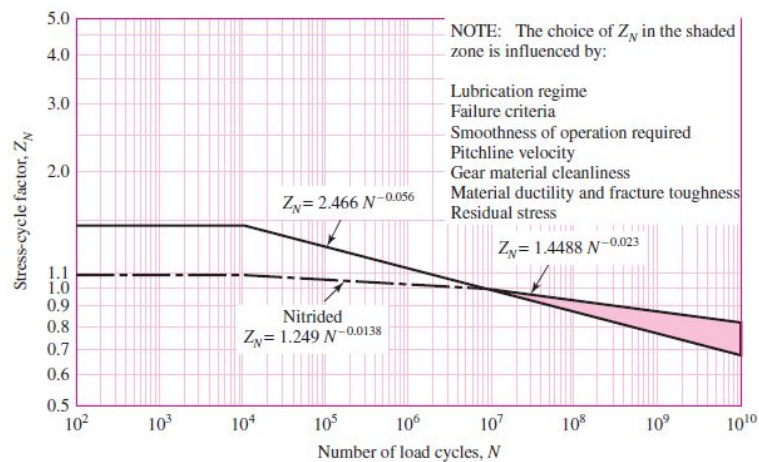


Figure 10: Stress Cycle Factor  $Z_N$

-Hardness Ratio Factor,  $C_H$

The hardness ratio factor considers the difference in hardness of the mating surfaces and only applies to the gear. The pinion and gear have the same Brinell Hardness so  $C_H = 1$ .

Using the working stresses and the allowable stresses for contact and bending, the factors of safety can be calculated using:

$$n_{bending} = \frac{\sigma_{bending,allowable}}{\sigma_{bending}}$$

$$n_{contact} = \frac{\sigma_{contact,allowable}}{\sigma_{contact}}$$

The final factors of safety for all three chosen gears can be seen in the spreadsheet.

## Bearings

The bearings were chosen to allow for 200 hours of life, which for the Baja car is an eternity. With the hours of life and the rpm the bearings spin at, the  $L_{10}$  life (in millions of revolutions) can be calculated using the following equation:

$$L_{10} = h(60)(n)/10^6$$

Where  $h$  is the hours of life and  $n$  is the rotation speed in rpm. The basic load rating ( $C$ ) of a bearing can be found with the following equation:

$$L = \left(\frac{C}{P}\right)^m$$

Where  $P$  is the equivalent load in Newtons,  $L$  is the  $L_{10}$  life in millions of revolutions, and  $m = 3$  for ball bearings and  $m = 10/3$  for roller bearings. For pure radial loading  $P$  can be calculated with:

$$P = A_p P_r$$

Where  $A_p$  is the application factor which can be chosen from Figure 11 from Shigley's Mechanical Design Textbook shown below.

**Table 11-5**

Load-Application Factors

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

Figure 11: Bearing Application Factors

$P_r$  is the resultant bearing radial reaction force which can be found using free body diagrams. In these free body diagrams(FBD), each of the gears consists of a radial and tangential force acting on the shaft and the input shaft has an additional force from the CVT. The spur gears used in this design do not produce any thrust forces, so only radial forces must be accounted for. To create a FBD, I first created a coordinate system in which the XZ-plane contains the axes of all three shafts and the y-axis is perpendicular to that plane, which can be seen in Figure 12.

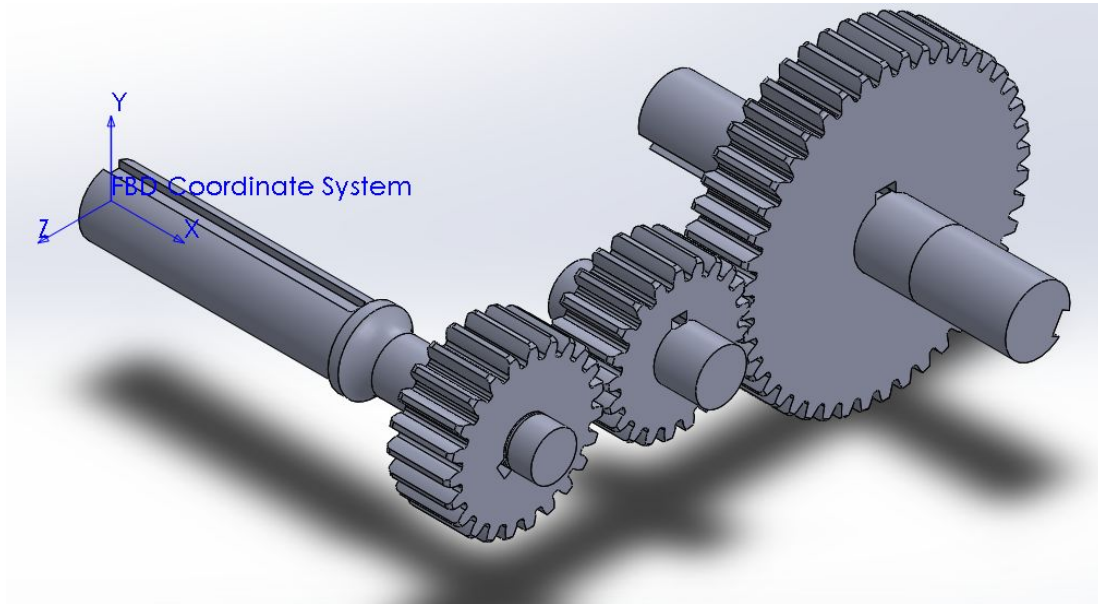


Figure 12: Global Coordinate System

Bearings have number and letter designation. The number corresponds to what shaft it is on (1, 2, or 3) and the letter corresponds to what side of the shaft the bearing is on. Bearings with the letter *a* are on the input side of the gearbox and bearings with the letter *b* correspond to the opposite side of the input.

#### -Shaft 1 FBD

For shaft 1, the coordinate system is defined in Figure 13. This shaft consists of the radial and tangential forces from the gear and the Y and Z components of the CVT belt. In Figure 14/15, the distance between the CVT and first bearing is marked  $pb$ , between the first bearing and the gear marked  $bg$ , and between the gear and second bearing marked  $gb$ .

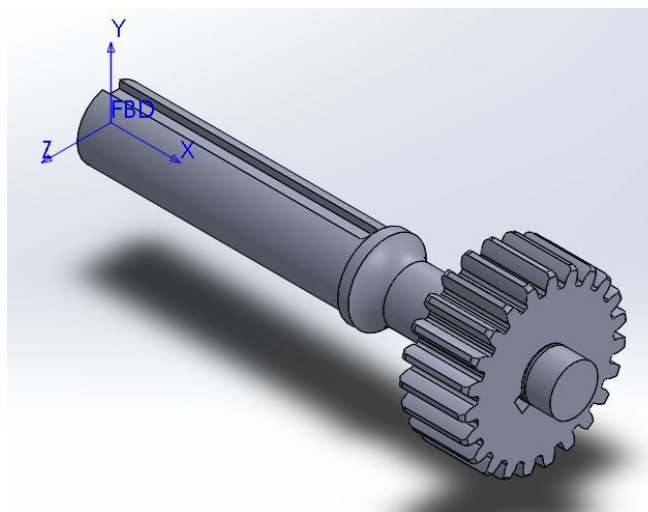


Figure 13: Shaft 1 Coordinate System

In the XY plane, show in Figure 14, taking the moments about the point where  $1_{by}$  is applied yields:

$$\Sigma M_{1by} = 0 \rightarrow 1_{ay}(bg + gb) = W_t(gb) + CVT_y(pg + bg + gb)$$

$$1_{ay} = \frac{W_t(gb) + CVT_y(pg + bg + gb)}{(bg + gb)}$$

Summing the forces in the Y direction yields:

$$\Sigma F_y = 0 \rightarrow CVT_y + W_t + 1_{by} = 1_{ay}$$

$$1_{by} = 1_{ay} - CVT_y - W_t$$

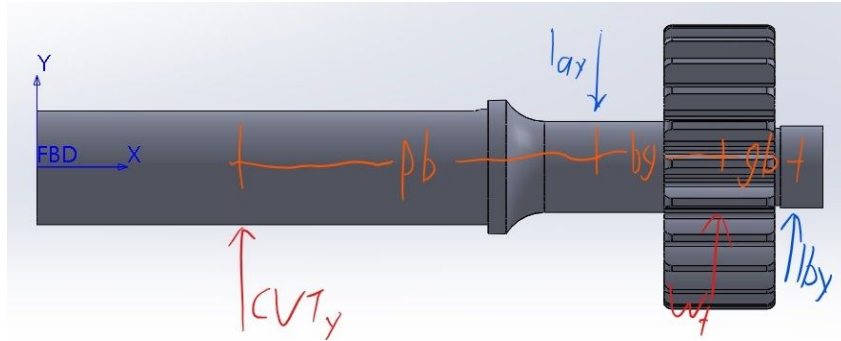


Figure 14: Shaft 1 XY Plane

In the XZ plane, shown in Figure 15, taking the moments about the point where  $1_{bz}$  is applied yields:

$$\Sigma M_{1bz} = 0 \rightarrow W_r(gb) = 1_{az}(bg + gb) + CVT_z(pg + bg + gb)$$

$$1_{az} = \frac{W_r(gb) - CVT_z(pg + bg + gb)}{(bg + gb)}$$

Summing the forces in the Z direction yields:

$$\Sigma F_z = 0 \rightarrow W_r = CVT_z + 1_{az} + 1_{bz}$$

$$1_{bz} = W_r - 1_{az} - CVT_z$$

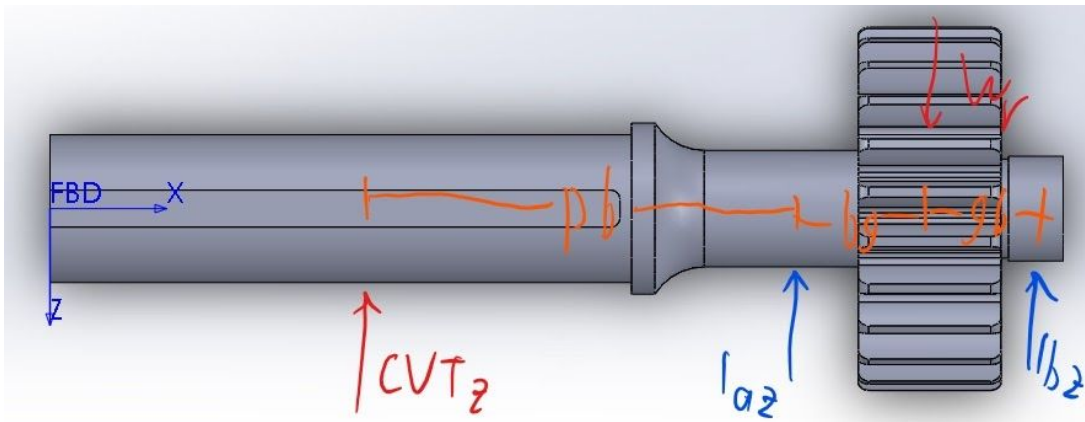


Figure 15: Shaft 1 XZ Plane

### -Shaft 2 FBD

For shaft 2, the coordinate system is defined in Figure 16. This shaft only consists of the radial and tangential forces from the gear. In Figure 17/18, the distance between the first bearing and the gear is marked  $bg$  and between the gear and second bearing is marked  $gb$ .

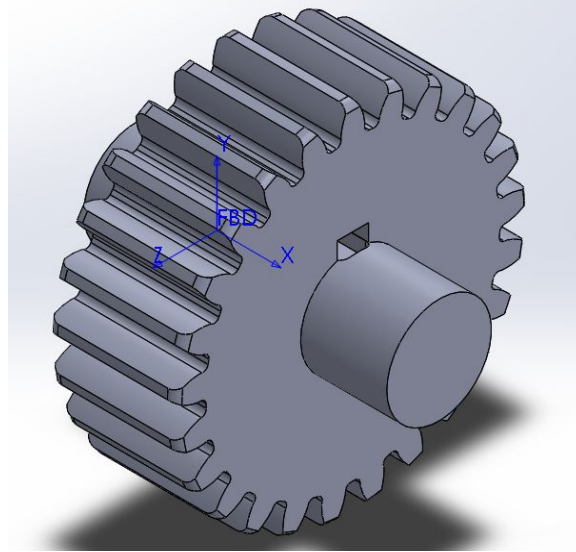


Figure 16: Shaft 2 Coordinate System

In the XY plane, shown in Figure 17, taking the moments about the point where  $2_{ay}$  is applied yields:

$$\begin{aligned}\Sigma M_{2ay} &= 0 \rightarrow 2 * W_t(bg) = 2_{by}(bg + gb) \\ 2_{by} &= \frac{2 * W_t(bg)}{(bg + gb)}\end{aligned}$$

Summing the forces in the Y direction yields:

$$\begin{aligned}\Sigma F_y &= 0 \rightarrow 2 * W_t = 2_{ay} + 2_{by} \\ 2_{ay} &= 2 * W_t - 2_{by}\end{aligned}$$



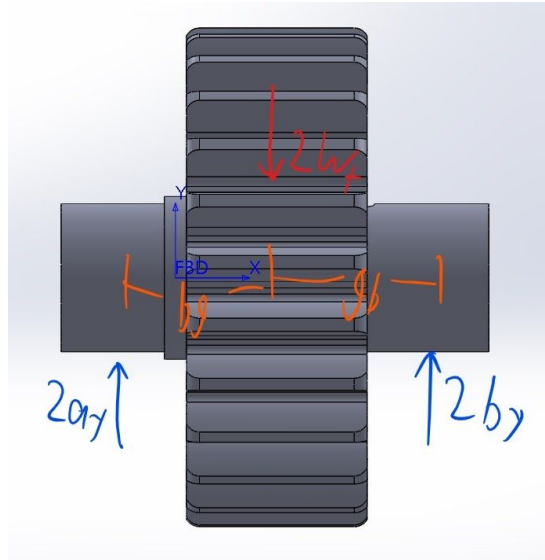


Figure 17: Shaft 2 XY Plane

In the XZ plane, shown in Figure 18, the radial gear forces cancel each other out because they both act purely in the XZ plane. Therefore the bearing reaction forces in the Z direction are 0.

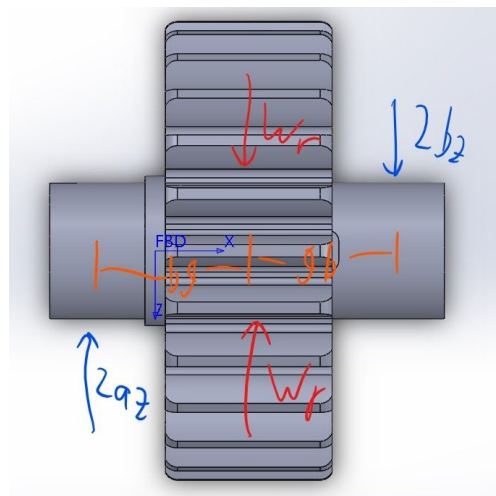


Figure 18: Shaft 2 XZ Plane

#### -Shaft 3 FBD

For shaft 3, the coordinate system is defined in Figure 19. This shaft only consists of the radial and tangential forces from the gear. In Figure 20/21, the distance between the first bearing and the gear is marked  $bg$  and between the gear and second bearing is marked  $gb$ .

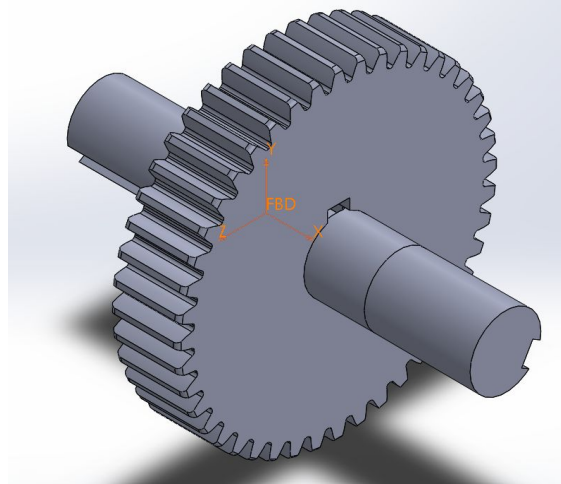


Figure 19: Shaft 3 Coordinate System

In the XY plane, shown in Figure 20, taking the moments about the point where  $2_{ay}$  is applied yields:

$$\Sigma M_{3ay} = 0 \rightarrow W_t(bg) = 3_{by}(bg + gb)$$

$$3_{by} = \frac{W_t(bg)}{(bg + gb)}$$

Summing the forces in the Y direction yields:

$$\Sigma F_y = 0 \rightarrow W_t = 3_{ay} + 3_{by}$$

$$3_{ay} = W_t - 3_{by}$$

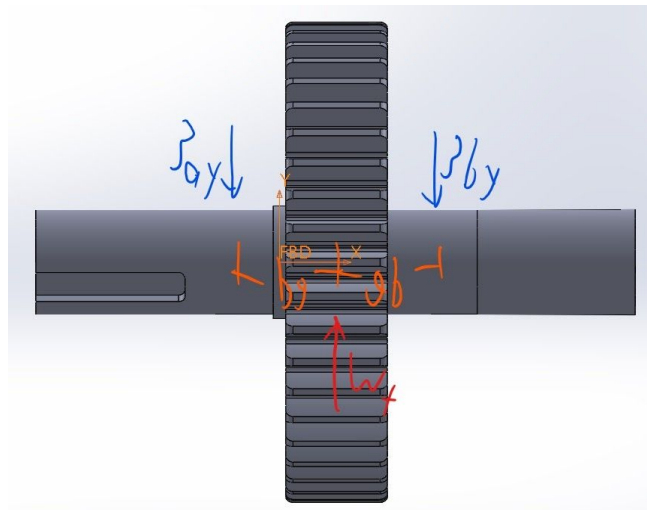


Figure 20: Shaft 3 XY Plane

In the XZ plane, shown in Figure 21, taking the moments about the point where  $2_{az}$  is applied yields:

$$\Sigma M_{3az} = 0 \rightarrow W_r(bg) = 3_{bz}(bg + gb)$$

$$3_{bz} = \frac{W_r(bg)}{(bg + gb)}$$

Summing the forces in the Z direction yields:

$$\Sigma F_z = 0 \rightarrow W_r = 3_{az} + 3_{bz}$$

$$3_{az} = W_r - 3_{bz}$$

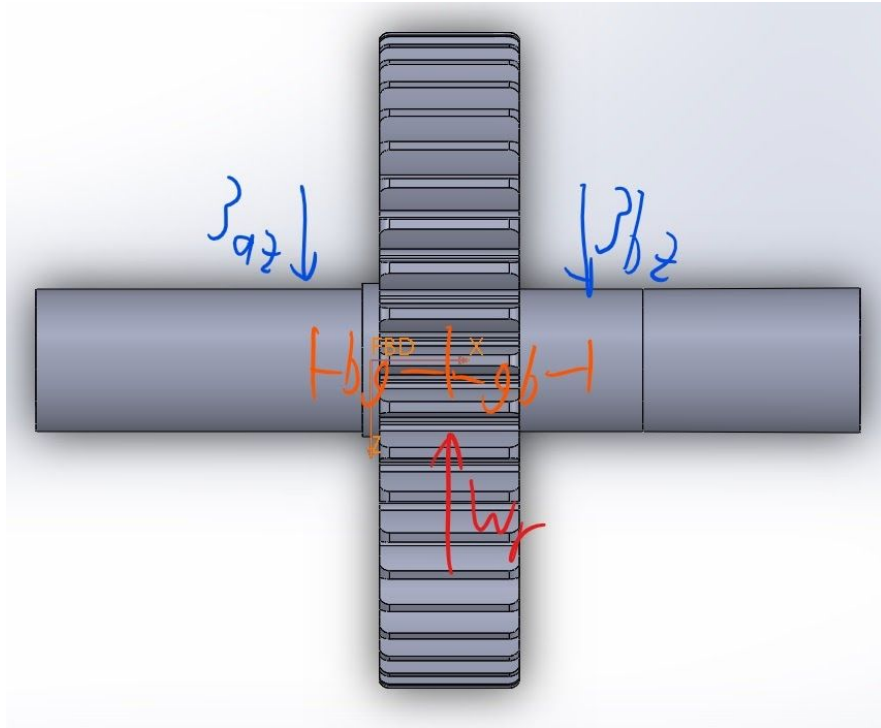


Figure 21: Shaft 3 XZ Plane

All of the variables in the above equations are taken from the spreadsheet to solve for the reaction forces. See the calculation spreadsheet for more details. With the bearing reaction forces in both directions for all the bearings, the resultant force acting on the bearing can be calculated and it can be used to calculate the basic load rating required for each bearing.

### Shafts

The shaft analysis depends on the diameter of the shaft chosen, which is limited to the bore that the gears can be purchased with. The endurance limit for ferrous materials can be found with the following equation:

$$S'_e = 0.5S_{ultimate-tensile}$$

This endurance limit of the material however is different from the endurance limit of the mechanical component. The endurance limit of the mechanical component can be found with the following equation:

$$S_e = k_f k_s k_r k_t k_m S'_e$$

This equation has an additional 5 K-factors which are different than those used for the gear analysis. These factors seek to modify the endurance limit, which is obtained from rotating-beam tests, based on specific properties of the mechanical component. Here  $k_f$  is the surface finish factor,  $k_s$  is the size factor,  $k_r$  is the reliability factor,  $k_t$  is the temperature factor, and  $k_m$  is the miscellaneous factor.

-Surface Finish Factor,  $k_f$

The surface finish factor considers the effect of surface finish on the component and can be found with the following equation:

$$k_f = e(S_{ut})^f$$

The shafts are machined on a lathe so  $e = 4.51$  and  $f = -0.265$ .

-Size Factor,  $k_s$

The geometry factor considers the effect of size on the mechanical component and can be found with the following equation:

$$k_s = 1.189(d)^{-0.112}$$

Where  $d$  is the shaft diameter in millimeters.

-Reliability Factor,  $k_r$

The reliability factor considers the uncertainty related to pre-existing material defects.

For a 99% probability of survival  $k_r = 0.82$ .

-Temperature Factor,  $k_t$

The temperature factor considers the effects of temperature on fatigue. With no special temperature requirements  $k_t = 1$ .

-Miscellaneous Factor,  $k_m$

The miscellaneous factor considers the myriad of other processes and conditions that can affect the fatigue of a component. With no other special considerations  $k_m = 1$ .

With the modified endurance limit, there are two more equations that provide fatigue failure criteria. The first is Goodman's line for design which can be found with the following equation:

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n_s}$$

Where  $S_e$  is the endurance limit of the mechanical component,  $S_{ut}$  is the ultimate tensile strength of the material,  $\sigma_a$  is the amplitude of the Von Mises stress,  $\sigma_m$  is the midrange Von Mises stress and  $n_s$  is the factor of safety. The second is the yield line for design which can be found with the following equation:

$$\frac{\sigma_{vm}}{S_y} = \frac{1}{n_s}$$

Where  $\sigma_{vm}$  is the Von Mises stress,  $S_y$  is the yield strength of the material, and  $n_s$  is the factor of safety. Both of these design criteria require the midrange and amplitude Von Mises stresses.

There are four components of Von Mises stress: the midrange bending stress, the amplitude of the bending stress, the midrange shear stress, and the amplitude of the shear stress. With a spur gear system, the midrange bending stress( $\sigma_{bm}$ ) and the amplitude of the shear stress( $\tau_a$ ) are negligible. This leaves the amplitude of the bending stress( $\sigma_{ba}$ ) and the midrange shear stress( $\tau_m$ ). The amplitude of the bending stress can be calculated with the following equation:

$$\sigma_{ba} = \frac{32M_c}{\pi d^3}$$

Where  $M_c$  is the combined maximum bending moment and  $d$  is the diameter of the shaft. The midrange shear stress can be calculated with:

$$\tau_m = \frac{16T}{\pi d^3}$$

Where  $T$  is the torque and  $d$  is the diameter of the shaft. The torque was calculated earlier in the gear analysis and the combined maximum bending moment can be found by analyzing the shear/moment diagrams that can be created from the bearing reaction force analysis. The forces acting on the shafts are those presented in the bearing analysis section. Using these forces the following moment diagrams can be created about the Z and Y axis as shown in Figures 22-27. The corresponding shear diagrams can be found in the calculation spreadsheet.

-Shaft 1

Shaft 1 - Moment about Z Axis Diagram

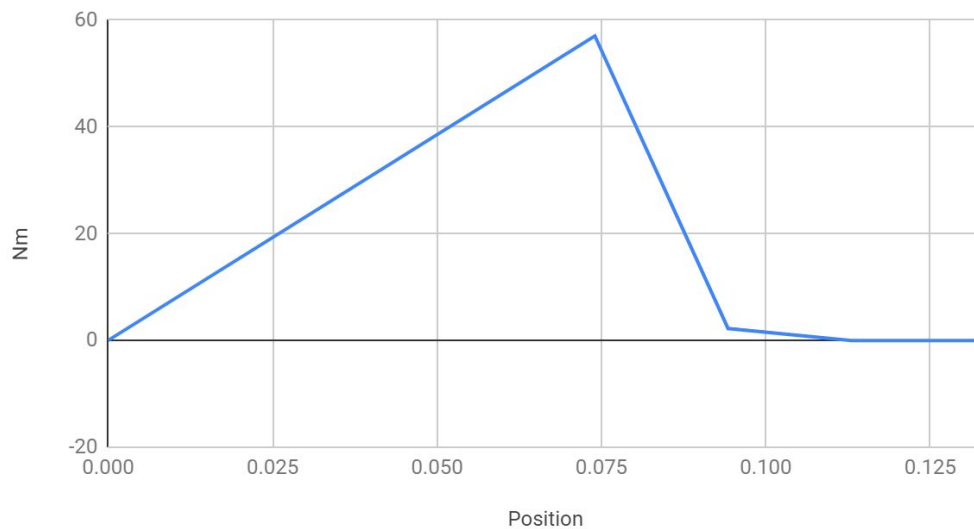


Figure 22: Shaft 1 Moment about the Z-Axis



Shaft 1 - Moment about Y Axis Diagram

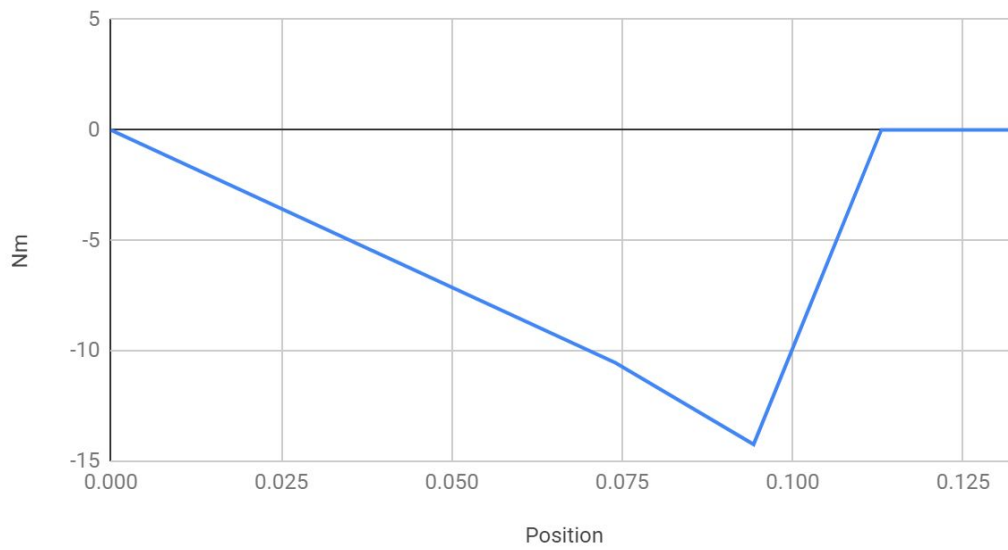


Figure 23: Shaft 1 Moment about the Y-Axis

-Shaft 2

Shaft 2 - Moment about Z Axis Diagram

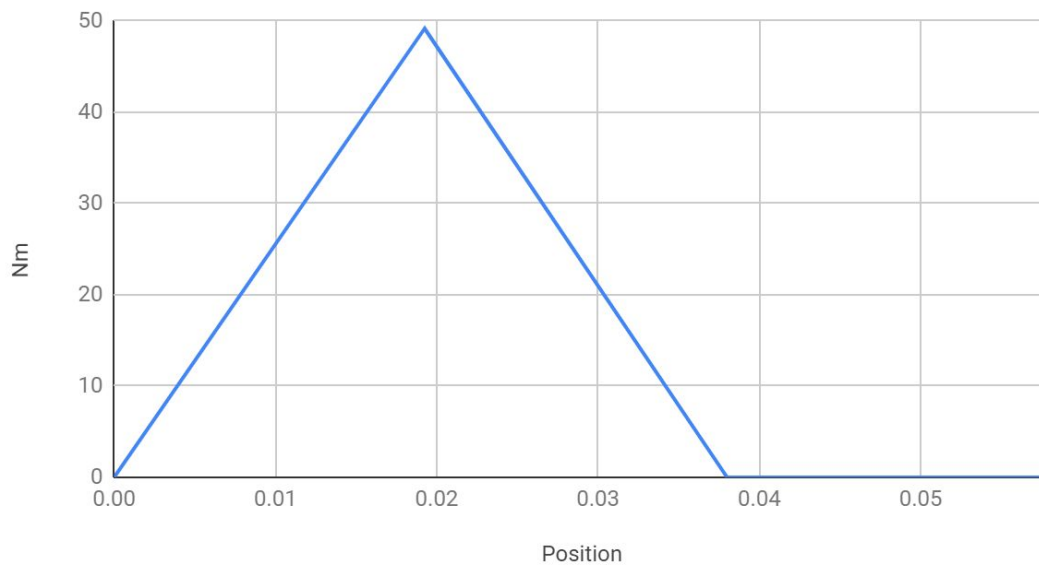


Figure 24: Shaft 2 Moment about the Z-Axis

Shaft 2 - Moment about Y Axis Diagram

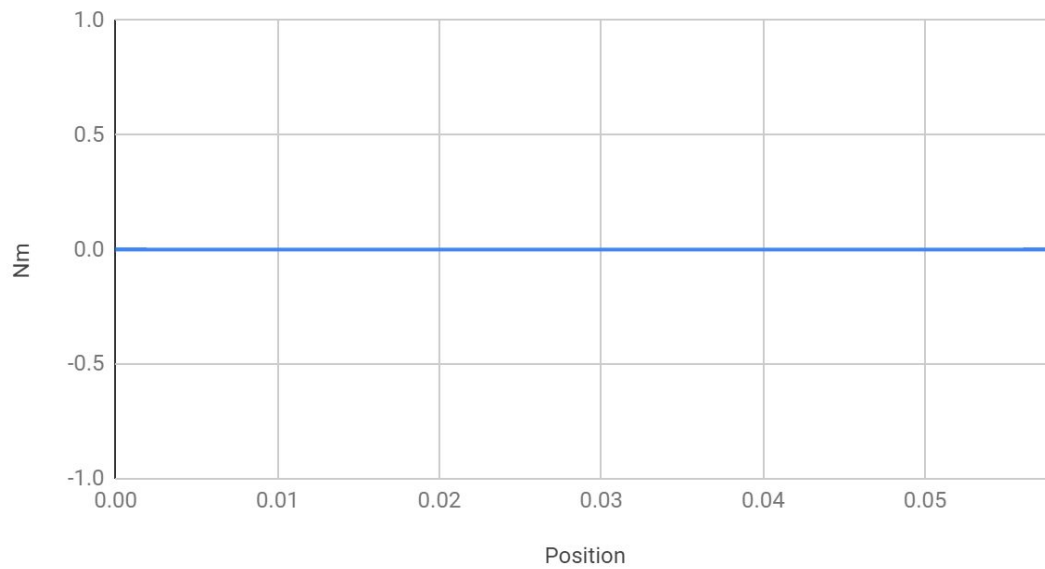


Figure 25: Shaft 2 Moment about the Y-Axis

-Shaft 3

Shaft 3 - Moment about Z Axis Diagram

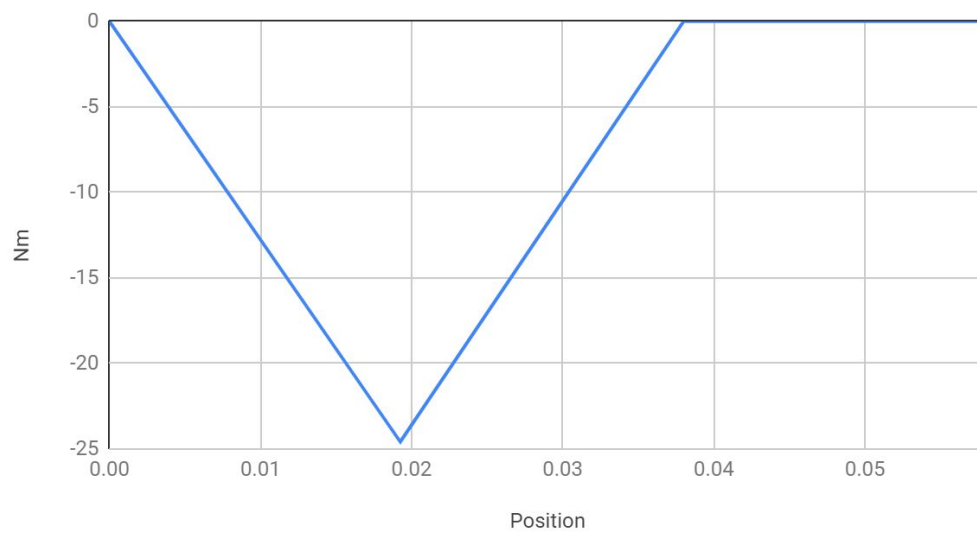


Figure 26: Shaft 3 Moment about the Z-Axis

### Shaft 3 - Moment about Y Axis Diagram

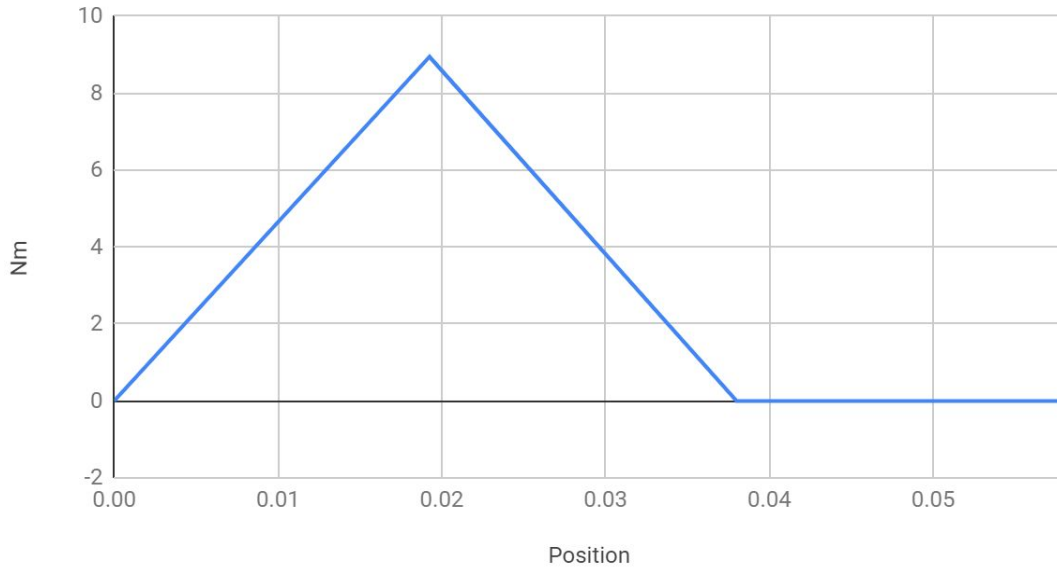


Figure 27: Shaft 3 Moment about the Y-Axis

From these diagrams the maximum moment about the Y and Z axis can be determined for each shaft and then combined with the following equation to calculate  $M_c$ :

$$M_c = \sqrt{M_{z,max}^2 + M_{y,max}^2}$$

For Goodman's line, the following equation can be used for the midrange Von Mises stress:

$$\sigma'_m = \sqrt{\sigma_{bm}^2 + 3\tau_m^2}$$

The following equation can be used for the amplitude of the Von Mises stress:

$$\sigma'_a = \sqrt{K_f \sigma_{ba}^2 + 3\tau_a^2}$$

There is one last K-factor present in this equation,  $K_f$ , the fatigue stress concentration factor. The fatigue stress concentration factor considers the stress concentration that occurs when a geometric discontinuity is present. It can be calculated with the following equation:

$$K_f = 1 + q(K_c - 1)$$

Where  $K_c$  is the theoretical stress concentration factor and  $q$  is the notch sensitivity. The following tables in Figures 28 and 29 from Shigley's Mechanical Design Textbook show the values for theoretical stress concentration factors in torsion and in bending respectively.

**Figure A-15-8**

Round shaft with shoulder fillet  
in torsion.  $\tau_0 = Tc/J$ , where  
 $c = d/2$  and  $J = \pi d^4/32$ .

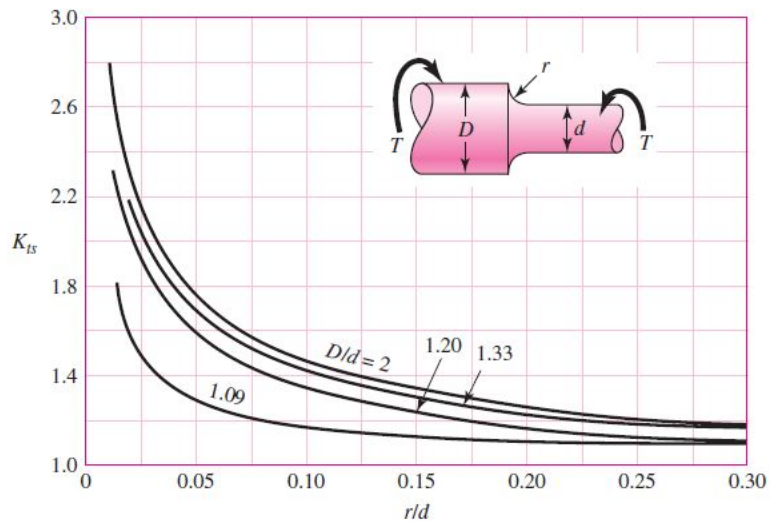


Figure 28: Theoretical Stress Concentration Factor in Torsion

**Figure A-15-9**

Round shaft with shoulder fillet  
in bending.  $\sigma_0 = Mc/I$ , where  
 $c = d/2$  and  $I = \pi d^4/64$ .

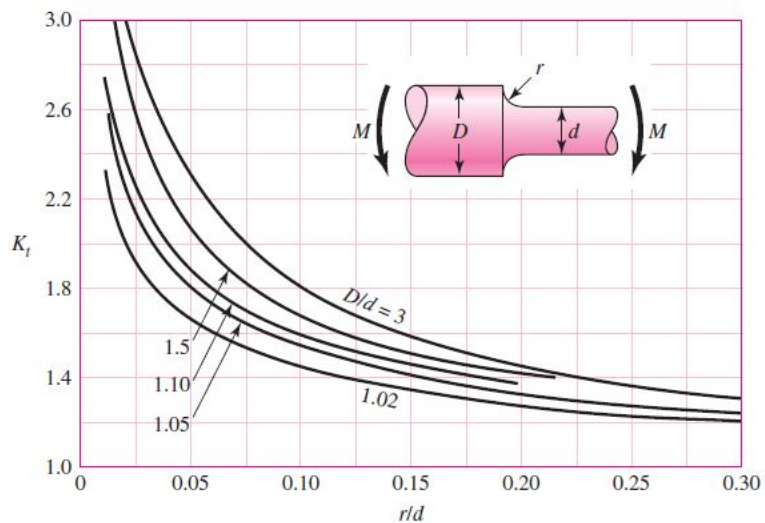


Figure 29: Theoretical Stress Concentration Factor in Bending

The notch sensitivity is between 0 and 1 and represents how sensitive the material is to notches with  $q = 0$  meaning no sensitivity to notches and  $q = 1$  meaning the material has full sensitivity to notches. Notch sensitivity can be found using the graph in Figure 30 from Shigley's Mechanical Design Textbook. For specific calculations refer to the calculation spreadsheet. Every variable in Goodman's line has now been defined and a factor of safety can be calculated.

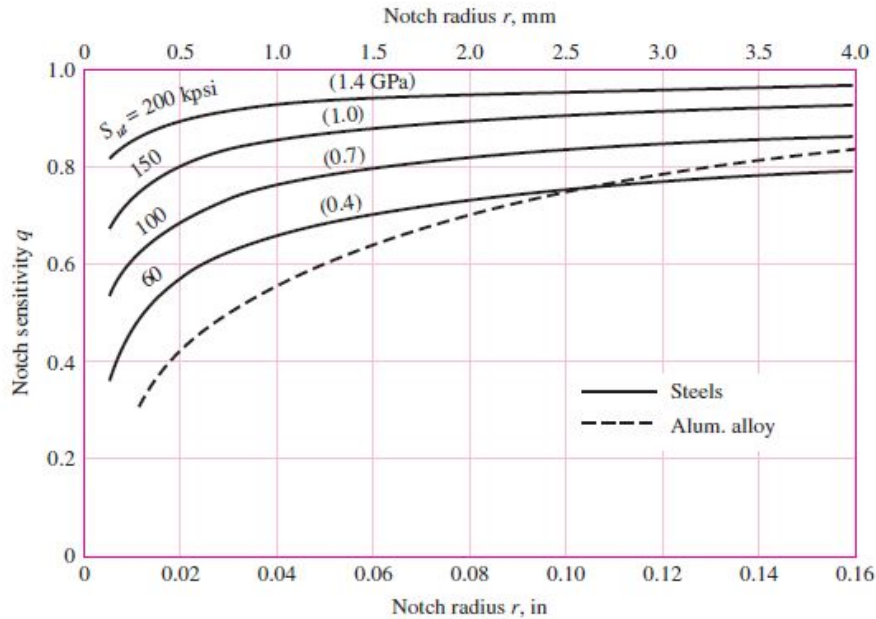


Figure 30: Notch Sensitivity

The yield line is also a necessary design criteria to check if yield would occur at peak stress. The Von Mises stress is also used in this calculation, however no K-factors should be used. The Von Mises stress can be found with the following equation:

$$\sigma_{vm} = \sqrt{(\sigma_{ba} + \sigma_{bm})^2 + 3(\tau_a + \tau_m)^2}$$

With this, every variable in the yield line equation has been defined and a second factor of safety for the shafts can be calculated.

## Component Selection

With all of the calculations in a Google Sheets spreadsheet, the parameters/properties of the gears, bearings, and shafts can be changed and all the calculations will propagate through the sheet. After much analysis, a general synopsis of the chosen components are below. For a more in depth look at the components view the calculation spreadsheet and go to the *BOM* tab.

### Gears

	Teeth	Module	Face Width	Bending FOS	Contact FOS
Gear 1(Pinion)	23	2.5	24.5 mm	2.42	1.51
Gear 2 (Idler)	25	2.5	24.5 mm	1.54	1.58
Gear 3 (Gear)	44	2.5	24.5 mm	2.79	2.21



Final Ratio = 1.913
---------------------

### Bearings

	Bearing Type	Dynamic Load Rating	FOS
1a	Roller Bearing	28.5 kN	2.20
1b	Ball Bearing	7.28 kN	2.25
2a	Roller Bearing	28.5 kN	3.32
2b	Roller Bearing	28.5 kN	3.23
3a	Ball Bearing	8.06 kN	1.90
3b	Ball Bearing	8.06 kN	1.85

### Shafts

	Diameter	Material	Goodman's FOS	Yield FOS
Shaft 1	18 mm	Ti 6AL-6V-2SN	1.51	6.43
Shaft 2	20 mm	1144 Steel	1.45	5.40
Shaft 3	25 mm	1144 Steel	4.07	7.55

### **Gearbox Design**

With all the components selected, I began the SolidWorks models. Firstly, I created subassemblies of each of the shafts and placed all components on each of their designated shafts including spacers, retaining rings, and seals. The shaft 1 subassembly is shown in Figure 31 and a section view is shown in Figure 32. This is the input shaft of the gearbox and is made out of a titanium alloy that is given to the team by a sponsor each year. This shaft consists of (from left to right) a 1 inch diameter keyed section for the CVT, a brown shaft seal, a roller bearing, the pinion gear, a retaining ring, and a ball bearing. In between the bearings and gear there are shoulders/spacers so the inner race of the bearing does not rub against the gear.

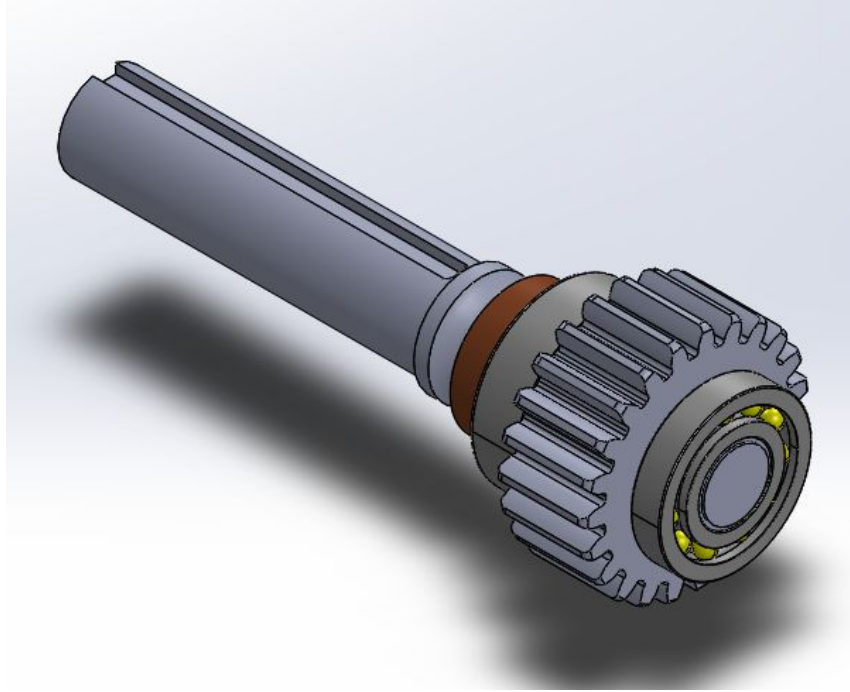


Figure 31: Shaft 1 Subassembly

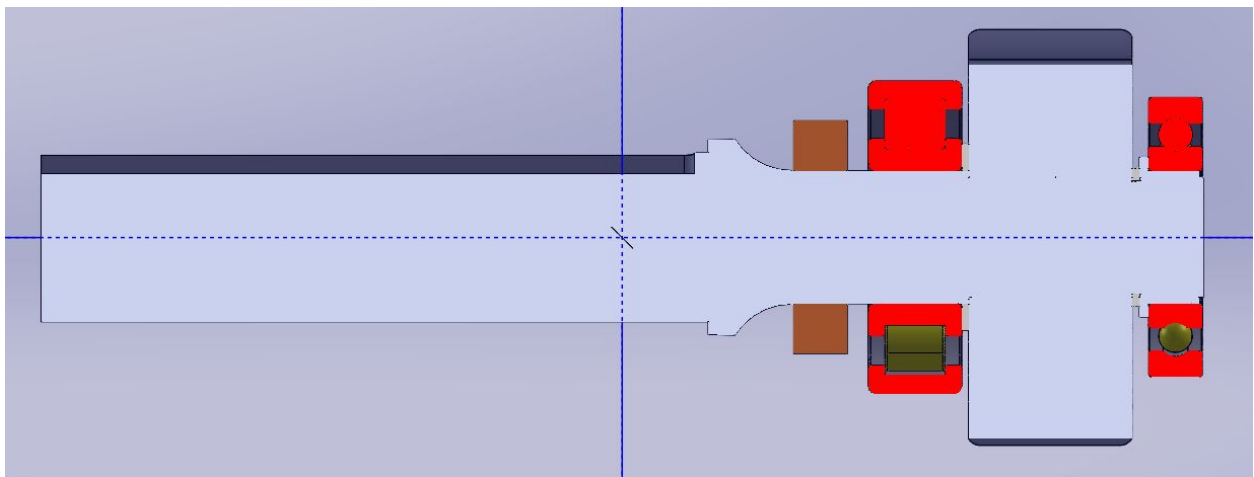


Figure 32: Shaft 1 Section View

The shaft 2 subassembly can be seen in Figure 33 and a section view is shown in Figure 34. Shaft 2 is the idler shaft of the gearbox and contains the idler gear. This shaft consists of (from left to right) a roller bearing, the idler gear, and another roller bearing. This shaft has no input/output outside of the gearbox so there is no need for any seals. Once again between the bearings and gears there are shoulders/spacers so the inner race of the bearing is spaced off from the gear.

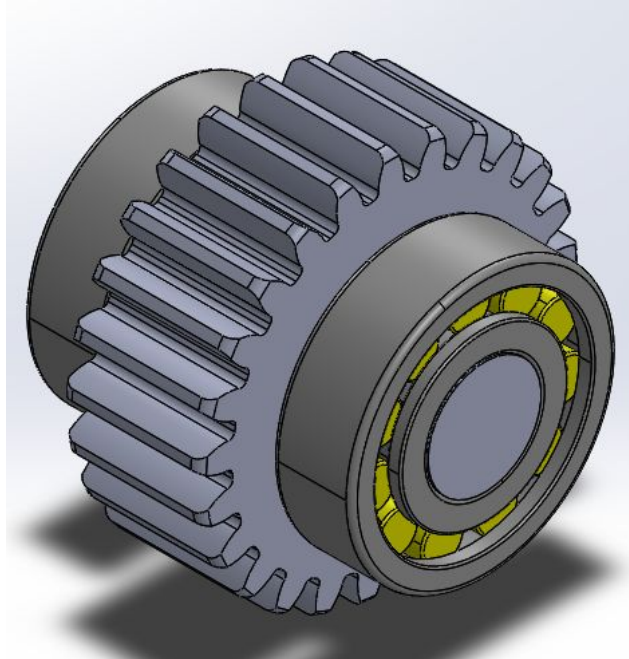


Figure 33: Shaft 2 Subassembly

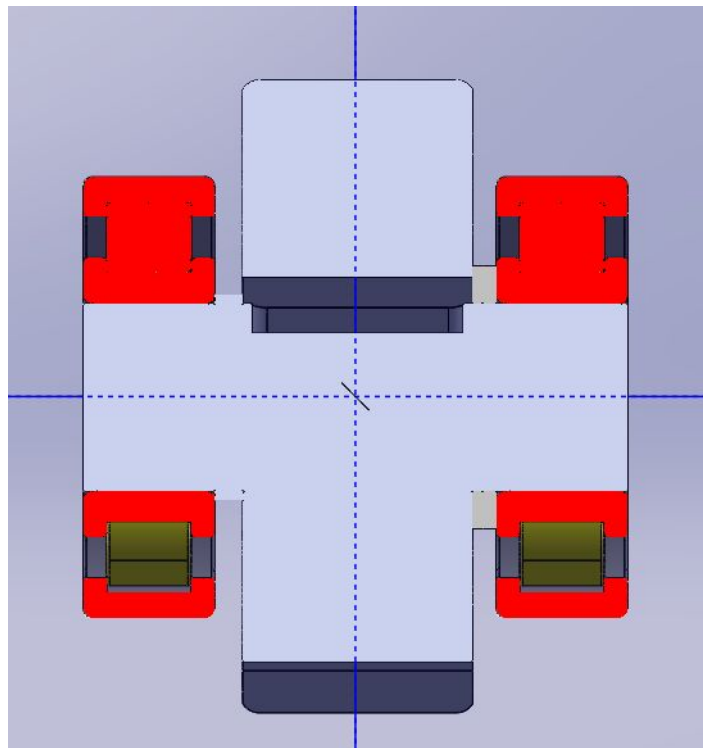


Figure 34: Shaft 2 Section View

The shaft 3 subassembly can be seen in Figure 35 and a section view is shown in Figure 36. This is the output shaft of the gear box and has two different couplings on each end of the shaft. This shaft consists of (from left to right) a shaft coupling from McMaster, a brown shaft seal, the first roller bearing, the gear, the second roller bearing, the second brown shaft seal, and a custom steel coupling that will be welded directly to a u-joint. Once again between the bearings and gears there are shoulders/spacers so the inner race of the bearing is spaced off from the gear.

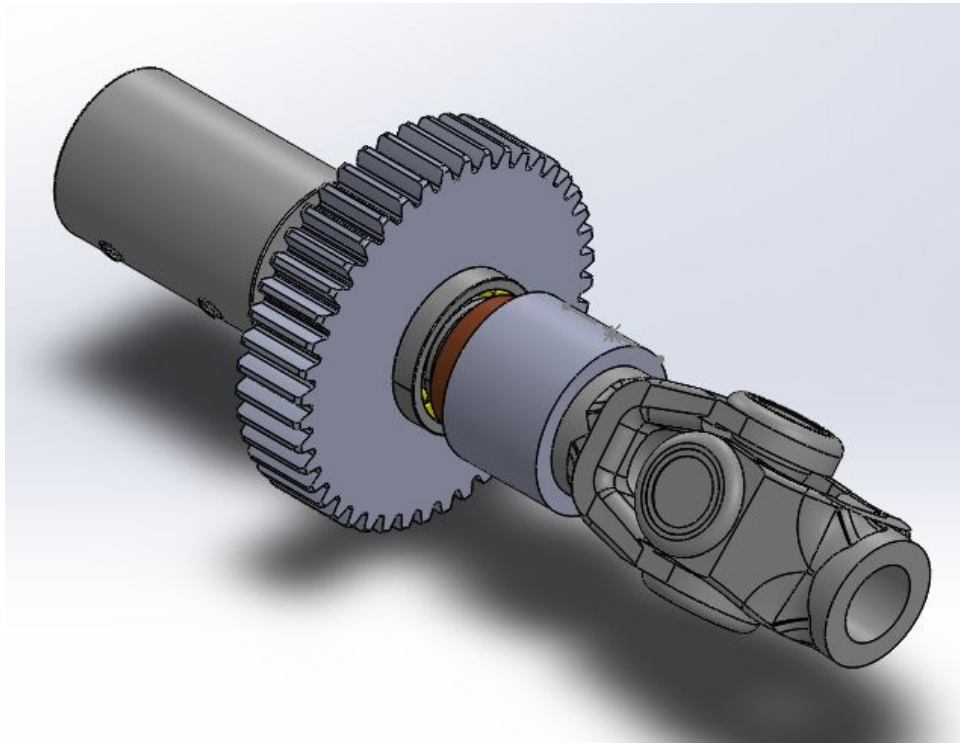


Figure 35: Shaft 3 Subassembly

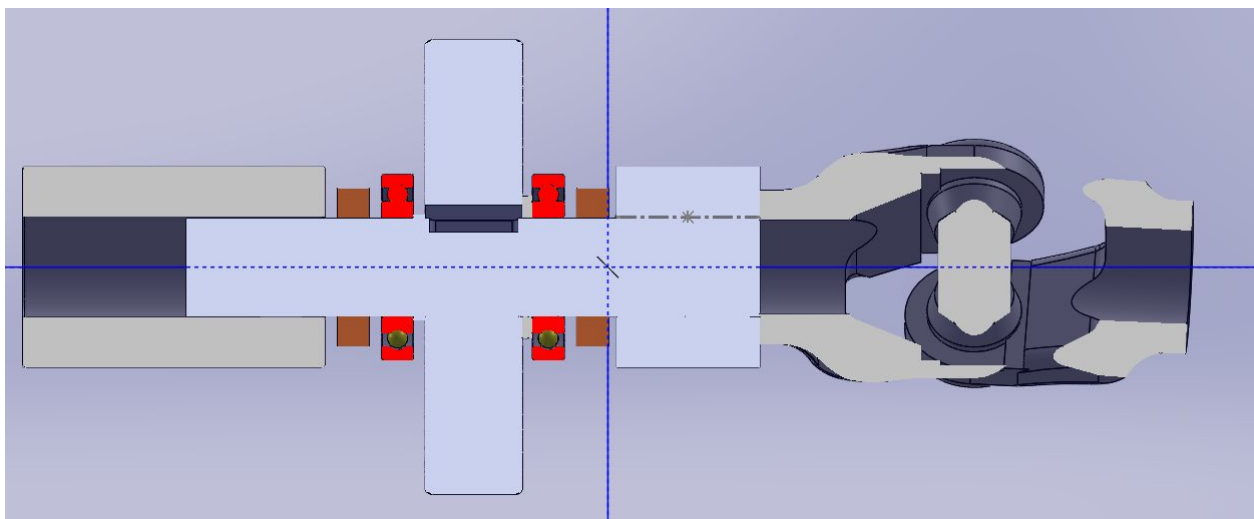


Figure 36: Shaft 3 Section View

All of the shaft assemblies rest in a custom made aluminum housing that is fabricated in the Ford shop with a CNC. The housing is subject to change once I can get back into the Ford shop and see what tooling is available. Housing side 1 is shown in Figures 37-39 and houses the first set of bearings and all the gears. Around the lip of both sides of the housing there is a groove for a rubber gasket which will seal the gearbox and prevent oil from leaking. Furthermore the lip consists of nine 8-32 bolts which hold the two sides of the housing together and three 5/16 bolt clearance holes for mounting. The outside of side 1 contains small pockets for which the shaft seals will be placed in.

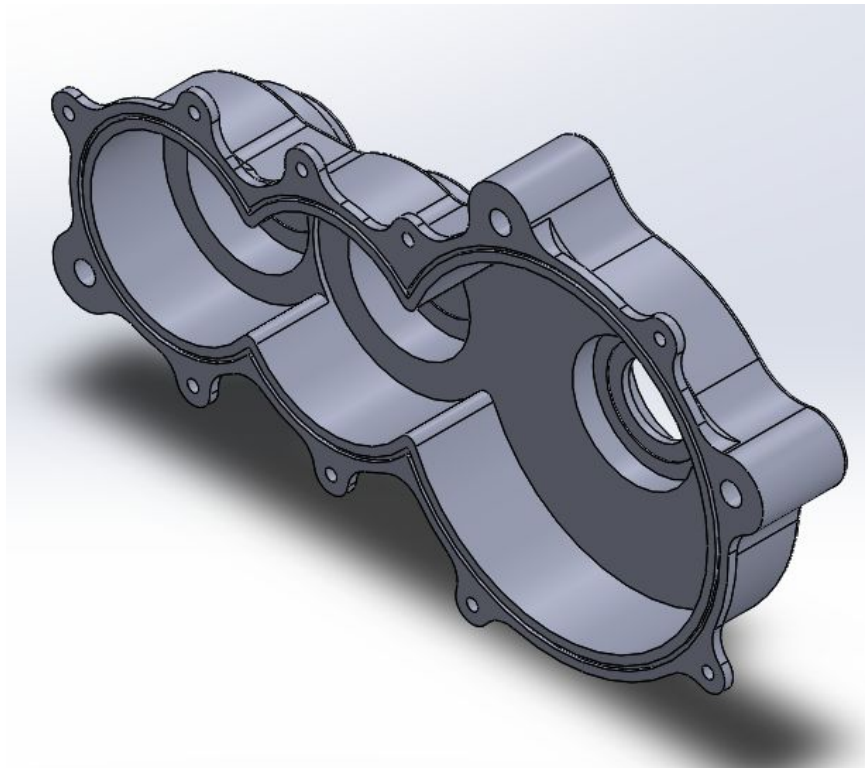


Figure 37: Gearbox Housing Side 1



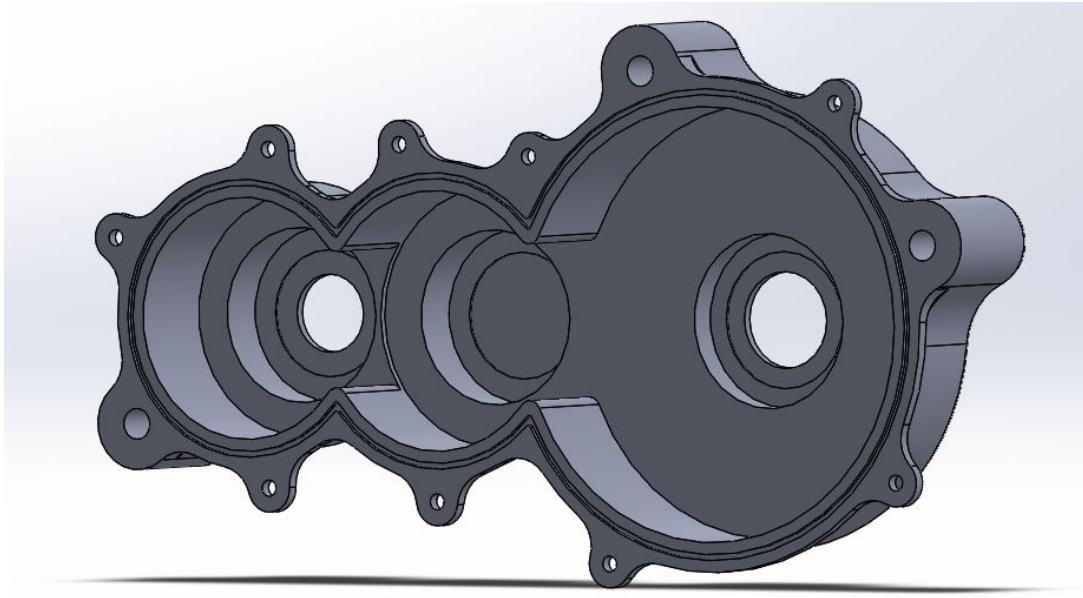


Figure 38: Gearbox Housing Side 1 - Inside

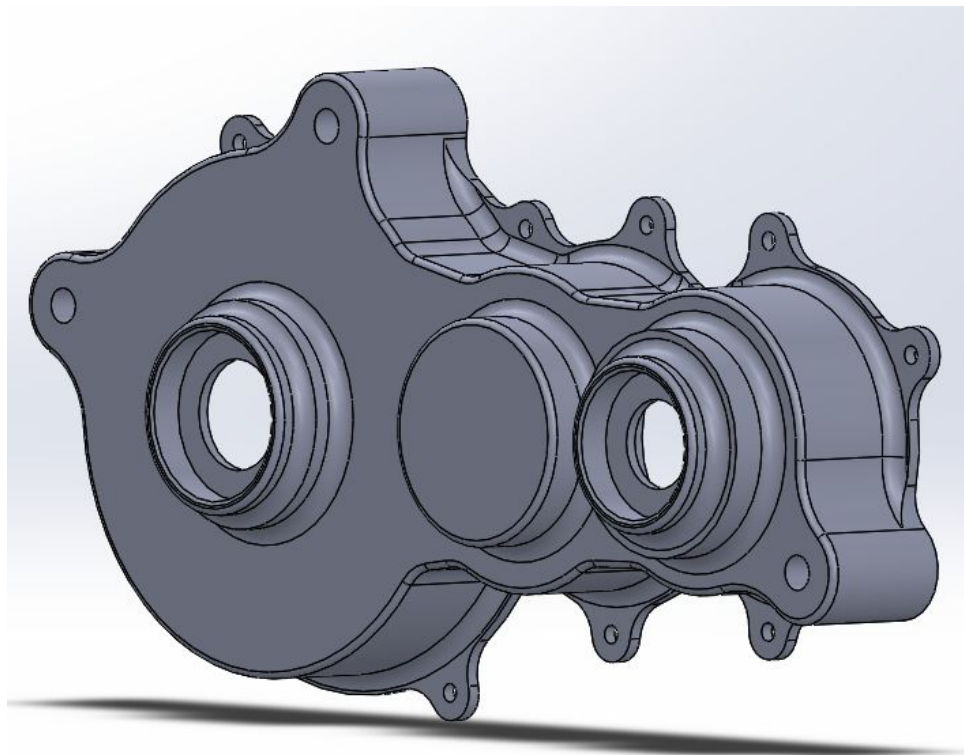


Figure 39: Gearbox Housing Side 1 - Outside

Housing Side 2 is shown in Figure 40 and 41. The gearbox is only 1 stage and therefore does not contain gears on both sides of the housing so the housing side 2 is essentially a lid to the gearbox.

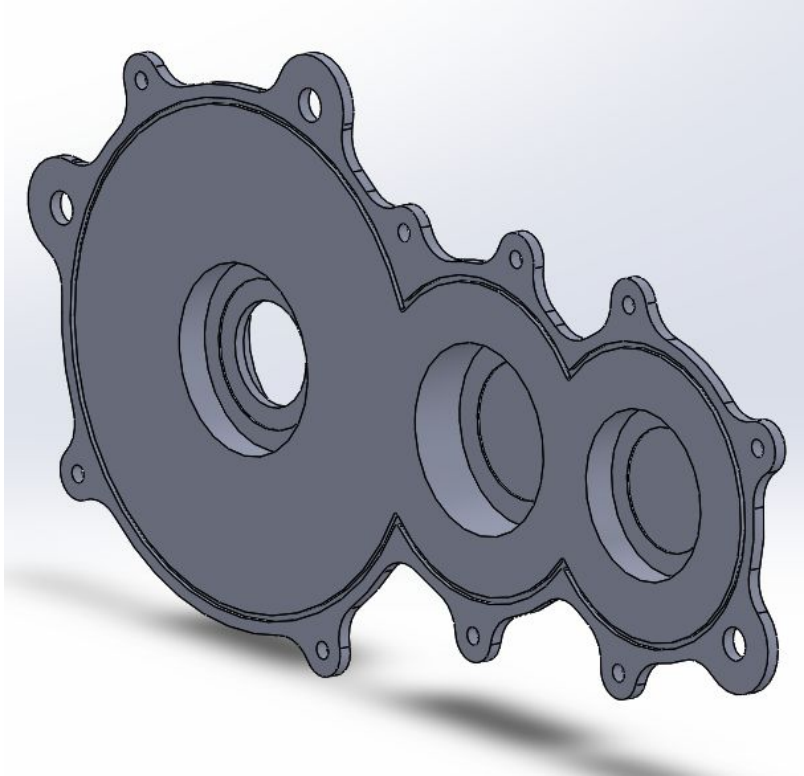


Figure 40: Gearbox Housing Side 2 - Inside

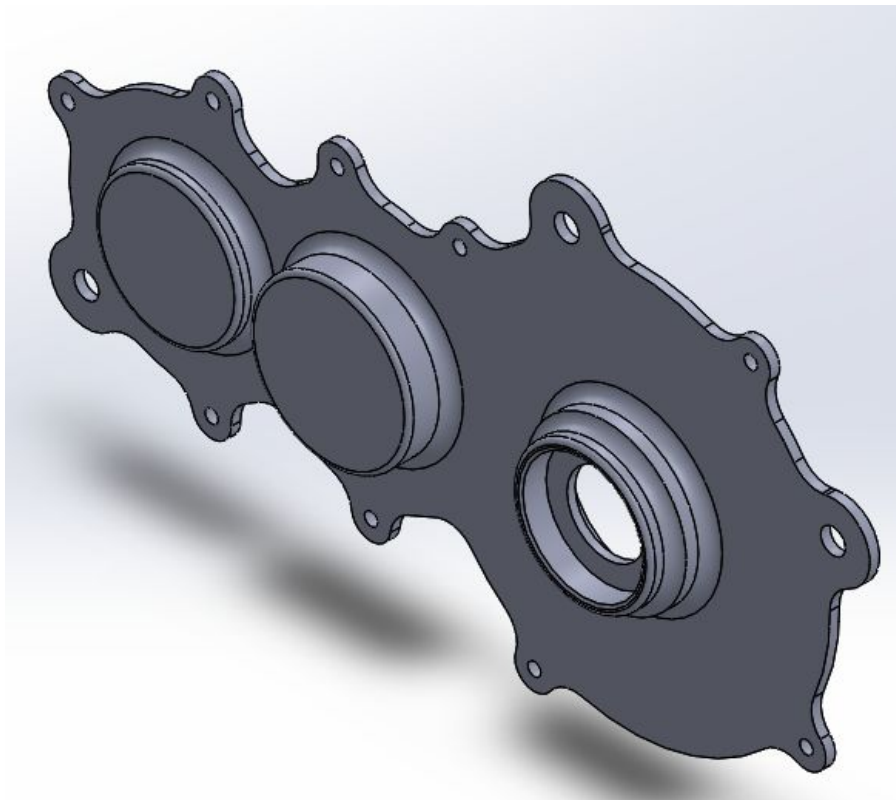


Figure 41: Gearbox Housing Side 2 - Outside

Furthermore, on the inner side of both housings where the bearings rest, there is a small divot to prevent the inner race of the bearing and shaft from rubbing against the housing. This divot can be seen on the right in Figure 42.

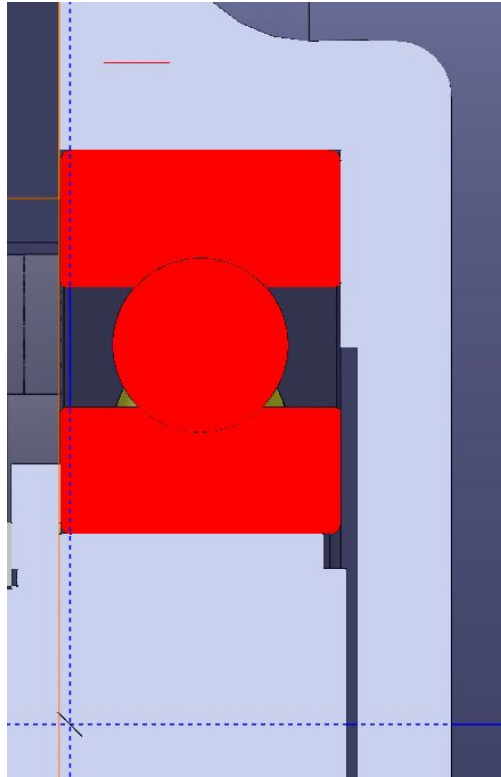


Figure 42: Inner Bearing Divots

Overall the housing is similar in size to previous years and should weigh slightly less due to the removal of an entire gear/with the use of smaller gears. The entire Assembly can be seen in Figure 43.

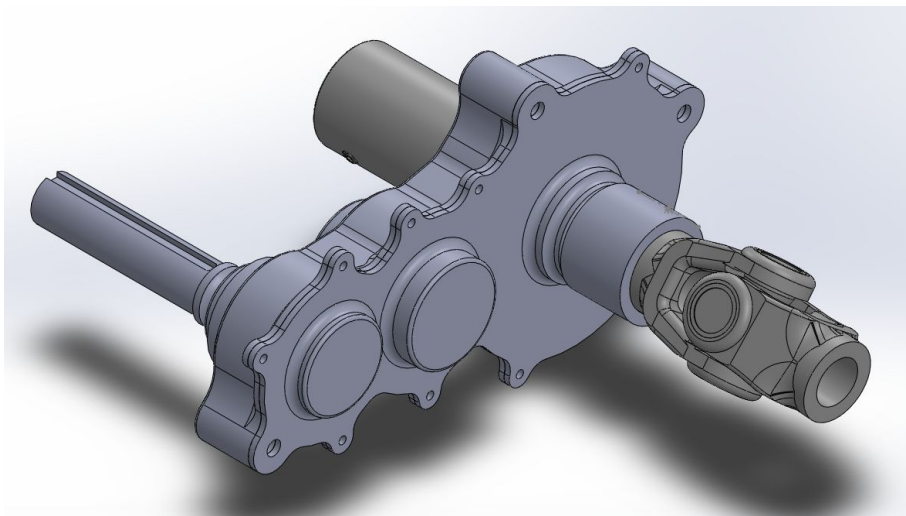


Figure 43: Complete Assembly

## Final System

With a designed gearbox I can now begin to lay out of the components in the frame of the car. While my work may be close to finishing, the frame for the 2021 season car is a work in progress so I chose to create a rough layout in the 2020 season car to give a frame designer a sense of space needed. The assembly is purposely rough because it is impractical to fully mount the powertrain system in an old frame that will have to be redone once more progress is made on the new frame. Figure 44-46 are various images of the powertrain layout in the 2020 season frame. A large amount of the misalignment in the system is handled by u-joints that go to the front and rear of the car, however misalignment will be minimized in the final layout. Not pictured in the images are the tubes that will act as shafts between the u-joints in the front section of the car. Assembly of this system will consist of temporarily securing all the components with tack welds to ensure fit before permanently welding to the frame. Overall this system is much more compact than the 2020 season powertrain design with significantly less components and therefore methods of failure. The engine is resting on a large steel plate that is held up by 1 inch square tubing. The square tubing is ideally coped to the frame members which will also help to locate them. The gearbox itself is also mounted to 1 inch square tubing with holes that match the 5/16 mounting holes on the lip of the gearbox. The differentials come stock with four mounting holes on the bottom which are attached to a pair of tabs that get welded to the frame.

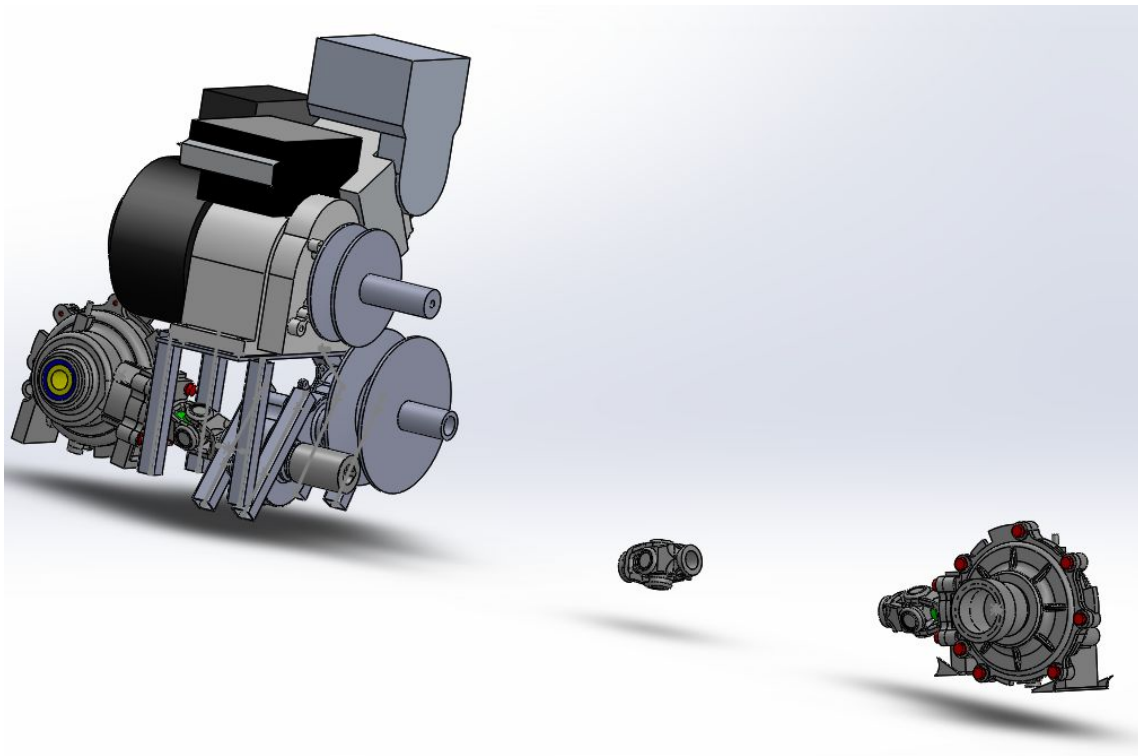


Figure 44: Layout without Frame



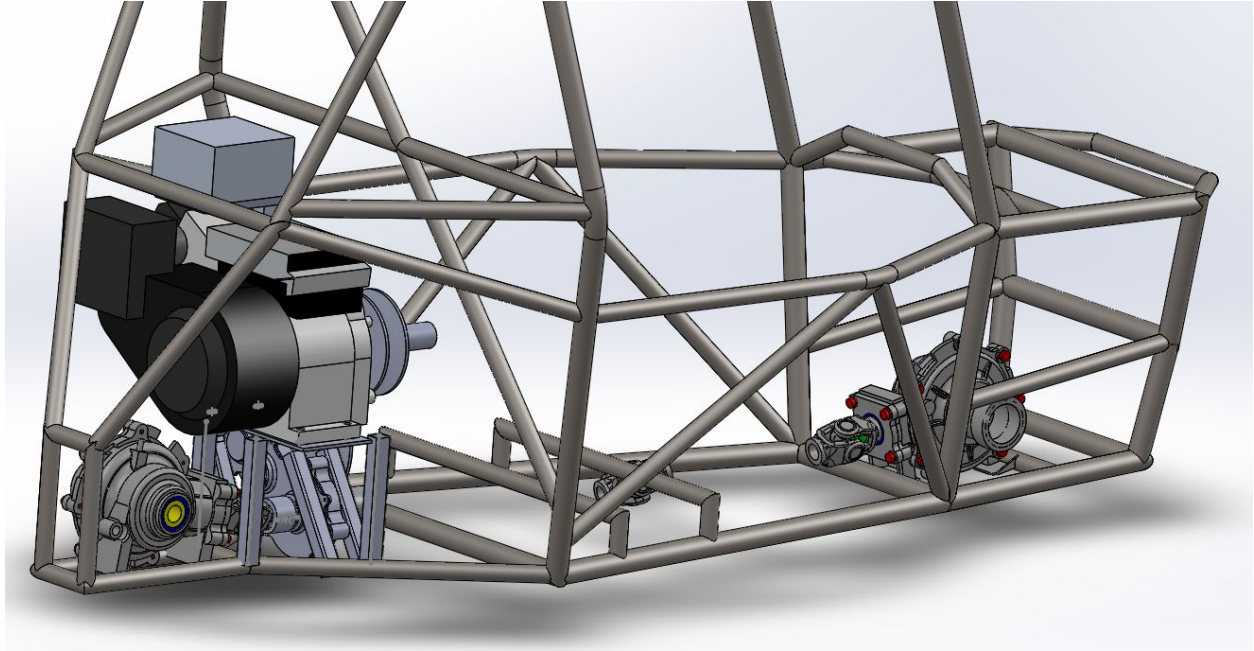


Figure 45: Layout with Frame

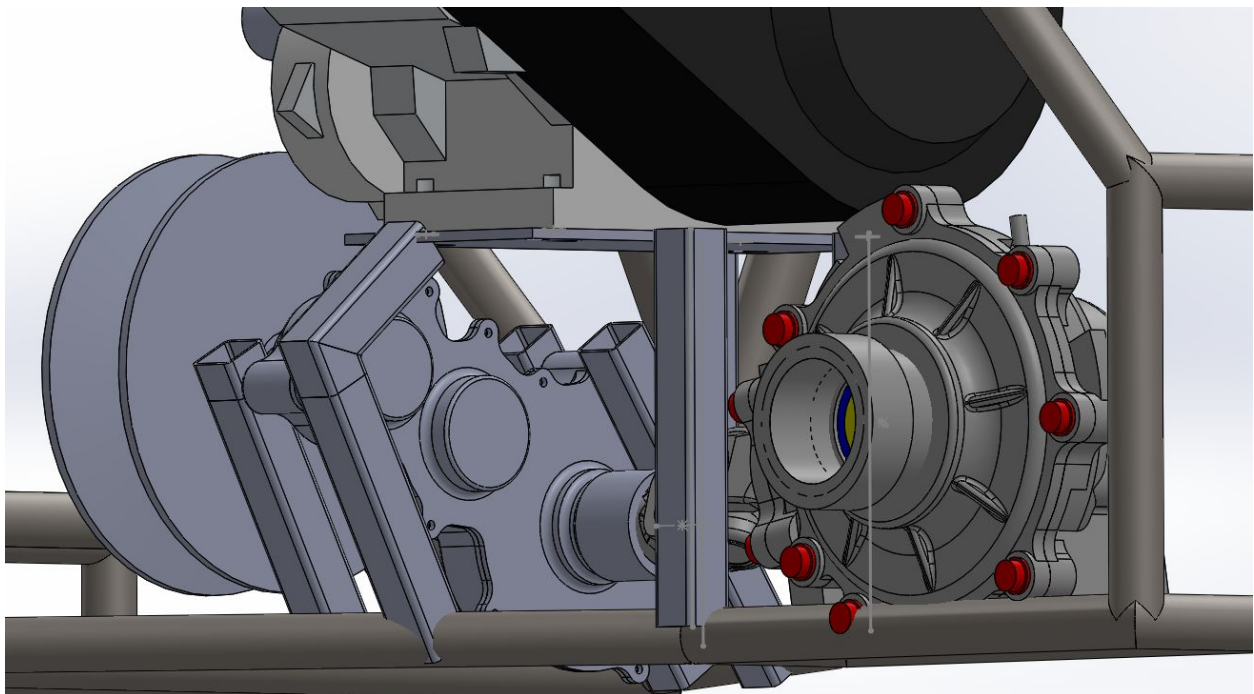


Figure 46: Close Up of Rear

## Conclusion

I believe the primary goal of removing all the redundant components from the 2020 car was achieved with this simple yet elegant design. There are significantly less parts to fabricate/purchase than last year and the overall system should be significantly lighter and much

more efficient. Furthermore, the double differential system would greatly improve the turning ability of the car. Throughout this independent study I learned the necessary analysis methods and equations to design a practical and functional gear driven powertrain system. I learned about many failure methods for gears, shafts, and bearings and the different failure theories that govern them. I can say with reasonable certainty that each of the components chosen will be able to survive for the entire life of the car and more. This project has given me the tools to become a better engineer and the experience working on a real world design challenge, and in the future I will be able to take everything I have learned here and put it to good use.